centers on the mirror. The former interpretation no doubt dominates for a clean mirror; the latter may be the more important in the case of a "dirty" mirror.

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Line Shape and Amplitude of Giant Quantum Oscillations in Ultrasonic Absorption

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The line shape of the giant quantum oscillations and the dependence of the amplitude of these oscillations on ultrasonic frequency, magnetic field, and temperature are calculated on the basis of the theory of Gurevich *et al.* Expressions for determining cross sections of the Fermi surface, effective masses, g factors, matrix elements of the electron-phonon interaction, and electron relaxation times, by the use of giant quantum oscillations, are given. Measurements of the line shape and amplitude of the giant quantum oscillations in gallium are presented and compared with theory.

I. INTRODUCTION

WHEN ultrasonic waves are propagated in a pure metal at very low temperatures, the attenuation coefficient may exhibit quantum oscillations as a function of the magnetic-field intensity. Under certain conditions these "oscillations" become a series of sharp absorption peaks separated by wide absorption minima. This series of absorption peaks, which has been termed "giant quantum oscillations" (GQO), was first predicted by Gurevich, Skobov, and Firsov (GSF).[1] An alternative theoretical treatment of this phenomenon has been given by Quinn and Rodriguez[2] and the theory was further extended by several workers.[3–10] The spike-like GQO were first observed in bismuth by Korolyuk.[11] Subsequently the present authors have carried out a quantitative investigation of the line shape of the GQO in gallium for the purpose of determining the effective mass and the g factor of the carriers which give rise to the "oscillations."[9,10] In the present paper the theory of GSF[1] and Gantsevich and Gurevich[3] (GG) is applied to calculate the line shape of the giant absorption peaks for the case of an arbitrary Fermi surface and arbitrary angle, but one which is not too close to 90°, between the direction of the magnetic field and the direction of sound propagation. The dependence of the amplitude of the absorption peaks on ultrasonic frequency, magnetic field intensity and temperature is also considered. Expressions for determining cross sections of the Fermi surface, effective masses, g factors, effective deformation potentials, and electron collision times by use of the GQO are given. Detailed experimental results in gallium are presented and compared with theory. The present discussion is concerned only with longitudinal sound waves. Furthermore, we restrict ourselves to the case of a sufficiently high magnetic field in which the energy separation between Landau levels is large enough to prevent the occurrence of transitions between different Landau levels due to the absorption of a phonon from the sound wave.

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1 V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Zh. Eksperim. i Teor. Fiz. **40**, 786 (1961) [English transl.: Soviet Phys.—JETP **13**, 552 (1961)].
2 J. J. Quinn and S. Rodriguez, Phys. Rev. **128**, 2487 (1962).
3 S. V. Gantsevich and V. L. Gurevich, Zh. Eksperim. i Teor. Fiz. **45**, 587 (1963) [English transl.: Soviet Phys.—JETP **18**, 403 (1964)].
4 D. N. Langenberg, J. J. Quinn, and S. Rodriguez, Phys. Rev. Letters **12**, 104 (1964).
5 M. S. Svirskii, Zh. Eksperim. i Teor. Fiz. **44**, 628 (1963) [English transl.: Soviet Phys.—JETP **17**, 426 (1963)].
6 S. Rodriguez, Phys. Rev. **130**, 929 (1963).
7 R. F. Kazarinov and V. G. Skobov, Zh. Eksperim. i Teor. Fiz. 43, 1496 (1962) [English transl.: Soviet Phys.—JETP **16**, 1057 (1963)].
8 V. G. Skobov, Zh. Eksperim. i Teor. Fiz. **40**, 1446 (1961) [English transl.: Soviet Phys.—JETP **13**, 1014 (1961)].
9 Y. Shapira and B. Lax, Phys. Rev. Letters **12**, 166 (1964).
10 Y. Shapira, Phys. Rev. Letters **13**, 162 (1964).
11 A. P. Korolyuk, Fiz. Tverd. Tela **5**, 3323 (1963) [English transl.: Soviet Phys.—Solid State **5**, 2433 (1964)].

II. THEORY

In the present section we review some of the theoretical results of GSF[1] and GG.[3] On the basis of these results we derive explicit formulas for the line shape of the GQO and for the dependence of the amplitude of the GQO on ultrasonic frequency, magnetic-field intensity, and temperature.

A. The Period of the GQO

It has been shown[1,3] that when a strong magnetic field $\mathbf{H}$ is applied along the z direction, the absorption of sound is restricted to electrons which satisfy the condition

$$E(n,\, k_z+q_z)-E(n,k_z)=\hbar V_s q\,. \tag{1}$$

Here E is the energy of the electron, n is the Landau level quantum number, $\hbar\mathbf{k}$ is the crystal momentum of the electron, $\hbar\mathbf{q}$ is the momentum of the sound phonon, and V_s is the sound velocity. Equation (1) leads to the "selection rule" $k_z=k_0$ where k_0 is given approximately by the relation

$$\frac{\cos\theta}{\hbar}\left(\frac{\partial E(n,k_z)}{\partial k_z}\right)_{k_z=k_0}=V_s\,. \tag{2}$$

Here θ is the angle between $\mathbf{H}$ and $\mathbf{q}$. Let A_0 be that cross-sectional area of the Fermi surface which is determined by the intersection of the Fermi surface (in $\hbar\mathbf{k}$ space) and the plane $k_z=k_0$; then one can show (cf. Ref. 3) that the giant absorption peaks are periodic in H^{-1} and that the period P is given by the expression

$$P=2\pi e\hbar/cA_0\,. \tag{3}$$

The derivation of Eq. (3) is similar to the derivation[12] of the relation for the de Haas–van Alphen (dHvA) period ΔH^{-1}, viz.,

$$\Delta H^{-1}=2\pi e\hbar/cA_{\mathrm{ex}}\,, \tag{4}$$

where A_{ex} is the extremal cross-sectional area of the Fermi surface which is perpendicular to the magnetic field. From Eqs. (3) and (4) we obtain

$$P/\Delta H^{-1}=A_{\mathrm{ex}}/A_0\,. \tag{5}$$

If θ is not too close to 90° then in most experimental situations A_0 is very close to A_{ex} so that the period P of the GQO is very close to the dHvA period ΔH^{-1}.

B. The Line Shape of the GQO

The energy-attenuation coefficient Γ for a longitudinal sound wave propagating in a direction which is parallel to the magnetic field was calculated by GSF.[1] In this calculation it was assumed that the energy E is an isotropic quadratic function of the crystal momentum, that the matrix elements which govern the phonon-absorption process are constant and that $\hbar\omega\ll kT$. After making some simple modifications, the result of this calculation can be written as

$$\Gamma=\Gamma_0\frac{\beta H}{8kT}\sum_{n,S_z}\cosh^{-2}\left[\frac{(n+\frac{1}{2})\beta H+\frac{1}{2}gS_z\mu_B H+(\hbar^2k_0^2/2m^*)-\zeta}{2kT}\right], \tag{6}$$

where Γ_0 is the attenuation coefficient at zero magnetic field, μ_B is the Bohr magneton, m^* is the effective mass, $\beta=e\hbar/m^*c$, T is the absolute temperature, ζ is the Fermi energy, g is the effective g factor, and n and S_z are the Landau level number and spin quantum number, respectively ($S_z=\pm1$). Γ and Γ_0 refer only to the absorption due to the particular group of electrons which give rise to the GQO.

The line shape of the absorption peaks was studied in Refs. 9 and 10. Strictly, the period ΔH^{-1} which appears in the formulas of these references should be replaced by P. However, this modification is quite insignificant in most cases. When a given absorption peak corresponds either to a single term in the double summation of Eq. (6) or to two terms having maxima at the same field, the line shape of the absorption peak takes the form

$$\Gamma_i=A_i\cosh^{-2}[\beta(H-H_i)/2kTH_iP]\,, \tag{7}$$

where H_i is the field at the center of the peak and A_i is a coefficient which varies slowly with H and which for a given absorption peak can be taken as a constant. The full width $(\delta H)_i$ at half-height is then given by

$$(\delta H)_i=3.53kTH_iP/\beta\,. \tag{8}$$

The line shape of an absorption peak for the case in which two terms in the double summation of Eq. (6) give rise to two peaks which overlap was shown to be given by the expression

$$\Gamma_i=A_i\left\{\cosh^{-2}\left[\beta\frac{(1+\alpha)H-\bar{H}}{2kT\bar{H}P}\right]+\cosh^{-2}\left[\beta\frac{(1-\alpha)H-\bar{H}}{2kT\bar{H}P}\right]\right\}, \tag{9}$$

where $\bar{H}$ is very nearly the field at the center of the peak and α is a parameter which is related to the g factor. If the absorption peak described by Eq. (9) is a superposition of two subpeaks having maxima at H_1 and H_2 and if the first subpeak is associated with Landau level n and spin up and the second subpeak with Landau level $n+l$ and spin down then $\bar{H}^{-1}=\frac{1}{2}(H_1^{-1}+H_2^{-1})$ and

$$\alpha=\tfrac{1}{2}\bar{H}\left(\frac{1}{H_1}-\frac{1}{H_2}\right)=\tfrac{1}{2}\bar{H}P[(g\mu_B/\beta)-l]\,. \tag{10}$$

[12] L. Onsager, Phil. Mag. 43, 1006 (1952); I. M. Lifshitz and A. M. Kosevich, Zh. Eksperim. i Teor. Fiz. 29, 730 (1955) [English transl.: Soviet Phys.—JETP 2, 636 (1956)]. See also Appendix A.

In deriving Eqs. (7)–(10) it was assumed that the energy E is a quadratic function of the crystal momentum and that the sound is propagating in a direction parallel to the magnetic field. However, by using the results of GG[3] and by employing Onsager's quantization[12] it is shown in Appendix A that these equations remain approximately valid for an arbitrary dependence of E on $\mathbf{k}$ and for any value of θ which is not too close to 90°. In this generalization the effective mass m^* which appears in the relation $\beta = e\hbar/m^*c$ is defined as

$$m^* = \frac{1}{2\pi}\left(\frac{dA(n,k_0)}{dE(n,k_0)}\right)_{E=\zeta}, \tag{11}$$

where $A(n,k_0)$ is the area of the orbit of an electron, which is at Landau level n and $k_z = k_0$, in momentum space. Equations (7)–(10) which describe the line shape of the GQO are valid even if the matrix elements of the electron-phonon coupling vary with H, provided this variation is small over the field interval in which one absorption peak occurs.

C. Amplitude of the GQO

We start by considering the amplitude of the GQO for a longitudinal sound wave propagating in a direction parallel to the magnetic field. If we assume that the electron energy is a quadratic function of the crystal momentum and that the matrix elements which govern the absorption process are constant, then the line shape of any absorption peak which is not a superposition of two subpeaks is given by Eq. (7). The height of the absorption peak in this case is given by A_i where

$$A_i = \Gamma_0(\beta H_i/8kT). \tag{12}$$

In the more general case when Eq. (9) holds, we have

$$A_i = \Gamma_0(\beta \bar{H}/8kT) \tag{13}$$

and the height of the absorption peak is given by the product of A_i and the maximum value of the sum of the two $\cosh^{-2}$ terms which are inside the curly brackets of Eq. (9). We proceed to study the dependence of A_i on various parameters.

We shall see (Sec. II D) that a necessary condition for the validity of the foregoing discussion concerning the absorption coefficient is that the electronic mean free path l is much larger than the ultrasonic wavelength λ. It is known[13] that in this case the absorption coefficient at zero field Γ_0 is proportional to the ultrasonic frequency and is essentially independent of the temperature. It follows that A_i should be proportional to the ultrasonic frequency and inversely proportional to the absolute temperature T. Furthermore, A_i should be proportional to the magnetic field at which the absorption peak occurs.

[13] R. W. Morse, *Progress in Cryogenics*, edited by K. Mendelssohn (Heywood and Company Ltd., London, 1959), Vol. I.

In the more general case when a longitudinal sound wave is propagating at an angle which is not too close to 90° with the magnetic field and when the dependence of the energy on the crystal momentum is arbitrary, the line shape near the absorption maxima is given by Eq. (A13). Using the results of GG[3] one can show that the maximum of the absorption peak is given by

$$A_i = \frac{eH_i|I_{n,n}|^2}{16\pi kT\hbar\rho u_0^2 V_s c|\Delta v_z|}, \tag{14}$$

where ρ is the density of the crystal, u_0 is the amplitude of the sound vibration, $I_{n,n}$ is a matrix element defined below and

$$\Delta v_z = \frac{1}{\hbar}\left[\frac{\partial E(n,k_z+q_z)}{\partial k_z} - \frac{\partial E(n,k_z)}{\partial k_z}\right]_{k_z=k_0, H=H_i}. \tag{15}$$

From Eq. (15) we have approximately

$$\Delta v_z = \hbar q \cos\theta / m_z^*, \tag{16}$$

where

$$\frac{1}{m_z^*} = \frac{1}{\hbar^2}\left(\frac{\partial^2 E(n,k_z)}{\partial k_z^2}\right)_{k_z=k_0, H=H_i}. \tag{17}$$

In the GG theory the perturbation caused by the interaction of an electron with the sound wave is taken to be

$$V = \tfrac{1}{2}(Ue^{-i\omega t} + U^\dagger e^{i\omega t}), \quad U = e^{i\mathbf{q}\cdot\mathbf{r}}\Lambda_{ik}u_{ik}^0, \tag{18}$$

where u_{ik}^0 is the maximum value of the strain tensor, Λ_{ik} is a second-rank tensor and $U^\dagger$ is the Hermitian adjoint of U. If we choose a coordinate system in which one of the axes is directed along $\mathbf{q}$, then Eq. (15) of Ref. 3 may be written as

$$I_{n,n} = qu_0 \int \varphi^*_{n,k_0+q_z} e^{iqx\sin\theta}\Lambda_{qq}\varphi_{n,k_0}dx, \tag{19}$$

where the functions $\varphi_{n,k}$ which are defined by Eq. (10) of Ref. 3 are to be evaluated at $H = H_i$. We define an effective deformation potential[14] $\bar{\Lambda}$ by the relation

$$\bar{\Lambda} = \int \varphi^*_{n,k_0+q_z} e^{iqx\sin\theta}\Lambda_{qq}\varphi_{n,k_0}dx \tag{20}$$

and obtain

$$A_i = \left|\frac{m_z^* eH_i\omega\bar{\Lambda}^2}{16\pi kT\hbar^2\rho V_s^2 c\cos\theta}\right|. \tag{21}$$

If we assume that for a given orientation of the magnetic field and a given direction of sound propagation $\bar{\Lambda}$ and m_z^* are constant, then it follows that A_i is

[14] In the present work the electron-phonon coupling is represented by Eq. (18). We do not inquire into the physical origin of the tensor Λ_{ik} but we show how the matrix elements of this tensor can be evaluated experimentally.

proportional to ω and to H_i and is inversely proportional to T. If two absorption peaks overlap and if the line shape of the combined absorption peak can be described by Eq. (9) (cf. Appendix A), then the foregoing conclusions concerning the amplitude factor A_i still apply. The absolute magnitude of A_i depends on the effective deformation potential $\bar{\Lambda}$ which represents the strength of the electron-phonon interaction.

We proceed to study the parameter $\bar{\Lambda}$ under the assumption that Λ_{qq} is a constant. If $\mathbf{q}\|\mathbf{H}$ (i.e., $\theta=0$) then Eq. (20) gives $\bar{\Lambda}=\Lambda_{qq}\int\varphi^*_{n,k_0+q}\varphi_{n,k_0}dx$. The magnitude of the functions $\varphi_{n,k_z}(x)$ is expected to be small for values of x which are large compared with the classical cyclotron radius $r_c=[(2n+1)\hbar c/eH]^{1/2}$. Therefore, when $qr_c\sin\theta\ll1$ we expect that $\bar{\Lambda}\cong\Lambda_{qq}\int\varphi^*_{n,k_0+q_z}\varphi_{n,k_0}dx$. This conclusion can be easily verified for a simple parabolic band for which φ_{n,k_z} is a normalized harmonic oscillator function which does not depend on k_z. In this case the evaluation of the integral in Eq. (20) leads to[15]

$$\bar{\Lambda}=\Lambda_{qq}\exp\left(\frac{-q^2\hbar c\sin^2\theta}{4eH}\right)L_n\left(\frac{q^2\hbar c\sin^2\theta}{2eH}\right), \qquad (22)$$

where $L_n(\xi)$ is a Laguerre polynomial normalized to unity at $\xi=0$. When

$$nq^2\hbar c\sin^2\theta/2eH\ll1, \qquad (23)$$

we have $\bar{\Lambda}\cong\Lambda_{qq}$.[16] Condition (23) is satisfied when $qr_c\sin\theta\ll1$. Since $n\approx[HP]^{-1}$, provided $n\gg1$, Eq. (23) may be written as

$$H\gg[\hbar c\omega^2\sin^2\theta/2eV_s^2P]^{1/2}. \qquad (24)$$

D. Conditions for the Validity of the Preceding Formulas

In order to observe the GQO it is required that the energy separation βH between two adjacent Landau levels exceeds the width $\sim kT$ of the "region of diffusivity" of the Fermi surface. Furthermore, the GQO will cease once all the Landau levels (or all but one of the Landau levels) have passed through the Fermi surface. Roughly speaking this means that in order to observe the oscillations we must have $\zeta>\beta H$. It is noteworthy that the parameters which enter into the last two requirements are independent of the mean free path l of the electron and of the ultrasonic wavelength, λ.

The effects of electron collisions on the GQO were considered by several authors.[1,5,8,17] Physically, the collision broadening of the Landau levels introduces some uncertainty in specifying the momentum k_z of the absorbing electrons [cf. Eq. (1)]. As a result, electrons with momenta k_z which differ somewhat from k_0 may also absorb phonons from the sound wave. This has the effect of broadening the absorption peaks. The effects of electron collisions on the line shape of the GQO can be ignored only if the ultrasonic frequency is so high that the collision broadening of the absorption peaks is small compared with the widths of the peaks in the absence of collisions. At present there is no unanimity of opinion as to the mathematical inequalities which must be satisfied in order that collisions may be ignored. It appears that a necessary (but perhaps not a sufficient) condition for neglecting collisions is[18]

$$B\equiv(2kT/m^*)^{1/2}q\tau\gg1 \qquad (25)$$

while sufficient (but perhaps not necessary) conditions are $\omega\tau\gg1$ and Eq. (25). Here τ is the electron collision time which may depend on H. In most cases the condition $\omega\tau\gg1$ is more stringent than condition (25), although the ratio of B to $\omega\tau$ is often less than ten. In our experiments $B/\omega\tau\sim7$ at 1.5°K. For a given ultrasonic frequency either of the conditions $B>1$ and $\omega\tau>1$ imposes a much stronger requirement on the purity of the sample than the condition $ql>1$ which must often be satisfied in ultrasonic work.

In Sec. II B it was assumed that the Fermi energy ζ is a constant independent of the magnetic field. In general, this assumption holds only for $n\gg1$. However, in many cases a series of GQO arises from electrons which are in a small pocket in momentum space—the majority of the electrons being in other larger pockets in this space. In these cases, the Fermi energy will remain virtually unchanged even for small values of n, i.e., in the "quantum limit" of this series of oscillations. Also, in the discussion of the line shape of the GQO for the case of an arbitrary Fermi surface (Appendix A), use was made of Onsager's results[12] which are based on the semiclassical quantization rules. The results which were obtained are applicable therefore only when these quantization rules are valid.

The expressions for the absorption coefficient near a giant absorption peak [Eqs. (7) and (9)] give only the "resonant-like" absorption due to those electrons which are responsible for that peak. Experimentally, one can in many cases separate the "resonant" absorption from the "nonresonant" absorption since the latter is a smooth function of H. If there are several periods of GQO, absorption peaks due to different series of oscillations may overlap and analysis of the experimental results may become difficult.

[15] cf., I. M. Ryshik and I. S. Gradstein, *Tables of Series, Products and Integrals* (VEB Deutscher Verlag der Wissenschaften, Berlin, 1963).

[16] The overlap integral $\int\varphi^*_{n,k_0+q_z}\varphi_{n,k_0}dx$ equals unity when φ_{n,k_z} is independent of k_z.

[17] J. J. Quinn, Phys. Rev. **137**, A889 (1965); S. H. Liu and A. M. Toxen (to be published).

[18] Condition (25) was first derived by GSF (Ref. 1) as a necessary and sufficient condition for ignoring collisions in the case of a simple parabolic band when $\mathbf{q}\|\mathbf{H}$. In the $\cosh^{-2}$ term which appears in Eq. (22) of Ref. 1, y should be replaced by y^2. This error by itself does not invalidate the derivation of condition (25) provided $m^*V_s^2/2kT\lesssim1$. The latter condition is satisfied when $m^*\lesssim m_0$ and $T\gtrsim1$°K, which is the case in most experimental situations. For a criticism of the GSF treatment of collisions see Ref. 17.

The foregoing theory neglects the broadening of Landau levels by the periodic lattice potential.[19] Such a broadening is expected to be small for small values of n[20] but may become appreciable for some large n. In fact, the GQO can be used as an experimental tool in studying Landau level broadening. Few other practical matters should be mentioned. In order for the preceding theory of the line shape to hold, the variation of the magnetic field across the sample must be small compared with the widths of the absorption peaks. Further, the spread in the orientation of the crystallites which constitute the specimen must be small. Finally, the present theory holds only when $\hbar\omega \ll kT$.

E. Information which Can Be Obtained from the Study of the GQO

The GQO can be used as a powerful tool in the investigation of the Fermi surface. From the period of a given series of giant absorption peaks one obtains a certain cross-sectional area of the Fermi surface [cf. Eq. (3)]. If the angle θ between the magnetic field and the direction of sound propagation is sufficiently different from 90° then this cross-sectional area will be, in most cases, close to the extremal cross-sectional area perpendicular to the magnetic field. By fitting the line shapes and/or widths of the absorption peaks to the theoretical expressions of Sec. II B, one can determine the effective mass and in addition one can obtain information concerning the g factor (cf. Refs. 9 and 10). The determination of the effective mass and the g factor can be carried out most easily if each absorption peak is split, due to spin, into two completely separated subpeaks with maxima at H_1 and H_2. The mass m^* can then be determined from the width of either subpeak [cf. Eq. (8)]. Also the difference $|H_2-H_1|$ can be measured directly and used to obtain information about the g factor. If the first subpeak is associated with Landau level n and spin up and the second subpeak with Landau level $n+l$ and spin down, then

$$g=2\frac{m_0}{m^*}\left[\frac{H_2-H_1}{PH_1H_2}+l\right], \tag{26}$$

where m_0 is the mass of a free electron. By using the value of $|H_2-H_1|$ one can then limit g to a certain discrete set of values. In general, the integer l and the sign of H_2-H_1 cannot be determined by this method so that the g factor cannot be obtained uniquely.

The electron collision time τ can be determined, in principle, by measuring the dependence of the line shape of the absorption peaks on the ultrasonic frequency and comparing it with a theory which takes electron collisions into account. According to GSF[1] the absorption peaks will tend to be broader than predicted in Sec. II B unless condition (25) is satisfied. This makes it possible to get a rough estimate of τ in the following way. As the ultrasonic frequency ω is increased from frequencies at which $B<1$ to frequencies at which $B\gg 1$, the width of any given absorption peak will first decrease and then approach a constant value. Let ω_0 be that frequency at which the width starts to level off then the parameter B must be of the order of few units at $\omega=\omega_0$. If we take $B_{(\omega_0)}\sim 5$ we obtain the rough estimate

$$\tau\sim 5(m^*/2kT)^{1/2}(V_s/\omega_0). \tag{27}$$

If we use the condition $\omega\tau>1$ rather than condition (25) (cf. Sec. II D) we obtain

$$\tau\sim 1/\omega_0. \tag{28}$$

In most cases the estimates (27) and (28) agree to within an order of magnitude.

The amplitude of the GQO is determined, among other things, by the effective deformation potential $\bar{\Lambda}$ which, in the present model, represents the strength of the electron-phonon interaction. This parameter, which has the dimensions of energy, can be determined by using Eq. (21) provided m_z^* is known. For a quadratic dependence of the energy E on the crystal momentum $\mathbf{k}$ the mass m_z^* can be calculated from the reciprocal mass tensor which can be obtained by the use of the GQO or otherwise. Moreover, if the mass anisotropy of the band is not too large one may obtain an estimate of $\bar{\Lambda}$ by setting m_z^* equal to m^*.

It should be noted that the various parameters (A_0, m^*, g, $\bar{\Lambda}$, and τ) which can be determined by a study of a given series of GQO all refer to the same group of electrons at the Fermi surface. This circumstance is advantageous for while it is possible to dedetermine masses, g factors, etc., by several independent experiments, it is difficult to associate a particular mass with a particular g factor, for example, when the Fermi surface is complicated. Moreover, since the absorption of sound phonons by electrons is limited to those electrons on the Fermi surface which satisfy Eq. (1), one can study the anisotropy of the various parameters by changing the direction of the magnetic field.

III. EXPERIMENTAL TECHNIQUE

Three single crystals of gallium were grown from high purity (99.9999%) bars supplied by Alcoa. The samples were grown in lucite molds, oriented with x rays and hand lapped for ultrasonic work. X-cut quartz transducers of 10-Mc/sec fundamental frequency were obtained from the Valpey Crystal Corporation. These crystals were polished for overtone excitation, plated with gold on both sides, and bonded to the

[19] This broadening of the Landau levels corresponds to the removal of the degeneracy of the energy with respect to the center of the electron orbit. This degeneracy was assumed in Ref. 3.

[20] J. Zak, Phys. Rev. **136**, A776 (1964).

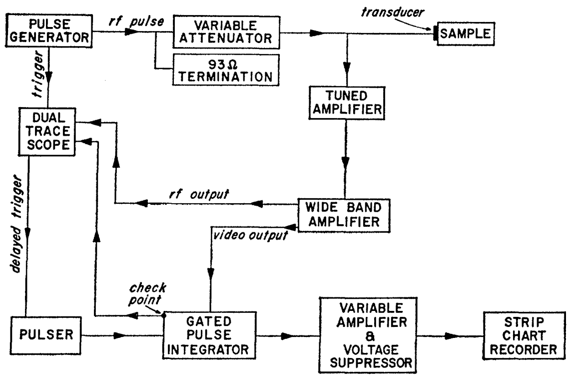

FIG. 1. Block diagram of the apparatus for measuring ultrasonic attenuation.

samples with a Dow Corning 200 silicone fluid or with a Nonaq stopcock grease. The transducers were then excited in their fundamental frequency or in their odd harmonics.

A block diagram of the ultrasonic apparatus is shown in Fig. 1. Radio-frequency pulses of about two microseconds duration were generated by an Arenberg PG 650 pulsed oscillator operated at a pulse repetition frequency of 60 pulses/sec. The rf frequency used varied between 9 and 90 Mc/sec. Each rf pulse passed through a variable attenuator and was then sent to a transducer which was bonded to the sample under investigation. Echoes received at the same transducer were amplified first by a tuned preamplifier and then by a wide-band amplifier. The video output of the wide-band amplifier was fed into a gated pulse integrator which integrated the envelope of one of the received echoes. The dc output of the integrator was amplified by a Keithley model 150 microvoltmeter (indicated in Fig. 1 as a "variable amplifier and voltage suppressor") and was then recorded by a Leeds and Northrop Speedomax G strip-chart recorder. By introducing 1 db, 2 db, $\cdots$ steps at the attenuator, while the magnetic field was held constant, and by measuring the resultant signals at the recorder, a scale was obtained against which all attenuation changes due to the magnetic field could be compared.[21]

Experiments were conducted in 2- and 1-in. bore water cooled Bitter solenoids, the former having a maximum field of 110 kG and the latter having a maximum field of 156 kG. The magnetic field at the center of each magnet, as a function of the current passing through the magnet, was measured prior to each experiment with a type-J Newport Magnetometer. During each run the current was monitored and the magnetic field intensity was deduced from the field versus current characteristic of the magnet.

Several Dewar systems of the standard type were used in the experiments. A typical Dewar system consists of an outer Dewar for nitrogen and an inner Dewar for helium. Temperatures below 4.2°K, down to 1.1°K, were obtained by pumping on the helium bath. By controlling the valves between the pump and the helium bath, it was possible to regulate the temperature to within 0.02°K. Since the samples were always in a direct contact with the helium, it is safe to assume that their temperature did not differ appreciably from that of the helium. The temperature was determined from the helium vapor pressure which was measured with a Wallace and Tierman pressure gauge.

IV. RESULTS

The attenuation of 9–90-Mc/sec longitudinal sound waves propagated along the b axis of gallium was measured as a function of magnetic field intensity. These measurements were performed at liquid-helium temperatures down to 1.1°K. Of the seven runs that were performed, five were carried out in a magnetic field up to 110 kG and two were carried out in a magnetic field up to 143 kG. In six runs the magnetic field was directed along the b axis while in one run the field was rotated in the ab plane. Oscillations in the ultrasonic attenuation as a function of the magnetic field intensity were observed in all cases.

We begin by describing the experimental results for the case in which the magnetic field is parallel to the b axis (and hence to $\mathbf{q}$). As H increases from zero the attenuation first shows marked oscillations at low fields ($H \lesssim$ few kG). These low-field oscillations were not studied in detail. At higher fields ($H \sim 5$ kG) the ultrasonic attenuation decreases substantially but then levels off. At still higher fields a series of absorption peaks which are periodic in H^{-1} appears. These are the giant quantum oscillations (GQO). As the temperature is lowered the field at which the GQO make their appearance decreases. At 4.2°K this field is $\sim$10 kG. At very high fields and low temperatures other series of GQO (subsidiary periods) appear. The most pronounced subsidiary oscillation has a period of 14

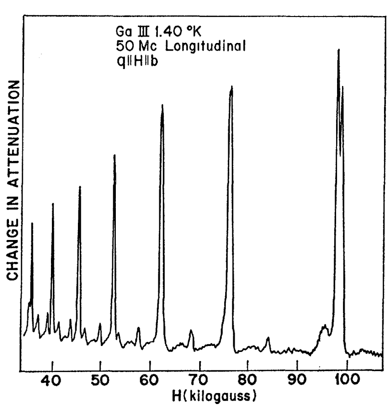

FIG. 2. Recorder tracing of the attenuation of 50-Mc/sec longitudinal waves in gallium showing the GQO at 1.40°K with $\mathbf{q}\|\mathbf{H}\|b$. The recorder response to the change in attenuation is slightly nonlinear.

[21] It was found that this procedure is sufficiently accurate if a 10-db attenuation step is always present in addition to any other attenuation steps which are introduced.

$\times 10^{-7}$ G^{-1}. Few other periods of the order of $10^{-7}-10^{-8}$ G^{-1} were also observed. These subsidiary periods, most of which have been observed earlier by Condon,[22] were not investigated in detail and attention was focused on the dominant series of oscillations which has a measured period of $(29.8\pm 1.5)\times 10^{-7}$ G^{-1}. This period was first observed by Shoenberg[23] and later by Goldstein[24] and by Condon.[22] The remainder of the discussion is confined to this series of absorption peaks. As the temperature was lowered, the width of each absorption peak decreased as expected [cf. Eq. (8)] and the spike-like character of the GQO became apparent. Figure 2 shows a recorder tracing of the oscillation pattern for a 50-Mc/sec sound wave at 1.40°K. At the lowest temperatures ($T\approx 1.1$°K) the spin splitting of the absorption peaks at 136, 96, and 75 kG was clearly observed. Figure 3 shows the spin splitting of the absorption peak at 136 kG. The height of each subpeak in Fig. 3 is $\sim$6.5 db/cm.

The line shapes of the peaks at 136, 96, and 75 kG, measured at various temperatures between 1.66 and 1.14°K, were fitted to Eq. (9) by a computer which determined, by the least-squares method, the best values of the parameters β and α. In this fashion we obtained $\beta=(28.7\pm 2.1)\times 10^{-20}$ cgs units and $2|\alpha|/\bar{H}P=(0.0305\pm 0.0016)$. The quoted uncertainties represent only the uncertainties due to random errors and do not include systematic errors. The best value for β corresponds to $m^*=(0.065\pm 0.005)m_0$. For the effective g factor we obtain $g=2(m_0/m^*)[l\pm(0.0305\pm 0.0016)]$. The most likely possibilities for l are $l=0$ and $|l|=1$. For $l=0$ we obtain, using our value for m^*, $|g|=0.94\pm 0.08$, while for $|l|=1$ we obtain $|g|=30\pm 2$ or $|g|=32\pm 2$. The two subpeaks at 136 kG were sufficiently separated at the lowest temperatures to allow the use of Eqs. (8) and (26). The effective mass and g value

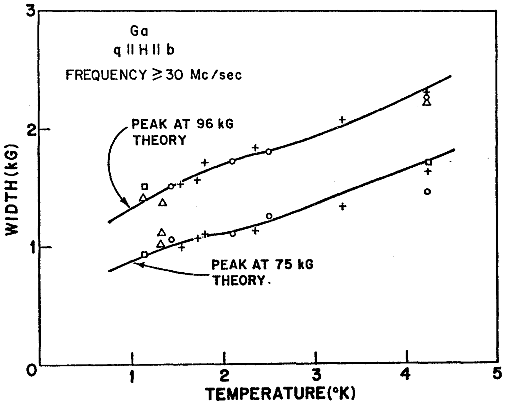

FIG. 4. Temperature variation of the widths of the absorption peaks at 96 and 75 kG. The solid curves are calculated on the basis of Eq. (9).

obtained in this fashion are close to those given above. The value for β lies between the values quoted by Shoenberg[23] and by Condon.[22] Thus Shoenberg has measured $\beta=31.6$ and 33.8×10^{-20} cgs units for the magnetic field oriented 10° from b in the bc plane and 11.1° from b in the ab plane. Condon's closest measurement near the b axis ($\sim$7° from b in the ab plane) gave $m^*=0.069m_0(\beta=26.9\times 10^{-20}$ cgs units). It should be noted that experimental factors, such as field inhomogeneity, tend to broaden the giant absorption peaks. As a result, the value of β, as determined from the line shape of the absorption peaks, tends to be smaller than the true value. Using typical values for the inhomogeneity and time instability of the magnetic field it was estimated that the systematic error, due to these factors, in the determination of β was less than 5%.

In one run the magnetic field was rotated in the ab plane. It was found that the period, the effective mass, and the g factor did not change by more than a few percent when the direction of the magnetic field made an angle of less than 20° with the b axis. At much larger angles the pattern of the oscillations in the ultrasonic absorption became complicated and a detailed analysis was not attempted.

The widths (at half-height) of the absorption peaks were measured at various temperatures. These widths, which refer to the observed absorption peaks each of which being a superposition of two subpeaks, were found to be independent of the ultrasonic frequency provided the latter was not lower than 30 Mc/sec. At 10 Mc/sec the absorption peaks were, in some cases, slightly broader.[25] Figure 4 shows the results for the peaks at 96 and 75 kG. The solid curves in this figure are theoretical curves calculated on the basis of Eq. (9) and the best values of the parameters P, α, and β. As can be seen, the agreement between theory and experiment is quite satisfactory.

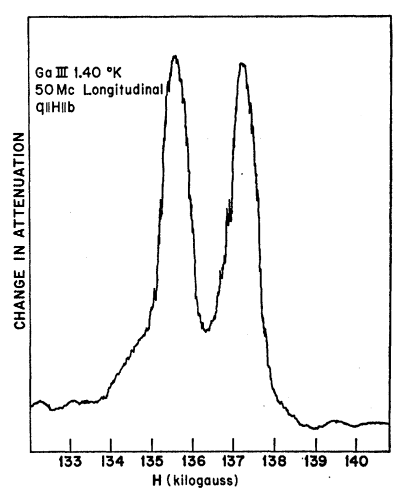

FIG. 3. Recorder tracing of the attenuation of 50-Mc/sec longitudinal waves in gallium showing the spin splitting of the peak at 136 kG. The recorder response to the change in attenuation is slightly nonlinear.

[22] J. H. Condon, Bull. Am. Phys. Soc. **9**, 239 (1964).
[23] D. Shoenberg, Phil. Trans. Roy. Soc. London **A245**, 1 (1952).
[24] A. Goldstein (private communication).

[25] In determining the best values for β and α the data for 10-Mc/sec waves were not used.

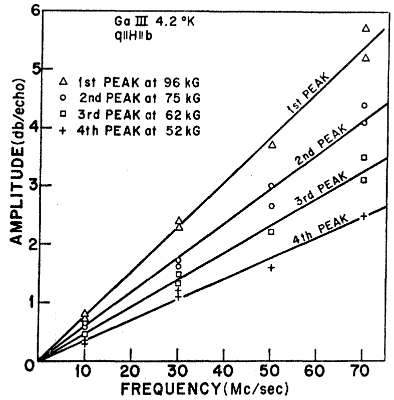

FIG. 5. Frequency dependence of the amplitude of the absorption peaks at 4.2°K.

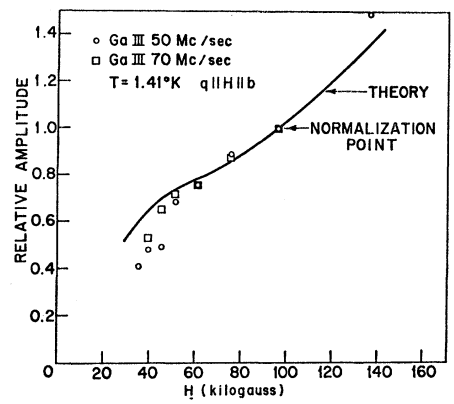

FIG. 6. Field dependence of the amplitude of the absorption peaks at 1.41°K. The solid curve is calculated on the basis of Eq. (9) with $A_i \propto \bar{H}$.

The amplitude (height) of the absorption peaks was measured as a function of ultrasonic frequency, magnetic field, and temperature. The results indicate that the amplitude of the GQO is proportional to ω, as predicted by the theory. Figure 5 shows the dependence of the height of the absorption peaks on the ultrasonic frequency at 4.2°K.

The magnetic field dependence of the amplitude of the GQO was measured at various temperatures. The results indicate that the amplitude increases with H somewhat faster than predicted by Eq. (9) with $A_i \propto \bar{H}$. Figure 6 shows the results at 1.41°K. Interpreted within the framework of the present theory these results seem to indicate that the product $m_z^* \bar{\Lambda}^2$ increases slightly when the magnetic field increases [cf. Eq. (21)]. The results for the temperature dependence of the amplitude of the GQO are in a qualitative agreement with the predictions of the theory, but the data are insufficient for a quantitative comparison.

The effective deformation potential $\bar{\Lambda}$ was estimated from the amplitude of the absorption peaks at 136 and 96 kG measured between 1.14 and 1.66°K. Here use was made of Eq. (21) and m_z^* was set equal to m^*. The results gave $\bar{\Lambda} \approx 11.5$ eV. However, as noted above, $\bar{\Lambda}$ may depend slightly on the magnetic field intensity.

It was remarked previously that the widths of the absorption peaks were found to be independent of the ultrasonic frequency provided the latter was not lower than 30 Mc/sec. Using this fact and Eq. (27) we estimate a lower limit of $\sim 4 \times 10^{-9}$ sec for the electron relaxation time τ in all three samples. In the sample Ga-III the widths of the absorption peaks for 10 and 29-Mc/sec waves were found to be slightly broader than for higher frequency waves. We therefore estimate the order of magnitude of the relaxation time in this sample, by using Eq. (27) or Eq. (28), as $\tau \sim 5 \times 10^{-9}$ sec. It should be emphasized that this relaxation time refers only to those electrons which give rise to the absorption peaks at $\sim 10^5$ G. In general, τ may be different for different groups of electrons and it may also depend on H. The sample Ga-III was grown from a gallium ingot obtained from Professor J. F. Cochran who has performed dc resistivity measurements on samples grown from similar ingots. From these measurements, which were carried out at zero magnetic field, the electron mean free path l at 1–2°K was estimated as ~ 2 cm.[26] If we assume that this value of l applies also to the particular group of electrons which give rise to the absorption peaks at $\sim 10^5$ G and if we let $l = V_F \tau$, where V_F is the Fermi velocity, we obtain $\tau \sim 4 \times 10^{-8}$ sec. The samples Ga-I and Ga-II were grown from an ingot supplied by Dr. L. M. Foster of Alcoa. Roberts[27] has observed ultrasonic cyclotron resonance in gallium crystals obtained from the same source and has estimated $\tau \sim 10^{-8}$ sec for his samples. We therefore conclude that our estimate for the electron relaxation time τ is quite reasonable.

V. CONCLUSION

The comparison of the experimental results in gallium with the theoretical predictions indicates that the theory of GSF and GG gives an adequate description of the line shape of the GQO. In particular, it is demonstrated that a study of the shape of the absorption peaks can be quite useful in obtaining quantitative information about the electrons in a metal. It is also shown that the amplitude of the absorption peaks can be used to determine the effective deformation potential $\bar{\Lambda}$ which represents the strength of the electron-phonon coupling.

In the present experiments advantage was taken of the unusually long electronic mean free path in gallium. Thus, the rather stringent requirements for the validity of Eqs. (7)–(10) for the line shape of the GQO were met at relatively low ultrasonic frequencies. The use of higher ultrasonic frequencies should permit similar studies in other metals and semimetals which have a shorter electronic mean free path.[28] Furthermore, it

[26] J. F. Cochran (private communication).
[27] B. W. Roberts, Phys. Rev. Letters 6, 453 (1961).
[28] It should be noted, however, that the theory of GSF and GG assumes that $\hbar\omega \ll kT$. When this inequality is not satisfied the theory must be modified.

may be possible to observe other types of GQO which arise from electron-phonon collisions in which the Landau level quantum number n is changed.[3,4] The study of the line shape of the absorption peaks in this case may turn out to be quite informative as well.

ACKNOWLEDGMENTS

We wish to thank J. Zak, S. Williamson, A. Goldstein, and H. Praddaude for useful discussions, M. Kelly for technical assistance and Mrs. R. Sheshinsky for programming.

APPENDIX A

In this appendix we derive an explicit formula for the line shape of the GQO in the case of an arbitrary dependence of the electron energy E on the crystal momentum $\hbar\mathbf{k}$. This formula applies to any angle θ between $\mathbf{H}$ and $\mathbf{q}$ which is not too close to 90°.

According to the GG theory[3] the line shape of any absorption peak which is not a superposition of two overlapping subpeaks is given by

$$\Gamma_i = A_i \cosh^{-2}\left[\frac{E(n,k_0)\pm\frac{1}{2}g\mu_B H-\zeta}{2kT}\right]. \tag{A1}$$

Here $E(n,k_0)$ is the energy of an electron, which is at the Landau level n and at $k_z=k_0$, in the absence of spin splitting. The (+) sign in Eq. (A1) refers to an electron with spin up and the (−) sign to an electron with spin down. The energy $E(n,k_0)$ depends on H and the maximum of the absorption peak occurs at the field H_i where

$$E(n,k_0,H_i)\pm\tfrac{1}{2}g\mu_B H_i=\zeta. \tag{A2}$$

For fields H which are very close to H_i we have

$$E(n,k_0)\pm\tfrac{1}{2}g\mu_B H-\zeta \cong\left[\left(\frac{dE(n,k_0)}{dH}\right)_{H=H_i}\pm\tfrac{1}{2}g\mu_B\right](H-H_i). \tag{A3}$$

But

$$\left(\frac{dE(n,k_0)}{dH}\right)_{H=H_i}=\left[\left(\frac{dE(n,k_0)}{dA(n,k_0)}\right)\left(\frac{dA(n,k_0)}{dH}\right)\right]_{H=H_i}, \tag{A4}$$

where $A(n,k_0)$ is the area of the orbit in $\hbar\mathbf{k}$ space. Using the results of Onsager[12] we have

$$dA(n,k_0)/dH=e(n+\gamma)h/c, \tag{A5}$$

where γ is a constant between zero and one. From Eqs. (A3)–(A5) it follows that

$$E(n,k_0)\pm\tfrac{1}{2}g\mu_B H-\zeta\cong\left(\frac{e(n+\gamma)h}{m^*c}\pm\tfrac{1}{2}g\mu_B\right)(H-H_i), \tag{A6}$$

where

$$m^*=\frac{1}{2\pi}\left(\frac{dA(n,k_0)}{dE(n,k_0)}\right)_{H=H_i}. \tag{A7}$$

The condition for a maximum in the absorption coefficient is given by Eq. (A2) which can be written as

$$E_{[A(n,k_0,H_i)]}-E_{(A_0)}=\mp\tfrac{1}{2}g\mu_B H_i, \tag{A8}$$

where A_0 is the value of $A(n,k_0,H)$ when $E=\zeta$. To a sufficient approximation we then have

$$A(n,k_0,H_i)-A_0=\left(\frac{dA(n,k_0)}{dE(n,k_0)}\right)_{H=H_i}\times(E_{[A(n,k_0,H_i)]}-E_{(A_0)})=\mp\pi m^* g\mu_B H_i. \tag{A9}$$

From Onsager we have

$$A(n,k_0,H_i)=e(n+\gamma)hH_i/c, \tag{A10}$$

which gives

$$e(n+\gamma)h/c=(A_0/2\pi H_i)\mp\tfrac{1}{2}gm^*\mu_B. \tag{A11}$$

Equation (A11) leads immediately to Eq. (3). Combining (A6) and (A11) we have

$$E(n,k_0)\pm\tfrac{1}{2}g\mu_B H-\zeta\cong\frac{A_0(H-H_i)}{2\pi H_i m^*}=\frac{\beta(H-H_i)}{PH_i}, \tag{A12}$$

where $\beta=eh/m^*c$. The line shape of the absorption peak at fields close to H_i is then given by

$$\Gamma_i\cong A_i\cosh^{-2}\left[\frac{\beta(H-H_i)}{2kTPH_i}\right]. \tag{A13}$$

If Eq. (A13) holds for fields which satisfy $|H-H_i|<1.76kTH_iP/\beta$ then[29]

$$(\delta H)_i\cong 3.53kTH_iP/\beta. \tag{A14}$$

One can show that in the case of an absorption peak which is a superposition of two subpeaks, the line shape is given by Eq. (9) provided Eq. (A13) with $\beta=2\pi e\hbar c^{-1}[dE(n,k_0)/dA(n,k_0)]_{H=\bar{H}}$ is an adequate approximation to the line shape of either subpeak over the relevant range of H. Furthermore, the parameter α in this case is still given by Eq. (10).

[29] One can show that in general $(\delta H)_i=3.53kTH_iP/\bar{\beta}$. Here $\bar{\beta}=2\pi e\hbar c^{-1}[dE(n,k_0)/dA(n,k_0)]_{H=\eta}$ where η is a certain (generally unknown) value of H inside the field interval at which the absorption is greater than half its maximum value. In most cases the difference between β and $\bar{\beta}$ is negligible and either can be taken to be equal to $2\pi e\hbar c^{-1}[dE(n,k_0)/dA(n,k_0)]_{E=\zeta}$.