Helium Afterglow and the Decay of the Electron Energy*

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The electron energy decay in the afterglow of a pulsed dc discharge in helium has been studied over a pressure range from 0.03 mm to 5 mm Hg. The energy decay is found to be influenced by cooling due to electron-atom elastic recoil and heating due either to electron-metastable superelastic collisions at low pressures or to the ionizing collision between two metastable atoms at high pressures. The analysis allows for the deviation of the electron velocity distribution from Maxwellian resulting from these heating effects.

I. INTRODUCTION

THE electron energy decay in the afterglow of a pulsed discharge is determined by the combined effect of the mechanisms capable of increasing or decreasing the average electron energy. A study of the electron energy decay as a function of plasma parameters such as gas pressure, electron density, added impurity concentration, and, in some cases, the applied dc magnetic field, will allow a separation of these effects and evaluation of their magnitude.

To interpret properly the observed electron energy decay it is necessary to know the nature of the electron velocity distribution. The problem is straightforward if the electrons have a Maxwellian velocity distribution, but this may not be the case if heating and cooling mechanisms act to add or remove electrons preferentially from some portion of the velocity distribution.

The electron energy is experimentally determined by the measurement of the plasma radiation temperature T_R which[1–3] is equal to the electron temperature T if the electrons have a Maxwellian velocity distribution.

The following presents the results and interpretation of measurements of the decay of the electron radiation temperature in the afterglow of a pulsed dc discharge in helium. The theory used accounts in an approximate way for the deviation of the electron velocity distribution from Maxwellian.

II. THEORY

In this section the electron velocity distribution is calculated as a function of time in the afterglow under the influence of electron-atom elastic recoil, electron-electron collisions, ambipolar diffusion, electron-metastable superelastic collisions, and metastable-metastable ionizing collisions. This distribution function is then used to calculate the time variation of the electron average energy u and the electron density n.

Other mechanisms which can affect the electron energy have been considered and have been found to be either negligibly small or not to explain the measured electron energy decay.[3] Those effects which can act to increase the electron energy are: dissociative recombination involving an electron and the helium molecule ion,[4] and a superelastic collision involving a free electron and a helium atom in an excited state other than a metastable state. The ionizing collision between two helium atoms, one being initially in an excited state within 1.5 eV of the ionization level,[5] which results in the production of the helium molecule-ion and an electron, acts to raise the average electron energy only if the energy of the electron produced exceeds the average energy of the plasma electrons. For the electron densities involved in the experiments, electron-ion elastic-recoil collisions are not important in determining the electron energy decay. Below 10 000°K the effect of inelastic collisions may also be neglected in helium.

Ambipolar diffusion generally tends to cool the electrons, but may, in some cases, when a dc magnetic field is applied to the plasma, actually heat the electrons.[3] In any case its direct cooling or heating effect on the electrons may be neglected for the conditions of the experiment. However, its effect on the electron density decay and on the velocity distribution cannot be ignored.

With these approximations the Boltzmann equation for an isotropic electron velocity distribution f, with no externally applied electric field, is

$$\frac{\partial}{\partial t}(nf)=\left(\frac{\partial}{\partial t}nf\right)_D+\left(\frac{\partial}{\partial t}nf\right)_{ee}+\frac{gn}{\epsilon^{1/2}}\frac{\partial}{\partial\epsilon}\epsilon^{3/2}\nu_c\left(f+kT_g\frac{\partial f}{\partial\epsilon}\right)+nH,\quad(1)$$

where n = electron density, g = the average fractional energy exchange in an electron-gas atom collision, ν_c = electron-atom collision frequency, ϵ = electron energy,

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[1] G. Bekefi, J. L. Hirshfield, and S. C. Brown, Phys. Rev. 122, 1037 (1961).

[2] G. Bekefi and S. C. Brown, J. Appl. Phys. 32, 25 (1961).

[3] J. C. Ingraham, Ph.D. thesis, Department of Physics, MIT, Cambridge, Massachussets, 1963 (unpublished).

[4] E. P. Gray and D. E. Kerr, Bull. Am. Phys. Soc. 5, 372 (1960).

[5] J. A. Hornbeck and J. P. Molnar, Phys. Rev. 84, 621 (1951).

m = electrons mass, k = Boltzmann constant, T_g = temperature of the gas atoms. The velocity distribution f is normalized such that

$$\int f4\pi v^2 dv = (2/m^3)^{1/2} 4\pi \int f\epsilon^{1/2} d\epsilon = 1.$$

The first term on the right of Eq. (1) represents the removal and redistribution of electrons due to diffusion; the second term represents electron-electron collisions; the third term is the electron-atom elastic recoil term[6]; and the last term is a heating term resulting from the presence of metastable atoms.

A. Heating Due to Superelastic De-excitation of the Metastables to the Ground State

Electron-metastable superelastic collisions act to remove electrons from one region of energy ϵ, $\epsilon+d\epsilon$, to another region $\epsilon+\epsilon_m$, $\epsilon+\epsilon_m+d\epsilon$, where $\epsilon_m \simeq 20$ eV is the metastable energy. With an excess of electrons present at these elevated energies due to the superelastic collisions, it is also necessary to include the inverse of the above process, the inelastic collision.

If superelastic collisions are the dominant heating mechanism, H becomes for $\epsilon < \epsilon_m$

$$H = -\sigma_{se}(2\epsilon/m)^{1/2} n_m f + \sigma_x[\epsilon+\epsilon_m](\epsilon+\epsilon_m)(2/m\epsilon)^{1/2} N f[\epsilon+\epsilon_m] \quad (2)$$

and for $\epsilon > \epsilon_m$

$$H = \sigma_{se}[\epsilon-\epsilon_m](\epsilon-\epsilon_m)(2/m\epsilon)^{1/2} n_m f[\epsilon-\epsilon_m] - \sigma_x(2\epsilon/m)^{1/2} N f, \quad (3)$$

where σ_{se} is the superelastic-collision cross section, σ_x is the inelastic-collision cross section, n_m is the metastable atom density, and N is the gas atom density. Whenever a quantity is not a function of ϵ but of $\epsilon+\epsilon_m$ or $\epsilon-\epsilon_m$, its functional dependence is included in square brackets (for example, $\sigma_x[\epsilon+\epsilon_m]$ in Eq. (3)). The effects of the singlet and triplet metastable atoms are combined into one effective cross section, since their electron-collision excitation functions have about the same form[7] and their energy separation is small relative to their excitation energy. The inelastic cross section σ_x has been accurately measured as a function of electron energy by Schulz and Fox.[8]

Boltzmann's equation is now solved for f by making a number of approximations for the collision terms in the vicinity of $\epsilon \approx \epsilon_m$. These approximations are made possible in part by the fact that the energy of an electron following a superelastic collision is much greater than the average electron energy, $u(\leqslant 1$ eV), and in part by the fact that the excess number of electrons in the region is small compared to the total number.

The diffusion term is approximated by $-(2\epsilon_m/3\nu_{cm} m \Lambda_B^2) n f$ in this region of energy, where[9]

$$\nu_{cm} \cong 2.3 \times 10^9 p \text{ sec}^{-1}, \quad (4a)$$

and

$$\frac{1}{\Lambda_B^2} = \left(\frac{2.4}{R}\right)^2 \frac{1}{1+(eB/m\nu_{cm})^2}, \quad (4b)$$

and where p is the gas pressure in mm Hg, B the applied dc magnetic field, and R the tube radius. The expression for the diffusion term is obtained simply by averaging the Boltzmann equation over all electron energies for a distribution function containing a small peak f_1 at $\epsilon = \epsilon_m$, and then inspecting this average to see which term containing f_1 is dominant. The next larger term in f_1 is of order u/ϵ_m.

The electron-electron term in this region is approximated by $-(f-f_0)/\tau_{ee}$ where f_0 is a Maxwellian distribution with the same average energy as f and τ_{ee} is a relaxation time calculated using the formula given by Spitzer[10] into which has been substituted an electron energy $\epsilon = \epsilon_m$. This gives an approximation for the rate at which the energetic electrons are re-thermalized with the bulk of the electrons.

The electron-atom collision term is approximated by $-(g\nu_{cm} n f)$ in this region. This does not now conserve electrons but does give the correct energy decay. Since we assume the number of electrons in this region of energy to be a small fraction of the whole, this approximation for the collision term should be reasonable.

The Boltzmann equation for $\epsilon \geqslant \epsilon_m$ becomes, then,

$$\frac{\partial}{\partial t} n f = -\frac{2\epsilon_m}{3\nu_c m \Lambda_B^2} n f - n\frac{(f-f_0)}{\tau_{ee}} - g\nu_{cm} n f + nH, \quad (5)$$

where H is given by Eq. (3). An approximate solution of this equation may immediately be obtained if $(\partial/\partial t) n f$ may be assumed negligibly small. Since this term is equal to the difference between the effect which adds electrons to this region and the effects which remove electrons, it is possible that it may be small compared to either of the two types of effects. The solution thus obtained for superelastic heating is, for $\epsilon \simeq \epsilon_m$,

$$f = \frac{\tau_{ee}\sigma_x \left(\frac{2\epsilon}{m}\right)^{1/2} n_m f[\epsilon-\epsilon_m] + f_0}{1+\tau_{ee}\left[g\nu_{cm} + \frac{2\epsilon_m}{3m\nu_{cm}\Lambda_B^2} + \sigma_x\left(\frac{2\epsilon}{m}\right)^{1/2} N\right]}, \quad (6)$$

[6] W. P. Allis, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. XXI, pp. 383–444.

[7] H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Clarendon Press, Oxford, England, 1952), p. 77.

[8] G. J. Schulz and R. E. Fox, Phys. Rev. **106**, 1179 (1957).

[9] S. C. Brown, *Basic Data of Plasma Physics* (John Wiley & Sons, Inc., New York, 1959), p. 5.

[10] L. Spitzer, *Physics of Fully Ionized Gases* (Interscience Publishers, Inc., New York, 1956), p. 80.

where σ_{se} has been eliminated using the relation[11]

$$\sigma_{se}[\epsilon-\epsilon_m]=(\epsilon/\epsilon-\epsilon_m)\sigma_x[\epsilon]. \tag{7}$$

The distribution function in the vicinity of $\epsilon\simeq u$ is assumed to be Maxwellian in form with a temperature T. This is not incompatible with the fact that the distribution may be strongly non-Maxwellian in the region of $\epsilon\simeq\epsilon_m$ since u is much less than ϵ_m and the electron-electron interaction will consequently be much stronger for the electrons of near average energy than for the energetic electrons. Since we assumed the fraction of energetic electrons to be small, this implies that the bulk of electrons will not be greatly disturbed.

The total distribution function is then composed of a Maxwellian distribution of temperature T and a non-Maxwellian part as given by Eq. (6). We can determine the equation for the time rate of change of the average electron energy u by performing the proper integral of the Boltzmann equation. Neglecting cooling or heating effects from diffusion and requiring that the electron-electron collision term give zero contribution, we find

$$\frac{du}{dt}=-a(T/T_g)^{1/2}\tfrac{3}{2}k(T-T_g)+H_{se}, \tag{8}$$

where

$$H_{se}=\frac{8\pi}{m^2}n_m\epsilon_m \times\int_0^\infty \frac{\sigma_{se}\epsilon f\left(1-g\nu_{cm}\tau_{ee}\dfrac{\epsilon}{\epsilon_m}\right)d\epsilon}{1+\tau_{ee}\left[g\nu_{cm}+\dfrac{2\epsilon_m}{3m\nu_{cm}\Lambda_B^2}+\sigma_{se}\epsilon\left(\dfrac{2}{m(\epsilon+\epsilon_m)}\right)^{1/2}N\right]}. \tag{9}$$

The first term on the right of Eq. (8) represents the electron-atom elastic-recoil energy loss for the Maxwellian portion of the velocity distribution valid for electron temperatures below 10 000°, where $\nu_c=20\nu p$ $\sec^{-1}$ in helium,[9] and, as a result, $a=7.87\times10^4p$ $\sec^{-1}$.

Order of magnitude estimates of the quantities in the denominator of the integrand of Eq. (9) show that for all conditions of the experiment the term proportional to σ_{se} will be large compared to the other terms over most of the range of the integral. Neglecting the other terms we obtain

$$H_{se}=\frac{1}{\tau_{ee}}\left(\frac{2kT}{\pi^{1/2}}\frac{n_m}{N}\left(\frac{\epsilon_m}{kT}\right)^{3/2}\right). \tag{10}$$

This limiting expression for H_{se} corresponds to a very weak electron-electron interaction for electrons of energies $\epsilon\approx\epsilon_m$ where the steady-state shape of the disturbed distribution function is determined by a balance between superelastic collisions and the inverse inelastic collisions. The other limit of very strong electron-electron interactions in this region of energies is obtained from Eq. (9) by setting $\tau_{ee}=0$.

The assumption in Eq. (5) that $\partial/\partial t(nf)$ was negligible may, with the help of Eq. (6), be shown to be valid provided the time constants for metastable density decay, electron density decay, and the decay of the temperature of the Maxwellian part of the velocity distribution are all long compared to the shortest of the relaxation time constants τ_{ee}, $1/g\nu_{cm}$, $3m\nu_{cm}\Lambda_B^2/2\epsilon_m$, and $1/\sigma_x(2\epsilon/m)^{1/2}N$. These provisions are satisfied for the conditions of the experiment.

B. Heating Due to Metastable-Metastable Ionizing Collisions Alone

The collision between two metastable atoms can result in the production of an electron, an atomic ion, and an unexcited atom.[12,13] The maximum energy available to the electron thus produced is the difference between twice the metastable energy and the atomic ionization energy, or about 14.5 eV. Since the final state involves three bodies, the electron produced can have energies ranging from zero to 14.5 eV. The probability per unit interval of energy of producing an electron of energy ϵ is found to be proportional to $\epsilon^{1/2}(\epsilon_m'-\epsilon)^{3/2}$, where $\epsilon_m'=14.5$ eV. This expression is determined by considering the joint probability of producing each of the three particles with momentum vectors terminating in respective incremental volume elements in momentum space, and, for a fixed range of electron energies, averaging over-all possible electron directions of motion, and all allowable atom and ion momenta subject to the constraints of conservation of total momentum and energy. The average energy of these electrons is found to be $5/8\epsilon_m'\approx9$ eV. In calculating the heating due to metastable-metastable ionizing collisions alone, it is assumed that the hot electrons are all produced at an energy $\epsilon_m''=9$ eV. The inverse process which would be a three-body collision between an electron, an ion, and a gas atom may be neglected.

The heating term for this case then takes the form

$$H=(\beta n_m^2/n)\delta(\epsilon-\epsilon_m''). \tag{11}$$

If the helium molecule ion is produced in the collision[14] instead of the atomic ion and an atom, the electrons produced will have a single energy of about 15 eV. A value of $\epsilon_m''=15$ eV in Eq. (11) would represent this effect. The inverse process may be important but no attempt will be made here to account for it.

Proceeding exactly as in Sec. IIA, an expression is obtained for the perturbed part of the velocity distribu-

[11] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1955), p. 206.

[12] M. A. Biondi, Phys. Rev. **82**, 453 (1951).

[13] A. V. Phelps and S. C. Brown, Phys. Rev. **86**, 102 (1952).

[14] A. R. Tynes and J. J. Brady, IEEE Trans. on Nucl. Sci. **11**, 231 (1964).

tion f_1 and the rate of change of the electron average energy

$$f_1=\frac{\tau_{ee}\beta n_m^2\delta(\epsilon-\epsilon_m'')}{n\left(1+\tau_{ee}\left[g\nu_{cm}+\dfrac{2\epsilon_m''}{3m\nu_{cm}\Lambda_B^2}\right]\right)}, \tag{12}$$

$$\frac{du}{dt}=-a\left(\frac{T}{T_g}\right)^{1/2}\tfrac{3}{2}k(T-T_g)+H_{mm}, \tag{13}$$

where

$$H_{mm}=\frac{\beta n_m^2\epsilon_m''}{n\left(1+\tau_{ee}\left[g\nu_{cm}+\dfrac{2\epsilon_m''}{3m\nu_{cm}\Lambda_B^2}\right]\right)}. \tag{14}$$

C. Heating Due to Superelastic De-excitation of the Singlet Metastable to the Triplet Metastable

The superelastic collision between an electron and a helium singlet metastable producing the triplet metastable releases 0.8 eV of energy to the electron and has a a cross section of $\sigma_{13}=3\times10^{-14}$ cm^2 as measured by Phelps[15] for room-temperature electrons. If this is an exchange collision[15] the cross section for a given electron velocity will be proportional to the square of the electron de Broglie wavelength. Hence, when the product of the cross section and the electron velocity is averaged over the electron velocity distribution, the result will be inversely proportional to the square root of the electron temperature.

Assuming the electrons to be in an undisturbed Maxwellian, the following contributions to the singlet density decay and the electron heating are obtained:

$$\frac{1}{n_s}\frac{dn_s}{dt}=-3.5\times10^{-7}\left(\frac{T_g}{T}\right)^{1/2}n \tag{15}$$

and

$$H_{13}=7.2\times10^{-6}\left(\frac{T_g}{T}\right)^{1/2}(\tfrac{3}{2}kT_gn_s), \tag{16}$$

where n_s is the singlet metastable density and H_{13} represents the heating term for the superelastic conversion of the singlet to the triplet. The inverse process whereby the triplet is converted into the singlet through an inelastic collision with an electron should also be included in these expressions. For electron temperatures below 4000°K, it will be less than a 10% correction, assuming the triplet metastable density to be no greater than the singlet metastable density.

The assumption that the electron velocity distribution is not appreciably disturbed by this heating mechanism is reasonable because the energetic electrons produced have energies low compared to the electrons produced by the heating effects of the preceding sections and hence will have a much faster rate of thermalization. Comparing the rate at which the superelastic collisions disturb the electrons from the Maxwellian distribution with the rate with which they are rethermalized shows the above assumption to be valid.

III. RESULTS AND INTERPRETATION

A. Pure Helium Afterglow

On the Fig. 1 is shown the temperature decay calculated from Eq. (8) with $H_{se}=0$ for a pressure $p=0.5$ mm as well as the measured temperature decay for these conditions. It is clear from this that there is some heating effect present in the afterglow. Temperature measurements such as those shown in Fig. 1 were made for pressures varying from 0.03 mm–5 mm in helium. The following analysis is done for the range of pressures from 0.1–1.0 mm for which the heating effect was most pronounced. At a pressure of 0.06 mm the measured temperature decay appeared free from heating effects and at a pressure of 5 mm the measured temperature decay seemed nearly free of heating effects for discharge currents greater than 250 mA except in the later afterglow where the temperature remained above room temperature for a few hundred microseconds. The electron densities given with the figures are the electron densities at the initial time of the afterglow determined as described in Sec. V.

We now make the approximation that $u\simeq\frac{3}{2}kT\simeq\frac{3}{2}kT_R$. The validity of this assumption is discussed in Sec. VIA. Equations (8) and (13) may be written in the form

$$\bar{H}=a\left(\frac{T}{T_g}\right)^{1/2}\tfrac{3}{2}k(T-T_g)+\frac{d}{dt}\tfrac{3}{2}kT, \tag{17}$$

where $\bar{H}$ may represent either H_{mm}, H_{se}, or H_{13}, or a combination thereof.

The right-hand side of Eq. (17) is completely determined by the measured variation of T with time and hence $\bar{H}$ as a function of time can be calculated from the

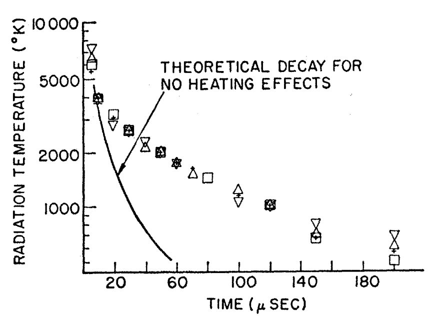

FIG. 1. Four measured radiation temperature decays in the helium afterglow at a pressure of 0.50 mm Hg and a discharge current of 100 mA measured on different days to show reproducibility of measurements. Magnetic field is 1920 G. Also shown is a calculated temperature decay assuming that only electron-atom elastic recoil influences the temperature decay. The estimated initial electron density is 1.5×10^{11} cm^{-3}.

[15] A. V. Phelps, Phys. Rev. 99, 1307 (1955).

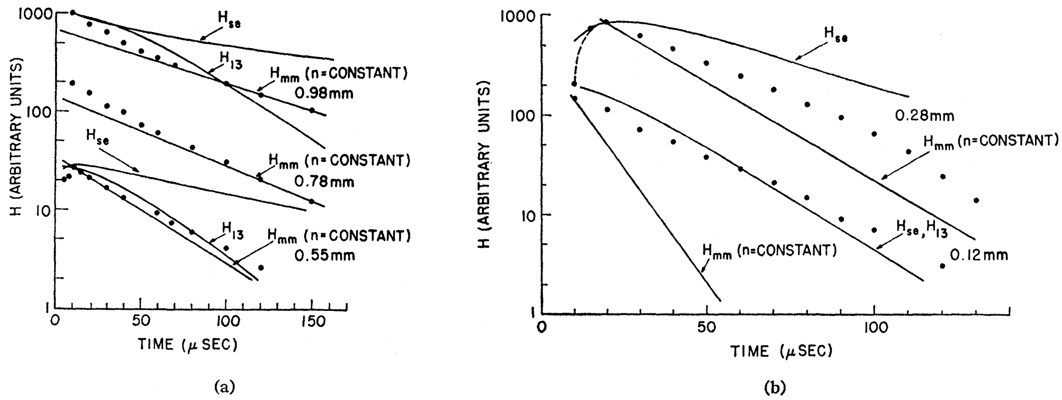

FIG. 2. Experimentally determined variation of $\bar{H}$ (points) and comparison with theoretical variation for pressures of 0.98, 0.78, 0.55, 0.28, and 0.12 mm. The transition from metastable-metastable heating (H_{mm}) at higher pressures to superelastic heating (H_{se} and H_{13}) at lower pressures is apparent.

Experimental Conditions

p (mm)	Discharge current (mA)	Initial electron density$\times 10^{-11}$ cm^{-3}	Magnetic field (G)
0.98	60	1.7	1530
0.78	75	1.6	1900
0.55	100	1.5	1900
0.28	100	0.8	1900
0.12	200	0.68	1900

temperature decay curves. Figure 2 displays this experimentally determined variation of $\bar{H}$ for pressures of 0.98, 0.78, 0.55, 0.28, and 0.12 mm. Also shown on this figure are the theoretically determined variations of H_{mm}, H_{se}, and H_{13} as given by Eqs. (10), (14), and (16), respectively. H_{mm} is calculated using the metastable diffusion coefficient[15] $D_m=470/p$ cm^2/sec and assuming the electron density is constant. The electron density decay in the expression for $H_{se}(\tau_{ee}\sim 1/n)$ is calculated assuming the electrons are lost by ambipolar diffusion and assuming that they have a Maxwellian distribution. Only the atomic ion is assumed present and $D_+=270/p$ cm^2/sec is used.[13] H_{13} is calculated by numerically integrating Eq. (15) with an additional loss term due to diffusion and substituting into Eq. (16). The electron density in Eq. (15) is assumed constant and equal to the value determined from the discharge current and voltage. The theoretical curves are fitted at one point to each experimental curve.

The figure indicates that at higher pressures the heating is due to H_{mm} and at lower pressures to H_{13} or H_{se}. The assumption that the electron density is constant when only metastable-metastable heating is present is discussed in Sec. IVB and a first-order calculation of the electron density variation is shown to improve the agreement between the experimental and theoretical variations of H_{mm}.

Two further pieces of experimental evidence indicate that H_{mm} is the dominant heating term at the higher pressures. At 0.98 mm measurements of temperature decay were analyzed for discharge currents of 10 mA and 120 mA in addition to the 60-mA current of Fig. 2(a). The observed time dependence of $\bar{H}$ was the same for each of these runs which is consistent with H_{mm} being the dominant heating term but inconsistent with H_{13} being dominant in view of the strong dependence of H_{13} on electron density through the term given in Eq. (15). At 0.78 mm it was assumed, tentatively, that $\bar{H}=H_{13}$ and the required electron density variation with time was calculated as predicted by Eq. (16) coupled with the equation for the singlet metastable density decay. This calculated electron density decayed at a rate two to three times faster than the theoretical rate for ambipolar diffusion in the presence of the atomic ion, thus indicating the assumption that H_{13} is dominant to be incorrect.

At the lower pressures the variations of H_{se} and H_{13} with time become indistinguishable. It is likely that H_{13} is the more important heating term as can be verified by direct calculation from Eqs. (10) and (16). H_{13} will be greater than H_{se} at the higher pressures, also, so that the arguments of the preceding paragraphs may be used to substantiate the evidence that H_{se}, as well as H_{13}, are negligible compared to H_{mm} at higher pressures.

It is possible for H_{mm} to dominate over H_{13} because each electron produced in the reaction contributing to H_{mm} has about ten times the energy of each electron produced in the reaction contributing to H_{13}. Furthermore, the cross section for the H_{mm} reaction is very large, being equal to about 10^{-14} cm^2 as determined by Phelps and Molnar.[16]

Since the metastable atoms are produced during the discharge pulse by electron-atom collisions and lost predominantly by diffusion, the steady-state metastable density for sufficiently long discharge pulses will be

[16] A. V. Phelps and J. P. Molnar, Phys. Rev. 89, 1202 (1953).

proportional to the cube of the pressure for a given discharge current. The initial metastable density of the afterglow period decreases rapidly with decreasing pressure as a result of this, and it is not surprising to find that at low pressures the heating terms which are linear in the metastable density become dominant over the quadratic heating term. The steady-state electron temperature during the discharge pulse increases with decreasing pressure, tending to partially offset the decrease of metastable density with pressure.

B. The Effect of Added Argon and Neon Impurities

Near $p=0.5$ mm the H_{mm} and H_{13} curves are both close to $\bar{H}$. By adding measured amounts of argon or neon impurities to the helium at this pressure and observing how the time dependence of $\bar{H}$ is altered, it is possible to calculate a value for the metastable destruction cross section due to the added impurity provided that one of the heating terms may be assumed to dominate. Such calculations were carried out assuming that $\bar{H}=H_{mm}$ and $\bar{H}=H_{13}$.

Assuming first that $\bar{H}=H_{mm}$, the following destruction cross sections are calculated:

$$\text{for argon} \quad \sigma_D=1\pm0.2\times10^{-16}\ \text{cm}^2 \tag{18a}$$

$$\text{and for neon} \quad \sigma_D=0.3\pm0.1\times10^{-16}\ \text{cm}^2. \tag{18b}$$

These values are in agreement with Biondi's[17] value of $0.93\pm0.08\times10^{-16}$ cm^2 for argon and Javan's[18] value of $0.37\pm0.05\times10^{-16}$ cm^2 for the destruction of the triplet metastable in a collision with a neon atom.

If $\bar{H}$ is now assumed equal to H_{13}, values for the singlet destruction cross section can be calculated. For argon $\sigma_D=1.6\times10^{-16}$ cm^2 and for neon $\sigma_D=0.06\times10^{-16}$ cm^2 which values compare poorly with values measured by Benton[19] for the destruction of the singlet metastable of $\sigma_D=55\times10^{-16}$ cm^2 for argon and $\sigma_D=4.1\times10^{-16}$ for neon.

The conclusion is that H_{mm} is still the dominant heating mechanism at 0.5 mm and that it is probably the triplet-triplet ionizing collision which contributes most strongly to H_{mm}.

IV. THE ELECTRON DENSITY DECAY AND METASTABLE–METASTABLE HEATING

A. Experiment

The electron density decay was measured at $p=0.515$ mm and $B=1150$ G using a phase-shift measurement of a microwave signal transmitted through the plasma and the result is given in Fig. 3. It is seen that the electron density does not decay but actually increases during the early afterglow. The decay of the density commences after about the time constant for the decay of the square of the metastable density ($\tau_{mD}/2$), where this time constant is calculated assuming the metastables are lost by diffusion and that the metastable diffusion coefficient has the value quoted in Sec. IIIA. This indicates that metastable-metastable collisions are contributing appreciably to the electron density. Measurements of the electron density decay for magnetic field values in the vicinity of cyclotron resonance were prohibited by the strong attenuation of the microwave signal in this region. Useful measurements at higher pressures were prevented by signal absorption in the plasma.

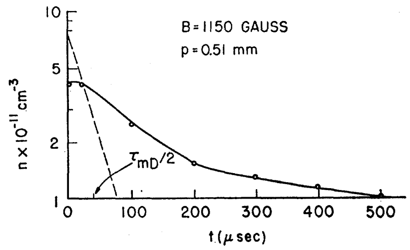

FIG. 3. Experimentally determined electron density decay showing effects of metastable-metastable ionizing collisions in the early afterglow. The dotted line is the theoretical decay for the square of the metastable density assuming loss by diffusion. The discharge current was 200 mA.

B. Correction to H_{mm} for Electron Density Variation

It is possible to improve the agreement between the experimental and theoretical values of H_{mm} in Fig. 2(a) by taking into account the time variation of the electron density in calculating the theoretical H_{mm}.

To obtain an equation for the electron density variation, we integrate Boltzmann's equation assuming that H is given by Eq. (11) and that the distribution function is a Maxwellian part plus a non-Maxwellian part given by Eq. (12). We may also eliminate the average of H, H_{mm}, from the resulting equation by using the expression for H_{mm} as given by Eq. (17) with $d/dt(\frac{3}{2}kT)$ set equal to zero. This approximation introduces an error of less than 10% for times in the afterglow later than 10 μsec. The equation finally obtained for the electron density is

$$\frac{\partial n}{\partial t}=-\frac{n}{\tau_D}\left[(1+g\nu_{cm}\tau_{ee}) -1.3\left(1+\tau_{ee}\left[g\nu_{cm}+\frac{2\epsilon_m}{3m\nu_{cm}\Lambda_B{}^2}\right]\right)\right], \tag{19}$$

where $1/\tau_D\cong254(T/T_g)^{3/2}$ sec^{-1}. Values corresponding to the conditions of the temperature decay at 0.98-mm pressure of $B=1530$ G, $R=0.65$ cm and $D_+=270/p$ cm^2/sec have been used to obtain this expression. The only unknown quantity in this expression is τ_{ee} which

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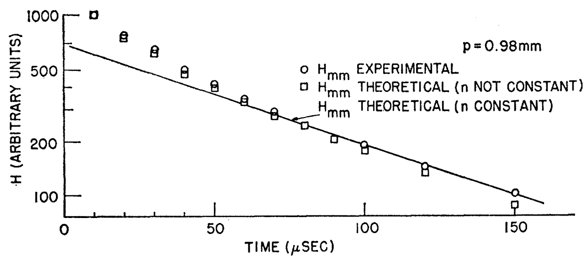

FIG. 4. Plot of experimental H and the theoretical $H(H_{mm})$ for the case where metastable-metastable ionizing collisions are the dominant heating mechanisms showing the improved agreement obtained between theory and experiment when a correction (theoretical) is made for the electron-density variation.

varies as the reciprocal of the electron density. Hence if τ_{ee} is determined at one point in the afterglow, it can be determined at another by numerical integration of Eq. (19). Such a value can be determined for τ_{ee} using Eq. (14), requiring that the time derivative of the experimental H_{mm} at $t=10$ μsec yield the same value for $\partial n/\partial t$ as does Eq. (19) at the same time in the afterglow. This gives a value of

$$\tau_{ee}=1.4\times10^{-6}\text{ sec} \tag{20}$$

at $t=10$ μsec in the afterglow where values corresponding to the experimental conditions of

$$g\nu_{cm}=0.63\times10^{-6}\text{ sec} \tag{21}$$

and

$$\frac{2\epsilon_m}{3m\nu_c\Lambda_B^2}=0.46\times10^{-6}\text{ sec}=0.72g\nu_{cm} \tag{22}$$

have been used.

It is now possible to calculate n for subsequent times using Eq. (19) and from this to calculate a theoretical H_{mm} for a nonconstant electron density using Eq. (14). The result is shown in Fig. 4 where the theoretical curves for H_{mm}, both with n constant and n varying with time, are compared against the experimental points for H_{mm}.

A theoretical estimate for τ_{ee} at $t=10$ μsec can be obtained from Spitzer's formula[10] using an electron energy of 9 eV and an electron density equal to the steady-state discharge value of $n=1.7\times10^{11}$ cm^{-3} as determined from the current and voltage of the discharge. The value is found to be

$$\tau_{ee}=4.0\times10^{-6}\text{ sec} \tag{23}$$

in reasonable agreement with Eq. (20).

If, instead, an electron energy of 15 eV is used, corresponding to the production of the helium molecule ion in the metastable-metastable collision, then a theoretical value of $\tau_{ee}\cong8\times10^{-6}$ sec is found.

V. EXPERIMENT

The 75-cm by 13-mm cylindrical quartz discharge tube was mounted on the axis of symmetry of a meter-long solenoid magnet capable of producing dc magnetic fields up to 2000 G. The center 30 cm of the discharge tube was contained in a 1-$\times$2-in. rectangular waveguide so as to allow detection of the microwave emission from the plasma at 5500 megacycles. The magnetic field in this region was uniform to better than 1.0%. The discharge was pulsed at a rate of 200 sec^{-1}. The discharge-current pulse could be varied from 10–300 mA in magnitude and from 10 to 1000 μsec in duration. The electron density was estimated from the tube current and voltage and the known value of the electron-atom collision frequency in helium.

The applied magnetic field served to inhibit the diffusional decay of the electron density, to reduce diffusion cooling effects and to increase the plasma opacity to the microwave radiation, thereby improving the plasma emission especially at low gas pressures and low electron densities. Also, by observing the dependence of kT_R on magnetic field, the deviation of the electron velocity distribution from Maxwellian could be measured as discussed in Sec. VI.

After baking the vacuum system for four days at 250°C, the apparent leak rate was less than 2×10^{-6} mm Hg per h with the discharge-tube cathode (either a tungsten spiral or an oxide-coated nickel mesh) in operation, and the system isolated from the vacuum pump. The measured temperature decays were repeatable over many consecutive runs and could be reproduced from one day to the next using a gas sample which had been allowed to remain in the system overnight, indicating the temperature decay did not depend strongly on very small amounts of impurity. Four consecutive runs made with the same gas sample at 0.98-mm pressure over a 2-h period showed good reproducibility and an accuracy of about ±100°K. Shown on Fig. 1 are curves indicating the day-to-day reproducibility of the measurements. These curves are for pressures near 0.5 mm for different gas samples on different days but approximately the same discharge current.

The waveguide containing the discharge tube was terminated with a reflecting short at one end so that the plasma was free to emit only in one direction. The plasma radiation temperature was measured by comparing electronically the power emitted per frequency interval from a radiation standard whose temperature T_s was known, with the plasma emission plus that part of the standard radiation which was reflected from the plasma and waveguide termination, $\epsilon kT_R+\Gamma kT_s$, where ϵ is the emission coefficient and Γ the reflection coefficient of the plasma and reflecting waveguide termination. If losses in the region of the waveguide near the plasma, other than those due to the plasma itself, are negligible, then from considerations of detailed balance $\epsilon=1-\Gamma$, and when T_s is adjusted (using a calibrated attenuator) so that the signals being compared are equal, $T_s=T_R$, independent of Γ. This method proved very useful since it is not limited only to those cases where the plasma is well-matched to the waveguide. The radiation standard was a neon gas discharge calibrated by the Bendix Corporation to have a temperature of $17\,500 \pm 680$°K.

The time resolution for the measurement of the electron radiation temperature was obtained by energizing the microwave receiver with a 1-μsec gating signal such that in one cycle the receiver would detect the signal $\epsilon kT_R + \Gamma kT_z$ at the desired time in the afterglow and one-half cycle later detect the signal from the standard alone, kT_z. Since this whole process was repetitive, it was possible to employ synchronous detection, thus reducing the time required to make a given measurement.

VI. DISCUSSION OF ASSUMPTIONS

By studying the variation of the radiation temperature of the plasma as the magnetic field is varied through cyclotron resonance,[1] an estimate of the error made in assuming that $\frac{3}{2}kT \simeq \frac{3}{2}kT_R \simeq u$ is obtained. Such measurements show the error made in making this assumption for the experiments here reported is never greater than 20%.

The validity of the assumption that the number of electrons in the perturbed part of the distribution function represent a small fraction of the whole may now be directly verified by integrating Eq. (12) over all electron velocities and substituting the appropriate quantities from Sec. IVB to obtain this fraction for the particular case therein discussed. The value thus found for this fraction is 1/20, thus justifying the assumption.

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Recombination of Ions and Electrons in a Highly Ionized Hydrogen Plasma*

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A hydrogenic plasma with an electron density near 5×10^{15} cm^{-3} and a temperature between 1 and 3 eV is produced by a powerful transient discharge in a tube of 14-cm diam and 86-cm length. Spectroscopic techniques are used to follow the subsequent decay of both electron density and temperature for about 300 μsec. A strong uniform axial magnetic field suppresses rapid radial particle losses, but does not appreciably reduce the radial energy transport. The axial transport apparently results merely in the slow growth of relatively thin dense boundary layers at the tube ends. In the interior the ionization is shown to decay primarily by volume recombination, and the rate coefficient agrees well with that calculated by Bates, Kingston, and McWhirter for plasmas that are virtually opaque to the Lyman-line radiation. Evidently, the plasma starts out essentially fully ionized and seems to remain close to local thermal (Saha) equilibrium for 120 μsec. The inferred energy-loss rate, on the other hand, initially exceeds that expected for kinetic transport by perhaps an order of magnitude, indicating that early in the afterglow radiative transfer may be significant.

I. INTRODUCTION

SEVERAL authors, beginning with Lord Rayleigh in 1944, have observed recombination in dense hydrogen plasmas.[1–6] The recombination was at first attributed to "radiative recombination," described by

$$H^+ + e \rightarrow H^* + h\nu\,, \tag{1}$$

with an associated recombination coefficient α defined by

$$dN_i/dt = -\alpha(T)N_iN_e\,, \tag{2}$$

where N_i is the ion density, N_e is the electron density, and T is the temperature. Measured values of α have been too large, sometimes by more than a factor of 10, to be explained by radiative recombination alone. D'Angelo showed that another process, "three-body recombination," could contribute significantly to the recombination rate of a dense plasma.[7] This process is simply the inverse of electron collisional ionization:

$$H^+ + e + e \rightarrow H^* + e\,. \tag{3}$$

Several recent publications by Bates and Kingston,[8,9]

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