Threshold Behavior of Regge Trajectories

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(Received 1 October 1964)

The object of this paper is to point out, with simple examples, that the expected threshold behavior of Regge trajectories need be of no practical significance. The term giving the leading order behavior at low energies may be small compared with other terms, for energies at which the phase shifts are still of order 10^{-6}.

THE threshold behavior of the position, $\alpha(k^2)$, of a Regge pole is given[1] by

$$\alpha(k^2)=\alpha(0)+Ak^2-B(k^2)^{\alpha(k^2)+\frac{1}{2}}e^{-i\pi[\alpha(k^2)+\frac{1}{2}]}+O(k^4)\,, \quad (1)$$

where A and B are real constants, with $B>0$. If $-\frac{1}{2}<\alpha(0)<0$ we thus see that the trajectory must leave the real l axis at $k^2=0$ in a direction making an acute angle with the negative l axis. It is difficult to see how such a trajectory can give rise to a narrow resonance at $l=0$; indeed, this fact has been used[2] to explain, from the Regge pole point of view, why such resonances do not occur for potentials such as a single Yukawa.

A potential was therefore chosen which would give an $l=0$ resonance. This was the sum of two Yukawas: a short-range attraction, plus a long-range repulsion. The position of the leading Regge pole was found by numerical integration of the Schrödinger equation, with error of the order of 10^{-6}. It was found that the trajectory left the real l axis as a tangent in the forward direction, so that there was no difficulty in producing the required resonance, as the pole passed close to $l=0$. However, this result appears to contradict the above theoretical result on threshold behavior.

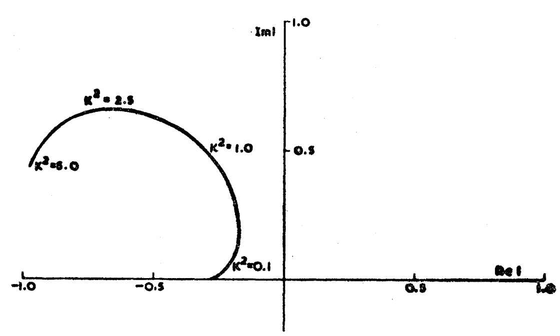

FIG. 1. The leading trajectory for $V=-2e^{-r}/r+0.5e^{-r/10}/r$.

To investigate the paradox, the repulsive tail of the potential was gradually weakened, and, at the same time, the attractive core was decreased so as to keep $\alpha(0)$ in the range $(-\frac{1}{2}, 0)$. An intermediate situation was reached where a small "hook" could be seen at the low-energy end of the trajectory. (Figs. 1 and 2.) This was for energies less than 10^{-4}, at which energy the phase shifts, of order $\mathrm{Im}\alpha(k^2)$, were of the order of 10^{-4}. This potential did not support a resonance. As the potential was further weakened, the hook became larger and larger, until the familiar results for the single Yukawa potential were obtained. Presumably, for stronger potentials, the hook was still present, giving the correct threshold behavior, but was too small to be seen. It is remarkable that the hook is negligible even before a resonance has been produced.

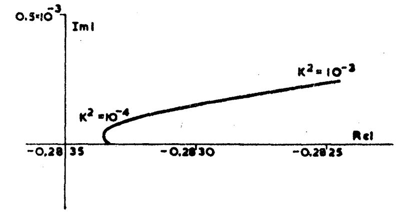

FIG. 2. The threshold behavior in detail.

It was found possible to vary the potential so that $\alpha(0)$ lay in the range $(0, +\frac{1}{2})$, when the trajectory was again apparently a tangent. We thus expect that all resonance-producing trajectories will leave the l axis effectively as tangents.

The result is expected to carry over to relativistic theory, since we are dealing with small energies, and the region $\mathrm{Re}l>-\frac{1}{2}$. Thus, doubt is cast as to the validity of the use of Eq. (1) to describe, for example, pion-pion scattering, as is done by Barut and Au.[3]

[1] A. O. Barut and D. E. Zwanziger, Phys. Rev. **127**, 974 (1962).
[2] R. G. Newton, J. Math. Phys. **3**, 867 (1962).
[3] A. O. Barut and W. S. Au, Phys. Rev. Letters **13**, 165 (1964).