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Non-Regge Terms in the Vector-Spinor Theory*

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The vector-spinor theory is examined to find whether there are any Kronecker-delta terms in the angular-momentum plane or, in other words, to find whether all particles are on Regge trajectories. It had previously been shown in lowest-order perturbation theory that a remarkable cancellation resulted in the vanishing of the Kronecker-delta term from the spinor channel. This conclusion is re-established by general reasoning which is independent of perturbation theory. The channel with the quantum numbers of the vector particle is examined, and here there is no cancellation. The vector particle is not on a Regge trajectory. It is concluded that the absence of Kronecker-delta terms in the j plane may still be used as a criterion for a "bootstrap" system.

I. INTRODUCTION

WHEN scattering amplitudes calculated by Lagrangian field theory are expressed as a Regge representation, it usually happens that the representation must be modified by the addition of polynomials in z, the cosine of the scattering angle. A polynomial $P_n(z)$ will only affect the nth partial wave and will therefore leave the Regge continuation $a(l,s)$ unchanged. The existence of such a term will mean however that the function $a(l,s)$ is not equal to the physical partial wave at $l=n$. In other words, the Regge function will have a "Kronecker-delta" singularity at $l=n$ if it is to represent all physical partial waves correctly. Since the channels in which the Kronecker-delta singularities occur depend on the elementary particles in the Lagrangian, the existence of such singularities has been suggested as a criterion for determining which particles are elementary. We shall see that one cannot obtain a criterion for determining unambiguously in all cases whether a particular channel possesses elementary particles. However, we shall be able to determine whether a theory has *any* elementary particles or, in other words, to obtain a criterion for a "bootstrap" solution. Such a solution will be one in which there are no Kronecker-delta singularities in any channel.

Kronecker-delta singularities occur in the following instances: (1) If a theory has an elementary particle of spin σ, a Kronecker-delta singularity will occur at $j=\sigma$ in the channel with the quantum numbers of the elementary particle. To show this we need merely consider the integral equation one has to solve to obtain the scattering amplitude. Generally the "potential"[1] will be an analytic function of j, so that we can hope to prove that the scattering amplitude is a meromorphic function of j. When there is an elementary particle of spin σ, however, there will be a term in the integral equations which affects only the partial wave $j=\sigma$. The integral equation for this partial wave does not therefore have the same analytic form as the integral equation for the other partial waves, and the scattering amplitude for $j=\sigma$ will not be equal to that obtained from other values of j by analytic continuation. There is thus a Kronecker-delta singularity at $j=\sigma$.

(2) In a theory with two elementary particles of spin σ_1 and σ_2, a Kronecker-delta singularity will occur at $j=\sigma_1+\sigma_2-1$ in the channel with quantum numbers equal to the sum of the quantum numbers of the elementary particles.[2] The reason for this singularity is the presence of unphysical or "nonsense" states. If we expand the scattering amplitude in orbital angular momenta l, the smallest possible value of l is $j-\sigma_1-\sigma_2$. At $j=\sigma_1+\sigma_2-1$, there will thus be a state with $l=-1$. If we first consider the equations for the scattering amplitude at high values of j and then continue to $j=\sigma_1+\sigma_2-1$, the nonsense state will be coupled to the physical state. In the equations for the correct physical scattering amplitude, however, the coupling to the nonsense state should not be included. Thus, the Regge continuation of the amplitude to $j=\sigma_1+\sigma_2-1$ will not be equal to the physical partial-wave amplitude at this value of j.

Gell-Mann and Goldberger[3] have raised the question whether there might be cancellations, so that the Kronecker-delta singularities would not occur at the positions just stated. They have suggested in particular that the cancellation might occur in a theory of nucleons interacting with vector mesons, and that the singularity might be absent from the channel with the quantum numbers of the nucleon. Such a channel has both an elementary particle (the nucleon) with $\sigma=\frac{1}{2}$, and a pair of elementary particles (the nucleon and meson) with $\sigma_1+\sigma_2-1=\frac{1}{2}$. The suggestion of Gell-Mann and Goldberger has been examined further,[4] and a factorization criterion has been given for the cancellation to occur.

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[1] By "potential" we simply mean the input to whatever integral equations are to be used. We do not imply that we are inserting the potential into a Schrödinger equation.

[2] S. Mandelstam, Nuovo Cimento **30**, 1113 (1963).

[3] M. Gell-Mann and M. L. Goldberger, Phys. Rev. Letters **9**, 275 (1962).

[4] M. Gell-Mann, M. L. Goldberger, F. E. Low, and F. Zachariasen, Phys. Letters 4, 265 (1963). M. Gell-Mann, M. L. Goldberger, F. E. Low, E. Marx, and F. Zachariasen, Phys. Rev. **133**, B145 (1964). M. Gell-Mann, M. L. Goldberger, F. E. Low, V. Singh, and F. Zachariasen, *ibid.* **133**, B161 (1964).

When the criterion was applied to the nucleon channel of the spinor-vector theory, it was found that the cancellation did indeed occur in the lowest order of perturbation theory. In other words, the nucleon lies on the Regge trajectory in lowest order. The cancellation can only occur if nonsense channels are present, but it certainly does not always occur in such cases. For instance, it does not occur in the theory of scalar "nucleons" interacting with vector mesons.

The fact that the lowest-order perturbation amplitude in the vector-spinor theory satisfied the factorization criterion appeared to be somewhat of a miracle. The perturbation terms were just such as to produce a Regge behavior (without a Kronecker-delta singularity), but no deeper reason for this was evident. It is the aim of the present paper to show directly, without detailed calculation, that there is no Kronecker-delta singularity at $j=\frac{1}{2}$ in the nucleon channel. We shall thus be able to confirm the conjecture of Gell-Mann *et al.* that the cancellation of the Kronecker-delta singularity is not confined to the lowest order of perturbation theory, but is general.

We shall also examine the channel with the quantum numbers of the vector meson in the vector-spinor theory. There is of course an elementary particle in this channel at $j=1$ and, while there does not exist a pair of particles with $\sigma_1+\sigma_2-1=1$, there exists a triplet of particles, namely three vector mesons, with $\sigma_1+\sigma_2+\sigma_c-2=1$. It has therefore been suggested that the cancellation of the Kronecker-delta singularity might occur in this channel as well. The theory would then be free of such singularities. We shall find, however, that the singularity is in fact present. We shall make the approximation of neglecting intermediate states with more than three particles, but such an approximation appears to possess all the essentials of the problem.

The characteristic feature of the nucleon channel in the vector-spinor theory is that the diagram Fig. 1(a), with a one-nucleon intermediate state, cannot exist by itself but must be taken together with Fig. 1(b). Either of these diagrams taken separately would give an amplitude, the asymptotic behavior of which is in conflict with unitarity. Stated more loosely, the individual diagrams are not gauge-invariant. The vector-meson channel does not have this feature. The diagram Fig. 1(c) is gauge-invariant and can certainly exist alone. It had been suspected that this difference between the nucleon and meson channels might lead to a difference in their Regge behavior, and we shall show that such is the case. We should emphasize, however, that the necessity of adding a diagram like Fig. 1(a) to one like Fig. 1(b) does not in itself guarantee a Regge behavior. One must compare the number of threshold conditions with the number of free parameters in a partial-wave integral equation. The scalar-vector theory also has a diagram Fig. 1(b) which goes with Fig. 1(a), but it is not of Regge-type.

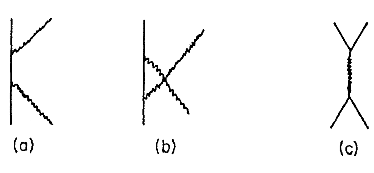

FIG. 1. Lowest-order perturbation diagrams in the nucleon on vector-meson channels.

From the remarks made in this section, it is clear that one cannot use the Kronecker-delta singularities in the j plane as a universal criterion for distinguishing elementary from nonelementary particles. Even in theories without vector mesons one cannot obtain an unambiguous criterion. If a theory has elementary fermions of spin $\frac{1}{2}$, for instance, one cannot use such a method to test for the existence of an elementary particle with spin zero in the two-fermion channel. This channel has a pair of particles, the two fermions, with $\sigma_1+\sigma_2-1=0$, and there will therefore be a Kronecker-delta singularity whether or not there is an elementary particle present. One can still use the criterion to test whether there are any elementary particles present in a theory. The vector-spinor system does not provide a counterexample, since there is a Kronecker-delta singularity in the meson channel even though there is no such singularity in the nucleon channel.

II. THE BEHAVIOR OF THE POTENTIAL AND OF THE SCATTERING AMPLITUDE NEAR $j=\sigma_1+\sigma_2-1$

Before treating our problem it will be necessary to review some properties of the amplitude in the j-plane near $j=\sigma_1+\sigma_2-1$. We write the scattering amplitude as

$$A=A^L+A^R, \tag{2.1}$$

where A^L is the integral over the left-hand cut and A^R the integral over the right-hand cut. In general we would know A^L, either explicitly or from a previous iteration, and we would then find A^R by unitarity. We shall refer to A^L as the "potential" and shall denote it by the symbol V. We shall use the subscripts s and n to denote sense and nonsense states.

When the angular momentum j approaches the value $\sigma_1+\sigma_2-1$, the matrix elements of the *potential* will behave as follows:

$$V_{ss}\to \text{finite}, \tag{2.2a}$$

$$V_{sn}\to b/\sqrt{(j-\sigma_1-\sigma_2+1)}, \tag{2.2b}$$

$$V_{nn}\to c/(j-\sigma_1-\sigma_2+1). \tag{2.2c}$$

The pole in the element V_{nn} arises from the fact that a nonsense amplitude involves a value of l equal to -1 (or another negative integer). We recall that the Froissart-Gribov formula for the analytically continued amplitude contains the function Q_l, which has poles when l is a negative integer. The sense-nonsense amplitude will also include a factor Q_l with $l=-1$ but, on the other hand, such an amplitude always has a factor $\sqrt{(j-\sigma_1-\sigma_2+1)}$ from the Clebsch-Gordan coefficient, and this factor vanishes at the integer in question. Thus V_{sn} only has a factor $\sqrt{(j-\sigma_1-\sigma_2+1)}$ in the denominator.

If we examine the *unitary scattering amplitude* instead of the potentials we must bear in mind that each matrix element of the scattering amplitude will be bounded by unitarity. The square-root branch point at $j=\sigma_1+\sigma_2-1$ will still be present, so the amplitude will have the behavior

$$A_{sn}\to B\sqrt{(j-\sigma_1-\sigma_2+1)}\,, \tag{2.3a}$$

$$A_{nn}\to C\,. \tag{2.3b}$$

Equations (2.2) and (2.3) indicate a difference between the behavior of the potential and the unitary scattering amplitude near $j=\sigma_1+\sigma_2-1$.

A question closely related to the singularities of the potential is the behavior of the Regge trajectories as the coupling is turned on. In a theory without spin, the leading trajectory begins to move from the value $j=-1$ as the coupling is increased. If, however, there is a nonsense channel at $j=n$, where n is a positive integer or zero, a trajectory will move from $j=n$. This is an immediate consequence of the pole in the potential at $j=n$ since, when the coupling is very small, the scattering amplitude must be almost equal to the potential and must have a pole very near $j=n$. Similarly, if there are r nonsense channels at $j=n$, there will be r separates poles in the potential at $j=n$, so that r Regge trajectories will move from the value $j=n$. In general, there will *not* be particles or resonances at the points where these trajectories cross the lines $j=n$, because the physical partial waves at $j=n$ are not given by the Regge function. In the exceptional case of Gell-Mann *et al.*, however, the nucleon will lie at the point where the trajectory crosses the line $j=\frac{1}{2}$.

III. THE PHYSICAL PARTIAL WAVES COMPARED WITH THE REGGE CONTINUATION

Before we treat the spinor-vector system, let us examine a general channel which has a pair of elmentary particles with spin σ_1 and σ_2, and in which the special features of the spinor-vector system are not present. We shall compare the correct physical partial wave at $j=\sigma_1+\sigma_2-1$ with the Regge continuation and shall investigate how they differ. We shall find that the effect of the coupling to the nonsense states can be replaced by a fictitious elementary particle. In Sec. IV we shall apply this result to the nucleon channel of the vector-spinor theory, and shall deduce the results stated in Sec. I.

The scattering amplitude can be determined from the potential by solving the integral equation

$$N(j,s)=V(j,s)+\frac{1}{\pi}\int ds'\frac{V(j,s')-V(j,s)}{s'-s}\times k(s')N(j,s')\,, \tag{3.1a}$$

$$D(j,s)=1-\frac{1}{\pi}\int ds'\frac{1}{s'-s}k(s')N(s')\,, \tag{3.1b}$$

where k is a kinematic factor. The quantities N, D, and V will be matrices of order equal to the number of channels. If we are calculating the physical partial wave we must include only the sense channels, whereas we must include all channels for the Regge continuation. Since the matrix elements of V involving the nonsense channels are certainly not zero, the physical partial wave will differ from the Regge continuation.

To investigate the matter further, we shall examine the conditions which led to Eq. (3.1). The equation is a consequence of the following three requirements:

(i) analyticity,
(ii) unitarity,
(iii) the choice of the lowest Castillejo-Dalitz-Dyson (C.D.D.) solution,

and we must determine whether these conditions when applied to the physical partial wave are different from when they are applied to the Regge continuation.

The analyticity requirements (in the s plane) for the physical partial wave and the Regge continuation are of course identical. The unitarity equations are

$$\mathrm{Im}A_{ab}=kA_{ac}{}^*A_{cb}\,, \tag{3.2}$$

where the summation is over sense intermediate states c in the physical partial wave, and over sense and nonsense intermediate states c in the Regge continuation. We have already seen however that the scattering amplitude between a sense and a nonsense state is zero at $j=\sigma_1+\sigma_2-1$ [Eq. (2.3)]. If, therefore, a and b are sense states, the summation c need be taken over sense states alone, even in the Regge continuation. The unitarity condition for the physical partial wave is, therefore, the same as for the Regge continuation.

The difference between the physical partial wave and the Regge continuation must, therefore, lie in the C.D.D. ambiguity. We have mentioned in Sec. I that r Regge trajectories start from the value $j=n$ in question, where r is the number of nonsense states. For very small coupling, some or all of these trajectories may pass through the value $j=n$ at a particular value of s. The solution of the equations for the coupled sense and nonsense states will have particles or resonances at this value of s. If, therefore, we take the solution and examine only the sense states, which we have seen to satisfy the unitarity equation among themselves, we may find that they represent any C.D.D. solution up to the $(r+1)$th.

One may ask whether Levinson's theorem does not forbid the higher C.D.D. solutions if elementary particles are not introduced explicitly into the calculations. However, Levinson's theorem applied to the partial wave $j=n$ would refer to the physical partial wave, not the Regge continuation. One could also apply

Levinson's theorem to values of j other than $j=n$, and then proceed to $j=n$ by analytic continuation. In that case the theorem would involve the sum of the phase shifts in the sense and nonsense channels. The theorem does not say anything about the phase shift of the sense channels alone in the Regge continuation. Thus, if we solve the equations for the coupled sense and nonsense channels and examine the sense channel of the solution, we may find that it is a higher C.D.D. solution, even if elementary particles have not been introduced into the calculation.

To summarize, the sense states of the Regge continuation may represent any of the C.D.D. solutions up to the $(r+1)$th where r is the number of nonsense states. In other words, there may be up to $r+1$ fictitious elementary particles in the sense states of the Regge continuation. The parameters corresponding to these particles cannot be found without solving the equations with the nonsense states included, but, once they are known, the nonsense states can be ignored. The difference between the physical partial wave and the Regge continuation will, therefore, be that the physical partial wave involves the real elementary particles which are introduced explicitly into the calculation, while the Regge continuation involves the fictitious elementary particles which replace the nonsense states. In general, the number of elementary particles for the two cases and the parameters associated with them will be different, so that the physical partial wave will be different from the Regge continuation.

IV. THE NUCLEON CHANNEL IN THE VECTOR-SPINOR THEORY

For the nucleon channel of the vector-spinor theory, the above results must be supplemented by our knowledge that the parameters associated with the elementary nucleon cannot be varied arbitrarily if the analyticity-unitarity equations are to be soluble. We have pointed out that a potential represented by Fig. 1(b) has an asymptotic behavior which violates the unitarity limit, and that the equations will not be soluble unless Fig. 1(a) is also included. Further, the parameters associated with Fig. 1(a) cannot be varied arbitrarily if the asymptotic behavior of the terms associated with the two diagrams is to cancel. The question, therefore, arises whether the parameters associated with Fig. 1(a), i.e., the position and residues of the pole, can be varied at all without destroying the cancellation between the two diagrams. If they cannot, we may conclude that the parameters associated with the real elementary particle in the physical partial wave are the same as those associated with the fictitious elementary particle in the Regge continuation. The two amplitudes then cannot differ, and there will be no Kronecker-delta term in the j plane. If on the other hand, we may vary the parameters associated with the elementary particle and still obtain a cancellation between the asymptotic behaviors, we cannot conclude that the Kronecker-delta term must vanish

We shall proceed to investigate this question by examining the dispersion relation for the partial waves. For simplicity we shall confine ourselves to the equal-mass case. We shall find that the parameters cannot be varied in the vector-spinor theory but that they can in the vector-scalar theory. Our reasoning will not depend upon the precise value of the discontinuity across the left-hand cut, so that we will not be limited to an input corresponding to Fig. 1(b). The conclusions are therefore, independent of perturbation theory. The partial wave of interest is that in the nucleon vector-meson channel with $j=\frac{1}{2}$ and positive parity. There are two sense states and one nonsense state involved:

$$\text{Sense: } S=\tfrac{3}{2},\ l=1;\quad \text{Nonsense: } S=\tfrac{3}{2},\ l=-1.$$
$$S=\tfrac{1}{2},\ l=1.$$

The variable S is the total spin, l is the orbital angular momentum. When we write down partial-wave dispersion relations in a channel with half-integral spin, we have to take w, the square root of s, as out variable, otherwise the amplitudes will contain kinematical singularities at $s=0$. On going from w to $-w$, the partial waves will go into linear combinations of the partial waves with the same value of j but with opposite parity. The possible states are then:

$$\text{Sense: } S=\tfrac{3}{2},\ l=2;\quad \text{Nonsense: } S=\tfrac{3}{2},\ l=0.$$
$$S=\tfrac{1}{2},\ l=0.$$

We now have to investigate the threshold and asymptotic behavior of the partial waves. We shall normalize the partial waves so that the unitarity condition contains a factor q/w, where q is the center-of-mass momentum. The amplitudes will then be free of kinematical singularities at $w=0$ (in the equal-mass case). There will be three sense-sense amplitudes corresponding to the two sense states, and they will all have a constant asymptotic behavior at infinity. At the thresholds $w=2m$ (for positive parity) and $w=2m$ (for negative parity) they will have the behavior:

$w=2m$: all three waves are P-waves, which behave like $w=m$;

$$w=2m:\qquad DD\sim(w+m)^2,$$
$$DS\sim(w+m),$$
$$SS\sim\text{const}.$$

If we set up an N/D calculation for the sense states with a given left-hand cut, we can put in three subtraction constants corresponding to the constant asymptotic behavior of the three partial waves. On the other hand, we have six threshold conditions to satisfy. We therefore have too few parameters available and unless the left-hand cut has special features, we shall not be able to solve the problem.

We can overcome this difficulty by introducing an elementary particle into the channel. As there are two sense states, we shall have three further parameters at our disposal, and we can just solve the problem. None of the parameters will be free; they must all be given particular values if the threshold behaviors are to come out right. This conclusion cannot be modified by assuming the left-hand cut to have special features. The N/D equations for the problem with an undermined elementary-particle pole are linear integral equations with no free parameters and, therefore, have a unique solution, the case where the homogeneous equation is soluble being easily accounted for. With suitable left-hand cuts the residues at the pole could turn out to be zero, though this does not occur in the problem under consideration. The equation would then be soluble without any elementary-particle pole, but it cannot happen that the equations are soluble with a variable elementary-particle pole.

We can now conclude that the physical partial wave must be identical with the Regge continuation in the nucleon channel. The physical partial wave has one elementary particle, and the Regge continuation has one fictitious elementary particle which replaces the effect of the single nonsense state. Since we have just proved that the parameters associated with a single elementary particle are fixed by the threshold conditions, it follows that the parameters associated with the real and the fictitious elementary particle must be the same. The physical partial wave is thus identical with the Regge continuation.

We can also treat the scalar-vector problem, and we shall examine the amplitude for the scattering of scalar particles by vector mesons. In the positive-parity state with $j=0$ there is one sense state ($l=1$) and one nonsense state ($l=-1$). The sense state must have a constant asymptotic behavior at infinity and must satisfy one threshold condition. The N/D integral equations are thus soluble without any free parameters whatever the position and residue of the elementary-particle pole. The parameters of the elementary particle are not determined by the remainder of the problem, and the Regge continuation need not be the same as the physical partial wave.[5] Such reasoning does not of course prove that the two amplitudes will not be equal, but direct examination shows that they are in fact unequal.

[5] The above reasoning would appear to indicate that one could obtain a solution with arbitrary values of both the position and residue of the elementary-particle pole. Actually only one of these parameters is arbitrary. The reason is that there is a relation between the helicity amplitudes at $s=0$, analogous to Eq. (7.15a) in the paper of Goldberger, Grisaru, MacDowell and Wong, Phys. Rev. **120**, 2250 (1960). This relation must hold for the Kronecker-delta term, and will reduce the number of parameters from two to one.

V. VECTOR-MESON CHANNEL IN THE VECTOR-SPINOR THEORY

We have remarked that there is no correlation between diagrams in the vector-meson channel analogous to that between Figs. 1(a) and 1(b) in the nucleon channel. The arguments which we have given above for the cancellation of the Kronecker-delta term in the nucleon channel cannot, therefore, apply here. We now wish to examine the amplitude in that channel in order to show that the cancellation does not occur and that there is a non-Regge term.

We have already mentioned that the simplest intermediate state which might Reggeize the vector meson is the three-meson state. A complete treatment of the problem will, therefore, require a knowledge of complex angular momentum in the three-body systems. We hope to give a discussion of that topic in a subsequent paper. In the present paper we shall assume some general results concerning the three-body problem, and shall then show that there is a Kronecker-delta term in the vector-meson channel at $j=1$.

We first summarize some results of Ref. 4. It is shown there that the potential has the following behavior in the j plane near $j=n$, where n is an integer at which non-Regge terms occur:

$$\begin{aligned} V_{ss}&=-a\delta_{ss}\,,\\ V_{sn}&=b/\sqrt{(j-n)}\,,\\ V_{nn}&=c/(j-n)\,. \end{aligned} \tag{5.1}$$

The quantities a, b, and c are independent of the energy. If there are several sense and nonsense channels, a and c will be square matrices, b a diagonal matrix. We note that the type of singularity in the sense-sense amplitude is completely different from that in the nonsense amplitudes. In the sense-sense amplitude it is a Kronecker-delta term, and it may arise from diagrams such as Fig. 1(a) or Fig. 1(c) which affect only one value of j. In the nonsense amplitude it is an infinity, and it arises from the fact that the function Q_l has a pole when l is a negative integer.

Gell-Mann *et al.* showed that the Kronecker-delta term in the scattering amplitude cancels provided that

$$a=b^Tc^{-1}b\,. \tag{5.2}$$

They did not show conclusively that (5.2) was a sufficient condition, as there were some uncertainties regarding subtraction terms in dispersion relations. However, (5.2) was definitely a necessary condition, and it was obtained from the factorization property of residues at Regge poles. Note that (5.2) provides a separate equation for each sense state, so that we may examine only those sense state which are convenient if we are trying to prove that the equation is untrue. On the other hand, the equation involves a sum over all the nonsense states.

If the nonsense states in question are three-particle states, the quantities a, b, and c will be integral operators instead of finite matrices, and the question arises whether the expression c^{-1} has a meaning. We hope to show in our subsequent paper in the three-body problem that the operator c only spans a finite number of the three-particle states, even though an infinite number of three-particle states exists (at given values of s and j). Thus, the equations (5.1) remain finite matrix equations.

Let us, therefore, investigate a nucleon-antinucleon sense state and the three-meson nonsense states. The potentials V_{ss} and V_{sn} will be given in lowest order by Fig. 2(a) and (b). Since they are of order g^2 and g^3, respectively, the potential V_{nn} will have to be of order g^4 if (5.2) is to hold. The only diagram of order g^4 for the process 3 mesons $\rightarrow$ 3 mesons is the disconnected diagram Fig. 2(c), and it has been suggested that this diagram might provide a contribution to c in (5.1) which fulfills the criterion (5.2).[6]

We can conclude, however, that the diagram Fig. 2(c) does not provide a contribution to the quantity c in (5.1). Let us take as our angular variables the helicities λ and λ' of the incoming system AB and of the outgoing system $A'B'$, together with the total angular momentum j. The angle corresponding to j will be the angle of scattering of C. The scattering amplitude will then contain a factor $\delta(z)$, so that its angular-momentum projection will be $\delta_{\lambda\lambda'}$, which is independent of j. This diagram, therefore, has no pole at $j=1$, and it will not contribute to the quantity c of (5.1).

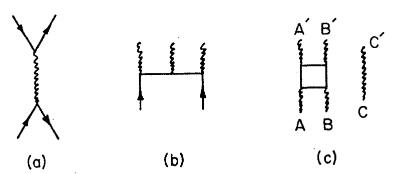

FIG. 2. Perturbation diagrams in the vector-meson channel.

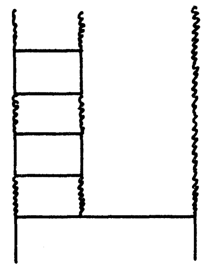

FIG. 3. A diagram obtained by iteration of those in Fig. 2.

We can see in another way that the potential represented by Fig. 2(c) cannot give a contribution to the quantity c. Figure 2(b), together with repeated iteration of Fig. 2(a), would give Fig. 3. Now the reasoning in Ref. 4 shows that repeated iteration of the potential diagrams must give increasing powers of the logarithm of the momentum transfer if they are to result in cancellation of the Kronecker-delta term. However, Fig. 3 was precisely the type of diagram considered by Mandelstam[7] and, by a slight variation of the arguments used in Sec. 2 and Appendix I of that paper, it can be shown that the increasing powers of the logarithm do not occur.

Our conclusion is, therefore, that there is no contribution to c of order g^4, so that the Kronecker-delta term in the vector-meson amplitude at $j=1$ is not cancelled. As a matter of fact, the three-meson states do not give any pole at all in the potential at $j=1$, and c is zero to all orders of g^2. There is thus no Regge trajectory which moves from $j=1$ as the coupling is turned on. We shall prove this result when discussing the three-body problem. The conclusions of the present paper are, of course, independent of the last assertion.

[6] The assertion that c must be of order g^4 for (4.2) to hold could possibly be wrong if the matrix $b^Tc^{-1}b$ vanished in lowest order. Thus if $b=b_3g^3$, $c=c_6g^6+c_8g^8$ and $b_3{}^Tc_6{}^{-1}b_3=0$, the right-hand side of (4.2) would be of order g^2. We shall show in our paper on the three-body problem that this does not occur for the case in question.

[7] S. Mandelstam, Nuovo Cimento **20**, 1127 (1963).