Effect of Electromagnetic Corrections to Low-Energy Nucleon-Nucleon Scattering on Charge Independence*

R. E. SCHNEIDER† AND R. M. THALER
Case Institute of Technology, Cleveland, Ohio
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The low-energy nucleon-nucleon scattering lengths are calculated, taking into account the electric charge and magnetic-moment distributions of the nucleons. Corrections in the one-pion exchange part of the effective nuclear interaction due to the charged and uncharged pion mass difference are also included. The singlet proton-proton, proton-neutron, and neutron-neutron scattering lengths are calculated and compared with experiment. These corrections reduce the discrepancy to be explained by other effects in order to maintain charge independence. We conclude that recent measurements of the neutron-neutron scattering length do not contradict the hypothesis of charge independence.

I. INTRODUCTION

THE specifically nuclear force operating between two nucleons has frequently been considered to be independent of the charge state of the two nucleons. One quantitative test of this hypothesis may be made by comparing the low-energy proton-proton, proton-neutron, and neutron-neutron scatterings in the singlet state. Such experiments are interpreted in terms of the effective range expansion. The measured values of the scattering lengths are

$a_{pp}=-7.68$ F for the proton-proton scattering length;
$a_{np}=-23.74$ F for the neutron-proton scattering length.

Recent measurements[1] indicate that the neutron-neutron scattering length is $a_{nn}=-17\pm5$ F.

The hypothesis of charge independence requires that the specifically nuclear force be the same for the three cases mentioned above. The scattering lengths are quite different. The electrostatic repulsion between the two protons has the effect of reducing the attractive force between the two particles. When the Coulomb repulsion between point charges is taken into account, a_{pp} becomes $\simeq-16$ F while the effective range is unchanged.

We here ask whether the discrepancy between this value of the Coulomb-corrected proton-proton scattering length and the observed value of the neutron-proton scattering length, $a_{np}=-23.74$ F violates charge independence. Schwinger[2] pointed out some time ago that the additional electromagnetic effects arising from the magnetic moments of the nucleons must also be considered. By taking these effects into account Schwinger[2] obtained agreement with charge independence, provided the nuclear interaction was taken to be singular at the origin. Subsequently, Salpeter[3] pointed out that this calculation was no longer valid if the nuclear force possessed a hard core, in which case the effect due to the point magnetic moments vanishes.

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† Present address: University of Akron, Akron, Ohio.
[1] J. W. Ryan, Phys. Rev. Letters **12**, 564 (1964).
[2] J. Schwinger, Phys. Rev. **78**, 135 (1950).
[3] E. Salpeter, Phys. Rev. **91**, 994 (1953).

II. ELECTROMAGNETIC CORRECTIONS

In this paper we consider further the electromagnetic corrections due to the charge and magnetic moment distributions of the nucleons as taken from the experiments of Hofstadter *et al.*[4] and Wilson *et al.*[5]

The experimental results may be interpreted to yield the Fourier transforms of the charge and magnetic-moment distributions. We may calculate electromagnetic potentials for use in the Schrödinger equation by writing

$$V(\mathbf{r})=\frac{1}{(2\pi)^3}\int e^{i\mathbf{q}\cdot\mathbf{r}}V(\mathbf{q})d\mathbf{q}\,, \tag{1.1}$$

and

$$V(\mathbf{q})=f_1(\mathbf{q})f_2(\mathbf{q})v(\mathbf{q})\,, \tag{1.2}$$

where $f_1(\mathbf{q})$ and $f_2(\mathbf{q})$ are the Fourier transforms of the two interacting charge or magnetic moment distributions and $v(\mathbf{q})$ is the Fourier transform of the interaction between two point sources.

Following Hofstadter and Herman,[4] we relate the Dirac form factors of the neutron and proton, F_{1n} and F_{1p}, and the Pauli form factors for the neutron and proton, F_{2n} and F_{2p}, to the vector and scalar form factors as

$$F_{1S}=F_{1p}+F_{1n}\,, \tag{2.1}$$

$$F_{1V}=F_{1p}-F_{1n}\,, \tag{2.2}$$

$$F_{2S}=\{\mu_p'F_{2p}+\mu_n'F_{2n}\}/\{\mu_p'+\mu_n'\}\,, \tag{2.3}$$

$$F_{2V}=\{\mu_p'F_{2p}-\mu_n'F_{2n}\}/\{\mu_p'-\mu_n'\}\,, \tag{2.4}$$

where $\mu_p'=1.79$ and $\mu_n'=-1.91$ are the anomalous magnetic moments in nuclear magnetons of the proton and the neutron, respectively. The quantities F_{1S}, F_{1V}, F_{2S}, F_{2V} so defined are each of a form such that their Fourier transform is a delta function plus a Yukawa cloud. The spatial interpretation is then a Yukawa cloud plus a point core.

The explicit forms assumed by deVries, Hofstadter,

[4] R. Hofstadter and R. Herman, Phys. Rev. Letters **6**, 293 (1961).
[5] D. N. Olson, H. F. Schopper, and R. R. Wilson, Phys. Rev. Letters **6**, 286 (1961).

and Herman[6] are

$$F_{1S}=\frac{s_1}{1+2.04q^2/M_S^2}+(1-s_1)\,, \tag{3.1}$$

$$F_{2S}=\frac{s_2}{1+2.04q^2/M_S^2}+(1-s_2)\,, \tag{3.2}$$

$$F_{1V}=\frac{v_1}{1+2.04q^2/M_V^2}+(1-v_1)\,, \tag{3.3}$$

$$F_{2V}=\frac{v_2}{1+2.04q^2/M_V^2}+(1-v_2)\,. \tag{3.4}$$

The system of units employed is such that $\hbar=c=1$.

Use of the experimentally determined values[6] for the parameters in Eq. (3) above, yields the explicit formulas

$$F_{1p}=\tfrac{1}{2}\left\{-0.09+\frac{1.17}{1+0.0887q^2}+\frac{0.92}{1+0.113q^2}\right\}, \tag{4.1}$$

$$F_{1n}=\tfrac{1}{2}\left\{-0.25+\frac{1.17}{1+0.0887q^2}-\frac{0.92}{1+0.113q^2}\right\}, \tag{4.2}$$

$$F_{2p}=\frac{1}{3.58}\left\{-0.55+\frac{0.06}{1+0.0887q^2}+\frac{4.07}{1+0.113q^2}\right\}, \tag{4.3}$$

$$F_{2n}=\frac{1}{3.82}\left\{-0.19-\frac{0.06}{1+0.0887q^2}+\frac{4.07}{1+0.113q^2}\right\}. \tag{4.4}$$

These form factors are related to the electric G_E and magnetic G_M form factors given by Wilson[7] according to

$$G_E=F_1-(q^2/4m^2)\mu' F_2\,, \tag{5.1}$$

$$G_M=F_1+\mu' F_2\,, \tag{5.2}$$

where m refers to the nucleon mass. The functions G_E and G_M are the ones which are then used to compute $V(\mathbf{q})$ of Eq. (1.2) and hence, finally the desired electromagnetic potential, $V(\mathbf{r})$.

The potentials calculated in this way are given below and are shown graphically in Fig. 1.

$$V_{pp}^{\mathrm{el}}(r)=(e^2/r)\{1-e^{-3.36r}(0.582r-2.776) \\ -e^{-2.97r}(0.644r+3.639)\}\,, \tag{6.1}$$

$$V_{pp}^{\mathrm{mag}}(r)=(\mu_0^2/r)\{e^{-3.36r}(14.32r-258.0) \\ +e^{-2.97r}(163.15r+220.9)\}\,, \tag{6.2}$$

$$V_{np}^{\mathrm{el}}(r)=(e^2/r)\{e^{-3.36r}(-0.229-0.582r) \\ +e^{-2.97r}(0.215+0.644r)\}\,, \tag{6.3}$$

$$V_{np}^{\mathrm{mag}}(r)=(\mu_0^2/r)\{e^{-3.36r}(14.32r-4.854) \\ +e^{-2.97r}(12.77-163.15r)\}\,, \tag{6.4}$$

$$V_{nn}^{\mathrm{el}}(r)=(e^2/r)\{-e^{-3.36r}(0.567r+1.047) \\ +e^{-2.97r}(0.527r+1.161)\}\,, \tag{6.5}$$

$$V_{nn}^{\mathrm{mag}}(r)=(\mu_0^2/r)\{e^{-3.36r}(14.32r+248.23) \\ -e^{-2.97r}(246.73-164.16)\}\,, \tag{6.6}$$

where μ_0 is the nuclear magneton. The terms which have a delta function behavior at the origin are not shown. These terms are of no interest in this calculation, be-

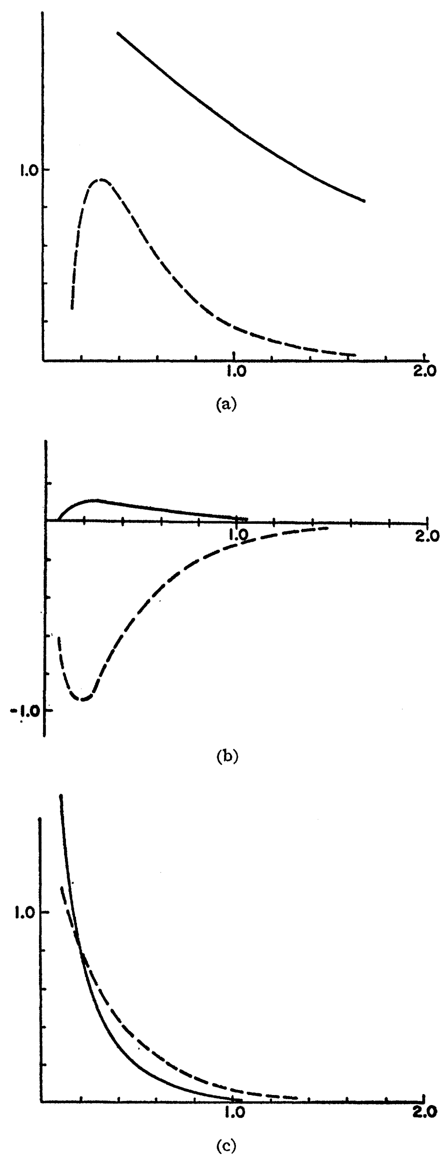

FIG. 1. Nucleon-nucleon electromagnetic potentials, as a function of radial distance as derived from the Hofstadter electron scattering data. The ordinates are in MeV, the abscissae in fermis. The electric potentials are indicated by solid curves, the magnetic potentials are indicated by dashed curves. Panel (a) shows the proton-proton electromagnetic potentials; panel (b) shows the neutron-proton electromagnetic potentials; panel (c) shows the neutron-neutron electromagnetic potentials.

[6] C. deVries, R. Hofstadter, and R. Herman, Phys. Rev. Letters 8, 381 (1962).

[7] L. N. Hand, D. G. Miller, and R. Wilson, Phys. Rev. Letters 8, 110 (1962).

cause we assume that the nucleon-nucleon wave function vanishes near the origin.

III. CALCULATIONS AND RESULTS

In the computational program a nuclear potential with two adjustable parameters, a depth parameter V_0, and a range parameter b, was assumed. The $l=0$ radial equation for the proton-proton system

$$[d^2u_p(r)/dr^2]+(M/\hbar^2)\times[E-V_N(r)+V_{pp}{}^e(r)]u_p(r)=0 \quad (7)$$

was then solved, and the parameters V_0 and b of V_N adjusted, so as to produce the proton-proton singlet scattering length of $a_{pp}=-7.68$ F and an effective range of $\rho_{pp}=2.65$ F. Analyses[8] of recent experimental data suggest that the proton-proton effective range may be larger than the value $\rho_{pp}=2.65$ F used here by about 5%. The use of a larger value for the proton-proton effective range would not affect the conclusions here drawn. The electromagnetic contribution to the interaction is $V_{pp}{}^e$. The phenomenological nuclear interaction V_N, so determined was then used to calculate the neutron-proton effective-range parameters.

A Yukawa potential with a hard core was assumed for the nuclear interaction V_N:

$$\begin{aligned} V_N &= \infty\,, & r&<r_{\text{core}} \\ &=-V_0(e^{-r/b}/r/b)\,, & r&>r_{\text{core}}. \end{aligned} \quad (8)$$

The core radius[9] used in this calculation was $r_{\text{core}}=0.388$ F. In carrying out this particular program of calculations four cases were considered. The first three are presented solely to aid in interpreting the results. In the first case the interaction for the proton-proton system was assumed to consist of the phenomenological Yukawa potential with a hard core plus the point charge Coulomb repulsion between the two protons. The parameters in Eq. (8) were adjusted to fit the proton-proton scattering length $a_{pp}=-7.68$ F and effective range, $\rho_{pp}=2.65$ F. The neutron-proton data was then calculated using the nuclear interaction so determined, with all electromagnetic effects ignored. In the second case the proton was considered to be a point charge, and the magnetic interaction due to the distributed moments of the two nucleons were taken into account. In the third case the electric charge but not the magnetic-moment distributions were taken into account. In the fourth case the electric charges and magnetic moments were considered to be distributed as in Eq. (6). The fitting parameters and calculated data are given in Table I. The neutron-neutron effective range parameters were also calculated and likewise appear in Table I.

Another effect which may be considered arises from the difference in mass of the charged and uncharged pions. The contribution to the nuclear potential due to single-pion exchange has been calculated in a variety of ways and the coupling constant reasonably well determined experimentally.[10]

TABLE I. Summary of calculated electromagnetic effects on the proton-proton, neutron-proton and neutron-neutron scattering lengths and effective ranges. All the quantities shown are lengths measured in fermis, except V_0, which is in MeV. In all cases the nuclear force was assumed to be that of Eq. (8) with a hard-core radius of 0.388 F. For Case I, point charges and point magnetic moments were assumed for the interacting particles; for Case II, point charges and distributed magnetic moments; for Case III, distributed charges and point magnetic moments. In Case IV, both the charge and the magnetic moment are distributed.

	Case I	Case II	Case III	Case IV	Exp.
V_0	402.71	403.56	401.75	402.58	⋯
b	0.6960	0.6960	0.6960	0.6960	⋯
a_{pp}	−7.68	−7.68	−7.68	−7.68	−7.68
ρ_{pp}	2.65	2.65	2.65	2.65	
a_{np}	−16.44	−17.33	−15.82	−16.64	−23.74
ρ_{np}	2.79	2.76	2.80	2.79	
a_{nn}	−16.44	−17.21	−15.76	−16.46	∼−17
ρ_{nn}	2.79	2.78	2.80	2.79	⋯

The procedure followed in order to take into account one-pion exchange was very similar to that discussed above except that V_N of Eq. (7) consists of two parts. The first part is the phenomenological Yukawa potential of Eq. (8) and the second part is the one-pion exchange potential with the strength determined by meson scattering.[10] It is this second part which is slightly different for the proton-proton and the neutron-proton systems. For symmetric pseudoscalar meson theory with pseudoscalar coupling, the second-order potential is given by

$$V=\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}V_2(\mu_c,r)-\tau_3{}^{(1)}\tau_3{}^{(2)}[V_2(\mu_c,r)-V_2(\mu_0,r)], \quad (9)$$

where μ_c and μ_0 refer to the masses of the charged and uncharged pions, respectively, and $\boldsymbol{\tau}^{(1)}$ and $\boldsymbol{\tau}^{(2)}$ refer to the isotopic spins of the nucleons. The potential due to the exchange of a charged pion is $V_2(\mu_c)$ and the potential due to the exchange of an uncharged pion is $V_2(\mu_0)$. For the proton-proton and neutron-neutron case one finds

$$V=V_2(\mu_0,r); \quad (10.1)$$

and for the neutron-proton case

$$V_{np}=2V_2(\mu_c,r)-V_2(\mu_0,r)\,, \quad (10.2)$$

with

$$V_2(\mu,r)=(-g^2/4\pi)(\mu/2M)^2(e^{-\mu r}/r)\,, \quad (10.3)$$

where g is the coupling constant and M is the nucleon mass in units chosen such that $\hbar=c=1$. The explicit expression for the proton-proton and neutron-neutron one-pion exchange potential is

$$V=-W(e^{-\beta r}/\beta r). \quad (11.1)$$

The neutron-proton one-pion exchange potential is

$$V_{np}=-(W/\beta r)(2\Delta^2e^{-\Delta\beta r}-e^{-\beta r}). \quad (11.2)$$

In Eqs. (11.1)–(11.2) above, the quantities V, β, and Δ

[8] H. P. Noyes, Phys. Rev. Letters **12**, 171 (1964).
[9] D. Giltinan and R. M. Thaler, Phys. Rev. **131**, 805 (1963).
[10] See, for example, P. Cziffra, M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **114**, 880 (1959).

are given by

$$W=(g^2/4\pi)(\mu_0/2M)^2\mu_0, \quad (11.3)$$

$$\beta=\mu_0, \quad (11.4)$$

and

$$\Delta=1+[(\mu_c-\mu_0)/\mu_0]. \quad (11.5)$$

The value of the coupling constant is such that $W=11.17$ MeV. The pion masses yield $\beta=0.7074\ \mathrm{F}^{-1}$ and $\Delta=1.032$.

If the one-pion exchange contribution to the scattering is treated as a Yukawa potential, we believe it is unrealistic to carry this potential all the way in to the origin, since the residual interaction must itself in some part include this very effect. Further, the over-all phenomenological potential, especially near the origin, contains multiple-pion exchanges which tend to cancel each other. To single out one-pion exchange in this region is, perhaps, a serious mistake. For this reason the one-pion exchange potential was cut off at various radii in studying its effect on the scattering parameters. The cutoff was smoothed by using

$$F(r)=\{1+\exp[-32(r-r_{\mathrm{cutoff}})]\}^{-1}.$$

The resulting one-pion potential used was $F(r)V(r)$. In addition the effect of varying the radius of the hard core was also studied. The numerical results are shown in Table II.

Moving the cutoff and core values in, to smaller radii, brings the calculated neutron-proton scattering lengths into closer agreement with the experimental values. However, we believe that this should not be taken too seriously, one way or the other, until more reliable estimates of the two-pion exchange effects, for example, are made.

Precision low-energy neutron-proton scattering data[11] indicate that the singlet neutron-proton effective range is about 10% smaller than the values obtained in this calculation.

We have not studied the significance of this discrepancy.

TABLE II. Summary of calculations of mesonic corrections with a hard-core radius of 0.388 F. All quantities shown are lengths measured in fermis except V_0 which is given in MeV.

r_{cutoff}	0.100	0.300	0.500	0.700	0.900
V_0	894	895.96	900.33	926.45	974.40
b	0.5071	0.506	0.506	0.503	0.498
a_{pp}	−7.68	−7.68	−7.68	−7.68	−7.68
ρ_{pp}	2.65	2.65	2.65	2.65	2.65
a_{np}	−19.3	−19.3	−19.3	−19.0	−18.5
ρ_{np}	2.70	2.70	2.70	2.70	2.71

[11] C. E. Engelke, R. E. Benenson, E. Melkonian, and J. M. Lebowitz, Phys. Rev. **129**, 324 (1963).

TABLE III. Summary of mesonic corrections with different hard-core radii. All quantities shown are lengths measured in fermis except V_0 which is in MeV.

r_{core}	0.388	0.288	0.188
V_0	894	355	156
b	0.507	0.617	0.745
a_{pp}	−7.68	−7.68	−7.68
ρ_{pp}	2.65	2.65	2.65
a_{np}	−19.3	−20.14	−21.3
ρ_{np}	2.70	2.68	2.66

IV. CONCLUSION

The nucleon-nucleon scattering lengths give no indication that charge independence of nuclear forces need be abandoned. On the contrary, the corrections we have made are all in the right direction, and in this way of proceeding do not suggest the presence of large self-canceling contributions.

Wong and Noyes[12] have calculated the neutron-neutron scattering length using the methods of dispersion theory. They find that the electromagnetic corrections then make the neutron-neutron scattering length very much larger in magnitude than either the value we have found or the experimental value. On the basis of this calculation charge symmetry has been questioned. However, what Wong and Noyes[12] have done is essentially to make the momentum space analog to our coordinate space argument. In terms of our coordinate space discussion, above, we believe that the essential difference between their calculation and ours is that they leave out the possibility of there being a hard core, so that the corrections may become large, as can be seen from Table III.

We might rephrase these arguments to say, that the experimental value of the neutron-neutron scattering length, although uncertain, tends to support the idea of the hard core on the assumption that charge symmetry of nuclear forces is not in question. While this manuscript was in preparation the authors received a preprint of a paper by Signell, Yoder, and Heller[13] who arrive at similar conclusions regarding the neutron-neutron scattering length.

ACKNOWLEDGMENTS

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[12] D. Y. Wong and H. P. Noyes, Phys. Rev. **126**, 1866 (1962).

[13] L. Heller, P. Signell, and N. R. Yoder, Phys. Rev. Letters **13**, 577 (1964). We are grateful to these authors for making their results available to us prior to publication.