Isotope Shift in the Spectrum of Erbium I*

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Using enriched samples, the isotope shift in seven lines of the arc spectrum of erbium was measured with a Fabry-Perot interferometer. The average relative isotope shift for the even isotopic pairs $\Delta\nu(170\text{–}168):\Delta\nu(168\text{–}166):\Delta\nu(166\text{–}164):\Delta\nu(164\text{–}162)$ was found to be $1.00:0.96\pm0.02:0.94\pm0.03:1.23\pm0.04$, showing a large change in nuclear deformation between the isotopes with neutron numbers 96 and 94. This results in a predicted intrinsic quadrupole moment for Er^{162} of 6.30 ± 0.30 b. No pronounced even-odd staggering was observed for the odd isotope Er^{167}, since the average value for $\Delta\nu(168\text{–}166):\Delta\nu(168\text{–}167cg)$ was found to be $1.00:0.52\pm0.03$.

INTRODUCTION

THE optical isotope shift in erbium ($Z=68$) was first reported by Wilets and Bradley,[1] who found that in a natural sample the ratio $\Delta\nu(170\text{–}168)/\Delta\nu(168\text{–}166)$ was unity, within the accuracy of their observations. No attempt was made to account for the disturbing effects of the only natural odd isotope, Er^{167}, which is now known[2–5] to have a spin of $\frac{7}{2}$, a magnetic moment of ±0.58 nm, and a quadrupole moment of ±11 b. A later investigation by Murakawa and Suwa,[6] also using natural erbium, showed that $\Delta\nu(170\text{–}168):\Delta\nu(168\text{–}166):\Delta\nu(166\text{–}164)$ was equal to $1.00:0.92\pm0.05:0.88\pm0.07$, where the disturbing components of 167, which were distributed between Er^{168} and Er^{166}, were taken into account in considering the limits of error.

With the present availability of enriched erbium samples it has been possible to make more accurate measurements of the isotope shift ratio by minimizing the influence of 167, to extend the measurements to include the isotope 162, and to estimate the intrinsic quadrupole moment of this isotope.

EXPERIMENTAL PROCEDURE

The enriched samples[7] of erbium listed in Table I were mixed to provide the compositions as shown in Table II. Excitation was achieved in a liquid-nitrogen-cooled hollow-cathode discharge tube, using either neon or argon as the carrier gas. Photographs were obtained with a Fabry-Perot interferometer mounted external to a Hilger E-495 spectrograph. Two pairs of interferometer plates were utilized, each with a 7-layer dielectric coating having 95% reflectance. One pair covered the range 4000 to 5200 Å and the other pair from 5200 to 7000 Å. The comparator readings were reduced on an IBM 1620 computer.

TABLE I. Isotopic constitution of enriched samples of erbium, in percent.

Sample	Isotope 162	164	166	167	168	170
Natural	0.14	1.56	33.41	22.94	27.07	14.88
162	14.10	9.00	40.00	17.10	14.60	5.20
164	0.10	15.60	60.40	14.30	7.80	1.90
166	0.01	0.01	99.97	0.02	0.01	0.01
167	0.05	0.10	5.81	87.20	6.42	0.53
168	0.05	0.05	3.40	11.40	76.90	8.30
170 I	0.05	0.05	1.68	2.10	9.00	87.30
170 II	0.05	0.10	1.03	0.96	1.94	96.07

TABLE II. Isotopic constitution of mixed samples of erbium, in percent.

Mixed sample	Isotope 162	164	166	167	168	170
162+170 I	12.33	7.87	35.16	15.12	13.89	15.57
164+170 I	0.09	13.02	50.48	12.38	8.00	16.04
166+170 II	0.03	0.05	50.99	0.49	0.97	47.56
167+170 I	0.05	0.09	4.51	60.25	7.23	27.90
168+170 I	0.05	0.05	2.56	6.78	43.94	46.65

RESULTS

Since well over one hundred lines in the visible spectrum of erbium show observable isotope shift, survey spectrograms were taken with the enriched 167 sample to determine the extent to which the hyperfine structure of this isotope would disturb measurements of the even positions. Using etalon spacers up to 30 mm, it was found that all of the lines having a measurable isotope shift also exhibited unresolved hyperfine complexes, or at least some line broadening. The seven lines which were selected for study were those which were easily excited, well isolated from neighboring lines, and which had well defined hyperfine structure complexes in the region around 166 and 168 with no disturbing components near 162, 164, or 170. These lines all belong to the Er I spectrum and exhibit self-reversal.[8,9] The only

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information on the classification of the arc spectrum indicates that λ4409 represents a $J=5$ to $J=6$ transition,[10] and that λ4607 is a transition to the ground state $^3H_6$ from the upper level 21701.88 ($J=6$).[9] All of these lines exhibit a positive isotope shift in which the component from the heaviest isotope lies on the high-frequency side.

The isotope 170 was chosen as the standard and the location of all other isotopes was made with reference to it. It can be seen that the 166+170 II mixture was ideal for measuring the 166 position. The other isotopic samples were mixed in such a manner as to give compatible intensities for the particular isotope under consideration, compared with the intensity of 170. No correction was made for those components of 167 which lie close to 168 in the 168+170 I mixture, since any correction is considerably less than the error in reading the position of 168 itself. The isotopes 164 and 162 could be measured directly in their respective mixtures.

The values for the observed positions of the even isotopes are listed in Table III. The error values represent three times the standard deviation of the mean, derived from readings made on spectrograms with different spacers.

TABLE III. Positions of the even isotopes in the spectra of Er I (10^{-3} cm^{-1}.)

Wavelength (Å)	Isotope 162	164	166	168	170
4409.341	−184.2±0.5	−129.2±0.6	−87.6±0.3	−44.8±0.3	0
4426.769	−200.4±0.8	−140.5±0.8	−95.7±0.3	−48.5±0.4	0
4606.605	−188.5±0.5	−132.1±0.6	−89.2±0.2	−46.0±0.3	0
4673.162	−206.3±0.7	−144.6±0.7	−97.5±0.2	−49.9±0.4	0
4722.693	−211.5±0.8	−147.9±0.6	−99.7±0.5	−50.9±0.3	0
6308.773	−202.5±0.4	−142.3±0.5	−94.9±0.4	−49.0±0.5	0
6583.480	−201.6±0.4	−141.4±0.4	−95.2±0.4	−48.4±0.2	0

In the set of lines which was chosen for this study the 167 isotope shows two complexes due to the unresolved hyperfine structure components. In order to locate the center of gravity of the 167 isotope, only those lines which showed these two complexes to be symmetrical in shape were used. Their line profiles and intensities were measured on a Jarrell-Ash microphotometer. The positions were measured on spectrograms taken with the mixed sample, 167+170 I, and these positions were corrected for the presence of the 166 and 168 isotopes in this mixture. Table IV lists the positions of these complexes with reference to 170, their measured relative intensities, and the resulting centers of gravity.

TABLE IV. Hyperfine complex positions in Er^{167} relative to the isotope 170 (10^{-3} cm^{-1}).

Wavelength (Å)	167A	167B	Intensity A:B	167 cg	Δν(168–167 cg) / Δν(168–166)
4409	−90.0±0.4	−42.8±0.4	100:100	−66.4±0.8	0.50±0.03
4427	−97.3±0.5	−46.6±0.5	100:95	−72.6±1.0	0.51±0.03
4607	−87.0±0.7	−48.6±0.4	100:90	−68.8±1.1	0.53±0.04
4673	−95.2±0.5	−53.6±0.5	100:85	−76.1±1.0	0.55±0.02
				Average	0.52±0.03

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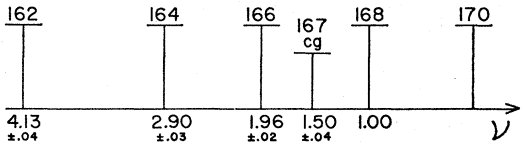

FIG. 1. Relative isotope positions in the Er I spectrum.

DISCUSSION

The erbium lines which were investigated all show the same general even and odd isotope shift pattern. Since these lines are unclassified and all exhibit a positive shift, it is assumed that the relative isotope positions are independent of the transitions involved. The resulting average positions are shown in Fig. 1, where Δν(170–168) has been taken as unity and a weighted average is obtained for the relative positions of the other even isotope shift ratios and the center of gravity of 167. Our results confirm the trend of decreasing ratios from 170 to 164 as found by Murakawa and Suwa,[6] although our values are less than theirs.

The effect that the components due to 167 have upon the positions of the even components, especially the 166 position, can be seen from Table V. Here a comparison is made between the measurements made by previous investigators[1,6] using natural erbium (22.9% of 167) and our results with the separated samples. Except for the 166 position in λ4607, these differences can be explained by the presence of the 167 complexes.

TABLE V. Comparison of even isotope positions in erbium (10^{-3} cm^{-1}).

Wavelength (Å)	Δν(170–168) Wilets and Bradley	Murakawa and Suwa	Our data	Δν(170–166) Wilets and Bradley	Murakawa and Suwa	Our data
4409	45.1	···	44.8	91.3	···	87.6
4427	47.2	···	48.5	97.2	···	95.7
4607	45.6	48.2	46.0	91.2	92.7	89.2
4673	49.5	···	49.9	96.4	···	97.5
4723	50.6	52.7	50.9	101.9	100.7	99.7

Table VI shows the relationship of the relative even-even isotope shifts in erbium to those in its neighboring elements. They seem to agree in general trend. There is a corresponding increase in the shifts in erbium and dysprosium[11] between neutron pairs 94–96 compared to 96–98. For neutron pairs 98–100 compared to 100–102 one finds the shift decreases for erbium and increases slightly for ytterbium.[12] Considering the limits of experimental accuracy it is difficult to determine whether this latter difference is significant.

[11] A. R. Striganov, A. F. Golovin, and M. P. Gerasimova, Opt. i Spektry. 14, 7 (1963) [English transl.: Opt. Spectry 14, 3 (1963)].
[12] J. S. Ross, J. Opt. Soc. Am. 53, 299 (1963).

TABLE VI. Comparison of even-even isotope shift ratios in dysprosium, erbium, and ytterbium.

Neutron pair	Element / Atomic number	Dy 66	Er 68	Yb 70
90–92		1.57±0.04		
92–94		0.99±0.02		
94–96		1.10±0.01	1.23±0.04	
96–98		1.00	0.94±0.03	
98–100			0.96±0.02	1.43±0.03
100–102			1.00	1.38±0.03
102–104				1.05±0.02
104–106				1.00

As is illustrated in Table IV, the center of gravity of the two complexes due to the hyperfine structure components of 167 is only slightly shifted toward the lighter isotope. Thus erbium does not exhibit the usual even-odd staggering effect which is found in most odd-isotope positions in the heavy elements.

The even stable isotopes of erbium, with neutron numbers from 94 to 102, lie near the peak of the curve of nuclear deformation values as determined from Coulomb excitation studies.[13] Comparison of experimental measurements and theoretical calculations has shown that the most important contributions to the isotope shift phenomena in the heavy elements comes from the nuclear volume and nuclear deformation effects. From the relative position of the erbium isotope 162, as illustrated in Fig. 1, a large change in nuclear deformation is expected as one goes from 96 to 94 neutrons.

Several relationships have been developed relating isotope shift constants and nuclear deformation parameters.[14–20] According to Ionesco-Pallas,[19] the following equation holds for a given element, between isotopes differing by two neutrons:

$$C_{\text{exp}}=C_{\text{th}}[S_1+\tfrac{3}{2}A(\delta\alpha^2/\delta N)_Z S_2],$$

where C_{exp} is the experimental isotope shift constant defined by Kopfermann[21] and C_{th} is a theoretical volume-dependent constant based upon a homogeneous sphere of radius $R=1.2\times A^{1/3}\times 10^{-13}$ cm. The coefficients S_1 and S_2 are functions of the atomic weight A and are tabulated. α is the distortion parameter, and $\delta\alpha^2=\alpha_{A+2}{}^2-\alpha_A{}^2$. Ionesco-Pallas has derived an expression (9.3) relating the distortion parameter to the intrinsic quadrupole moment Q_0. In addition he provides an expression (7.10) for A as a function of Z, obtained from the relation connecting Z and A for the most stable nuclei. If one knows the two α values for an isotopic pair (N and $N+2$) and the relative isotope shift between this pair and an adjacent pair (N and $N-2$), then it is possible to predict the α and Q_0 values for the third isotope, without explicit knowledge of C_{exp}.

Since it is not possible to determine C_{exp} for erbium until a more complete classification is available, the procedure outlined above has been used to predict Q_0 for Er^{162}. The Q_0 values for the erbium isotopes 166 and 164 have been determined with a standard error of less than 2.5% by Elbek, Olesen, and Skilbreid,[22] from measurements of the Coulomb excitation cross sections for the first excited state. Table VII lists these measured Q_0 values and that predicted for Er^{162} from our relative isotope-shift data. The 5% error attached to this predicted value was determined by using this procedure to predict Q_0 values from relative isotope shifts of other elements ($60\leqq Z\leqq 70$) for which measured Coulomb excitation values were available for comparison.

TABLE VII. Intrinsic quadrupole moments and nuclear deformation parameters for erbium.

Isotope	Coulomb excitation measurements Q_0 (b)	α	Prediction from isotope shift Q_0 (b)	α
166	7.56	0.2185		
164	7.14	0.2074		
162			6.30±0.30	0.1851

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