that flux conservation questions should be considered. I am grateful to R. M. Drisko for numerous discussions which helped in the preparation of the article.

APPENDIX

Equations (26) for the square well lattice model are a set of differential-difference equations. Despite the complications of these equations they possess solutions in normal modes. For normal mode number m the wave function in region l is

$$\psi_{l,m}(x)=B_{l,m}\exp[i\lambda_m(x-l\xi)],$$

where a convenient phase factor has been inserted into the definition. The secular equations are found to be

$$0=(k^2-\lambda_m^2)B_{1,m}+\tfrac{1}{2}\kappa^2B_{2,m}, \quad \text{for } l=1,$$

$$0=(k^2-\lambda_m^2)B_{n,m}+\tfrac{1}{2}\kappa^2B_{n-1,m}, \quad \text{for } l=n,$$

$$0=(k^2-\lambda_m^2)B_{l,m}+\tfrac{1}{2}\kappa^2(B_{l-1,m}+B_{l+1,m}), \quad \text{for } l\neq 1,n.$$

Although these equations are solved very easily in any explicit case, no solution for general n has yet been found.

The actual solution function $\psi(x)$ for the lattice model is a linear combination of the normal modes. The combination coefficients must be chosen so that $\psi(x)$ and $\psi'(x)$ are continuous across the boundaries between subintervals.

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Possible Source Mechanism for Low-Energy Galactic Electrons

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A calculation is made of the expected secondary electron flux resulting from the knock-on collisions of the primary nuclear beam with the interstellar gas. The model includes ionization losses and a statistical Fermi-mechanism energy gain. Comparison is made with recent satellite experimental data.

INTRODUCTION

RECENT interest in cosmic-ray electrons has been confined largely to higher energies. Specifically, experimental results[1–3] in the energy region of the order of 100 MeV to several BeV have been of interest because of their bearing on the problem of galactic radio emission. The study of lower energy electrons, although probably not of direct importance to the radio emission question, is of importance because of its relationship to the higher energy electron spectrum, and because of its bearing upon the questions of solar modulation and energetic electron production.

Several workers in the field have arrived at the conclusion that the primary cosmic-ray beam must traverse several g/cm^2 of interstellar material prior to being sampled at or near the earth.[4–6] This necessarily implies a flux of low-energy electrons in equilibrium with the primary beam due to the knock-on process in the interstellar gas. This problem has been extensively studied for knock-on electrons due to muons in various substances.[7–9] The equilibrium problem in the interstellar gas is somewhat different from the laboratory experiments described in Refs. 7 and 8 because of the absence of the cascading process in the interstellar gas and the enhanced ionization loss rate in the partially ionized hydrogen.[10] In addition, there is the possibility of further acceleration of the secondary electrons in the interstellar material.[11]

It is not clear that these galactic electrons of low rigidity could penetrate into the solar cavity; however, recent work by Palmeira and Balasubrahmanyan[12] suggests that, at least during solar minimum, they can. This question is not considered here. The question of solar modulation is a separate one. By considering the knock-on flux as expected in the absence of solar influence and comparing with experimental data obtained outside the magnetosphere, new information concerning solar influence may be inferred.

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PROCEDURE

A model is adopted in which the knock-on electrons, once produced, lose energy due to the ionization effect and gain energy due to a statistical Fermi mechanism. It is further assumed that the electrons tend to remain in the somewhat localized regions in which they are produced and that the losses due to diffusion out of the galaxy are negligible at these low rigidities. In addition, synchrotron losses are neglected at the energies in question here. Then, assuming a source of knock-on electrons and the predominance of the ionization loss and statistical gain mechanisms, a calculation of the low-energy electron spectrum is made.

Assuming a primary proton beam not varying appreciably with time, one can write the equation for the density of knock-on electrons as

$$\partial N(E,t)/\partial t+\alpha N(E,t)-(k-\alpha E)(\partial N(E,t)/\partial E)=Q(E)$$

with $N(E,0)\equiv 0$, where

$N(E,t)$ = electron density at energy E and time t in electrons/m^3-MeV,

$(dE/dt)_{\text{Fermi}}=\alpha(E+M_ec^2)$ defines α,

$k=|dE/ds|\rho c-\alpha M_ec^2$, dE/ds being the ionization loss rate, and

$Q(E)$ = production rate in electrons/m^3-MeV-sec.

It is possible to solve the differential equation for arbitrary production rate $Q(E)$. The solution is found to be

$$N(E,t)=\sum_{n=0}^{\infty}\frac{(k-\alpha E)^n(1-e^{-\alpha t})^{n+1}}{(n+1)!\alpha^{n+1}}\frac{d^n}{dE^n}Q(E).$$

Adopting the Bhabha[9] cross section for knock-on production and the rigidity spectrum of McDonald and Webber[13] for the galactic proton beam, we may write for the production rate

$$Q(E)=\varphi(E)/E^2-\gamma(E)/E,$$

where

$$\varphi(E)=8\pi\rho CM_ec^2a\left(\frac{Mc^2}{Ze}\right)^{-1.25}\times\int_{(1-2M_ec^2/E)^{1/2}}^{\beta_{\max}}\beta^{-4.25}(1-\beta^2)^{-0.375}d\beta,$$

$$\gamma(E)=4\pi\rho Ca\left(\frac{Mc^2}{Ze}\right)^{-1.25}\times\int_{(1-2M_ec^2/E)^{1/2}}^{\beta_{\max}}\beta^{-4.25}(1-\beta^2)^{0.625}d\beta,$$

$$C=0.150\ \text{cm}^2/\text{g},$$

and

$$a(Mc^2/Ze)^{-1.25}=5420\ (\text{m}^2\text{-sr-sec})^{-1}.$$

The dependence of φ and γ upon E makes this rigorous approach impractical. Instead, by use of the mean-value theorem, one finds that $Q(E)$ may be approximated by

$$Q(E)\cong AE^\delta,$$

where A and δ are readily evaluated. Substitution of $Q(E)=AE^\delta$ into the differential equation for $N(E,t)$ enables one to use the method of characteristics to solve the equation to yield

$$N(E,t)=\frac{1}{k-\alpha E}\left(\frac{A}{\delta+1}\right)\left[\frac{1}{\alpha^{\delta+1}}\left(k-\frac{k-\alpha E}{e^{\alpha t}}\right)^{\delta+1}-E^{\delta+1}\right].$$

Taking $\rho=2\times10^{-26}$ g/cm^2,[14] and letting $t\to\infty$, we get, setting $dJ/dE=(c/4\pi)N(E)$,

$$\frac{dJ}{dE}=\frac{2.48\times10^{-13}}{1.625\alpha}\frac{E^{-1.625}-(k/\alpha)^{-1.625}}{(k/\alpha)-E}.$$

In addition to this flux calculated for the primary proton on hydrogen interaction, there will be a significant contribution from the heavier nuclei in the cosmic-ray beam. The knock-on production rate at a given primary velocity is very nearly a function of Z^2.[15] We then write the relation for the contribution of nuclei of charge Z_i as

$$\left(\frac{dJ}{dE}\right)_i=Z_i^2\frac{J_i(>\beta)}{J_p(>\beta)}\left(\frac{dJ}{dE}\right)_p.$$

Using relative fluxes as given in the review by Ginzburg and Syrovatsky,[14] we arrive at the conclusion that the

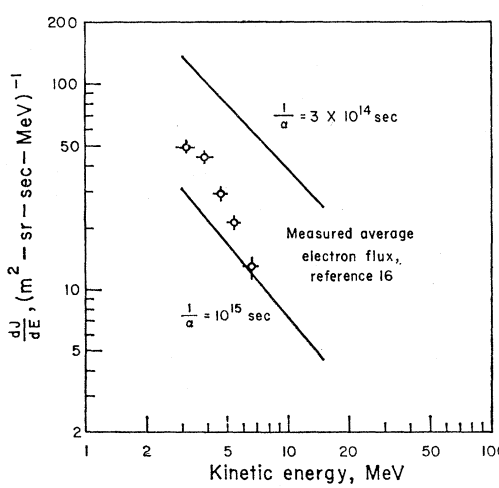

FIG. 1. Calculated spectra for values of α indicated. Circles: average electron flux from Ref. 16.

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knock-on contribution from primaries of charge $Z \geq 2$ will be approximately 0.75 times the proton contribution. The total expected knock-on flux is then approximately 1.75 times the proton contribution.

The ionization loss rate for electrons of 3 to 15 MeV is nearly independent of energy for materials of low Z. In the interstellar hydrogen gas, however, it is fairly strongly a function of the degree of ionization. A degree of ionization of 10% with a corresponding dE/ds value of 5 MeV/g/cm² has been taken.[10]

The calculated electron fluxes for different values of α are plotted in Figs. 1 and 2. It is seen that the resultant intensity is a strong function of α, the parameter in the statistical acceleration mechanism. Typical electron fluxes as measured with IMP-A satellite are also shown in the figures.[16] It is seen that the range of α values selected, $1/\alpha = 3\times10^{14}$ sec to $1/\alpha = 3\times10^{15}$ sec, allows a fairly good matching of the theoretical and experimental fluxes. The value $\alpha \sim 10^{-15}$ sec^{-1} does not appear unreasonable.[17,18]

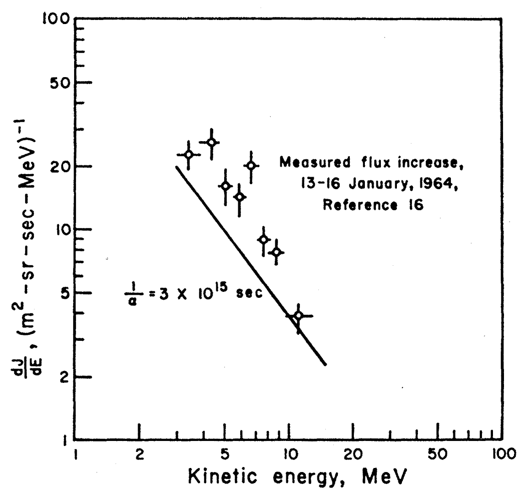

FIG. 2. Calculated spectrum for $1/\alpha = 3\times10^{15}$ sec. Circles: a typical flux increase, taken from Ref. 16.

It is not clear at this time whether the measured flux increase or the entire measured flux can be attributed to the knock-on process. Both possibilities are suggested by the reasonableness of the α values required.

CONCLUSION

The electron-positron flux resulting from the proton-proton interactions in the interstellar material has been discussed by several authors.[19,20] DeShong, Hildebrand, and Meyer[3] conclude, based on electron-positron ratios, that a substantial portion of the electron flux above 50 MeV must have an origin other than proton-proton collisions. It is speculated that a substantial portion of the lower energy electron flux seen in space may be attributed to knock-on electrons acted upon primarily by ionization losses in the interstellar gas and a Fermi-type acceleration process. This, of course, requires that the low-energy electron flux should be composed largely of negative electrons. In addition, any long term solar modulation should be of an inverse solar activity dependence, similar to the primary nuclear beam. Both of these expectations will be subjected to experimental test by proposed experiments during the next solar half-cycle.

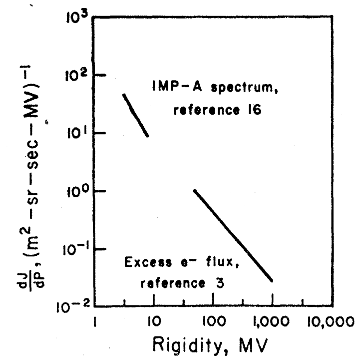

FIG. 3. The average electron flux from Ref. 16 shown along with the excess electron flux arrived at in Ref. 3.

The knock-on process should produce secondary electrons in the BeV energy range also. The theoretical cross section in this case contains spin-dependent terms, and one does not feel as trusting of it as in the low-energy case where the interaction is one of Coulomb force only. In addition, these higher energy electrons may diffuse out of the galactic disk more readily and will also be subject to synchrotron losses. It is nevertheless interesting to plot the low-energy electron flux along with the higher energy flux as has been done in Fig. 3. It is suggested by Fig. 3 that the knock-on process at higher energies may also be of significance.

Adopting for the moment the conclusion that the low-energy electrons as seen with IMP-A are due to the knock-on process, leads to the conclusion that the Fermi mechanism must be moderately effective for these low-energy electrons and that the parameter α has the value $\alpha \sim 10^{-15}$ sec^{-1}.

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