PHYSICAL REVIEW VOLUME 137, NUMBER 1B 11 JANUARY 1965

Short-Range Correlations and the Three-Body Binding Energy*

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(Received 21 August 1964)

The separable-potential approach of Mitra is extended by introducing an additional interaction which yields two-nucleon S-wave phase shifts to 340 MeV. A calculation of the *exact* binding energy and wave function of a simplified three-body system is used to determine the role of short-range correlations and the associated off-energy-shell matrix elements of the interaction. We find that the three-body binding energy ranges from 9.33 to 8.40 MeV; the larger value is calculated using a two-body interaction fitting low-energy data only, while the smaller value results from including a hard-shell repulsion. A smooth repulsion, which is equivalent to the hard shell in the two-body problem, yields a three-body binding energy of 8.79 MeV.

I. INTRODUCTION

RECENTLY, Mitra *et al.* have elegantly demonstrated that for separable two-body forces, it is possible to reduce the three-body problem to a finite number of coupled one-dimensional equations.[1] Assuming a potential which fits only low-energy two-nucleon scattering data, one finds that a numerical solution of these coupled integral equations yields a reasonable triton binding energy and provides some hope of resolving the neutron-deuteron scattering-length ambiguity.[1,2] More fundamentally, Lovelace has recently shown that the existence of the low-energy $^1S_0$ resonance and the $^3S_1$ bound state for the two-nucleon system justify the separable potential approach.[3] Therefore, one is encouraged to believe that a separable nucleon-nucleon interaction is a meaningful model which can provide notable improvements over the usual variational methods necessary for treating local potentials.

However, it is well known that high-energy nucleon-nucleon scattering requires a repulsion, usually represented by a hard core, which can significantly alter the off-energy-shell matrix elements of the interaction.[4] Nuclear matter calculations with local, nonlocal, and velocity-dependent pair interactions have not yet uniquely determined these off-energy-shell elements and the associated short-range correlations.[5,6] Whereas, the two-body and perhaps even the many-body systems determine only on-energy-shell elements, one suspects that the evaluation of three-body properties, sufficiently sensitive to short-range correlations, can be used to uniquely specify the off-energy shell behavior. Unfortunately, three-nucleon variational calculations using hard core interactions have not yet determined a sufficiently accurate upper and lower bound for the binding energy to specify a unique two-body force.[7] For example, Ohmura and Ohmura have used variational methods to investigate the effect of finite nucleon size on the Coulomb energy of He^3 and conclude that it is possible to have correct Coulomb and binding energies with or *even without a hard core*.[8] It is therefore of interest to study the role of short-range correlations in the three-body problem, independent of variational methods.

In this paper we extend the separable potential approach of Mitra to higher collision energies by introducing an additional separable potential which is adjusted to fit nucleon-nucleon S-state phase shifts to 340 MeV.[9] We restrict our study to the pair correlation dependence of the three-body binding energy without recourse to variational methods in the hope of gaining insight into the possibility of simulating or perhaps replacing the hard core by a separable potential. To isolate this pair correlation aspect of the three-particle system, we consider a simplified, smooth, two-body interaction which is taken to be a central, spin-, and isospin-independent S-state force (Sec. II). This model interaction matches an average of the singlet and triplet data. We then construct an equivalent hard-shell potential which is experimentally indistinguishable from the model potential on the energy shell (i.e., it has the same phase shifts up to 340 MeV) but differs in having strong short-range correlations. A third potential, constructed to match the model potential phase shifts only at low and medium energies, does not have the required repulsion; this case is used to test Mitra's assumption[1] that "one needs only a potential which fits scattering data at low and medium energies alone."

The *exact* binding energies of this simplified three-body system are found by solving two coupled integral

* Supported in part by the U. S. Atomic Energy Commission under Contract AT (30-1)-1934.

[1] A. N. Mitra, Nucl. Phys. **32**, 529 (1962); A. N. Mitra and V. S. Bhasin, Phys. Rev. **131**, 1265 (1963); B. S. Bhakar, Nucl. Phys. **46**, 572 (1963); A. N. Mitra, Phys. Rev. **131**, 832 (1963).

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[5] S. A. Moszkowski, Phys. Rev. **129**, 1901 (1963); K. A. Brueckner and K. S. Masterson, Jr., *ibid.* **128**, 2267 (1962); R. K. Bhaduri and M. A. Preston, Can. J. Phys. **42**, 696 (1964).

[6] F. Tabakin, Ann. Phys. (N. Y.) **30**, 51 (1964).

[7] J. M. Blatt and L. M. Delves, Phys. Rev. Letters **12**, 544 (1964); J. M. Blatt and G. Derrick, Nucl. Phys. **8**, 602 (1958); B. K. Srivastava, Phys. Rev. **133**, B545 (1964); Y. C. Tang, R. C. Herndon, and E. W. Schmid, *ibid.* **134**, B743 (1964).

[8] H. Ohmura and T. Ohmura, Phys. Rev. **128**, 729 (1962).

[9] G. Breit, N. H. Hull, Jr., K. E. Lasella, K. D. Pryatt, Jr., F. A. McDonald, and H. M. Ruppel, Phys. Rev. **128**, 826, 830 (1962).

TABLE I. Potential parameters, including two (ϵ_B) and three-body (E_B) binding energies.

	V_α (MeV)	a^{-1} (Fermi)	V_β (MeV)	b^{-1} (Fermi)	d^{-1} (Fermi)	a_s (Fermi)	r (Fermi)	ϵ_B (MeV)	E_B (MeV)
1A	115.9	0.834	235.6	0.901	0.694	−21.25	2.71		1.49
1B	181.5	0.834	235.6	0.801	0.694	5.68	2.09	2.24	19.54
1	148.7	0.834	235.6	0.801	0.694	11.20	2.34	0.43	8.79
2	129.8	0.870	0			11.20	2.34	0.43	9.33
3	267.8	0.755	44×10^3	0.182		11.20	2.34	0.43	8.40

equations for the model, the hard shell, and the low-energy pair interactions, respectively (Sec. III). We discuss the results in Sec. IV.

II. TWO-BODY SEPARABLE POTENTIAL

Let us define a central S-state separable potential[1,10] in momentum space as

$$\langle \mathbf{k}|V|\mathbf{k}'\rangle=(\lambda/2\pi^2)[-g(k)g(k')+h(k)h(k')], \quad (1)$$

where $\langle \mathbf{r}|\mathbf{k}\rangle$ are plane waves, $\mathbf{k}$, $\mathbf{k}'$ are relative coordinates $2\mathbf{k}=\mathbf{k}_1-\mathbf{k}_2$, and $\lambda=\hbar^2/m$. The potential function $h(k)$ introduces the repulsion required by high-energy ${}^1S_0$ scattering data.[4,9] For the attraction $g(k)$ we choose

$$g(k)=\alpha/(k^2+a^2), \quad (2)$$

corresponding to a Yukawa shape with a range a^{-1} F and a potential strength $V_\alpha=(\lambda\alpha^2/a)$ MeV. Defining the reaction matrix R as

$$\langle \mathbf{k}|R|\mathbf{k}'\rangle=(2\pi^2/\lambda)\langle \mathbf{k}|V|\Psi_{k'}\rangle, \quad (3)$$

where $|\Psi_k\rangle$ is the exact standing wave solution, we find that the R matrix is expressed exactly in terms of the potential functions by

$$\langle \mathbf{k}|R|\mathbf{k}'\rangle=N(k|k')/D(k'), \quad (4)$$

where

$$N(k|k')=-g(k)g(k')[1+H(k')]+h(k)h(k')[1-G(k')] +[g(k)h(k')+g(k')h(k)]M(k') \quad (5)$$

and

$$D(k)=[1+H(k)][1-G(k)]+M^2(k). \quad (6)$$

The functions $G(k)$, $H(k)$, and $M(k)$ are given by the principal value integrals

$$G(k)=\frac{P}{\pi}\int_{-\infty}^{\infty}\frac{dk'k'^2}{k'^2-k^2}g^2(k'),$$
$$H(k)=\frac{P}{\pi}\int_{-\infty}^{\infty}\frac{dk'k'^2}{k'^2-k^2}h^2(k'), \quad (7)$$
$$M(k)=\frac{P}{\pi}\int_{-\infty}^{\infty}\frac{dk'k'^2}{k'^2-k^2}g(k')h(k').$$

The two-particle wave function of relative motion in the S state is related to the R matrix by

$$rU(r)=\frac{\sin kr}{k}-\frac{P}{\pi i}\int_{-\infty}^{\infty}\frac{dk'k'}{k'^2-k^2}e^{ik'r}\langle \mathbf{k}'|R|\mathbf{k}\rangle, \quad (8)$$

for the standing wave boundary condition. At large distances this wave function approaches $k^{-1}\sin kr-\langle k|R|k\rangle\cos kr$, which shows that on the energy shell ($k=k'$), the R matrix is related to the asymptotic wave function or phase shifts by

$$\langle \mathbf{k}|R|\mathbf{k}\rangle=-(\tan\delta)/k. \quad (9)$$

Short-range correlations given by Eq. (8) are clearly determined by the off-diagonal R-matrix elements. It is therefore possible for two potentials to have the same phase shifts, over an energy range, with radically different off-energy-shell elements or, correspondingly, radically different wave functions.

With this possibility in mind, we first define the repulsion $h(k)$ by

$$h(k)=\frac{\beta k^2}{[(k-d)^2+b^2][(k+d)^2+b^2]}. \quad (10)$$

This function peaks at higher energies $k_{\max}{}^2=b^2+d^2$,

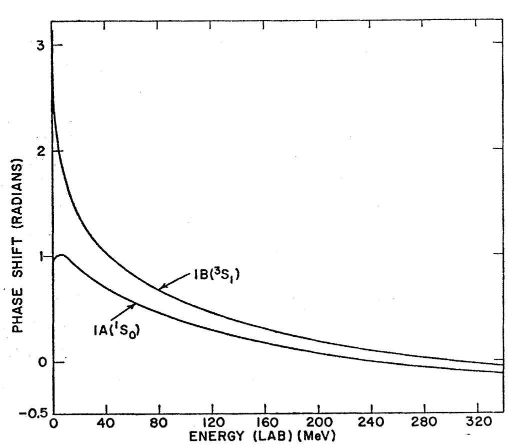

FIG. 1. The phase shifts for potentials 1A and 1B defined by Eqs. (1), (2), and (10), with parameters given in Table I. Potential 1A matches the YLAM ${}^1S_0$ phase shifts of Ref. 9 to 340 MeV. Potential 1B results from increasing the attraction V_α to fit low-energy ${}^3S_1$ phase shifts.

[10] Y. Yamaguchi, Phys. Rev. 95, 1628, 1635 (1954); A. N. Mitra and J. H. Naqvi, Nucl. Phys. 25, 307 (1961); see Ref. 3 for a more complete list of papers on separable potentials.

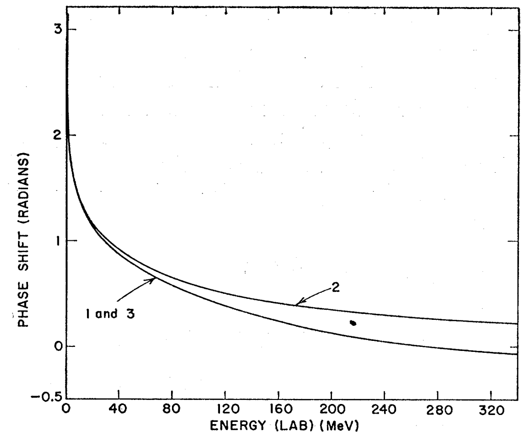

FIG. 2. The phase shifts for potentials 1, 2, and 3 defined by Eqs. (1) to (11), with parameters given in Table I. Potential 1 is an average of potentials 1A and 1B and is taken as our standard or model potential. Potential 2 has no repulsion and accurately fits the scattering length and effective range of the model potential. Potential 3 is the hard-shell [Eq. (11)] interaction adjusted to fit the model potential phase shifts to within 0.01 rad.

whereas $g(k)$ peaks at zero energy. Consequently, the off-diagonal elements of R are reduced, and a *smooth* two-body wave function is generated.[6] In this way, one can approach the situation assumed by Mitra, for which off-diagonal elements in the low-energy region are isolated from the behavior of $\langle k|R|k'\rangle$ at higher energies.

The set of parameters V_α, a^{-1}, $V_\beta\equiv\lambda\beta^2/b$, b^{-1}, and d^{-1} are first adjusted to fit the ${}^1S_0$ YLAM phase shifts[9] to 340 MeV (Table I and Fig. 1, potential 1A). Assuming the same ${}^3S_1$ repulsion and keeping the same range a^{-1}, we find that increasing V_α yields an approximate fit to ${}^3S_1$ low-energy data (potential 1B). Following an argument given in Ref. 11, we take potential 1 to be an average of the singlet and triplet forces (Fig. 2). In reality, the ${}^3S_1$ interaction includes a tensor force; our procedure merely serves as a guide for defining a model interaction which only approximates the real pair interaction in H^3. However, having defined the model potential by parameter set 1 and Eqs. (1)–(10), we now consider it as our *standard, fixed* interaction. One should therefore not expect precise agreement with the triton binding energy (-8.482 MeV) but rather view the model as a means of evaluating the *change* in binding energy generated by modified off-energy-shell behavior for *fixed* diagonal elements.

As a second potential we assume no repulsion ($\beta\rightarrow0$) and adjust V_α and a^{-1} (potential 2) to fit only the scattering length a_s and effective range r of the model potential. A third potential which corresponds to a hard-shell interaction[12] is defined by

$$h(k)=(\beta/bk)\sin(k/b). \tag{11}$$

[11] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), 1st ed., Chap. 4, pp. 192 and 195.

[12] R. D. Puff, Ann. Phys. (N. Y.) **13**, 317 (1961).

Fortunately, it is possible to match the model potential phase shifts to 340 MeV with this hard-shell repulsion (potential 3). At short distances, the hard-shell wave function is strongly correlated (Fig. 3) and, correspondingly, it has large off-diagonal elements. The repulsion strength of 44 BeV is balanced by an increase of V_α to 267.8 MeV. The potential parameters, phase shifts, and wave functions are given in Table I and Figs. 2–3 for the model potential, the no-repulsion low-energy interaction, and the hard-shell repulsion. These three S-state central interactions, which have different off-diagonal characteristics, are now used to test the pair correlation dependence of the three-body binding energy.

III. THREE-BODY SYSTEM

The spatial part of the triton's ground-state wave function is known to be predominantly symmetric with the spin state $2^{-1/2}[\alpha(1)\beta(2)-\alpha(2)\beta(1)]\alpha(3)$ having the required antisymmetry.[13] By averaging the singlet and triplet interactions, and ignoring the tensor force and isotopic spin of the nucleons, one reduces the triton problem to that of three identical, spin-zero particles.[2] For such a system, one can write the totally symmetric ground state in a form originally suggested by Eyges[14]:

$$\Psi=\psi(\mathbf{k}_{12},\mathbf{K}_3)+\psi(\mathbf{k}_{13},\mathbf{K}_2)+\psi(\mathbf{k}_{23},\mathbf{K}_1), \tag{12}$$

where

$$\psi(-\mathbf{k},\,\mathbf{K})=\psi(\mathbf{k},\mathbf{K}). \tag{13}$$

The relative momenta $\mathbf{k}_{12}$, $\mathbf{K}_3$ and the total momentum $\boldsymbol{\kappa}$ are expressed in terms of the particle momenta $\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_3$, by

$$\begin{aligned} 2\mathbf{k}_{12}&=\mathbf{k}_1-\mathbf{k}_2,\\ \mathbf{K}_3&=\tfrac{2}{3}\mathbf{k}_3-\tfrac{1}{3}(\mathbf{k}_1+\mathbf{k}_2),\\ \boldsymbol{\kappa}&=\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3, \end{aligned} \tag{14}$$

with similar expressions for $\mathbf{k}_{13}$, $\mathbf{K}_2$ and $\mathbf{k}_{23}$, $\mathbf{K}_1$. Ex-

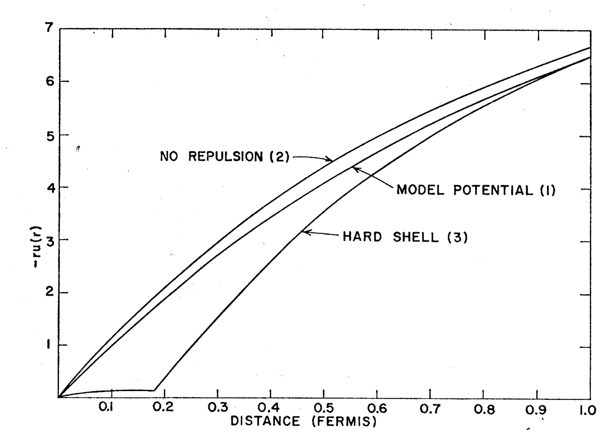

FIG. 3. The two-particle wave function of relative motion in the S state for the model, no repulsion, and hard-shell potentials; this case is for $E_{\mathrm{LAB}}=0$ MeV and $\delta=\pi$ rad. All three wave functions approach the same asymptote $r-a_s$ with $a_s=11.2$ F but differ significantly at short distances.

[13] M. Verde, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 34, p. 144.

[14] L. Eyges, Phys. Rev. **121**, 1744 (1961); **115**, 1643 (1959).

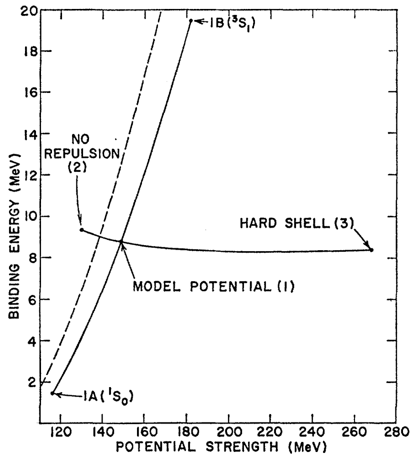

FIG. 4. The three-body binding energy E_B as a function of the strength V_α of the two-body attraction for potentials 1A, 1B, and the model, no repulsion, and hard-shell pair interactions. The dashed curve gives the binding energy for an attraction of range $\alpha^{-1}=0.834$ F with no repulsion; the difference between the solid and dashed curves gives the contribution of the model repulsion to the binding energy. Binding energies were calculated to an accuracy of 0.01 MeV.

pressed in these momenta, the kinetic energy operator is

$$T=\lambda[k^2+\tfrac{3}{4}K^2+\tfrac{1}{6}\kappa^2],$$

with (15)

$$k_{12}^2+\tfrac{3}{4}K_3^2=k_{13}^2+\tfrac{3}{4}K_2^2=k_{23}^2+\tfrac{3}{4}K_1^2.$$

The potential energy operator for separable pair interactions, neglecting possible three-body forces, is written as

$$\begin{aligned}\langle\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3|V|\mathbf{k}_1'\mathbf{k}_2'\mathbf{k}_3'\rangle &=\delta(\boldsymbol{\kappa}-\boldsymbol{\kappa}')[\delta(\mathbf{K}_3-\mathbf{K}_3')\langle\mathbf{k}_{12}|V_{12}|\mathbf{k}_{12}'\rangle\\ &\quad\times\delta(\mathbf{K}_2-\mathbf{K}_2')\langle\mathbf{k}_{13}|V_{13}|\mathbf{k}_{13}'\rangle\\ &\qquad+\delta(\mathbf{K}_1-\mathbf{K}_1')\langle\mathbf{k}_{23}|V_{23}|\mathbf{k}_{23}'\rangle],\end{aligned}\quad(16)$$

where $\langle\mathbf{k}|V|\mathbf{k}'\rangle$ is given by Eq. (1) and the delta function $\delta(\boldsymbol{\kappa}-\boldsymbol{\kappa}')$ ensures conservation of the total center-of-mass momentum for the three-body system. The assumption of three identical particles, which interact via potentials 1, 2, or 3 (Table I), requires equal pair interactions $V_{12}=V_{13}=V_{23}=V$.

Introducing this potential operator into the Schrödinger equation, we find an exact relative-wave function given by

$$\psi(\mathbf{k},\mathbf{K})=\frac{g(k)\chi(\mathbf{K})-h(k)\varphi(\mathbf{K})}{k^2+\tfrac{3}{4}K^2+k_B^2},\quad(17)$$

for three identical particles with binding energy $-E_B(E_B=\lambda k_B^2)$. The extremely convenient structure of this three-body wave function[1] is simply a "two-body wave function" $g(k)[k^2+\Delta^2]^{-1}$ multiplied by a spectator function $\chi(\mathbf{K})$ which is a single-particle function for the third or spectator particle in the presence of the other two. ($\Delta^2=\tfrac{3}{4}K^2+k_B^2$ plays the role of a binding energy in the presence of the third particle.) Our extension to higher energies preserves this convenient structure and merely introduces the additional spectator function $\varphi(\mathbf{K})$ for the repulsion. For S-state interactions, the spectator functions $\chi(\mathbf{K})$ and $\varphi(\mathbf{K})$ depend only on the magnitude of $\mathbf{K}$ and we find that they are determined by two coupled, homogeneous, linear, integral equations which are

$$\begin{aligned}[1-\hat{G}(K)]\chi(K)+\hat{M}(K)\varphi(K)&=\int_0^\infty dK'K'^2[\mathcal{G}(K|K')\chi(K')\\&\qquad-\mathfrak{M}(K|K')\varphi(K')],\end{aligned}\quad(18)$$

$$\begin{aligned}[1+\hat{H}(K)]\varphi(K)-\hat{M}(K)\chi(K)&=\int_0^\infty dK'K'^2[\mathfrak{M}(K'|K)\chi(K')\\&\qquad-\mathcal{H}(K|K')\varphi(K')].\end{aligned}\quad(19)$$

The kernals $\mathcal{G}$, $\mathcal{H}$, and $\mathfrak{M}$ are related to the potential and the binding energy by evaluating

$$\begin{aligned}\mathcal{G}(K|K')&=\frac{2}{\pi}\int_{-1}^{1}\frac{dx\,g(s)g(t)}{K'^2+K^2+k_B^2-KK'x},\\ \mathcal{H}(K|K')&=\frac{2}{\pi}\int_{-1}^{1}\frac{dx\,h(s)h(t)}{K'^2+K^2+k_B^2-KK'x},\\ \mathfrak{M}(K|K')&=\frac{2}{\pi}\int_{-1}^{1}\frac{dx\,g(s)h(t)}{K'^2+K^2+k_B^2-KK'x},\end{aligned}\quad(20)$$

with $s^2=K'^2+\tfrac{1}{4}K^2-KK'x$ and $t^2=K^2+\tfrac{1}{4}K'^2-KK'x$. Finally, the functions $\hat{G}$, $\hat{H}$, and $\hat{M}$ are given by

$$\begin{aligned}\hat{G}(K)&=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{dk\,k^2g^2(k)}{k^2+\Delta^2},\\ \hat{H}(K)&=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{dk\,k^2h^2(k)}{k^2+\Delta^2},\\ \hat{M}(K)&=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{dk\,k^2g(k)h(k)}{k^2+\Delta^2},\end{aligned}\quad(21)$$

with $\Delta^2=\tfrac{3}{4}K^2+k_B^2$.

These coupled equations yield the three-body binding energy λk_B^2 and the spectator functions $\chi(K)$, $\varphi(K)$ for given potential functions $h(k)$ and $g(k)$. In the special case of small $g(k)h(k')$ overlap, corresponding to the model interaction (potential 1), Eqs. (18) and (19) tend to decouple and one should then expect a binding energy close to the no-repulsion (potential 2) result. For large $g(k)h(k')$ overlap (potential 3) corresponding to large off-diagonal R-matrix elements and strong short-range correlations, Eqs. (18) and (19) become more tightly coupled, and we might expect considerable modification of the binding energy. However, there are two competing effects: The kinetic energy increases with short-range curvature, while the attraction strength V_α also increases to give the fixed phase shifts. It is difficult to predict intuitively the binding energy which will result from the balance of these effects.

IV. DISCUSSION OF RESULTS

The three-body binding energies $E_B=\lambda k_B^2$ and the spectator functions $\chi(K)$, $\varphi(K)$ for the model, the no repulsion, and the hard shell pair interactions were found by numerically solving the coupled-integral equations [Eqs. (18), (19)]. The results are given in Figs. 4, 5 and Table I. Figure 5 shows how the repulsion and the associated short-range correlations reduce the spectator functions at higher momenta; above $K=3$ F^{-1}, the hard-shell spectator functions oscillate as $K^{-1}\sin(K/b)$, indicating correlations similar to those of the corresponding two-body wave functions (Fig. 3).

Fixing on-energy-shell elements, we find that the kinetic and potential energies stay in approximate balance even for radically different off-diagonal behavior; hence, the three-particle binding energy is determined predominantly by the on- and near-energy-shell R-matrix elements. Nevertheless, the binding energy differences are sufficiently accurate to differentiate between the various pair interactions. For example, the low-energy potential of the form assumed by Mitra[1] (potential 2) yields a binding energy of 9.33 MeV, whereas the introduction of a model repulsion (potential 1), matching the same low-energy data, reduces the binding to 8.79 MeV. The hard-shell strongly-correlated repulsion (potential 3) precisely fits the model potential phase shifts to 340 MeV and further reduces the binding energy to 8.40 MeV.

Taking the hard shell as the most realistic potential considered, we conclude that it is not valid to use a low-energy potential alone if a binding energy accuracy of better than 0.93 MeV is required. It is not surprising that this is of the order of relativistic corrections[11] since the nonlocality or velocity dependence of the separable interaction originates in part from relativistic effects. The three-body binding energies for the model and hard-shell pair interactions differ by 0.39 MeV. This sensitivity to short-range correlations leads us to conclude that a unique determination of the two-body force requires an accuracy of better than 5% in a complete triton calculation. Correspondingly, correction terms generated by transforming a "realistic" hard core into a smooth effective potential should contribute about 5% to the triton's binding energy.[6,15]

These preliminary considerations indicate that Mitra's separable potential approach to the three-body eigenvalue problem can and should be extended to include higher energy two-nucleon scattering data.

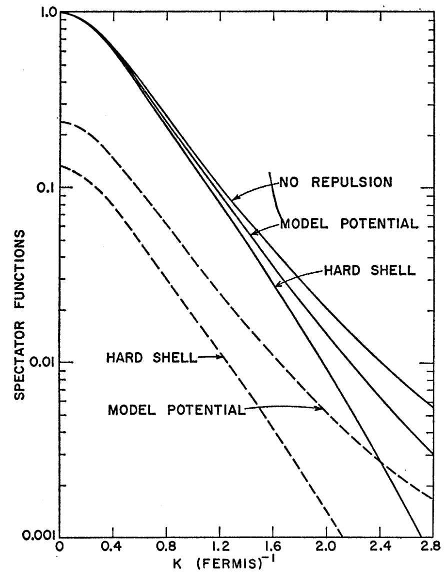

FIG. 5. The spectator functions as determined by solving Eqs. (18) and (19) for the model, no repulsion, and hard shell pair interactions. Solid curves indicate $\chi(K)$, dashed curves indicate $\varphi(K)$.

ACKNOWLEDGMENT

The author wishes to thank Professor F. Villars for helpful discussions.

[15] F. Villars, in *Proceedings of the International School of Physics, "Enrico Fermi"* (Academic Press Inc., New York, 1963), p. 24; J. S. Bell, in *Proceedings of the International School of Physics, University of Bergen* (W. A. Benjamin, Inc., New York, 1962), p. 214.