that they would not have to be "quarks" to permit the construction of a theory of baryons. The great disadvantage, from our point of view, is that with $n=4$ (see Table I), we would be forced to start with an even higher symmetry group, namely $\bar{W}_4$ (or $S\bar{W}_4$) which when broken would lead to U_4 (or SU_4) rather than to the thus-far-successful U_3 (or SU_3) group. We consider it preferable to hold the "baryon-lepton" principle in abeyance at this stage and to seek experimental evidence for broken $\bar{W}_3$ (or $S\bar{W}_3$) among the hadrons.

Finally, we note that there is no very strong reason for restricting oneself to a four-fermion nonlinear interaction model. One might inquire whether postulating in addition, say, a six-fermion interaction would alter the chief conclusions. The answer is negative if one considers some simple forms of the six-fermion interaction. One can write down a six-fermion interaction among three "basic" fields[30] having the structure

$$\sum_{1}^{3} \epsilon^{lmn}\epsilon^{ijk}(\psi_l C^{-1}Q_1\psi_i) \times(\psi_m C^{-1}Q_2\psi_j)(\psi_n C^{-1}Q_3\psi_k)+\text{H.c.} \quad (40)$$

which is invariant under SU_3, but not under U_3 [in (40), Q_1, Q_2, and Q_3 are some appropriate Dirac matrices]. A small admixture of such an interaction in addition to the main four-fermion interaction may serve to break both U_3 and $\bar{W}_3$ symmetry. However, it is premature to discuss this possibility further at this time.

[30] For a quartet of "basic" fields, one may write down a four-fermion interaction which is invariant under SU_4 but not under U_4, namely:

$$\sum_{1}^{4} \epsilon^{ijkl}(\psi_i C^{-1}Q\psi_j)(\psi_k C^{-1}Q\psi_l).$$

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Some Speculations Concerning High-Energy Large Momentum Transfer Processes*

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It is speculated that the sharp decrease with increasing energy of differential cross sections at large angles is due to a mechanism independent of the method of excitation. Some consequences of such a possibility are discussed.

IT has been known for some time that (i) the total pp cross section remains essentially constant at high energies, and that (ii) above 300 MeV of excitation energy the nucleon has many excited states. More recently, experiments[1] have shown that the *large-angle* elastic pp cross section drops down spectacularly with energy. For example, when the center-of-mass momentum of each proton is 3.8 BeV/c, the differential cross section at 90° is only about 10^{-36} cm²/sr.

These facts together suggest that the nucleon is an extended object with an internal structure having a "rigidity" characterized by an excitation energy of the order of a few hundred MeV. For hard collisions where the available energy is much larger than this, many degrees of freedom are excited in the nucleons, resulting in general in the emission of many particles.

Such a picture is more or less common to various statistical discussions[2] of high-energy collisions.

The spectacular drop mentioned above has been put in a more quantitative form by Orear.[3] His result is that (iii) the elastic pp differential cross section for large θ in the center-of-mass coordinate system is given by

$$(d\sigma/d\Omega)(\theta, pp\rightarrow pp)\sim Ae^{-p_\perp/0.15}, \quad (1)$$

where $p_\perp$ is the transverse momentum transfer in units of BeV/c.

Guided by these facts (i)–(iii), we attempt to speculate about the high-energy behavior of other processes. We observe that in picturing the nucleon as an extended object the difficulty in making large transverse mo-

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[1] G. Cocconi, V. T. Cocconi, A. D. Krisch, J. Orear, R. Rubinstein *et al.*, Phys. Rev. Letters **11**, 499 (1963) and W. F. Baker, E. W. Jenkins, A. L. Read, G. Cocconi, V. T. Cocconi *et al.*, *ibid.* **12**, 132 (1964).

[2] H. W. Lewis, J. R. Oppenheimer, and S. A. Wouthuysen, Phys. Rev. **73**, 127 (1948); E. Fermi, Progr. Theoret. Phys. (Kyoto) **5**, 570 (1950); G. Fast and R. Hagedorn, Nuovo Cimento **27**, 208 (1963); L. van Hove, Rev. Mod. Phys. **36**, 655 (1964); G. Cocconi, Nuovo Cimento **33**, 643 (1964); A. Bialas and V. F. Weisskopf, CERN (to be published); and many other papers. Notice that for very small angle elastic scattering, the many modes of excitation contribute in phase so that one has an enormous "diffraction peak." See R. Serber, Rev. Mod. Phys. **36**, 649 (1964) and earlier papers.

[3] J. Orear, Phys. Rev. Letters **12**, 112 (1964). See also A. D. Krisch, *ibid.* **11**, 217 (1963), and D. S. Narayan and K. V. L. Sarma, Phys. Letters **5**, 365 (1963).

mentum transfers could naturally be due to the *difficulty in accelerating the various parts of a nucleon without breaking it up*. If this is the case, such a difficulty is presumably present in all high-energy collisions. Furthermore, the dominant effect of such a difficulty is to contribute a rapidly decreasing factor to the appropriate differential cross sections *independent of the specific process*. (The specific process, i.e., method of excitation, may give rise to more slowly varying factors such as polynomials of the energy.) In particular, we speculate as follows:

1. In a nucleon-nucleon collision,

$$N+N \to A+B, \tag{2}$$

with large θ, so many degrees of freedom are excited in the collision that the emergent particles A and B assume various excited forms of the nucleons with *relative* probabilities that do not have such a precipitous dependence on energy as (1). In other words, with a suitable change of the factor in front of the exponential, (1) holds not only for pp collision but also for any process of the form (2), where A and B are any nucleonic states satisfying all conservation laws. For example,[3a] in the high-energy limit, with fixed $\theta \neq 0, \pi$,

$$\ln\frac{d\sigma}{d\Omega}(\theta, pn \to pn) \Big/ \ln\frac{d\sigma}{d\Omega}(\theta, pp \to pp) \to 1\,, \tag{3a}$$

$$\ln\frac{d\sigma}{d\Omega}(\pi-\theta, pn \to pn) \Big/ \ln\frac{d\sigma}{d\Omega}(\theta, pn \to pn) \to 1\,, \tag{3b}$$

and

$$\ln\frac{d\sigma}{d\Omega}(\theta, pp \to pp^*) \Big/ \ln\frac{d\sigma}{d\Omega}(\theta, pp \to pp) \to 1\,. \tag{3c}$$

For some recent experimental data relating to (3c), see Ref. 4.

2. We make the same speculation for the process $p+p \to \pi^+ + D$. Similar to (3), for $\theta \neq 0, \pi$,

$$\ln\frac{d\sigma}{d\Omega}(\theta, pp \to \pi D) \Big/ \ln\frac{d\sigma}{d\Omega}(\theta, pp \to pp) \to 1\,, \tag{4}$$

in the high-energy limit. Ulrich[5] has recently extended the Orear fit to this process.

3. Again, similar statements may be made for the πp processes. For example, in the same limit,

$$\ln\frac{d\sigma}{d\Omega}(\theta, \pi^+ p \to \pi^+ p) \Big/ \ln\frac{d\sigma}{d\Omega}(\theta, \pi^- p \to \pi^- p) \to 1\,, \tag{5a}$$

$$\ln\frac{d\sigma}{d\Omega}(\theta, \pi^- p \to \pi^0 n) \Big/ \ln\frac{d\sigma}{d\Omega}(\theta, \pi^- p \to \pi^- p) \to 1\,, \tag{5b}$$

$$\ln\frac{d\sigma}{d\Omega}(\theta, \pi p \to \pi p^*) \Big/ \ln\frac{d\sigma}{d\Omega}(\theta, \pi p \to \pi p) \to 1\,, \tag{5c}$$

and

$$\ln\frac{d\sigma}{d\Omega}(\theta, \pi p \to K\Lambda) \Big/ \ln\frac{d\sigma}{d\Omega}(\theta, \pi p \to \pi p) \to 1\,. \tag{5d}$$

4. The possible validity of (3)–(5) is not dependent on the strict exponential form exhibited in (1). What is truly relevant is that existing experimental data *suggest* that at high energies, at a fixed angle θ in the center-of-mass system, *the differential cross sections all $\to 0$ faster than any power law*. We speculate that this fast approach to zero is independent of the specific process.

5. We observe that, in pp elastic collision, *both* of the two final protons receive large $p_\perp$ without breaking up. Consider next electron-nucleon elastic scattering at high energies. Here, it is also difficult to transfer large momentum to the nucleon without the emission of many pions. On the other hand, no such difficulty is to be associated with the electron. Therefore, it is perhaps not unreasonable to relate the electron-nucleon differential cross section at large angles to the *square root* of that of pp scattering. In other words, one would try to relate the form factors G in ep scattering to the fourth root of the elastic pp scattering differential cross section. To pursue this line of speculation one must identify the variable q^2 of $G(q^2)$ with the proper variables in pp scattering.

In order to do this, it is necessary to form a picture describing the remarkable fact implicit in (1) that large longitudinal momentum transfers in pp scattering are *not* costly, as large $p_\perp$ is. We argue that this fact is "understandable" for two reasons: (1) Since each proton is an extended object, pieces of the two may be exchanged, leading to large longitudinal momentum transfers. (2) Different parts of a proton possess instantaneous momenta relative to each other. In the laboratory system, these momenta acquire large longitudinal components.

Now, for an ep collision, the above reasons do not apply (since no exchange of "pieces" can take place between e and p), and "longitudinal" momentum transfer must be treated on a similar footing as "transverse" momentum transfer. Thus we argue that we should

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[3a] *Note added in proof*. Equations (3)–(5) are valid in any unit chosen for $d\sigma/d\Omega$. For practical purposes, it is more convenient to write e.g., instead of (3a), as $E \to \infty$

$$\frac{\left[\ln\frac{d\sigma}{d\Omega}(\theta, pn \to pn)\right]_E - \left[\ln\frac{d\sigma}{d\Omega}(\theta, pn \to pn)\right]_{E_0}}{\left[\ln\frac{d\sigma}{d\Omega}(\theta, pp \to pp)\right]_E - \left[\ln\frac{d\sigma}{d\Omega}(\theta, pp \to pp)\right]_{E_0}} \to 1, \tag{3a'}$$

where E is the energy of the incoming system, and E_0 is an arbitrary fixed value for E.

[4] C. M. Ankenbrandt, A. R. Clyde, B. Cork, D. Keefe, L. T. Kerth, W. M. Layson, and W. A. Wenzel, University of California, Radiation Laboratory Report No. UCRL-11423, 1964 (to be published).

[5] B. T. Ulrich (to be published).

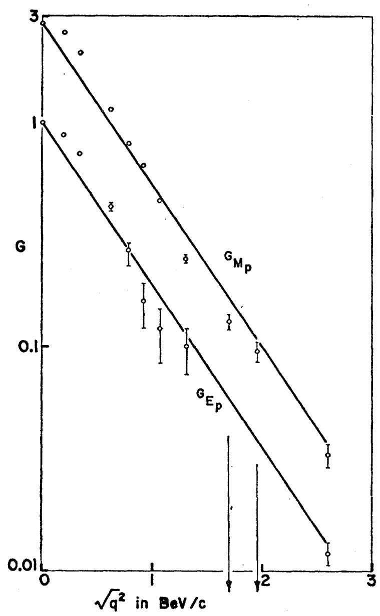

FIG. 1. Electromagnetic form factors of the proton. The straight line represents Eq. (6) with B = constant. The data are from references in footnote 7.

replace the $p_\perp$ of (1) by $(q^2)^{1/2}$, obtaining[6]

$$G(q^2) \sim B \exp[-(q^2)^{1/2}/0.6], \qquad (6)$$

where q^2 is measured in $(\text{BeV}/c)^2$. Equation (6) is applicable only for sufficiently large q^2. Whether it should be applied to both form factors, or only to the one that contributes dominantly to the cross section at large q^2, is unclear to us. In Fig. 1 we plot the experimental form factors[7] G_E and G_M and compare them with (6). We notice that if the measurements are extended to higher q^2, (6) yields form factors very different from any power law.

Since the mechanism of interaction in ep and pp collisions are quite different, the factor B in (6) may vary slowly with q^2, say like a power of q^2. Our speculation, more precisely, is then

$$\frac{\ln G(q^2)}{\ln(d\sigma/d\Omega)(90°, pp \to pp)} \to \tfrac{1}{4} \quad \text{as} \quad q^2 \to \infty\,, \qquad (7)$$

where $(d\sigma/d\Omega)(90°, pp \to pp)$ is taken at a center-of-mass momentum $= (q^2)^{1/2}$ for each proton. Again (7) may be valid even if (1) is not strictly true.

6. Similar considerations apply to processes

$$eN \to eN^*\,,$$
$$\nu n \to \mu p\,,$$
$$\nu N \to \mu N^*\,,$$

all of which involve cross sections falling off with energy like that of $ep \to ep$. Notice, however, that these rapid falloffs do not occur in processes involving only leptons, e.g.,

$$\nu e \to \nu e \quad \text{and} \quad \mu e \to \mu e\,.$$

7. We have no cogent arguments for the exponential form (1). We observe, however, that it is consistent with the following idea: Different regions of an extended object (the proton) contribute *independently* to the factor that describes the probability for the object not to break up. Such a line of reasoning also suggests that for a process such as $A+B \to C+D+E+F$ where all particles are strongly interacting particles, the rapid falloff factor at high energies is

$$\exp[-(\textstyle\sum |p_\perp|)/0.3],$$

where the sum extends to all final particles.

8. The above considerations are only concerned with the rapid falloff factors for various differential cross sections at large transverse momentum transfers. Now, for high-energy collisions where many degrees of freedom are excited, it is tempting to speculate about the possibility of further statistical properties for these processes. For example, the elastic differential cross section in various isotopic spin channels may have on the average[8] the same absolute amplitude with random relative phases. This assumption is similar to the one used by Fermi[9] in his discussion of the charge distribution of multiple pion production (and is quite independent of our discussions in the previous sections). Consequences of this assumption on the large angle elastic and charge exchange differential cross sections will be discussed in the Appendix. Also discussed there are similar considerations concerning spin correlations in such scatterings.

The above arguments are of course highly speculative. It is quite possible, nonetheless, that the main point is correct, viz., that the dominant fall-off factor (at high energies) of the large-angle elastic differential cross sections is independent of the excitation process. If so, it would be rather *difficult to extract*, unfortunately, from

[6] Equation (6) is not inconsistent with the required analyticity of $G(q^2)$ in the cut plane. If, furthermore, $G(q^2)$ is bounded by a polynomial in q^2 in this cut plane, then (6) implies that the discontinuity of $G(q^2)$ across the cut oscillates an infinite number of times about zero.

[7] Experiments performed at Stanford, Cornell, Paris, and Cambridge, Massachusetts have been fully reported in the literature. See review article by L. N. Hand, D. G. Miller, and R. Wilson, Rev. Mod. Phys. **35**, 335 (1963). For recent data, see K. W. Chen, A. A. Cone, J. R. Dunning, Jr., S. G. F. Frank, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. Letters **11**, 561 (1963); also R. Wilson and K. W. Chen, private communication. For an earlier discussion of possible connections between ep and pp scatterings, see R. Wilson (unpublished).

[8] Here "average" means average over small energy and angular intervals.

[9] E. Fermi, Phys. Rev. **92**, 452 (1953).

high-energy large momentum transfer processes, *intrinsic information pertaining to very small distance interactions.*

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APPENDIX

1. Consider the process

$$A+B\to C+D.$$

Let the matrix element for the process with a total isotopic spin I be denoted by a_I. The statistical hypothesis means that

$$\langle a_I a_{I'}^*\rangle=\delta_{II'}a\,, \tag{8}$$

where the average $\langle\cdots\rangle$ is defined in footnote 8. This is essentially the assumption made by Fermi.[9] Using (8), it is clear that the differential cross section on the average is proportional to

$$\textstyle\sum_I|\langle I_3(A),I_3(B)|I\rangle\langle I|I_3(C),I_3(D)\rangle|^2\,, \tag{9}$$

where $I_3(A)$ etc., are the I_3 component of the isotopic spin of A etc., I is the total isotopic spin, and the $\langle|\rangle$ symbols are the appropriate Clebsch-Gordan coefficients. Application of (9) to pp and πp large-angle scattering yields

$$\begin{aligned}\frac{d\sigma}{d\Omega}(\theta,\,pp\to pp)&=2\frac{d\sigma}{d\Omega}(\theta,\,pn\to pn)\\&=2\frac{d\sigma}{d\Omega}(\pi-\theta,\,pn\to pn)\,,\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{d\sigma}{d\Omega}(\theta,\,\pi^+p\to\pi^+p)&=\frac{9}{5}\frac{d\sigma}{d\Omega}(\theta,\,\pi^+n\to\pi^+n)\\&=\frac{9}{4}\frac{d\sigma}{d\Omega}(\theta,\,\pi^+n\to\pi^0p)\,,\\&=\frac{9}{5}\frac{d\sigma}{d\Omega}(\theta,\,\pi^0p\to\pi^0p)\,.\end{aligned} \tag{11}$$

2. One could discuss the spin dependence in a similar way. For example, consider large-angle $p+n\to p+n$. Denote the spin components of a particle in a direction perpendicular to the scattering plane by u (for up) and d (for down). There are the following possibilities[10] of spin arrangements:

$$uu\to uu\,,\quad uu\to dd\,,\quad ud\to ud\,,\quad ud\to du\,,$$
$$du\to ud\,,\quad du\to du\,,\quad dd\to uu\,,\quad dd\to dd\,.$$

The statistical hypothesis requires that they all have on the average[8] the same amplitude, and random phase differences.

[10] A. Bohr, Nucl. Phys. **10**, 486 (1959).

PHYSICAL REVIEW VOLUME 137, NUMBER 3B 8 FEBRUARY 1965

Uncoupled-Phase Method in the Multichannel N/D Formalism*

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The uncoupled-phase method is a nonperturbative formalism, developed by Ross and Shaw, relating the scattering amplitudes describing n strongly coupled two-body channels to the "uncoupled" amplitudes describing n-1 channels alone. The "uncoupled" scattering amplitudes are defined to be those that would exist if the couplings to the nth channel were switched off while the interactions among the n-1 channels remain unchanged. The uncoupled-phase method, previously based on the potential model, is extended to the relativistic problem by considering a set of n coupled N/D partial-wave dispersion relations. For the situation in which the left-hand cut is approximated by the form $g/(s+m)$ where g is an $n\times n$ matrix of constants and s is square of the total energy in the center-of-mass system, the uncoupled-phase method is exact. The quantitative validity of the uncoupled-phase method for more complicated left-hand singularities is tested by performing a two-channel computer experiment. A full numerical solution of the coupled integral equations for the N functions is obtained by the matrix-inversion technique. We consider the situations in which (a) the left-hand cut is replaced by a set of dipoles and (b) the left-hand cut is assumed to be given by exchange of a scalar particle in the corresponding "crossed" t channel of any given reaction. The coupled-phase method is found to be quantiatively accurate under a wide range of conditions. The range parameter of the coupled-phase method is directly given by a principal-value integral, and an estimate of it can be made *a priori.*

I. INTRODUCTION

THE uncoupled-phase method was developed[1] to confront certain theories of strong interactions with experiment. Consider a physical situation where the scattering is described by a set of n strongly coupled two-body channels. The "uncoupled" scattering ampli-

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[1] M. Ross and G. Shaw, Ann. Phys. (N. Y.) **9**, 391 (1960); G. Shaw and M. Ross, Phys. Rev. **126**, 806 (1962).