Three-Field Model and Higher Symmetry Group W_3 with Parity Interchange*

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A theory of hadrons (strongly interacting particles) based on the higher symmetry group $W_3=U_3\otimes U_3$ (or $SW_3=SU_3\otimes SU_3$) *with parity interchange* between the two U_3 (or SU_3) groups has been developed. The four-fermion vector model with three massless Dirac fields has been used to guide the construction of the meson and baryon particles. A parity doublet structure in the W_3 (or SW_3) limit is found for the mesons but not for the baryons. Mass relations between scalar (axial vector) and pseudoscalar (vector) octets or nonets of mesons are derived under various assumptions concerning the underlying group, the tensorial behavior of the symmetry-breaking terms, and the particle representations. Some experimental tests of these mass relations are noted. The possibility of incorporating the leptons into the theory is discussed.

1. INTRODUCTION

THE number of particles (and resonances) which undergo strong interactions is vastly greater than the number of invariance principles which characterize these interactions. If we put aside the discrete transformations P, C, T, we find that the strong interactions are invariant under two independent gauge transformations (baryon and hypercharge) and the isospin rotation group. Expressed in terms of group theory, we say that the strong interactions are invariant under[1] $(U_1\otimes U_2)$. Some time ago, Thirring[2] showed that at least three Weyl (2-component) fields are required to yield this group structure and that six Weyl fields are needed in order to obtain the multiplicative constants of motion which follow from P, C, T invariance.

The Sakata[3] model satisfied Thirring's conditions since three Dirac (4-component) fields were associated with the observed particles (p,n,Λ). On the other hand, Heisenberg's model[4] did not satisfy Thirring's conditions since he attempted to develop a unified theory of hadrons on the basis of one "basic[5]" massless Dirac field (equivalent to two Weyl fields). By a judicious choice of the nonlinear four-fermion interaction (axial vector), Heisenberg *et al.* could enlarge the underlying group to U_2 and take care of baryon conservation as well as isospin invariance; however, they were compelled to postulate a degenerate vacuum, spurions, etc., in order to understand the role of hypercharge in the strong interactions and further they could not treat the discrete operations in the usual fashion.

The success of broken SU_3 symmetry as exemplified by the Gell-Mann–Okubo (GMO) mass formula for octets and decuplets has raised new difficulties for the Heisenberg theory[6] and has also ruled out the identification of the three Dirac fields in the Sakata model with the known particles (p,n,Λ). However, if the three Dirac fields are taken with equal mass and proper choices are made for the internal quantum numbers,[7] then the underlying group structure becomes SU_3 and it is possible to break this symmetry in such a way that one obtains the GMO mass formula for the hadrons. The mass difference between the Λ and nucleon (as well as the mass differences within any unitary multiplet) is then due to the SU_3-symmetry breaking term, and the U_2 group structure takes over. From these remarks, it follows that within the framework of a nonlinear four-fermion theory, the choice of three Dirac fields with equal bare mass involves a higher group structure, namely U_3 (or SU_3).

In a quite different context, the choice of three Dirac fields is suggested by the existence of two (4-component) charged leptons (e and μ) and two (2-component) neutrinos (ν_e and ν_μ). If we define a 4-component neutrino ν by[8] $\begin{pmatrix}\nu_e\\ \bar{\nu}_\mu\end{pmatrix}$, then ν_e and $\bar{\nu}_\mu$ are, respectively, the positive and negative chirality projections of ν, and (ν,e,μ) comprise a triplet of 4-component Dirac fields. Since there are no strong interactions among leptons, there should be a close correspondence between particles and fields for leptons, a point recently reemphasized by Schwinger.[9] In accordance with an old-

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[1] U_2 represents a combination of the isospin and hypercharge groups while U_1 represents the baryon gauge group.

[2] W. E. Thirring, Nucl. Phys. **10**, 97 (1959); see also J. E. Wess, Nuovo Cimento **15**, 52 (1960).

[3] S. Sakata, Progr. Theoret. Phys. (Kyoto) **16**, 686 (1956); see also M. M. Lévy and R. E. Marshak, Nuovo Cimento Suppl. **11**, 366 (1954) and M. A. Markov, Proc. Acad. Sci. USSR (English transl.) **101**, No. 1, 51 (1955).

[4] H. P. Dürr, W. Heisenberg, H. Mitter, S. Schlieder, and K. Yamazaki, Z. Naturforsch. **14a**, 441 (1959) and subsequent papers.

[5] We shall sometimes refer to a massless Dirac (4-component) field as a "basic" field.

[6] Heisenberg *et al.* have succeeded in deriving the GMO mass formula for octets (despite the underlying U_2 group) by utilizing some special properties of the spurions; it is not clear whether the general GMO formula can be derived in this manner. One of the authors (R.E.M.) is indebted to Professor Heisenberg and Professor Dürr for interesting conversations.

[7] M. Ikeda, S. Ogawa, and Y. Ohnuki, Progr. Theoret. Phys. (Kyoto) **22**, 715 (1959); see also H. Goldberg and Y. Ne'eman, Nuovo Cimento **27**, 1 (1963), M. Gell-Mann, Phys. Letters **8**, 214 (1964) and G. Zweig, CERN report (unpublished).

[8] This definition implies opposite lepton number for ν_e and ν_μ and hence opposite lepton number for e and μ; as is well-known such as assignment can explain the failure to observe processes ike $\mu\to e+\gamma$, $\mu\to 3e$, etc. [see C. Iso, Nuovo Cimento **25**, 456, (1962)].

[9] J. Schwinger, Phys. Rev. **135**, B816 (1964).

baryon-lepton symmetry principle,[10] this would imply that a triplet of Dirac fields is a reasonable starting point for a theory of baryons (and of mesons).

If it is granted that a possible starting point for a theory of hadrons is one triplet of (4-component) Dirac fields with equal bare mass interacting via four-fermion terms, the question is still open whether one should begin with zero or finite bare mass. In a previous paper,[11] two of the authors showed that with the choice of zero bare mass for all three Dirac fields, the four-fermion interaction would be consistent with an underlying group structure of a higher symmetry than the obvious U_3 group, the particular higher symmetry group depending on whether the interaction is scalar (S), pseudoscalar (P), vector (V), axial vector (A), or tensor (T). More generally, it was proved[11a] that if ψ_i $(i=1,\cdots n)$ are n "basic" fields and ψ_i is decomposed into the positive and negative (2-component) chirality projections ϕ_i and ξ_i, respectively, then one finds the higher symmetry group for each type of interaction shown in Table I.

In Table I, R_n is the rotation group of dimension n, U_n is, as usual, the unitary group of dimension n, and $USp(n)$ is the unitary symplectic group of dimension n. The $\bar{W}_n$ group in Table I is not to be confused with the W_n group studied by several authors recently (see Ref. 16 below); the $\bar{W}_n$ group is W_n *with parity interchange* in the sense that the parity operation interchanges the two U_n groups comprising $\bar{W}_n$ since the first U_n group arises from the n ϕ's, the second U_n group from the n ξ's, and the parity operation transforms ϕ into ξ, and conversely (see Ref. 11 and Sec. 2 below). From Table I, it follows that for $n=3$, the S or P four-fermion interaction is governed by the R_6 group, the V interaction by the $\bar{W}_3$ group, the A interaction by the U_6 group, and the T interaction by the $USp(6)$ group.

Table I shows clearly why one "basic" field (i.e. $n=1$) is insufficient for a theory of hadrons. It is apparent that the largest group corresponding to $n=1$ is U_2 and, as mentioned earlier, the success of broken SU_3 symmetry requires a larger group than U_2. We must therefore look beyond $n=1$ within the framework of the nonlinear four-fermion theory. Since $n=2$ will not accommodate hypercharge and isospin in a natural way, the next simplest choice is $n=3$, which is our choice. We now try to give some arguments for selecting one of the four possible groups corresponding to $n=3$: R_6, $\bar{W}_3$, U_6, and $USp(6)$.

[10] A. Gamba, R. E. Marshak, and S. Okubo, Proc. Natl. Acad. Sci. U. S. **45**, 881 (1959); in that paper, the neutrino was treated as a 2-component particle and the correspondence was set up between the lepton triplet and the triplet (p,n,Λ); the baryon-lepton symmetry principle would now postulate a correspondence between the triplet of 4-component lepton fields and the triplet of "basic" fields whose interactions produce the hadrons [see also C. Iso, (Ref. 8)].

[11] R. E. Marshak and S. Okubo, Nuovo Cimento **19**, 1226 (1961) (see Appendix); see also A. Salam and J. C. Ward, *ibid.* **20**, 419 (1961).

[11a] *Note added in proof.* In our original paper (Ref. 11), we used ϕ and $\chi=\sigma_2\xi^*$; ϕ and χ are $(\frac{1}{2},0)$ representations whereas ξ is a $(0,\frac{1}{2})$ representation of the Lorentz group.

TABLE I. Hidden group symmetries for n "basic" fields on the four-fermion model.

Four-fermion interaction	Higher symmetry group
S or P	R_{2n}
V	$\bar{W}_n=U_n\otimes U_n$
A	U_{2n}
T	$USp(2n)$

The group R_6 has the smallest number of generators and possesses the interesting property that a finite bare mass term (it is a scalar) does not destroy the R_6 invariance of the S or P four-fermion interaction; in addition, the parity operation commutes with the R_6 group. However, the R_6 group has the unpleasant feature that a reduction at the U_3 (or SU_3) level of an irreducible representation of R_6 always leads to at least one irreducible representation of U_3 (or SU_3) with nonzero triality number. Thus, one finds

$$6=3\oplus 3^*,$$
$$15=8\oplus 1\oplus 3\oplus 3^*,$$
$$20=8\oplus 6\oplus 6^*,$$
$$50=10\oplus 10^*\oplus 15\oplus 15^*,$$

where the numbers on the left-hand side designate dimensions of irreducible representations of R_6 while those on the right-hand side are the appropriate reduction into irreducible representations of the U_3 (or SU_3) group. Since there is no evidence for unitary triplets or other multiplets of hadrons with nonzero triality associated with octets (which have zero triality), the R_6 group is not very attractive.

The last objection can also be raised against two of the other groups corresponding to $n=3$, namely U_6 and $USp(6)$. In the case[12] of U_6, some examples of the reduction are

$$6=3\oplus 3^*,$$
$$15=8\oplus 1\oplus 3\oplus 3^*,$$
$$35=8\oplus 8\oplus 1\oplus 6\oplus 6^*\oplus 3\oplus 3^*,$$

and for[13] $USp(6)$

$$6=3\oplus 3^*,$$
$$14=8\oplus 3\oplus 3^*,$$
$$21=8\oplus 6\oplus 6^*\oplus 1.$$

In all reductions of U_6 and $USp(6)$, there is always a contribution from a unitary multiplet with nonzero triality (together with an octet) and we have no evidence for such hadron multiplets. Moreover, the U_6 and $USp(6)$ groups listed in Table I have a further com-

[12] The U_6 group with a different rationale has been investigated by F. Gürsey and L. Radicati, Phys. Rev. Letters **13**, 173 (1964), A. Pais, *ibid.* **13**, 175 (1964), and by B. Sakita, Phys. Rev. **136**, B1756 (1964).

[13] A $USp(6)$ group based on two triplets has been studied by H. Bacry, J. Nuyts and L. Van Hove, Phys. Letters **9**, 279 (1964).

plicating feature in connection with the parity operation; for both of these groups, the parity can be represented by an operation similar to an antiunitary operator with the unitary part a member of the group. This produces some peculiar mixing effects which, in our opinion, justify the rejection of these two groups.

We are left with the $\bar{W}_3$ (or $S\bar{W}_3$)[14] group, some of whose irreducible representations contain only irreducible representations of U_3 (or SU_3) with zero triality number; this can be seen immediately if one recalls that at the U_3 (or SU_3) level, a representation of the product of the two U_3's (or SU_3's) comprising $\bar{W}_3$ (or $S\bar{W}_3$) becomes the ordinary product of two representations of the same U_3 (or SU_3) group. Furthermore, the parity operation involved in the $\bar{W}_3$ (or $S\bar{W}_3$) group is, so to speak, external to the group (in group-theoretic language, it is an Abelian group extension) and can easily be handled (see Sec. 2). The $\bar{W}_3$ (or $S\bar{W}_3$) group is the smallest group (corresponding to the choice $n=3$) which does not commute with the parity. The parity interchange aspect of the $\bar{W}_3$ (or $S\bar{W}_3$) group leads to parity degeneracy in the limit of $\bar{W}_3$ (or $S\bar{W}_3$) symmetry (see Sec. 2) and is especially intriguing because of the special role which parity plays in weak interactions[15] and the ultimate hope of developing a theory of leptons in conjunction with a theory of hadrons.

For all these reasons, we propose in this paper to explore rather fully the consequences of the $\bar{W}_3$ and $S\bar{W}_3$ groups.[16] In Sec. 2, we investigate the structure of the $\bar{W}_3$ and $S\bar{W}_3$ groups and the tensorial behavior of the very strong (V.S.) and medium strong (M.S.) interactions which can be expected to break $\bar{W}_3$ (or $S\bar{W}_3$) symmetry; the four-fermion vector model is used as a guide for this discussion. In Sec. 3, we work out the predictions—under various assumptions—for mesons of spin 0 and 1 on the basis of broken $\bar{W}_3$ (or $S\bar{W}_3$) symmetry. In Sec. 4, we sketch an extension of our theory to the more complicated case of baryons, and finally in Sec. 5, we briefly discuss our results and mention further directions in which the theory might be developed.

[14] We shall also be concerned with the $S\bar{W}_3$ group (see Sec. 2) which is defined by $S\bar{W}_3=SU_3\otimes SU_3$ *with parity interchange.*

[15] Riazuddin and R. E. Marshak [Phys. Letter **11**, 182 (1964)] have recently introduced scalar and axial vector meson octets on the basis of a generalized Goldberger-Treiman treatment of all partially conserved currents (see Sec. 3 below).

[16] M. Gell-Mann [Phys. Rev. **125**, 1067 (1962) and Physics **1**, 63 (1964)] came upon the $S\bar{W}_3$ group by looking for the group generated, under equal-time commutation, by the integrals of the time components of the vector and axial vector current octets. Despite the different starting points, there is a close connection between our theory and that of Gell-Mann; see also the more recent work of P. G. O. Freund and Y. Nambu [Phys. Rev. Letters **12**, 714 and **13**, 221 (1964)]. The work of Schwinger (Ref. 9) and of F. Gürsey, T. D. Lee, and M. Nauenberg [Phys. Rev. **135**, B467 (1964)] is based on the W_3 (or SW_3) group. Since Schwinger and Gürsey *et al.* postulate the existence of two triplets (in contradistinction to our one "basic" triplet), the parity operation does not interchange the two U_3 (or SU_3) groups whose product is W_3 (or SW_3) and the important feature of parity degeneracy does not enter the picture.

2. GROUP STRUCTURE OF $\bar{W}_3$ AND $S\bar{W}_3$ AND THE BREAKING OF THESE SYMMETRIES

While the essential predictions that follow from broken $\bar{W}_3$ (and $S\bar{W}_3$) symmetry do not depend upon the particular nonlinear four-fermion model (vector interaction of three "basic" fields) which motivated this study, it is convenient to begin with a discussion of this model. It will be recalled that the Lagrangian density describing three massless Dirac (4-component) fields with a quadrilinear interaction of the vector type is

$$\bar{\mathfrak{L}}=\sum_{i=1}^{3}\bar{\psi}_i\gamma_\mu\partial^\mu\psi_i+g\sum_{i,j=1}^{3}(\bar{\psi}_i\gamma_\mu\psi_i)(\bar{\psi}_j\gamma^\mu\psi_j)\,, \qquad (1)$$

where the coupling constant g has the dimension of (length)^2. This Lagrangian density $\bar{\mathfrak{L}}$ is invariant[11] under *separate* three-dimensional unitary transformations of the positive and negative chiral projections of the fields ψ_i; that is, under

$$\left.\begin{aligned}\phi_i&\equiv\tfrac{1}{2}(1+\gamma_5)\psi_i\to\sum_{j=1}^{3}a_i{}^j\tfrac{1}{2}(1+\gamma_5)\psi_j\equiv\sum_{j=1}^{3}a_i{}^j\phi_j\\ \xi_i&\equiv\tfrac{1}{2}(1-\gamma_5)\psi_i\to\sum_{j=1}^{3}b_i{}^j\tfrac{1}{2}(1-\gamma_5)\psi_j\equiv\sum_{j=1}^{3}b_i{}^j\xi_j\end{aligned}\right\} \quad (i=1,2,3) \qquad (2)$$

where $\|a_i{}^j\|$ and $\|b_i{}^j\|$ are independent unitary matrices. This invariance is characteristic of *vector* coupling,[17] and we have called this invariance group $\bar{W}_3$, $\bar{W}_3$ being the direct product of two U_3 groups (with parity interchange —see below).

Corresponding to the transformation (2), we have two sets of conserved currents

$$\begin{aligned}\partial^\mu\{\bar{\psi}_i\gamma_\mu(1+\gamma_5)\psi_j\}&=0\,,\\ \partial^\mu\{\bar{\psi}_i\gamma_\mu(1-\gamma_5)\psi_j\}&=0\,.\end{aligned} \qquad (3)$$

These currents are mixtures of vector and axial vector currents. We shall call the generators of the two separate U_3 groups, $A_i{}^j$ and $B_i{}^j$. These are the integrals (over space) of the time components of the corresponding conserved currents

$$\begin{aligned}A_i{}^j(t)&=-\frac{1}{2}\int d^3x\,\bar{\psi}_j\gamma_0(1+\gamma_5)\psi_i=(A_j{}^i)^\dagger\,,\\ B_i{}^j(t)&=-\frac{1}{2}\int d^3x\,\bar{\psi}_j\gamma_0(1-\gamma_5)\psi_i=(B_j{}^i)^\dagger\,,\end{aligned} \qquad (4)$$

and, as Gell-Mann has shown,[16] they obey the equal

[17] Actually, one can add an axial-vector four-fermion term to $\bar{\mathfrak{L}}$ without losing $\bar{W}_3$ invariance since $\bar{W}_3$ is a subgroup of U_6 (see Table I); a unique choice would be (V-A) since the $\bar{W}_3$ group structure is manifestly preserved for this combination of interactions under Fierz rearrangement (compare the four-fermion weak interaction).

time commutation relations (C.R.)

$$[A_i{}^j,A_k{}^l]=\delta_k{}^jA_i{}^l-\delta_i{}^lA_k{}^j,$$
$$[B_i{}^j,B_k{}^l]=\delta_k{}^jB_i{}^l-\delta_i{}^lB_k{}^j, \qquad (5)$$
$$[A_i{}^j,B_k{}^l]=0.$$

If the parity P and charge conjugation C operations are defined in the *conventional* way for the three basic fields ψ_i,

$$P:\ \psi_i(x,t)\rightarrow\gamma_0\psi_i(-x,t),$$
$$C:\ \psi_i(x,t)\rightarrow\psi_i{}^c(x,t), \qquad (6)$$

then the effect of P and C on the generators $A_i{}^j$, $B_i{}^j$ is as follows:

$$P:\ A_i{}^j\rightarrow B_i{}^j,\quad B_i{}^j\rightarrow A_i{}^j,$$
$$C:\ A_i{}^j\rightarrow -B_j{}^i,\quad B_i{}^j\rightarrow -A_j{}^i, \qquad (7)$$

The C.R.'s (5) are invariant under the P and C operations defined in (7). In the present theory, the parity operation interchanges the two U_3 groups of which $\bar{W}_3$ is composed (this is the reason why, as already explained in Sec. 1, we use $\bar{W}_3$ rather than the standard W_3 notation).

Consider next the addition to $\bar{\mathfrak{L}}$ of terms that gradually decrease the symmetry group of $\bar{\mathfrak{L}}$. First we add a common mass term $\mathfrak{L}'$:

$$\mathfrak{L}'=m_0\sum_{j=1}^{3}\bar{\psi}_i\psi_i. \qquad (8)$$

This immediately reduces the invariance group of the total Lagrangian, $\bar{\mathfrak{L}}+\mathfrak{L}'$, from $\bar{W}_3$ to U_3. $\mathfrak{L}'$ is invariant only under *common* unitary transformations of the two chiral projections of ψ_i. Correspondingly, only the *sum* of the two currents in (3) is conserved

$$\partial^\mu\{\bar{\psi}_i\gamma_\mu\psi_j\}=0, \qquad (9)$$

and whereas both $A_i{}^j$ and $B_i{}^j$ are constants of motion in the absence of $\mathfrak{L}'$, now it is only their sums

$$A_i{}^j+B_i{}^j$$

that are constants of the motion, and generate the group U_3.

Upon addition of a further term, $\mathfrak{L}''$, to $\bar{\mathfrak{L}}+\mathfrak{L}'$ of the form

$$\mathfrak{L}''=m_1\bar{\psi}_3\psi_3 \qquad (10)$$

we destroy U_3 invariance, and only maintain invariance under a group U_2. $\mathfrak{L}''$ corresponds to the usual (M.S.) symmetry-breaking term, and the resulting U_2 is a subgroup of U_3 and comprises the customary isotopic spin rotations and hypercharge gauge transformations.

The algebraic structure of the theory, embodied in Eqs. (5), (7), has here been derived from a specific three-field model. The model also prescribes definite tensorial behaviors, under $\bar{W}_3$, for the terms $\mathfrak{L}'$ and $\mathfrak{L}''$ that successively destroy first the $\bar{W}_3$ and then the U_3 symmetry (these will be described shortly). However, if we wish, we may retain just the group-theoretic structure given by (5) and (7), assume some definite transformation properties for $\mathfrak{L}'$ and $\mathfrak{L}''$, and then explore the consequences of assigning known (and unknown) particles to various representations of $\bar{W}_3$. We shall also consider the possibility that the underlying symmetry group is $S\bar{W}_3=SU_3\otimes SU_3$ *with parity interchange*, rather than $U_3\otimes U_3$ *with parity interchange*. In this case, we definitely move away from the three-field model, at least as set up above.

From now on, we shall distinguish between the two U_3 (or SU_3) groups by using unprimed tensor indices for the group generated by the $A_i{}^j$ and primed tensor indices for that generated by the $B_i{}^j$ (so that we shall henceforth write $B_{i'}{}^{j'}$). An irreducible representation of $\bar{W}_3$ is made up of an irreducible representation for each U_3 group. Denoting the latter by R_1, R_2, an irreducible representation of $\bar{W}_3$ may be labelled (R_1,R_2). Since we deal with only a few low-dimensional representations, we shall refer to them by their dimensionality. Under P and C, we have[18]

$$P:\ (R_1,R_2)\rightarrow(R_2,R_1),$$
$$C:\ (R_1,R_2)\rightarrow(R_2{}^*,R_1{}^*), \qquad (11)$$

where $R_1{}^*$ (or $R_2{}^*$) indicates the conjugate representation of R_1 (or R_2).

Now a tensor of type (3,1) obeys the C.R.'s

$$[A_i{}^j,T_k]=\delta_k{}^jT_i-\tfrac{1}{3}\delta_i{}^jT_k,$$
$$[B_{m'}{}^{n'},T_k]=0. \qquad (12)$$

[The term $-\frac{1}{3}\delta_i{}^jT_k$ in (12) and in all the subsequent equations is present or absent depending on whether we deal with $S\bar{W}_3$ or $\bar{W}_3$.] Similarly, a tensor T of type (1,3) obeys

$$[B_{m'}{}^{n'},T_{k'}]=\delta_{k'}{}^{n'}T_{m'}-\tfrac{1}{3}\delta_{m'}{}^{n'}T_{k'}$$
$$[A_i{}^j,T_{k'}]=0. \qquad (13)$$

A mixed tensor T of type (3,3*), say, is defined by

$$[A_j{}^k,T_l{}^{m'}]=\delta_l{}^kT_j{}^{m'}-\tfrac{1}{3}\delta_j{}^kT_l{}^{m'}$$
$$[B_{j'}{}^{k'},T_l{}^{m'}]=-\delta_{j'}{}^{m'}T_l{}^{k'}+\tfrac{1}{3}\delta_{j'}{}^{k'}T_l{}^{m'}. \qquad (14)$$

[18] The property (11) of the C operation at first sight seems to possess the following unpleasant feature. If a particle θ is assigned to the representation (R_1,R_2) of $\bar{W}_3$, and if we believe that the C operation takes θ into its antiparticle $\bar{\theta}$, then in general the annihilation of θ and $\bar{\theta}$ into photons or vacuum (for example) is forbidden if $R_1\neq R_2$. One may take the point of view that one must deal always with eigenstates of P, certainly in the presence of $\mathfrak{L}'$; a particle θ is then of the form $(R_1,R_2)\pm(R_2,R_1)$, $\bar{\theta}$ has the form $(R_2{}^*,R_1{}^*)\pm(R_1{}^*,R_2{}^*)$, and now the annihilation of θ by $\bar{\theta}$ is always allowed. This is similar to the situation for $K_1{}^0$ and $K_2{}^0$ where the (weak) interaction between K^0 and $\bar{K}^0$ requires us to define the decaying particles in terms of the CP eigenstates, namely, as $K_1{}^0$ and $K_2{}^0$ (assuming CP invariance for the time being).

As another example, a tensor of type (8,1) obeys

$$[A_j{}^k, T_l{}^m] = \delta_l{}^k T_j{}^m - \delta_j{}^m T_l{}^k, \quad [B_{j'}{}^{k'}, T_l{}^m] = 0. \tag{15}$$

By explicitly writing out both $\mathcal{L}'$ of (8) and $\mathcal{L}''$ of (10) in terms of ϕ_i, ξ_i [which transform like (3,1) and (1,3), respectively], we find that they are both made up of components of tensors of types (3,3*) and (3*,3), namely

$$\mathcal{L}' \sim m_0 \sum_{k=1}^{3} (T_k{}^{k'} + T_{k'}{}^k), \quad \mathcal{L}'' \sim m_1 (T_3{}^{3'} + T_{3'}{}^3). \tag{16}$$

Under transformations generated by the operators

$$A_k{}^l + B_{k'}{}^{l'},$$

that is, under the group U_3, the primed and unprimed indices transform in the same way, and the distinction between them disappears. $\mathcal{L}'$ is then clearly invariant under U_3, while $\mathcal{L}''$ has the tensor character under U_3 that is assumed in the usual derivation of the GMO mass formula. An alternative assignment of the behavior of $\mathcal{L}''$ under $\bar{W}_3$, leading to the same behavior under U_3 is

$$\mathcal{L}'' \sim (T_3{}^3 + T_{3'}{}^{3'}). \tag{17}$$

In (17), $\mathcal{L}''$ is made up of tensors of types (8,1) and (1,8), rather than (3,3*) and (3*,3), but at the U_3 level it exhibits the same tensorial behavior. We shall examine some consequences of both assignments[19] for $\mathcal{L}''$ under $\bar{W}_3$.

The definitions (7) of P and C lead to an interesting behavior under CP, for $\bar{W}_3$ multiplets. We shall discuss the (3,3*) and (8,1) representations of $\bar{W}_3$. Let $T_m{}^{n'}$ be a tensor of type (3,3*). [$T_m{}^{n'}$ could be a set of states, or a tensor operator.] Then under P and C we must have, according to (11),

$$PT_m{}^{n'}P^{-1} = S_{m'}{}^n, \quad CT_m{}^{n'}C^{-1} = U_n{}^{m'}, \tag{18}$$

where S and U are tensors of types (3*,3), (3,3*), respectively. Operating on S with C, or on U with P, we next obtain

$$CS_{m'}{}^nC^{-1} = PU_n{}^{m'}P^{-1} = V_{n'}{}^m. \tag{19}$$

[This is because $CP = PC$.] Thus starting with $T_m{}^{n'}$, we generate, in general, three different tensors S, U, V. In the limit of exact $\bar{W}_3$ symmetry, if $T_m{}^{n'}$ represents a multiplet of particle states, we should find, in general, three more multiplets degenerate (in mass and spin) with the given one.

We form eigenstates of C and P by setting

$$\begin{aligned}
\Theta_m{}^n &= T_m{}^{n'} + S_{m'}{}^n + U_m{}^{n'} + V_{m'}{}^n: \quad C = P = 1, \\
\Phi_m{}^n &= T_m{}^{n'} - S_{m'}{}^n + U_m{}^{n'} - V_{m'}{}^n: \quad C = -P = 1, \\
\Theta_m{}'^n &= T_m{}^{n'} + S_{m'}{}^n - U_m{}^{n'} - V_{m'}{}^n: \quad C = -P = -1, \\
\Phi_m{}'^n &= T_m{}^{n'} - S_{m'}{}^n - U_m{}^{n'} + V_{m'}{}^n: \quad C = P = -1.
\end{aligned} \tag{20}$$

By commuting the tensors Θ, Φ, Θ', Φ' with the generators $A_i{}^j$, $B_{k'}{}^{l'}$, we find that Θ and Φ are linked to one another, while Θ' and Φ' are similarly linked. In other words, an irreducible representation of C, P, and $\bar{W}_3$ is made up, in the case of the (3,3*) and (3*,3) representations for $\bar{W}_3$, of two multiplets with *opposite* values of CP. If we speak of a multiplet with $C = +1$ as being normal, then, for example, if we regard Θ and Φ (or Θ' and Φ') as referring to spin 0 mesons (see Sec. 3), both the scalar and pseudoscalar multiplets are normal (or both are abnormal). Further, it is possible to have

$$T_m{}^{n'} = U_m{}^{n'}, \quad S_{m'}{}^n = V_{m'}{}^n, \tag{21}$$

in which case the quantities (or states) Θ', Φ' in (20) will not exist. If we have negative signs in both equations in (21), then Θ, Φ will vanish but the above conclusions are unchanged.

The situation is opposite in the case of the representations (8,1) and (1,8) for $\bar{W}_3$. Here we find that the generators $A_i{}^j$, $B_{k'}{}^{l'}$ link operators (or states) with the *same* values of CP. Thus a normal pseudoscalar multiplet would be degenerate with an abnormal scalar multiplet or conversely.

3. MASS PREDICTIONS FOR MESONS OF SPIN 0 AND 1

In this section we shall discuss spin-0 and spin-1 mesons on the basis of our theory and, in the next section, treat the baryons. Consider first, the spin-0 mesons. If we think of them as built up from pairs of 2-component objects ϕ_i, ξ_j, and if we suppose that they are s wave bound states (i.e., no powers of momenta are involved), then the only spinless objects we can form are

$$\phi_i{}^\dagger \xi_{j'}, \quad \xi_{k'}{}^\dagger \phi_l. \tag{22}$$

This follows from the chirality properties of the 2-component objects ϕ, ξ. But the quantities (22) belong to the representations (3,3*) and (3*,3) of $\bar{W}_3$, and we are thus led to consider nonets of pseudoscalar and scalar particles. With nonets belonging to the (3,3*) and (3*,3) representations, both must have the same C quantum number (and therefore opposite CP, see Sec. 2) and we may assume them both to be normal.

Gell-Mann[16] has suggested another reason to assign the spin-0 mesons to the representations (3,3*) and (3*,3). He notes that, whereas in the $\bar{W}_3$ limit, the axial vector current [i.e. the difference of the two currents of (3)] is divergenceless, the presence of $\mathcal{L}'$ makes this

[19] It is worth noting that even though $\mathcal{L}''$ is weaker than $\mathcal{L}'$ and the effects of the latter must be calculated first, we may specify the tensor behavior of $\mathcal{L}''$ with respect to the group $\bar{W}_3$, invariance under which is destroyed by $\mathcal{L}'$. This is analogous to the situation for electromagnetic processes within the context of approximate SU_3 (or U_3) invariance for the strong interactions. There we specify the SU_3 quantum numbers of the electric current operator, for instance, even while taking account of the breakdown of SU_3 invariance to various orders in the M.S. interactions.

divergence nonzero; the specific tensor form (16) for $\mathcal{L}'$ requires this divergence also to belong to (3,3*), (3*,3). If we now think of these divergences as being approximately proportional to the pseudoscalar meson fields (in the sense of the Goldberger-Treiman relations), then we are led to the same assignment of representations for spin-0 mesons as was arrived at above by our more direct argument.

It should be emphasized that neither our argument nor Gell-Mann's is conclusive. If we are willing to form the spin-0 mesons out of p-wave bound states, then (22) would be replaced by[20]

$$\phi_i^\dagger\sigma_\mu\partial^\mu\phi_j\,,\quad \xi_{k'}^\dagger\tilde{\boldsymbol{\sigma}}_\mu\partial^\mu\xi_{l'}$$
$$\sigma_\mu=(\mathbf{1},\boldsymbol{\sigma})\,,\quad \tilde{\boldsymbol{\sigma}}_\mu=(1,-\boldsymbol{\sigma})\,, \tag{22a}$$

and the spin-0 mesons would belong to the representations (8,1), (1,8) of $\bar{W}_3$. Gell-Mann's argument would also change if we considered $\mathcal{L}'$ to have a tensor structure other than [(3,3*)+(3*,3)], e.g. (8,8); however, in the present paper, we shall only consider [(3,3*)+(3*,3)]. In one way, (22a) is preferable to (22) since we are more naturally dealing with octets rather than nonets and experimentally there is no clear evidence for a pseudoscalar nonet. Since the octets of scalar and pseudoscalar mesons would then belong to the (8,1), (1,8) representations of $\bar{W}_3$, they would have the same CP and therefore one octet would be abnormal (see Sec. 2).

For the vector mesons, we have to construct vectorial quantities out of ϕ,ξ. Again, if we assume that s-wave bound states are involved, the only vectors we can form are

$$\phi_i^\dagger\sigma_\mu\phi_j\,,\quad \xi_{k'}^\dagger\tilde{\boldsymbol{\sigma}}_\mu\xi_{l'}\,. \tag{23}$$

This suggests assigning the vector and axial vector mesons to the representations (8,1), (1,8), coinciding with the tensor properties of the currents conserved in the $\bar{W}_3$ limit. This time, if we assume that the vector particles have charge conjugation parity $C=-1$, the axial vector particles would have $C=+1$ (and hence be abnormal, see Sec. 2). If, instead, we built up spin-1 mesons out of p-wave bound states, then we could replace (23) by

$$\phi_i^\dagger\partial_\mu\xi_{j'}\,,\quad \xi_{k'}^\dagger\partial_\mu\phi_l\,, \tag{23a}$$

and the situation would be reversed and correspond to the s-wave bound states for spin-0 mesons [cf. (22)]. Henceforth, we shall assume only s-wave bound states, namely that (22) and (23) are the proper models for spin-0 and spin-1 mesons, respectively. The changes introduced by choosing (22a) in place of (22) or (23a) in place of (23) will be self-evident.

[20] Strictly speaking, (22a) will not generate $CP=-1$, $J=0$ meson octets because it leads to the combinations $\partial^\mu\bar{\psi}_i\gamma_\mu\psi_j$ and $\partial^\mu\bar{\psi}_i\gamma_5\gamma_\mu\psi_j$ which are identically zero ($CP=+1$, $J=0$ meson octets are permitted); however, it is possible to generate $CP=-1$, $J=0$ meson octets by means of multi-quark pair operators. We are indebted to Professor G. Feinberg for calling attention to this point.

When we start with a set of mesons assigned to the representations (3,3*), (3*,3), we have, in the $\bar{W}_3$ (or $S\bar{W}_3$) limit, two nonets S_9, P_9 of opposite parity (0^+ and 0^-, respectively) degenerate with one another. When the interaction $\mathcal{L}'$ is "switched on," each nonet breaks up into an octet with respect to U_3 (or SU_3) and a singlet; thus we have S_8, S_1, P_8, P_1, no longer degenerate with one another. The lowest order in which this separation is produced, will depend on which underlying group is used, $\bar{W}_3$ or $S\bar{W}_3$, and we shall consider both possibilities.

When the interaction $\mathcal{L}''$ is then also "switched on," the unitary octets break up into isomultiplets. If, in the U_3 limit, the unitary singlets are well separated from the octets, there may be reason to neglect mixing of octets and singlets when $\mathcal{L}''$ is present. In that case, we shall find the GMO mass formula to be valid in both octets. In addition, we may find additional relations connecting members of different octets such as K, π, K', π' (the primed particles are members of the scalar octet). If mixing of unitary octets and singlets must be allowed, then we lose the GMO formulas, but the remaining relations may still be valid.

In the case of the representations (1,8), (8,1), we have, in the $\bar{W}_3$ limit, only two octets of particles. However, since we assign the known vector mesons to this representation, we are led to *assume* the existence of two unitary singlets of mesons (1^- and 1^+). These need not be degenerate with the octets in the $\bar{W}_3$ limit. Also, in computing effects of $\mathcal{L}'$, $\mathcal{L}''$, these singlets can in principle behave quite differently from the octets. But past experience suggests that we treat them "dynamically in the same way" as the octets. It also seems clear that one should take proper account of mixing for the vector and axial vector mesons.

If we assume the tensor structure [(3,3*)+(3*,3)] for $\mathcal{L}'$, we have then to consider two possibilities for each of the following: underlying group, tensor structure of $\mathcal{L}''$, representations to which the particles belong, and mixing or no mixing. This leads to a total of sixteen cases, summarized in Table II. In the last two columns of Table II, we have written down the lowest orders, in a perturbation calculation, in which $\mathcal{L}'$, $\mathcal{L}''$ are effective.

Note that in all the odd-numbered cases (no mixing), the GMO formula is valid within the unitary octets, and in all the even-numbered cases (mixing), it is not. From now on, we shall not state explicitly in each case the validity (or otherwise), of this formula. As a sample calculation, we treat Case 3.

Case 3. We have a tensor (of states) $f_b{}^{a'}$, which goes into $f_{b'}{}^a$ under the parity operation P. We define scalar and pseudoscalar nonets by

$$S_b{}^a=((1+P)/\sqrt{2})f_b{}^{a'}=(1/\sqrt{2})(f_b{}^{a'}+f_{b'}{}^a)\,,$$
$$P_b{}^a=((1-P)/\sqrt{2})f_b{}^{a'}=(1/\sqrt{2})(f_b{}^{a'}-f_{b'}{}^a)\,. \tag{24}$$

TABLE II. Different cases considered for meson mass relations but always assuming $\mathcal{L}' \sim [(3,3^*)+(3^*,3)]$.

Case	Group	$\mathcal{L}''$	Particles	Mixing	Breakdown of $\bar{W}_3$ or $S\bar{W}_3$	Breakdown of U_3 or SU_3
1	$\bar{W}_3$	$(3,3^*)+(3^*,3)$	$(3,3^*),(3^*,3)$	No	$\mathcal{L}'\mathcal{L}'$	$\mathcal{L}'\mathcal{L}''$
2	$\bar{W}_3$	$(3,3^*)+(3^*,3)$	$(3,3^*),(3^*,3)$	Yes	$\mathcal{L}'\mathcal{L}'$	$\mathcal{L}'\mathcal{L}''$
3	$S\bar{W}_3$	$(3,3^*)+(3^*,3)$	$(3,3^*),(3^*,3)$	No	$\mathcal{L}'$	$\mathcal{L}''$
4	$S\bar{W}_3$	$(3,3^*)+(3^*,3)$	$(3,3^*),(3^*,3)$	Yes	$\mathcal{L}'$	$\mathcal{L}''$
5	$\bar{W}_3$	$(3,3^*)+(3^*,3)$	$(1,8),(8,1)$	No	$\mathcal{L}'\mathcal{L}'$	$\mathcal{L}'\mathcal{L}''$
6	$\bar{W}_3$	$(3,3^*)+(3^*,3)$	$(1,8),(8,1)$	Yes	$\mathcal{L}'\mathcal{L}'$	$\mathcal{L}'\mathcal{L}''$
7	$S\bar{W}_3$	$(3,3^*)+(3^*,3)$	$(1,8),(8,1)$	No	$\mathcal{L}'\mathcal{L}'$	$\mathcal{L}'\mathcal{L}''$
8	$S\bar{W}_3$	$(3,3^*)+(3^*,3)$	$(1,8),(8,1)$	Yes	$\mathcal{L}'\mathcal{L}'$	$\mathcal{L}'\mathcal{L}''$
9	$\bar{W}_3$	$(1,8)+(8,1)$	$(3,3^*),(3^*,3)$	No	$\mathcal{L}'\mathcal{L}'$	$\mathcal{L}''$
10	$\bar{W}_3$	$(1,8)+(8,1)$	$(3,3^*),(3^*,3)$	Yes	$\mathcal{L}'\mathcal{L}'$	$\mathcal{L}''$
11	$S\bar{W}_3$	$(1,8)+(8,1)$	$(3,3^*),(3^*,3)$	No	$\mathcal{L}'$	$\mathcal{L}''$
12	$S\bar{W}_3$	$(1,8)+(8,1)$	$(3,3^*),(3^*,3)$	Yes	$\mathcal{L}'$	$\mathcal{L}''$
13	$\bar{W}_3$	$(1,8)+(8,1)$	$(1,8),(8,1)$	No	$\mathcal{L}'\mathcal{L}'$	$\mathcal{L}''$
14	$\bar{W}_3$	$(1,8)+(8,1)$	$(1,8),(8,1)$	Yes	$\mathcal{L}'\mathcal{L}'$	$\mathcal{L}''$
15	$S\bar{W}_3$	$(1,8)+(8,1)$	$(1,8),(8,1)$	No	$\mathcal{L}'\mathcal{L}'$	$\mathcal{L}''$
16	$S\bar{W}_3$	$(1,8)+(8,1)$	$(1,8),(8,1)$	Yes	$\mathcal{L}'\mathcal{L}'$	$\mathcal{L}''$

The physical mesons are identified as

$$K^{+\prime}=S_1{}^3, \quad \pi^{+\prime}=S_1{}^2, \quad K^{0\prime}=S_2{}^3,$$
$$\eta'=(S_1{}^1+S_2{}^2-2S_3{}^3)/\sqrt{6}, \quad \pi^{0\prime}=(1/\sqrt{2})(S_1{}^1-S_2{}^2), \tag{25}$$

etc.

To evaluate the matrix elements of $\mathcal{L}'$ we have

$$\begin{aligned}\langle S_b{}^a|\mathcal{L}'|S_d{}^c\rangle &= \tfrac{1}{2}\langle f_b{}^{a'}|(1+P)\mathcal{L}'(1+P)|f_d{}^{c'}\rangle \\ &= \langle f_b{}^{a'}|\mathcal{L}'(1+P)|f_d{}^{c'}\rangle \\ &= \langle f_b{}^{a'}|(T_{\lambda'}{}^{\lambda}+T_{\lambda}{}^{\lambda'})|f_d{}^{c'}\rangle \\ &\quad +\langle f_b{}^{a'}|(T_{\lambda'}{}^{\lambda}+T_{\lambda}{}^{\lambda'})|f_{d'}{}^{c}\rangle. \end{aligned} \tag{26}$$

The two matrix elements in (26) must, by the separate SU_3 invariances of the underlying group $S\bar{W}_3$, be written as a sum of terms containing Kronecker deltas δ and epsilon symbols ϵ separately in the two kinds of indices. (The use of ϵ is allowed because the group here is $S\bar{W}_3$; if it were $\bar{W}_3$, we would be restricted to the use of δ alone as in Case 1, for example.) Thus we find

$$\begin{aligned}\langle f_b{}^{a'}|(T_{\lambda'}{}^{\lambda}+T_{\lambda}{}^{\lambda'})|f_d{}^{c'}\rangle &= 0 \\ \langle f_b{}^{a'}|(T_{\lambda'}{}^{\lambda}+T_{\lambda}{}^{\lambda'})|f_{d'}{}^{c}\rangle &= A\,\epsilon^{\lambda bc}\epsilon_{\lambda' d' a'}.\end{aligned} \tag{27}$$

Similarly, for the case of $\mathcal{L}''$,

$$\begin{aligned}\langle f_b{}^{a'}|\mathcal{L}''|f_d{}^{c'}\rangle = \langle f_b{}^{a'}|(U_{3'}{}^3+U_3{}^{3'})|f_d{}^{c'}\rangle &= 0, \\ \langle f_b{}^{a'}|(U_{3'}{}^3+U_3{}^{3'})|f_{d'}{}^{c}\rangle &= B\epsilon^{3bc}\epsilon_{3'd'a'}.\end{aligned} \tag{28}$$

Using (25), (26), (27), (28), and assuming a common $(\text{mass})^2$, μ^2, for all the mesons in the $S\bar{W}_3$ limit, we find the following extra relations (in addition to the GMO formulas) in lowest order, for Case 3

$$\begin{aligned} m_K{}^2+m_{K'}{}^2 &= m_\pi{}^2+m_{\pi'}{}^2, \\ m_{S_1}{}^2+m_{P_1}{}^2 &= m_K{}^2+m_{K'}{}^2, \\ m_{S_1}{}^2-m_{P_1}{}^2 &= \tfrac{4}{3}(m_K{}^2-m_{K'}{}^2)+\tfrac{2}{3}(m_\pi{}^2-m_{\pi'}{}^2). \end{aligned} \tag{29}$$

The last of Eqs. (29) reduces to $m_{S_1}{}^2-m_{P_1}{}^2=2(m_{P_8}{}^2-m_{S_8}{}^2)$ when $B=0$ (in agreement with Gell-Mann[16]).

We now summarize the extra relations obtained in the other cases:

Cases 1, 2, and 8. No relations.

Case 4. If the physical particles obtained after mixing are called S_1, η'; P_1, η, we get

$$\begin{aligned} m_K{}^2+m_{K'}{}^2 &= m_\pi{}^2+m_{\pi'}{}^2, \\ m_{S_1}{}^2+m_{\eta'}{}^2 &= \tfrac{3}{2}m_\pi{}^2+\tfrac{1}{2}m_{\pi'}{}^2, \\ m_{P_1}{}^2+m_\eta{}^2 &= \tfrac{3}{2}m_{\pi'}{}^2+\tfrac{1}{2}m_\pi{}^2. \end{aligned} \tag{30}$$

Cases 5 and 7. If the spin-1 octets and singlets possess unequal masses μ_8 and μ_1 in the $\bar{W}_3$ limit, then

$$2m_{K^*}{}^2+m_\rho{}^2-3m_{V_1}{}^2=2m_{K^{*\prime}}{}^2+m_{\rho'}{}^2-3m_{A_1}{}^2. \tag{31}$$

[In Case 5 alone, if $\mu_8=\mu_1$, both sides of (31) vanish.]

Case 6. Assuming $\mu_8=\mu_1$, we get

$$\begin{aligned} m_\rho{}^2 &= m_\omega{}^2, & 2m_{K^*}{}^2 &= m_\omega{}^2+m_\phi{}^2, \\ m_{\rho'}{}^2 &= m_{\omega'}{}^2, & 2m_{K^{*\prime}}{}^2 &= m_{\omega'}{}^2+m_{\phi'}{}^2. \end{aligned} \tag{32}$$

Cases 9, 10, and 16. Depending on whether we have spin-0 or spin-1 mesons, we get similar-looking equations

$$m_K{}^2-m_\pi{}^2=m_{K'}{}^2-m_{\pi'}{}^2, \tag{33}$$

$$m_{K^*}{}^2-m_\rho{}^2=m_{K^{*\prime}}{}^2-m_{\rho'}{}^2. \tag{34}$$

Case 11. In addition to (33), we get

$$\begin{aligned} m_{S_1}{}^2+m_{P_1}{}^2 &= \tfrac{2}{3}(m_K{}^2+m_{K'}{}^2)+\tfrac{1}{3}(m_\pi{}^2+m_{\pi'}{}^2), \\ m_{S_1}{}^2-m_{P_1}{}^2 &= 2(m_K{}^2-m_{K'}{}^2). \end{aligned} \tag{35}$$

Case 12. In addition to (33)

$$\begin{aligned} m_{S_1}{}^2+m_{\eta'}{}^2 &= \tfrac{3}{2}m_K{}^2+\tfrac{1}{2}m_{K'}{}^2, \\ m_{P_1}{}^2+m_\eta{}^2 &= \tfrac{3}{2}m_{K'}{}^2+\tfrac{1}{2}m_K{}^2. \end{aligned} \tag{36}$$

Cases 13 and 15. We find relations (31) and (34). [In Case 13 alone, both sides of (31) vanish if $\mu_8=\mu_1$.]

Case 14. We obtain relation (34), and (32) if $\mu_8=\mu_1$.

The relations (32) have been obtained previously by Okubo,[21] and by Gürsey *et al.*[16] All the remaining relations above connect meson multiplets of opposite parity and are characteristic of the present model. Equations (33) and (34), which are identical at the U_3 level, from the group-theoretic point of view, are remarkable in that they appear in all the cases from Case 9 onwards. In other words, they depend for their validity only on the tensor behavior [(8,1)+(1,8)] for $\mathcal{L}''$. In particular, they are independent of the tensor structure of $\mathcal{L}'$ as long as $\mathcal{L}'$ is invariant under U_3. [Note that in Cases 9 to 16 in Table II, breakdown of U_3 occurs with a term of the form $\mathcal{L}''$, not $\mathcal{L}'\mathcal{L}''$.]

It is interesting to see to what degree of approximation the above relations are maintained, and to compare their domain of validity with that for the GMO formula. Clearly, Eqs. (33) and (34) are true to all orders in $\mathcal{L}'$ alone, since $\mathcal{L}'$ affects m_K and m_π equally and $m_{K'}$ and $m_{\pi'}$ equally (similarly for K^*, ρ and $K^{*\prime}$, ρ'). They also turn out to be valid to all orders in $\mathcal{L}''$ by itself, as long as $\mathcal{L}''$ has the tensor character [(1,8)+(8,1)], independently of whether the particles are assigned to the representations $(3,3^*)\pm(3^*,3)$, or $(8,1)\pm(1,8)$. To prove this statement for Eq. (33), observe first that with $\mathcal{L}''$ of the form

$$\mathcal{L}''=U_3{}^3+U_{3'}{}^{3'},$$

$\mathcal{L}''$ conserves the hypercharges and isospins associated with the two independent U_3 groups, separately. {This is *not* true with $\mathcal{L}''\sim[(3,3^*)+(3^*,3)]$.} Now the contributions of a term of the form $(\mathcal{L}'')^n$ to $m_{K'}{}^2$ and $m_K{}^2$ are, respectively

$$\langle f_1{}^{3'}|(\mathcal{L}'')^n|(f_1{}^{3'}+f_{1'}{}^3)\rangle,$$
$$\langle f_1{}^{3'}|(\mathcal{L}'')^n|(f_1{}^{3'}-f_{1'}{}^3)\rangle.$$

The difference between these is

$$2\langle f_1{}^{3'}|(\mathcal{L}'')^n|f_{1'}{}^3\rangle;$$

this vanishes since the states on the two sides of $(\mathcal{L}'')^n$ have different isospins for the unprimed indices (or primed indices). Similarly, by considering the hypercharges corresponding to unprimed indices, say, one finds, in the case of $m_{\pi'}{}^2$ and $m_\pi{}^2$,

$$\langle f_1{}^{2'}|(\mathcal{L}'')^n|f_{1'}{}^2\rangle=0.$$

This shows that a term of type $(\mathcal{L}'')^n$ does not destroy Eq. (33). Similarly, we establish that Eq. (34) is maintained to all orders in $\mathcal{L}''$ alone.

If we consider cross terms in $\mathcal{L}'$ and $\mathcal{L}''$ in the case of the representations (8,1), (1,8) for the mesons [i.e., Eq. (34)], a term of the type $\mathcal{L}'\mathcal{L}''$ gives no contributions by triality considerations; and it is only in the next order $(\mathcal{L}')^2\mathcal{L}''$ that Eq. (34) breaks down. For the representations $(3,3^*)$,$(3^*,3)$ for the mesons [i.e., Eq. (33)], the term that destroys Eq. (33) is $\mathcal{L}'\mathcal{L}''$ or $(\mathcal{L}')^2\mathcal{L}''$, depending on whether we take $S\bar{W}_3$ or $\bar{W}_3$ as the underlying group. On the other hand, the GMO formula is valid (in the absence of mixing, of course!) to all orders in $\mathcal{L}'$ by itself, and for all values of n in cross terms of the type $(\mathcal{L}')^n\mathcal{L}''$. It breaks down where we have more than one power of $\mathcal{L}''$.

The generality of Eqs. (33) and (34) is encouraging and justifies, in our opinion, an intensive search for octets of scalar and axial vector mesons. We have seen that as long as the tensor character of $\mathcal{L}''$ is [(1,8) +(8,1)], these relations should hold regardless of whether the underlying group is $\bar{W}_3$ or $S\bar{W}_3$, the mesons belong to $(3,3^*)$, $(3^*,3)$ or (1,8), (8,1) representations of the underlying group, or there is mixing between a unitary octet and a unitary singlet. The independence of the relations (33) and (34) with respect to the choice of particle representation is particularly welcome since there is no way of really knowing at this stage whether the lowest mass mesons with a given spin are *s*- (or *p*-) wave bound states. If we admit a natural preference for *s*-wave bound states, then the π' and K' particles in Eq. (33) belong to a *normal* scalar octet and the ρ' and $K^{*\prime}$ particles in Eq. (34) belong to an *abnormal* axial vector octet (see above). But the point is that the mass relations (33) and (34) would hold equally well if the scalar octet were abnormal and/or the axial vector octet were normal. The feature of normality or abnormality will reflect itself in the production cross sections and decay modes of the scalar and axial vector octets but not in the masses. In order to make use of the mass relations (33) and (34), the mass of one member of the scalar or axial vector octet must be known (since a GMO formula holds within each octet). If we identify the K' in Eq. (33) with the 725-MeV resonance (an identification which is very tentative[22]), then we find $m_{\pi'}=555$ MeV and $m_{\eta'}=780$ MeV. For the axial vector octet, we prefer to make no mass predictions since the evidence for a $J=1^+$ resonance is even less certain.

We have laid special emphasis on the relations (33) and (34) because of the generality of these results. However, we cannot be sure that $\mathcal{L}''\sim[(1,8)+(8,1)]$, and some brief remarks are in order concerning $\mathcal{L}''\sim[(3,3^*)+(3^*,3)]$. While Cases 1–8 in Table II are all based on the $[(3,3^*)+(3^*,3)]$ tensor behavior of $\mathcal{L}''$, only two cases (Cases 3–4) lead to a mass relation analogous to (33), namely the first of Eqs. (29). Both of these cases require the underlying group to be $S\bar{W}_3$ and the mesons to be assigned to the $(3,3^*)$, $(3^*,3)$ representation of this group. We have already noted that this representation requires the spin-0 (1) mesons to be *s* (*p*) wave bound states and implies that both the scalar

[21] S. Okubo, Phys. Letters **5**, 165 (1963).

[22] G. Alexander, G. R. Kalbfleisch, D. H. Miller, and G. A. Smith, Phys. Rev. Letters **8**, 447 (1962); and M. Ferro-Luzzi, R. George, Y. Goldschmidt-Clermont, V. Henri, B. Jongejans *et al.*, Phys. Letters **12**, 255 (1964).

and axial vector octets will be normal (since the pseudoscalar and vector octets are taken as normal). If we choose $m_{K'}=725$ MeV (see above), then Eq. (29) would yield $m_{\pi'}=875$ MeV and the GMO formula (which only holds for Case 3, not for Case 4) would give $m_{\eta'}=680$ MeV.

4. BARYON MULTIPLETS

The discussion of spin-$\frac{1}{2}$ multiplets within the framework of our theory differs from that of the mesons on two counts: (i) the problem of constructing baryon multiplets from the basic fields ψ_i is more complicated; and (ii) the question of whether the existence of a $J^P=\frac{1}{2}^-$ multiplet is implied by the existence of a $J=\frac{1}{2}^+$ multiplet is more difficult to answer. We briefly discuss both points.

The simplest way to construct a spin-$\frac{1}{2}$ multiplet is to build up a multiplet trilinear in the ψ_i (such as $\psi\psi\psi$ or $\bar{\psi}\psi\psi$, say) such that, at the level of U_3, we obtain an octet of baryons. But this immediately restricts us to the choice $\psi\psi\psi$ since the product $\bar{\psi}\psi\psi\sim 3^*\otimes 3\otimes 3$ contains no octets in its reduction. Even the product $\psi\psi\psi\sim 3\otimes 3\otimes 3$ will lead to an octet representation with the desired hypercharge (and electric charge) contents only if we assign fractional values of hyper and electric quantum numbers to the basic fields ψ_i. This leads to the familiar "quark" scheme considered by several authors.[23] In addition, in order to obtain a baryon multiplet with unit baryon number, we must assign a baryon number of $\frac{1}{3}$ to each ψ field, this quantum number being disjoint from the U_3 (or SU_3) group and its quantum numbers that lead to the correct hyper and electric charges. If we assign a fractional baryon number to each ψ field, we are forced into an octet rather than a nonet representation for the baryons (if we wished to build up a baryon nonet, we could not assign any baryon number to the ψ's).

Turning away from an explicit construction of spin-$\frac{1}{2}$ baryons in terms of the ϕ_i, ξ_i but still retaining our original three-field model with $\bar{W}_3$ symmetry transformations defined on 2-component Weyl fields, let us next inquire into the parity doublet properties of the baryons. Suppose we assign the baryons to the representations (8,1), (1,8) of $\bar{W}_3$. We deal then with a tensor $f_a{}^b$ of type (8,1) transformed by the parity operation P into $f_{a'}{}^{b'}$ of type (1,8). The important question is whether these tensors f are 2-component Weyl fields or 4-component Dirac fields. If our three-field model is used as a guide, we should choose 2-component f's. If we do, we have an octet of baryons, with each baryon a Dirac 4-component object defined by

$$B_\nu{}^\mu=\begin{pmatrix} f_\nu{}^\mu \\ f_{\nu'}{}^{\mu'} \end{pmatrix}. \tag{37}$$

In (37), the parity operation interchanges, as usual, the upper two and lower two components of $B_\nu{}^\mu$. [Note that $f_\nu{}^\mu$ and $f_{\nu'}{}^{\mu'}$ transform contragrediently to one another under the proper homogeneous Lorentz group.] We have thus just *one* octet of baryons with J^P defined to be $\frac{1}{2}^+$ with the two chiral projections $\frac{1}{2}(1\pm\gamma_5)B_\nu{}^\mu$ of $B_\nu{}^\mu$ belonging to the two representations (8,1) and (1,8) of $\bar{W}_3$.

Our treatment of the baryons is similar to that of Gell-Mann[16] in some respects but differs in others. We agree in finding no parity degeneracy for baryons in the $\bar{W}_3$ (or $S\bar{W}_3$) limit and we also find vanishing masses for the baryons in the $\bar{W}_3$ (or $S\bar{W}_3$) limit.[24] We differ in that we are dealing with a baryon octet, whereas Gell-Mann ends up with a baryon nonet.[25] One disadvantage of the nonet representation for the baryons is that when $\mathcal{L}'$ is "switched on," the average mass of the unitary baryon octet is pushed above the zero mass $\bar{W}_3$ (or $S\bar{W}_3$) limit while the unitary singlet is pushed below. While it is possible, according to Freund and Nambu,[16] to reinterpret a negative mass $J=\frac{1}{2}^+$ baryon singlet as a positive mass $J=\frac{1}{2}^-$ particle, it is difficult to reconcile this inversion of parity and mass with the parity operation which we have used (see Sec. 2). This difficulty would not arise with the octet representation. There may be a further advantage in dealing with a baryon octet rather than a baryon nonet: the impossibility of writing down a $\bar{W}_3$ (or $S\bar{W}_3$)-invariant interaction between a baryon octet and a meson nonet (triality conservation). This implies the absence of a $\bar{W}_3$ (or $S\bar{W}_3$) invariant interaction between the baryon octet and a spin 0 meson nonet which could have the undesirable consequence of a very strong (unobserved) coupling of scalar mesons to baryons. [On the other hand, there could be a $\bar{W}_3$ (or $S\bar{W}_3$) invariant interaction between the baryon octet and the spin 1 meson octet.] It is therefore encouraging that our three-field model suggests a baryon octet without parity degeneracy.

If we choose to ignore the three-field model, as we have done on occasion (see Sec. 3), then it is possible to construct a parity doublet theory for the baryons. Thus, let us take an octet (8,1) of (4-component) Dirac fields $f_\nu{}^\mu$ which are transformed by P into $f_{\nu'}{}^{\mu'}$ as

$$P:\quad f_\nu{}^\mu(\mathbf{x},t)\rightarrow \gamma_0 f_{\nu'}{}^{\mu'}(-\mathbf{x},t).$$

The parity interchange aspect of $\bar{W}_3$ now suggests that a $\frac{1}{2}^-$ octet will accompany a $\frac{1}{2}^+$ octet; indeed, the two octets, denoted respectively by B' and B, are

$$\begin{aligned} B_\nu{}^\mu&=(1/\sqrt{2})(f_\nu{}^\mu+f_{\nu'}{}^{\mu'}),\\ B_\nu'{}^\mu&=(1/\sqrt{2})(f_\nu{}^\mu-f_{\nu'}{}^{\mu'}). \end{aligned} \tag{38}$$

[23] M. Gell-Mann and G. Zweig, Ref. 7.

[24] According to an argument of Y. Nambu [Phys. Rev. Letters 4, 380 (1960)], this is consistent with the finite masses for the mesons in the $\bar{W}_3$ (or $S\bar{W}_3$) limit which we have assumed (see Sec. 3).

[25] Gell-Mann (Ref. 16) considers but rejects the octet representation for the baryons on the ground that it leads to pure F-type coupling for the axial vector weak current in the $S\bar{W}_3$ limit; we do not consider this a strong argument because of the size of the $S\bar{W}_3$-symmetry breaking term.

With the definitions (38), we can expect that many of the mass relations for the mesons will find their counterparts for the baryons. As an illustration, if we assume, as in Cases 9 to 16 of Sec. 3, that $\mathcal{L}''\sim[(1,8)+(8,1)]$, then Eqs. (33) and (34) generalize into

$$m_N-m_{N'}=m_\Sigma-m_{\Sigma'}=m_\Xi-m_{\Xi'}, \tag{39}$$

where N', Σ', Ξ' denote the $\frac{1}{2}^-$ analogs of N, Σ, Ξ. Other baryon relations bearing a strong resemblance to the meson relations can easily be derived.

5. CONCLUDING REMARKS

In Secs. 2–4, we have explored the consequences of a very special type of symmetry group higher than U_3 (or SU_3), namely the product of two U_3 (or SU_3) groups with parity interchange. Initially (see Sec. 1), we were motivated in our study of this $\bar{W}_3$ (or $S\bar{W}_3$) group by a four-fermion vector model based on one triplet of "basic" fields. However, it should have been apparent from the treatment in Secs. 2–4 that while the four-fermion vector model served as a guide, the essential results did not depend on the details of the model, but followed from the group structure of $\bar{W}_3$ (or $S\bar{W}_3$) and the tensorial behavior of the V.S. and M.S. interactions which were presumed to break the $\bar{W}_3$ (or $S\bar{W}_3$) symmetry. Experimental tests of our mass predictions for scalar and axial vector mesons will be of crucial importance in deciding whether a group as large as $\bar{W}_3$ (or $S\bar{W}_3$) is a more useful ordering principle for the large number of observed particles (and resonances) than the present U_3 (or SU_3) theory. In view of the fact that the masses of the hadrons themselves [within the same U_3 (or SU_3) representation] are generally larger than the mass differences between the strange and nonstrange particles (this is always true for the baryons but not always true for the mesons), we cannot expect that our mass relations will hold as well as the GMO mass formula. Since there is so far such meager evidence for the existence of mesons of the "wrong parity" (i.e. scalar or axial vector mesons), the discovery of these objects with masses even near the predicted values would be a most promising development.

While confirmation of the mass predictions following from broken $\bar{W}_3$ (or $S\bar{W}_3$) symmetry would certainly not enable us to conclude that the three-field vector model is correct, it would increase interest in the Heisenberg-Nambu[26] type program for a triplet of "basic" fields. As was remarked in Sec. 1, a nonlinear three-field model offers the hope of providing a basis for a theory of leptons side by side with a theory of hadrons.[10,11] According to Heisenberg and Nambu, a nonlinear interaction among massless Dirac fields gives rise to a finite mass (as well as a zero mass) solution for fermions. From our present viewpoint, if we assign the finite mass to the baryons, then this common finite mass [i.e. the V.S. interaction $\mathcal{L}'$ in Eq. (8)] breaks the $\bar{W}_3$ (or $S\bar{W}_3$) symmetry for the baryons and reduces it to U_3 (or SU_3); the mass difference between the strange and nonstrange hadrons [i.e., the M.S. interaction in Eq. (10)] then breaks the U_3 (or SU_3) symmetry and reduces it to U_2 (or SU_2). The new version of the baryon-lepton symmetry[10] principle would then tell us to assign the common zero mass solution[11] to the lepton triplet (ν,e,μ) so that $\mathcal{L}'$ would vanish and the $\bar{W}_3$ (or $S\bar{W}_3$) symmetry would not be broken; it is only at the level of the U_3 (or SU_3) symmetry-breaking term, i.e., $\mathcal{L}''$, that the mass of the third component of the lepton triplet, namely μ, would be split away from the other two, namely ν and e, and given a finite value. This would imply that the muon mass has a nonelectromagnetic origin and is due to the M.S. interactions.[27] Since the electron presumably acquires a finite mass due to its electromagnetic interaction,[28] it would follow from the above line of argument that the 4-component neutrino ν (consisting of the positive chirality 2-component ν_e and the negative chirality 2-component $\bar{\nu}_\mu$) is a "manifestation" of one of the three "basic" fields with which we started.

These speculations concerning the leptons are suggestive but they run into one difficulty—as long as one holds on to a three-field model. We found in Sec. 4 that in order to accommodate the baryon octet within our theory, we were compelled to assume that the triplet of "basic" fields were objects with fractional electric charge, hypercharge and baryon number (i.e. "quarks"). While the failure thus far to observe baryon "quarks" does not constitute an argument against a theory of hadrons based on the three-field model (because the strong nonlinear interaction can, in principle, produce representations of particles with zero triality which are much lower in mass than those with nonzero triality), it does constitute an argument against the possibility of incorporating the leptons into such a three-field theory.

One way to incorporate the "baryon-lepton" symmetry principle within the framework of a nonlinear fermion model is to postulate a quartet of "basic" fields and to regard ν_e and ν_μ as themselves 4-component objects, a possibility noted by several authors.[29] This quartet of "basic" fields would also have the advantage

[26] Y. Nambu and G. Jona-Lasinio Phys. Rev. **122**, 345 and **124**, 246 (1960) as well as Heisenberg *et al.* (Ref. 4) have worked with one "basic" Dirac field, but the generalization of their arguments to three fields would be straightforward.

[27] One would then expect that an M.S. interaction of muons with (virtual) kaons—in contrast to electrons—should show up at momentum transfers of several BeV in muon-proton scattering. Thus far, no effect has been found up to 1-BeV momentum transfer [J. H. Tinlot (private communication)].

[28] See K. Johnson, M. Baker and R. Willey, Phys. Rev. Letters **11**, 518 (1963).

[29] Z. Maki, Progr. Theoret. Phys. (Kyoto) **31**, 331 (1964); and Y. Hara, Phys. Rev. **134**, B701 (1964).

that they would not have to be "quarks" to permit the construction of a theory of baryons. The great disadvantage, from our point of view, is that with $n=4$ (see Table I), we would be forced to start with an even higher symmetry group, namely $\bar{W}_4$ (or $S\bar{W}_4$) which when broken would lead to U_4 (or SU_4) rather than to the thus-far-successful U_3 (or SU_3) group. We consider it preferable to hold the "baryon-lepton" principle in abeyance at this stage and to seek experimental evidence for broken $\bar{W}_3$ (or $S\bar{W}_3$) among the hadrons.

Finally, we note that there is no very strong reason for restricting oneself to a four-fermion nonlinear interaction model. One might inquire whether postulating in addition, say, a six-fermion interaction would alter the chief conclusions. The answer is negative if one considers some simple forms of the six-fermion interaction. One can write down a six-fermion interaction among three "basic" fields[30] having the structure

$$\sum_1^3 \epsilon^{lmn}\epsilon^{ijk}(\psi_l C^{-1}Q_1\psi_i) \times(\psi_m C^{-1}Q_2\psi_j)(\psi_n C^{-1}Q_3\psi_k)+\text{H.c.} \quad (40)$$

which is invariant under SU_3, but not under U_3 [in (40), Q_1, Q_2, and Q_3 are some appropriate Dirac matrices]. A small admixture of such an interaction in addition to the main four-fermion interaction may serve to break both U_3 and $\bar{W}_3$ symmetry. However, it is premature to discuss this possibility further at this time.

[30] For a quartet of "basic" fields, one may write down a four-fermion interaction which is invariant under SU_4 but not under U_4, namely:

$$\sum_1^4 \epsilon^{ijkl}(\psi_i C^{-1}Q\psi_j)(\psi_k C^{-1}Q\psi_l).$$

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Some Speculations Concerning High-Energy Large Momentum Transfer Processes*

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It is speculated that the sharp decrease with increasing energy of differential cross sections at large angles is due to a mechanism independent of the method of excitation. Some consequences of such a possibility are discussed.

IT has been known for some time that (i) the total pp cross section remains essentially constant at high energies, and that (ii) above 300 MeV of excitation energy the nucleon has many excited states. More recently, experiments[1] have shown that the *large-angle* elastic pp cross section drops down spectacularly with energy. For example, when the center-of-mass momentum of each proton is 3.8 BeV/c, the differential cross section at 90° is only about 10^{-36} cm^2/sr.

These facts together suggest that the nucleon is an extended object with an internal structure having a "rigidity" characterized by an excitation energy of the order of a few hundred MeV. For hard collisions where the available energy is much larger than this, many degrees of freedom are excited in the nucleons, resulting in general in the emission of many particles.

Such a picture is more or less common to various statistical discussions[2] of high-energy collisions.

The spectacular drop mentioned above has been put in a more quantitative form by Orear.[3] His result is that (iii) the elastic pp differential cross section for large θ in the center-of-mass coordinate system is given by

$$(d\sigma/d\Omega)(\theta, pp\to pp)\sim Ae^{-p_\perp/0.15}, \quad (1)$$

where $p_\perp$ is the transverse momentum transfer in units of BeV/c.

Guided by these facts (i)–(iii), we attempt to speculate about the high-energy behavior of other processes. We observe that in picturing the nucleon as an extended object the difficulty in making large transverse mo-

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[1] G. Cocconi, V. T. Cocconi, A. D. Krisch, J. Orear, R. Rubinstein *et al.*, Phys. Rev. Letters **11**, 499 (1963) and W. F. Baker, E. W. Jenkins, A. L. Read, G. Cocconi, V. T. Cocconi *et al.*, *ibid.* **12**, 132 (1964).

[2] H. W. Lewis, J. R. Oppenheimer, and S. A. Wouthuysen, Phys. Rev. **73**, 127 (1948); E. Fermi, Progr. Theoret. Phys. (Kyoto) **5**, 570 (1950); G. Fast and R. Hagedorn, Nuovo Cimento **27**, 208 (1963); L. van Hove, Rev. Mod. Phys. **36**, 655 (1964); G. Cocconi, Nuovo Cimento **33**, 643 (1964); A. Bialas and V. F. Weisskopf, CERN (to be published); and many other papers. Notice that for very small angle elastic scattering, the many modes of excitation contribute in phase so that one has an enormous "diffraction peak." See R. Serber, Rev. Mod. Phys. **36**, 649 (1964) and earlier papers.

[3] J. Orear, Phys. Rev. Letters **12**, 112 (1964). See also A. D. Krisch, *ibid.* **11**, 217 (1963), and D. S. Narayan and K. V. L. Sarma, Phys. Letters **5**, 365 (1963).