As soon as the experimental data become available, a best fit of the coefficients a, b, c, d will lead to the evaluation of the ratios R, R_2 and ρ. It is in particular interesting to look at the forward-backward yields. From (29) indeed, one obtains, integrating separately in the forward and backward directions

$$\frac{\Gamma_b-\Gamma_f}{\Gamma_b+\Gamma_f}=\rho\times\frac{3}{2}\frac{J_3(1-2R_2)+2J_4}{2J_1(1-\frac{1}{2}R)+3J_2}=\rho X. \tag{32}$$

If, for instance, one makes the assumption that in the static limit f_1+f_2 and $f_1'+f_2'$ are nonzero and that the f's are of the order of unity (which seems not unreasonable), then the factor X in (32) is a positive slowly varying function of R and R_2, and $(\Gamma_b-\Gamma_f)/(\Gamma_b+\Gamma_f)$ depends linearly on ρ (which is substantially the ratio between the vector and the axial-vector coupling constant) thus allowing the determination of its sign and of its order of magnitude.

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Note added in proof: After the present paper was submitted, a work of J. Jellin about the electronic decay of the Ω^- has appeared in Phys. Rev. **135B**, 1203 (1964).

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Dirac Magnetic Poles Forbidden in S-Matrix Theory

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The residue of the one-photon exchange pole in the amplitude for the scattering of massive particles is calculated, using only generalized unitarity and the correspondence of particles to representations of the proper inhomogeneous Lorentz group. It is found that magnetic monopole coupling results in a residue which contains square-root singularities. Such a nonanalytic term is incompatible with the analyticity assumptions of S-matrix theory, and if it were present, the photon would appear in the annihilation channel as an intermediate state in all partial waves instead of only one. This behavior is theoretically implausible and discourages further experimental search for magnetic monopoles.

EVER since Dirac advanced the theory of magnetic monopoles to explain quantization of electric charge,[1] experimentalists have sought for monopole particles, but always with negative results.[2] We wish to show here that the existence of such particles contradicts our most elementary notions of the properties of scattering amplitudes, in particular their simplest analyticity properties. The argument relies upon (a) the identification of particles with irreducible representations of the connected Lorentz group (i.e., without parity or time reversal) and (b) the factorization of the photon pole in a scattering amplitude.

Consider the three-particle vertex at which a particle of momentum p_1 and mass $M\neq 0$ emits a photon of momentum k and is left with momentum p_2. The momentum four-vectors are defined on the complex manifold for which conservation and mass-shell conditions are satisfied identically, $k=p_1-p_2$, $p_1^2=p_2^2=M^2$, $k^2=0$. Then $P=p_1+p_2$ satisfies $P^2=4M^2$, $P\cdot k=0$. We suppose for simplicity that the massive particle has spin zero, although the argument becomes applicable to general spin by using the appropriate vertex function.[3]

It has been shown, using (a) only,[4] that a photon leg on an amplitude corresponds to antisymmetric tensor indices for which Maxwell's equations in a vacuum are satisfied. Since the massive particle has spin zero, the desired amplitude is just such a tensor $M_{\mu\nu}$ whose most general form[3] may be written

$$M_{\mu\nu}=\alpha(k_\mu P_\nu-P_\mu k_\nu)+\beta\epsilon_{\mu\nu\kappa\lambda}k^\kappa P^\lambda, \tag{1}$$

where α and β are complex constants representing, respectively, electric and magnetic monopole coupling. If the massive particle had nonzero spin, higher order multipole terms would also be present.

Any solution to Maxwell's equations may be written

$$M_{\mu\nu}=k_\mu J_\nu-k_\nu J_\mu, \tag{2}$$

where J is determined modulo k and satisfies $k\cdot J=0$. To find the J corresponding to Eq. (1), contract Eqs. (1) and (2) with an arbitrary four-vector a and equate the results, dropping terms proportional to k.

[1] P. A. M. Dirac, Proc. Roy. Soc. (London) **133**, 60 (1931).
[2] E. M. Purcell, G. B. Collins, T. Fujii, J. Hornbostel, and F. Turkot, Phys. Rev. **129**, 2326 (1963); E. Goto, H. H. Kolm, and K. W. Ford, Phys. Rev. **132**, 387 (1963); E. Amaldi *et al.*, N. C. **28**, 773 (1963); and E. Amaldi *et al.*, CERN Report 63–13, 1963 (unpublished). This latter work gives a general survey and bibliography of the subject.
[3] D. Zwanziger, *Proceedings of the Symposium on the Lorentz Group, June 1964* (University of Colorado Press, Boulder, Colorado, to be published).
[4] D. Zwanziger, Phys. Rev. **133**, B1036 (1964). S. Weinberg, *ibid.* **135**, B1049 (1964).

One finds

$$J_\mu = \alpha P_\mu - \beta \epsilon_{\mu\nu\kappa\lambda} k^\nu P^\kappa a^\lambda / a\cdot k. \tag{3}$$

Let us now consider the elastic scattering of two spinless particles of masses M and M'. The residue R of the photon exchange pole is well known to be

$$R = J'^\dagger \cdot J, \tag{4}$$

which may be obtained[4] using generalized unitarity in the channel where the photon is an intermediate state. The current $J^{\dagger\prime}$ of the second particle is obtained by transcription of Eq. (3)

$$J_\mu'^\dagger = \alpha'^* P'_\mu - \beta'^* \epsilon_{\mu\nu\kappa\lambda} k^\nu P'^\kappa a'^\lambda / a'\cdot k. \tag{5}$$

To calculate the residue it is convenient to choose a coordinate system, obtained by a complex Lorentz transformation if necessary, where $P=2M(1,0,0,0)$, $P'=2M'(\cosh\psi, \sinh\psi, 0,0)$, $k=(0,0,1,i)$ and to choose $a=a'=(0,0,0,1)$. Then $\epsilon_{\mu\nu\kappa\lambda}k^\nu P^\kappa a^\lambda/a\cdot k = 2iM(0,1,0,0)$ and $\epsilon_{\mu\nu\kappa\lambda}k^\nu P'^\kappa a'^\lambda/a'\cdot k = 2iM'(\sinh\psi, \cosh\psi, 0,0)$, so that Eq. (4) is

$$R = (\alpha'^*\alpha + \beta'^*\beta)4MM' \cosh\psi + i(\alpha'^*\beta - \beta'^*\alpha)4MM' \sinh\psi,$$

$$R = (\alpha'^*\alpha + \beta'^*\beta)P\cdot P' + i(\alpha'^*\beta - \beta'^*\alpha)[(P\cdot P')^2 - P^2P'^2]^{1/2}, \tag{6a}$$

or

$$R = (e_+'^* e_+ + e_-'^* e_-)P\cdot P' + (e_+'^* e_+ - e_-'^* e_-)[(P\cdot P')^2 - P^2P'^2]^{1/2}, \tag{6b}$$

where $e_\pm = (\alpha \pm i\beta)/\sqrt{2}$. By comparison with Eq. (1) we see that e_+ and e_- are the coupling strengths of the self-dual and anti-self-dual parts of the tensor $M_{\mu\nu}$ which are, within a phase factor, the couplings of the photon helicity eigenstates.

The crux of the present note is the observation that the second term on the right-hand side contains square-root singularities. This contradicts the basic analyticity property that location and nature of singularities are independent of spin, because the residue would be a constant if a spin-zero particle were exchanged. In general we expect the residue to be a polynomial whose degree equals the spin of the exchanged particle.

We consequently require that the coefficient of the nonanalytic term vanish so that we find for scattering of identical particles $\lambda = \beta/\alpha = \beta^*/\alpha^* = \lambda^*$ and in general $\lambda = \beta/\alpha = \beta'/\alpha' = \lambda'$. Thus α and β must be relatively real and their ratio λ is a universal constant, the same for all particles. In terms of the helicity couplings we have, for identical particles, $|e_+|^2 = |e_-|^2$, and in general $e_+'^*/e_-'^* = e_-'/e_+' = e_-/e_+$. So the coupling constants for the two helicity states are equal in strength and their ratio is a universal phase factor which is unobservable, since it may be absorbed into the definition of, say, the negative helicity eigenstate. The residue, Eq. (6), then takes the form

$$R = 2e_+'^* e_+ P\cdot P'. \tag{7}$$

By requiring that the amplitude be a Hermitian analytic function we find that the phase, modulo π, of e_+ is also universal and unobservable since it may likewise be absorbed in the definition of the positive helicity eigenstate. We consequently obtain a theory equivalent to the usual one where

$$J_\mu = eP_\mu, \quad e \text{ real}. \tag{8}$$

We conclude that the most general coupling scheme allowed by Lorentz invariance and analyticity is the usual one. In particular, magnetic monopoles, as an observable phenomenon, are excluded by these principles and the experimental search for them consequently seems a less pressing matter.

The argument presented here is of course subject to the uncertain fate of any demonstration in physics: nature may contradict the conclusion, thereby disproving at least one of the assumptions. In the present case, if magnetic poles were to be observed, the assumed analyticity properties would be violated, if not (a) or (b). Since many physicists are not accustomed to the invocation of analyticity properties as laws of nature, it is perhaps worthwhile emphasizing what would happen if the nonanalytic term of Eqs. (6) were in fact present. In the annihilation channel the photon would appear as an intermediate state in all angular momentum waves, instead of only one. In the elastic scattering channels, the photon pole lies in the forward direction, so that the residue must be real for energies above threshold (which means that e_+ and e_- must be real). But then the residue would no longer be real for forward scattering below threshold, in contradiction to what one expects from generalized unitarity. Lastly, if the photon had a small but finite mass, only the first term of Eq. (1) would be present (there is only one spin-0–spin-0–spin-1 vertex), so that the discovery of magnetic monopoles would be a qualitative proof that photons have zero mass.

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