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Leptonic Decay of Polarized Ω^- Particles*

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A previous calculation of the leptonic decay of the $\frac{3}{2}^+$ Ω^- particle assuming the most general interaction is extended to the case of a polarized Ω. The differential rate is given and the angular distribution of the Ξ is calculated for the decay $\Omega^- \to \Xi^0 + e + \bar{\nu}$. It is found to depend on three parameters which can be experimentally determined.

In a previous paper[1] (hereafter referred as I) the leptonic decay

$$\Omega^- \to \Xi^0 + l + \bar{\nu}, \quad l = \mu, e \tag{1}$$

of unpolarized Ω's has been calculated and some results of the calculation have been reported. It has furthermore been pointed out that more detailed information about the structure of the interaction can be obtained from the analysis of the decay of polarized Ω's.

It is the purpose of this paper to report the result of such a calculation.

The starting Hamiltonian is the same as in I

$$H' = \left[\bar{\Xi}\left(\frac{G_A'}{\sqrt{2}}O^{\nu\rho} + i\gamma_5 \frac{G_V'}{\sqrt{2}}Q^{\nu\rho}\right)\Omega_\nu\right][\bar{l}\gamma_\rho(1 - i\gamma_5)\nu], \tag{2}$$

with

$$O^{\nu\rho} = \left(f_1 + f_2 \frac{P^\lambda}{M}\frac{p_\lambda}{m}\right)g^{\nu\rho} + \left(f_3 \frac{P^\rho}{M} + f_4 \frac{p^\rho}{m} + f_5\gamma^\rho\right)\frac{p^\nu}{m},$$

$$Q^{\nu\rho} = \left(f_1' + f_2' \frac{P^\lambda}{M}\frac{p_\lambda}{m}\right)g^{\nu\rho} + \left(f_3' \frac{P^\rho}{M} + f_4' \frac{p^\rho}{m} + f_5'\gamma^\rho\right)\frac{p^\nu}{m}, \tag{3}$$

P, M and p, m being the four-momentum and the mass, respectively, of the Ω^- and of the Ξ particle; $f_i (i = 1$—$5)$ are form factors. Ξ, l, ν are usual $\frac{1}{2}$-spin operators, whereas Ω_ν is a $\frac{3}{2}$-spin operator.

Using the same technique and notation as in I, we adopt for the $\frac{3}{2}$-spinor the Rarita-Schwinger representation.[2] The orthonormal positive-energy spin state of polarization ξ is obtained[3,4] as a Clebsch-Gordan combination of the positive energy $\frac{1}{2}$-spinors $\Omega^\pm$ (spin parallel and antiparallel, respectively, to the quantization axis) with the polarization unit four-vectors

$$\begin{aligned} h_\nu^{(1)} &= (1/\sqrt{2})(1, i, 0, 0), \\ h_\nu^{(2)} &= (1/\sqrt{2})(1, -i, 0, 0), \\ h_\nu^{(3)} &= \quad (0,0,1,0). \end{aligned} \tag{4}$$

Thus, we have

$$\Omega_\nu^\xi = H_\nu^\xi \Omega^+ + F_\nu^\xi \Omega^- \tag{5}$$

with

$$\begin{aligned} &H_\nu^{(1)} = (1/\sqrt{2})(1, i, 0, 0), && F_\nu^{(1)} = 0, \\ &H_\nu^{(2)} = 0, && F_\nu^{(2)} = (1/\sqrt{2})(1, -i, 0, 0), \\ &H_\nu^{(3)} = (1/\sqrt{6})(1, -i, 0, 0), && F_\nu^{(3)} = (2/3)^{1/2}(0,0,1,0), \\ &H_\nu^{(4)} = (2/3)^{1/2}(0,0,1,0), && F_\nu^{(4)} = -(1/\sqrt{6})(1, i, 0, 0). \end{aligned} \tag{6}$$

To obtain transition probabilities, after squaring (2), we must put the products $\Omega_\nu^\xi \bar{\Omega}_\mu^\xi$ in a convenient form. With this aim we use the step operators (in the Ω operator lower indexes refer to energy and upper indexes refer to spin)

$$\begin{aligned} O_-\Omega_+^+ &= \Omega_+^- \\ O_+\Omega_+^- &= \Omega_+^+ \end{aligned} \tag{7}$$

which in our representation turn out to be

$$O^\mp = \frac{\gamma_5\gamma_0}{\pm n_1 + i n_2}(n_1\gamma_2 - n_2\gamma_1), \qquad \mathbf{n} = \mathbf{P}/|\mathbf{P}|. \tag{8}$$

Moreover, we will need spin projection operators

$$\begin{aligned} S_+\Omega_+ &= \Omega_+^+, \\ S_-\Omega_+ &= \Omega_+^-, \end{aligned} \tag{9}$$

which are given by

$$S_\pm = \tfrac{1}{2}(1 \pm i\gamma_5\gamma_0 \boldsymbol{\gamma}\cdot\mathbf{n}) \tag{10}$$

and the energy projection operator

$$\Lambda_+\Omega = \Omega_+, \tag{11}$$

given by

$$\Lambda_+ = (M + \gamma_\mu P_\mu)/2M. \tag{12}$$

Using (5), (7), (9), (11) we find

$$\Omega_\nu^\xi \bar{\Omega}_\mu^\xi = \sum_{i=1}^4 [(H_\nu^\xi H_\mu^{\xi*} + F_\nu^\xi H_\mu^{\xi*} O_-)S_+ + (F_\nu^\xi F_\mu^{\xi*} + H_\nu^\xi F_\mu^{\xi*} O_+)S_-]\Lambda_+ \Omega^i \bar{\Omega}^i. \tag{13}$$

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Now, with the help of (8), (10), (12), then going into the coordinate frame defined by $\mathbf{n}=(0,0,\mathbf{P}/|\mathbf{P}|)$ and finally going to the rest system of the Ω we obtain

$$\Omega_\nu{}^\xi\bar{\Omega}_\mu{}^\xi=\tfrac{1}{4}R_{\nu\mu}{}^\xi(1+\gamma_0)\textstyle\sum_{i=1}^4\Omega^i\bar{\Omega}^i \tag{14}$$

with

$$R_{\nu\mu}{}^\xi=L_{\nu\mu}{}^\xi-iM_{\nu\mu}{}^\xi\gamma_1\gamma_2+iN_{\nu\mu}{}^\xi\gamma_2\gamma_3+O_{\nu\mu}{}^\xi\gamma_3\gamma_1 \tag{15}$$

and

$$\begin{aligned}L_{\nu\mu}{}^\xi&=H_\nu{}^\xi H_\mu{}^{\xi*}+F_\nu{}^\xi F_\mu{}^{\xi*}, & M_{\nu\mu}{}^\xi&=F_\nu{}^\xi F_\mu{}^{\xi*}-H_\nu{}^\xi H_\mu{}^{\xi*},\\ N_{\nu\mu}{}^\xi&=H_\nu{}^\xi F_\mu{}^{\xi*}+F_\nu{}^\xi H_\mu{}^{\xi*}, & O_{\nu\mu}{}^\xi&=F_\nu{}^\xi H_\mu{}^{\xi*}-H_\nu{}^\xi F_\mu{}^{\xi*}.\end{aligned} \tag{16}$$

The expressions (14), (15), (16) solve completely the problem of expressing matrix elements involving arbitrarily polarized $\frac{3}{2}$-spin states in terms of matrix elements involving only normal $\frac{1}{2}$-spin states. Therefore, from this point on, the usual spin-$\frac{1}{2}$ projection and trace technique can be used, to calculate the matrix elements.

For a completely polarized Ω, (14) becomes

$$\Omega_\nu{}^1\bar{\Omega}_\mu{}^1=\tfrac{1}{4}R_{\nu\mu}{}^1(1+\gamma_0)\textstyle\sum_{i=1}^4\Omega^i\bar{\Omega}^i \tag{14'}$$

with $R_{\nu\mu}{}^1$ given by (15) and

$$\begin{gathered}L_{\nu\mu}{}^1=-M_{\nu\mu}{}^1=\tfrac{1}{2}(\delta_{\nu\mu}-\epsilon_{0\nu\mu3});\ \nu,\mu=1,2,\\ N_{\nu\mu}{}^1=O_{\nu\mu}{}^1=0.\end{gathered} \tag{17}$$

Introducing (14′) in the square matrix element, using the said techniques, and making the same approximation as in I (namely, neglecting terms of order p^2/m^2) we find

$$\frac{m\mu}{\omega w}|M_{fi}|^2=(2\pi)^5\Bigg\{K_1(w)\Bigg[F_0+\frac{2R}{m}F_1\Bigg]+K_1'(w)\Bigg[F_0+\frac{2R'}{m}F_1\Bigg]-\frac{K_2(w)}{m}\Bigg[p_3F_0-\frac{2R_2}{m}wF_2\Bigg]\Bigg\} \tag{18}$$

where

$$(2\pi)^5K_1(w)=\frac{m}{2w}G_A'^2\left(\frac{w}{m}+1\right)\left(f_1+f_2\frac{w}{m}\right)^2,$$

$$(2\pi)^5K_1'(w)=\frac{m}{2w}G_V'^2\left(\frac{w}{m}-1\right)\left(f_1'+f_2'\frac{w}{m}\right)^2, \tag{19}$$

$$(2\pi)^5K_2(w)=\frac{m}{2w}2G_A'G_V'\left(f_1+f_2\frac{w}{m}\right)\left(f_1'+f_2'\frac{w}{m}\right),$$

$$\begin{aligned}F_0&=2(\omega\omega'-k_3k_3')/\omega\omega',\\ F_1&=[\omega(\mathbf{k}'\cdot\mathbf{p}-k_3'p_3)+\omega'(\mathbf{k}\cdot\mathbf{p}-k_3p_3)]/\omega\omega',\\ F_2&=[k_3(\mathbf{k}'\cdot\mathbf{p}-k_3'p_3)+k_3'(\mathbf{k}\cdot\mathbf{p}-k_3p_3)]/\omega\omega',\end{aligned} \tag{20}$$

and

$$\begin{gathered}R=\frac{f_3+f_4(w/m)+f_5}{f_1+f_2(w/m)},\quad R'=\frac{f_3'+f_4'(w/m)+f_5'}{f_1'+f_2'(w/m)},\\ R_2=\frac{1}{2}\left(\frac{f_5}{f_1+f_2(w/m)}+\frac{f_5'}{f_1'+f_2'(w/m)}\right),\end{gathered} \tag{21}$$

m, w, $\mathbf{p}$ being the mass, energy, and momentum of the Ξ; μ, ω, $\mathbf{k}$ those of the lepton; and ω', $\mathbf{k}'$ the energy and momentum of the neutrino.

Now we note again that the ratio w/m is very close to 1 so that, in this approximation

$$\begin{gathered}(2\pi)^5K_1\simeq G_A'^2(f_1+f_2)^2\\ K_1'\simeq 0\\ (2\pi)^5K_2\simeq G_A'G_v'(f_1+f_2)(f_1'+f_2'),\end{gathered} \tag{22}$$

$$R=\frac{f_3+f_4+f_5}{f_1+f_2},\quad R_2=\frac{1}{2}\left(\frac{f_5}{f_1+f_2}+\frac{f_5'}{f_1'+f_2'}\right), \tag{23}$$

and (18) becomes

$$(\mu/\omega)|M_{fi}|^2\simeq(2\pi)^5K_1[(F_0+(2R/m)F_1)-\rho(p_3F_0-(2R_2/m)F_2)], \tag{24}$$

with

$$\rho=(K_2/K_1)=[G_V'(f_1'+f_2')]/[G_A'(f_1+f_2)]. \tag{25}$$

The decay rate is given by

$$\Gamma=\int d^3p\,k_1[I_0+(2R/m)I_1-\rho(p_3I_0-(2R_2/m)I_2)] \tag{26}$$

with

$$I_i=\int\delta^4(P-p-k-k')F_i\,d^3k\,d^3k'. \tag{27}$$

For the electronic decay

$$\Omega^-\to\Xi^0+e+\bar{\nu} \tag{28}$$

neglecting the electron mass and calling θ the angle between Ξ and the direction of polarization, the angular distribution is found to be of the form

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta}=a+b\cos\theta+c\cos^2\theta+d\cos^3\theta, \tag{29}$$

with

$$\begin{aligned}a&=\frac{3}{2}\frac{J_1(1-\frac{1}{2}R)+J_2}{2J_1(1-\frac{1}{2}R)+3J_2},\\ b&=-\frac{3}{2}\frac{2\rho[J_3(1-2R_2)+J_4]}{2J_1(1-\frac{1}{2}R)+3J_2},\\ c&=-\frac{3}{2}\frac{J_1(1-\frac{1}{2}R)}{2J_1(1-\frac{1}{2}R)+3J_2},\\ d&=\frac{3}{2}\frac{2\rho J_3(1-2R_2)}{2J_1(1-\frac{1}{2}R)+3J_2},\end{aligned} \tag{30}$$

where the J_i's are integrals which have been evaluated numerically and whose values are

$$\begin{aligned}J_1&=1.96, & J_2&=3.31,\\ J_3&=4.11\times10^{-1}, & J_4&=5.18\times10^{-1}.\end{aligned} \tag{31}$$

As soon as the experimental data become available, a best fit of the coefficients a, b, c, d will lead to the evaluation of the ratios R, R_2 and ρ. It is in particular interesting to look at the forward-backward yields. From (29) indeed, one obtains, integrating separately in the forward and backward directions

$$\frac{\Gamma_b-\Gamma_f}{\Gamma_b+\Gamma_f}=\rho\times\frac{3}{2}\frac{J_3(1-2R_2)+2J_4}{2J_1(1-\frac{1}{2}R)+3J_2}=\rho X. \quad (32)$$

If, for instance, one makes the assumption that in the static limit f_1+f_2 and $f_1'+f_2'$ are nonzero and that the f's are of the order of unity (which seems not unreasonable), then the factor X in (32) is a positive slowly varying function of R and R_2, and $(\Gamma_b-\Gamma_f)/(\Gamma_b+\Gamma_f)$ depends linearly on ρ (which is substantially the ratio between the vector and the axial-vector coupling constant) thus allowing the determination of its sign and of its order of magnitude.

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Note added in proof: After the present paper was submitted, a work of J. Jellin about the electronic decay of the Ω^- has appeared in Phys. Rev. **135B**, 1203 (1964).

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Dirac Magnetic Poles Forbidden in *S*-Matrix Theory

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The residue of the one-photon exchange pole in the amplitude for the scattering of massive particles is calculated, using only generalized unitarity and the correspondence of particles to representations of the proper inhomogeneous Lorentz group. It is found that magnetic monopole coupling results in a residue which contains square-root singularities. Such a nonanalytic term is incompatible with the analyticity assumptions of S-matrix theory, and if it were present, the photon would appear in the annihilation channel as an intermediate state in all partial waves instead of only one. This behavior is theoretically implausible and discourages further experimental search for magnetic monopoles.

EVER since Dirac advanced the theory of magnetic monopoles to explain quantization of electric charge,[1] experimentalists have sought for monopole particles, but always with negative results.[2] We wish to show here that the existence of such particles contradicts our most elementary notions of the properties of scattering amplitudes, in particular their simplest analyticity properties. The argument relies upon (a) the identification of particles with irreducible representations of the connected Lorentz group (i.e., without parity or time reversal) and (b) the factorization of the photon pole in a scattering amplitude.

Consider the three-particle vertex at which a particle of momentum p_1 and mass $M\neq 0$ emits a photon of momentum k and is left with momentum p_2. The momentum four-vectors are defined on the complex manifold for which conservation and mass-shell conditions are satisfied identically, $k=p_1-p_2$, $p_1^2=p_2^2=M^2$, $k^2=0$. Then $P=p_1+p_2$ satisfies $P^2=4M^2$, $P\cdot k=0$. We suppose for simplicity that the massive particle has spin zero, although the argument becomes applicable to general spin by using the appropriate vertex function.[3]

It has been shown, using (a) only,[4] that a photon leg on an amplitude corresponds to antisymmetric tensor indices for which Maxwell's equations in a vacuum are satisfied. Since the massive particle has spin zero, the desired amplitude is just such a tensor $M_{\mu\nu}$ whose most general form[3] may be written

$$M_{\mu\nu}=\alpha(k_\mu P_\nu-P_\mu k_\nu)+\beta\epsilon_{\mu\nu\kappa\lambda}k^\kappa P^\lambda, \quad (1)$$

where α and β are complex constants representing, respectively, electric and magnetic monopole coupling. If the massive particle had nonzero spin, higher order multipole terms would also be present.

Any solution to Maxwell's equations may be written

$$M_{\mu\nu}=k_\mu J_\nu-k_\nu J_\mu, \quad (2)$$

where J is determined modulo k and satisfies $k\cdot J=0$. To find the J corresponding to Eq. (1), contract Eqs. (1) and (2) with an arbitrary four-vector a and equate the results, dropping terms proportional to k.

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