Coulomb Disintegration of ^{6}Li†

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The disintegration of ^{6}Li projectiles into alpha particles and deuterons in the Coulomb field of a heavy target nucleus is studied theoretically by use of refined cluster model wave functions. Transitions to the assumed ^3D states in ^{6}Li, followed by alpha-deuteron breakup, as well as transitions to the alpha-deuteron continuum, are considered. A semiclassical, first-order time-dependent perturbation treatment is used. A very good agreement with measurements of the total breakup cross section with a gold target is obtained for bombarding energies below the Coulomb barrier. The long-range term in the relative-motion part of the cluster wave function is shown to be decisive for the magnitude of the breakup probability. No fitting of the nuclear model parameters is performed. The quadrupole transitions to the 3^+ state at 2.18 MeV are found to be dominating for sub-barrier energies. The theoretical value of the reduced matrix element for excitations to this state is calculated to be $B(E2;\ 1\rightarrow 3)=85\times10^{-62}e^2$ cm^4. This value is about 20 times the single-particle estimate, but is consistent with the alpha-deuteron disintegration experiments. This large number is interpreted as originating from the large diffuseness of the ^{6}Li nucleus.

I. INTRODUCTION

THE dissociation of ^{6}Li projectiles into alpha particles and deuterons by impact on several target nuclei has been observed in the last few years.[1,2] That this special mode of breakup is the dominating one can be roughly understood from a naive conception of the ^{6}Li nucleus as consisting of a deuteron loosely clinging to an alpha particle. In this picture the energy of binding of the deuteron to the alpha particle is remarkably small (1.47 MeV) and notably less than the internal binding energy of the deuteron. However, more profoundly considered and in view of the current strong interest in the structure of the ^{6}Li nucleus,[3–11] it is of great significance to get also a quantitative understanding of at least some of the mentioned measurements.

Whereas, for lighter target nuclei (carbon and nickel) the experiments seem to indicate a dissociation mechanism strongly influenced by nuclear interactions,[1] the experiments so far performed with heavy target material and bombarding energies of about 30 MeV support the notion of Coulomb excitation playing a major part.[2]

Total cross sections as well as differential cross sections for the breakup of ^{6}Li in the field of a gold nucleus were reported in Ref. 2. In a recent work[12] the total cross sections for this process were studied by application of the cluster model wave functions available at that time.[4] A simplified picture of the process was assumed. Transitions directly from the ^{6}Li ground state into a free alpha particle and a free deuteron in the continuum were considered. Furthermore, the relative motion of the fragments was approximated by a plane wave. The outcome of these computations show that the early version of the cluster model wave functions[4] as well as harmonic oscillator shell-model wave functions lead to total cross-section values much too small. In order to reproduce the observed magnitude of the breakup probability, an extreme long-range term was needed in the part of the cluster model wave function describing the relative motion of the alpha cluster and the deuteron cluster. This long-range term resulted in a too large root-mean-square radius of ^{6}Li. These results clearly indicate the necessity of further investigation, which is the aim of the present work. As in Ref. 12 we do not consider differential cross sections since their interpretation seems to be more involved than that of the total cross sections.[13,14]

The description of the relative motion of the fragments in the final state by plane waves, as made in Ref. 12, might be considered as a dubious assumption leading to the mentioned small value of the cross section. Still, insertion of a regular Coulomb wave function reduces the computed cross section even more.

The next obvious improvement would be the use of more realistic nuclear wave functions. Calculations show, however, that this alone is not sufficient to explain the observed cross sections.

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[13] E. M. Bernstein and E. Z. Skurnik, Phys. Rev. 121, 841 (1961).
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The way out of this dilemma is to re-examine the reaction mechanism. It turns out that the dominating reaction mode is Coulomb excitation to three quasi-bound states with isospin $T=0$ in ^{6}Li. These states are all situated above the threshold for alpha-deuteron breakup. Once excited, we can assume these states to disintegrate completely into free fragments. Besides this there are off-resonance contributions originating from transitions directly from the ground state to the continuum.

Refined cluster model wave functions for the ground state of ^{6}Li have been derived very recently by Schmid, Tang, and Wildermuth.[10] These wave functions are applied in the present work. In performing the calculations a few simplifying assumptions are introduced. These assumptions are shown to be of negligible significance for the results. Thus, the short-range two-body correlation factors in the projectile and the free-fragment wave functions are left out. Furthermore, the antisymmetrization between the clusters internal to ^{6}Li is partially omitted. Without any fitting of the nuclear model parameters, a very good agreement is obtained between the resulting theoretical cross section values and the existing measurements for bombarding energies below the Coulomb barrier. Cross sections calculated by insertion of simple harmonic oscillator shell-model wave functions are too small by an order of magnitude. These results are interpreted as convincingly showing the great usefulness of the refined cluster model wave functions for ^{6}Li.

It should be stressed that the long-range character of the Coulomb force is the reason why the cluster nature of ^{6}Li stands out so clearly in this special reaction.

In the following section (Sec. II), a broad outline is given of the method applied in the calculations and the various assumptions made concerning the reaction mechanism and the nuclear wave functions involved. Section III is devoted to details of the calculations and the numerical results. In the discussion in Sec. IV, we relate our results to a general point of view on nuclear models and nuclear reactions.

II. ASSUMPTIONS AND METHOD

A. Validity of a Semiclassical Treatment

It has been shown[15] that a necessary and sufficient condition for a classical description of the path of a charged projectile moving in the Coulomb field of a target nucleus, is that the inequality

$$\kappa=\frac{2Z_1Z_2e^2}{\hbar v}\gg 1 \qquad (1)$$

is well satisfied. In this inequality Z_1 and Z_2 are the charge numbers of the projectile and the target, respectively, and v the velocity of the projectile in the laboratory system. In fact, the classical description of the projectile path is exact only in the limit $\kappa\to\infty$.

We are concerned with ^{6}Li nuclei impinging on a gold target with energies below the Coulomb barrier. The height of the Coulomb barrier in this case is around 35 MeV. Hence, for such collisions we have $\kappa\gtrsim 31$. Furthermore, for the approximation to be valid the colliding nuclei should not penetrate into each other. For a bombarding energy of 25 MeV the classical distance of closest approach is about 14 F. This is to be compared with the distance between the mass centers when the nuclear surfaces just touch each other. This distance approximately equals 10 F. We are thus well outside the region where nuclear forces come into play. Consequently, these impacts may to a good approximation be described by classical hyperbolic orbits for the bombarding particle.

Let ΔE be the energy loss suffered by the projectile in the breakup process. One further condition for the applied classical description to be valid, is that the effect of the excitation or disintegration on the projectile motion can be neglected. Thus

$$\Delta E/E\ll 1, \qquad (2)$$

with E being the bombarding energy. From Eq. (2) is clearly seen a low-energy limitation on the present approach.

From the statements made above, it can be concluded that the semiclassical treatment of Coulomb excitation of nuclei can be adapted to the study of the ^{6}Li Coulomb disintegration process. This semiclassical method is broadly reviewed by Alder *et al.*[16]

It should be stressed that in this work we are only concerned with total breakup cross sections. Although the κ values for the collisions considered are finite, there is still reason to believe the semiclassical treatment to be adequate. For total Coulomb excitation cross sections, Alder and Winther have compared the results of a calculation based thoroughly on quantum mechanics with one using the semiclassical approach.[16,17] For $\kappa=30$ and for quadrupole transitions (which is the type of transition we are concerned with) the approximate calculation differs from the exact one by less than half of a percent (see for instance Fig. II.6 of Ref. 16 and also Ref. 13). This is in contrast to the differential cross sections. For these the corrections to the values from the semiclassical description of the Coulomb excitation process are shown to be larger.[14] The corrections in this case turn out to be of the order of several percent.

As usually done in Coulomb excitation studies, the unexcited particle is treated as a point charge. This pro-

[15] N. Bohr, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 18, No. 8 (1948).

[16] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. 28, 432 (1956).

[17] K. Alder and A. Winther, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 31, No. 1 (1956).

cedure is hard to avoid because of the complicated many-body problem represented by the charge distribution of the gold nucleus. No attempt has been made to correct for this (see also Sec. II.B).

B. Interaction Potential

In order to apply first-order time-dependent perturbation theory to the Coulomb disintegration process[12,16] an expression is needed for the Coulomb interaction. Assuming a spherical symmetric charge distribution in the target nucleus the Coulomb interaction operator can be written as

$$H_{\rm int}=Z_2e^2\sum_{l=1}^{6}\left(\frac{1}{|\mathbf{r}_l-\mathbf{r}_p(t)|}-\frac{1}{|\mathbf{r}_p(t)|}\right)\left(\frac{1+\tau_3{}^{(l)}}{2}\right). \quad (3)$$

The coordinate system is chosen such that $\mathbf{r}_l$ represent the individual nucleon distances from the projectile center-of-mass. Further $\mathbf{r}_p(t)$ denotes the time-dependent position vector of the projectile center-of-mass with respect to the target nucleus. The operators $\tau_3{}^{(l)}$ are the third components of the single nucleon isospin operators. In Eq. (3), the Coulomb interaction between the mass centers is subtracted, since this is accounted for by the hyperbolic path.

The operator $H_{\rm int}$ is symmetric in all the single nucleon coordinates. Hence, if $\varphi=A\psi_p$ is the wave function of the projectile, where ψ_p is not necessarily antisymmetric, the occurring projectile-target interaction integral can be written as

$$g_{p,t}=\int(A\psi_p)^*H_{\rm int}(A\psi_p)d\tau=6!\int(A\psi_p)^*H_{\rm int}\psi_p d\tau, \quad (4)$$

with the operator A denoting complete antisymmetrization and the nuclear wave function assumed normalized to unity.

A first nuclear model assumption is now introduced. We assume an alpha-deuteron cluster description of the ^{6}Li projectile,[4]

$$\psi_p=\varphi_\alpha(1234)\varphi_d(56)\chi_{\rm rel}\xi_{\sigma,\tau}(1234;56). \quad (5)$$

Here φ_α represents the alpha cluster, φ_d the deuteron cluster and $\chi_{\rm rel}$ the relative motion function of the clusters. The symbol $\xi_{\sigma,\tau}(1234;56)$ is a spin-isospin function chosen to give total spin $S=1$ and total isospin $T=0$.

On evaluating the quantity $H_{\rm int}\psi_p$ we make the further assumption that the charges of the clusters are concentrated in their respective mass centers. We then obtain

$$\left.\begin{aligned}&H_{\rm int}\psi_p=H_{\rm int}'\psi_p,\\ &\text{where}\\ &H_{\rm int}'(t)=Z_2e^2\left(\frac{Z^{(1)}}{|\mathbf{r}_1{}^{(cl)}|}+\frac{Z^{(2)}}{|\mathbf{r}_2{}^{(cl)}|}\right)-\frac{Z_1Z_2e^2}{|\mathbf{r}_p(t)|}\end{aligned}\right\}. \quad (6)$$

In Eq. (6), $\mathbf{r}_1{}^{(cl)}$ and $\mathbf{r}_2{}^{(cl)}$ are the distances from the target nucleus to the center-of-mass of each of the clusters. The symbols $Z^{(1)}$ and $Z^{(2)}$ stand for the charge numbers of the clusters, in our case the ones of the alpha cluster and deuteron cluster. The expression for $H_{\rm int}'(t)$ in (6) is identical to the one used in Ref. 12. In the further evaluation of $H_{\rm int}'(t)$, use is made of a multipole expansion. Only the leading term is considered.

$$H_{\rm int}'(t)\approx 4\pi Z_2e^2\sum_{\lambda,\mu}\frac{q_\lambda}{2\lambda+1}\frac{Y_{\lambda\mu}(\hat{r}_p(t))}{[r_p(t)]^{\lambda+1}}R^\lambda Y_{\lambda\mu}(\hat{R}). \quad (7)$$

In Eq. (7), λ is the multipole order. The quantity q_λ is a geometrical factor related to the cluster distances from the projectile center of mass (see also the Appendix). By $\mathbf{R}$ is meant the relative distance between the clusters.

It may be argued that the assumption of the clusters as point charges introduces extra uncertainties in the calculations. As a remedy for this, one could think of expanding the effective operator in terms of the actual proton distances from the center-of-mass of the alpha cluster and the deuteron cluster, respectively, thus obtaining correction terms.[18]

This might work well for the tightly packed alpha cluster, but not necessarily equally well for the more extended deuteron cluster. Also, even if an estimate of this kind were made, the effect of the extended target nucleus would still remain, its influence being probably of the same order of magnitude as the former effect. As long as no deformation of the gold target nucleus is taken into account, no quantitative estimate can be made of the errors thus introduced. However, the gold target nucleus is a big nucleus consisting of 79 protons and 118 neutrons. One is thus very near the closed proton shell at the magic number 82 and not far away from 126 for the neutrons. Hence, from a shell-model point of view one is inclined to consider the gold nucleus as being nearly a sphere and the point-charge assumption correspondingly good. Consequently, the corrections arising from the extended charge distribution probably are very small. We have reason to believe these corrections in total not to be bigger than 10% in the final result.

C. General Formulas

As outlined in Ref. 12 the center-of-mass of the unpolarized projectile is pictured as moving along its classical orbit in the repulsive Coulomb field of the point-charge target nucleus. In passing the target, the projectile feels a time-varying electromagnetic field. This field can cause transitions or complete disintegration into free fragments of the projectile. For particle

[18] This effect can also be looked at in the following way: The rigorous expression for $H'_{\rm int}$ contains terms like the one of Eq. (7) with the relative coordinate replaced by the internal coordinates, respectively. Contributions from these additional terms are present if excited states of the clusters are considered as intermediate states. For the process under consideration, therefore, they appear only as higher order terms.

velocities small compared with that of light, the main interaction is of Coulomb character.

By use of first-order time-dependent perturbation theory, one can derive a formula for the total Coulomb excitation cross section of the two-cluster system. For details of the evaluation see the Appendix and Ref. 12.

The total cross section for electric excitations of multipole order λ from an initial state i to a final state f is given in this picture by

$$\sigma_C^{(\lambda)}=\left(\frac{Z_2e}{\hbar v}\right)^2 d^{-2\lambda+2} f_{E\lambda}(\xi)\frac{e^2 q_\lambda^2}{4\pi} \times(2I_f+1)\langle l_i\lambda 00|l_f0\rangle^2 \vartheta_{\text{int,rel}}. \quad (8)$$

In this formula the orbital and nuclear parts of the cross section are separated. Only the very last term depends on nuclear properties. The notations are the following ones with $d=(Z_1Z_2e^2)/(M_0v^2)$ denoting half the distance of closest approach in a head-on collision where M_0 is the reduced mass of the projectile-target system. The functions $f_{E\lambda}(\xi)$ are orbital functions defined and tabulated by Alder and Winter[16,17] with $\xi=(d\Delta E)/(\hbar v)$. By I_f is meant the total angular momentum of the two-cluster system in the final state. The symbols l_i and l_f, both contained in a Wigner coefficient, stand for the initial and final-state relative angular momenta of the clusters. The remaining term $\vartheta_{\text{int,rel}}$ in the cross-section formula depends solely on the radial parts of the nuclear wave functions. This term is closely related to the interaction integral given in Eq. (4) and is defined by

$$\vartheta_{\text{int,rel}}=\left|\int(\psi_{\text{int}}^{(f)}\chi_{\text{rel}}^{(f)}(R))^* \times R^\lambda\psi_{\text{int}}^{(i)}\chi_{\text{rel}}^{(i)}(R)d\tau_{\text{int},R}\right|^2. \quad (9)$$

In Eq. (9), ψ_{int} denotes the internal cluster parts in the wave functions and χ_{rel} the relative motion part. The nuclear wave functions are assumed normalized to 1.

From the formula (8) and a comparison with the standard definitions, we get, using conventional notation, the following expression for the reduced transition probabilities for excitations[16,19]

$$B(E\lambda; I_i\rightarrow I_f)=e^2\frac{q^2{}_\lambda}{4\pi}(2I_f+1)\langle l_i\lambda 00|l_f0\rangle^2\vartheta_{\text{int,rel}}. \quad (10)$$

Obviously, cf. also Ref. 16,

$$\sigma_C^{(\lambda)}=\left(\frac{Z_2e}{\hbar v}\right)^2 d^{-2\lambda+2} f_{E\lambda}(\xi)B(E\lambda; I_i\rightarrow I_f). \quad (11)$$

[19] A. K. Kerman, *Nuclear Reactions*, edited by P. M. Endt and M. Demeur (North-Holland Publishing Company, Amsterdam, 1959), Vol. I, Chap. X.

Within the frame of our model we consider ratios between reduced transition probabilities. We limit ourselves to transitions of the same multipole order involving identical initial states. Let the final states considered be characterized by equal relative angular momenta between the clusters, but with different resulting total spins, I_f and I_f'. Under the assumption of equal radial dependence of the nuclear wave functions, we get

$$\frac{B(E\lambda; I_i\rightarrow I_f')}{B(E\lambda; I_i\rightarrow I_f)}=\frac{2I_f'+1}{2I_f+1}. \quad (12)$$

For calculations of total cross sections corresponding to transitions from the two-cluster ground state to free fragments in the continuum, expression (8) has to be modified. The density of final states must be brought in. This is given by $Vk^2dk/(2\pi)^3$, where V is the normalization volume and k the relative momentum of the clusters. An integration over all relative momenta has to be performed, and further a summation is necessary over all possible values of the unobserved total spin I_f in the final continuum state. In the general case the total cross section for transitions to states in the continuum is given by

$$\sigma_{C,\,\text{cont}}^{(\lambda)}=\frac{1}{2}\frac{Z_2^2e^4d^{-2\lambda+2}}{(2\pi)^4\hbar^2v^2}q_\lambda^2\sum_{I_f}(2I_f+1) \times\langle l_i\lambda 00|l_f0\rangle^2\int dkk^2 f_{E\lambda}(\xi)\vartheta_{\text{int,rel}}'. \quad (13)$$

Here the prime indicates that the normalization volume is contained in $\vartheta_{\text{int,rel}}'$. Thus, in close analogy with Eq. (9)

$$\vartheta_{\text{int,rel}}'=\left|\int(\psi_{\text{int}}^{(f)}\phi_{\text{rel}}^{(f)}(R))^* \times R^\lambda\psi_{\text{int}}^{(i)}\chi_{\text{rel}}^{(i)}(R)d\tau_{\text{int},R}\right|^2. \quad (14)$$

The only difference from Eq. (9) is the final-state relative motion function $\phi_{\text{rel}}^{(f)}$. It is taken to be a Coulomb wave function which is so normalized that it has the asymptotic amplitude 1. In the Appendix we give explicitly the cross section for quadrupole transitions to the continuum with the relative motion function approximated by a plane wave.

D. Reaction Mechanism

We want to go into details concerning the reaction mechanism. It turns out that the quadrupole Coulomb excitations to three excited states in the ^{6}Li projectile are the most important. This has already been assumed by Gluckstern and Breit in a preliminary study of this

breakup process.[20] The states mentioned can be considered as members of an $L=2$ triplet[4,21,22] and are all situated above the threshold for alpha-deuteron breakup. They have the energies 2.18, 4.52, and 5.5 MeV and the J^π assignments 3^+, 2^+, 1^+, respectively. They are all states with isospin $T=0$, cf. Ref. 23. The lifetimes for electromagnetic de-excitation of these states are much longer than the lifetimes against particle emission. Hence, these states will disintegrate almost completely by alpha-deuteron breakup.

The correct procedure would be to use scattering states as final states for the alpha-deuteron system. Here the effects of all the resonant states are included. However, the resonances are narrow. It has been shown experimentally[24] that the off-resonance phase shifts are to a good approximation 0° or 180°. This implies that except for the small region around the resonances, the continuum wave function behaves asymptotically like a regular Coulomb wave function. Consequently one can separate the resonance and off-resonance contributions. The contributions to the integral in Eq. (13) from the resonance regions calculated by use of regular Coulomb wave functions are negligible. This implies that the off-resonance part can be calculated by integration over all relative energies of the alpha-deuteron system, see Ref. 12. The resonance part to be added can be calculated by using bound-state wave functions which can be considered as a superposition of scattering states.

In summing up there are two possible ways to follow, either,

(i) using the scattering states as the description of the final state of the alpha-deuteron system;

(ii) using regular wave functions for the alpha-deuteron system and adding the resonant states.

We have followed the second procedure for the obvious reason that it does not involve the solution of the alpha-deuteron scattering problem.

The widths of the mentioned 2^+ and 1^+ levels are not as narrow as demanded by the chosen procedure. However, contributions to the total cross section from transitions to these levels are small anyway, cf. Sec. III.

E. Refined Cluster Wave Functions and Free Fragment Representation

The early version of the cluster-model wave functions for ^{6}Li is given in Ref. 4. These wave functions were of the general symbolic form

$$\Psi=A\{\varphi_\alpha\varphi_d\chi_{\rm rel}\xi_{\sigma,\tau}(1234;56)\}, \tag{15}$$

the notation in Eq. (15) being identical with the one used in Eq. (5) of Subsec. IIB. The alpha cluster and the deuteron cluster are in this case described by single Gaussians. The relative oscillation function was given the form

$$\chi_{\rm rel}(\mathbf{R})=R^2\exp(-\tfrac{2}{3}\beta R^2)Y_{LM}(\hat{R}), \tag{16}$$

with $\mathbf{R}$ representing the distance of separation of the mass centers of the two clusters. The form of these early cluster wave functions is such that for special values of the parameters they reduce to ordinary harmonic oscillator shell-model wave functions.[4,25] The values of the parameters occurring that give a minimum value of the total energy were determined from the Ritz variational principle in combination with a simple two-body nucleon force of Serber character.

The cluster model wave functions thus determined can only be considered as a first attempt in the right direction. These wave functions could account rather well for energy differences between observed levels in the ^{6}Li nucleus. However, the root-mean-square radius and the total binding energy of the ^{6}Li nucleus were not well reproduced.

Schmid, Tang, and Wildermuth[10] have derived refined cluster-model wave functions for ^{6}Li. Also these wave functions were the result of a variational calculation, but now with application of a central two-body interaction with hard core. The trial function was given a more complicated form than in the earlier calculations. It was chosen

$$\Psi=A\{\varphi_\alpha\varphi_d\chi_{\rm rel}(\mathbf{R}) \times \prod_{m=1,\cdots 4;\ n=5,6} f_1(r_{mn})\xi_{J,\tau}(1234;56)\}. \tag{17}$$

The function f_1 is a cutoff function which is equal to zero within the hard-core region. The symbols r_{mn} are internucleon distances. The alpha-cluster function φ_α is taken as a sum of two Gaussians times a cutoff function similar to f_1.

$$\varphi_\alpha=\{\exp[-\tfrac{1}{2}a_1(u^2+v^2+w^2)] +a_2\exp[-\tfrac{1}{2}a_3(u^2+v^2+w^2)]\}\prod_{i>k=1}^{4} f_2(r_{ik}). \tag{18}$$

The deuteron cluster part is more complicated. In the interior region, outside the hard core, it is generated directly from the Schrödinger equation. In the outside region it is given as a sum of two exponentials divided by the internucleon distance r within this cluster, i.e.,

[20] R. L. Gluckstern and G. Breit, in *Reactions Between Complex Nuclei*, edited by A. Zucker, F. T. Howard, and E. C. Halbert (John Wiley & Sons, Inc., New York, 1960), p. 77.

[21] J. P. Elliot and A. M. Lane, in *Handbuch der Physik*, edited by S. Flügge (Springer Verlag, Berlin, 1957), Vol. 39, p. 338.

[22] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 22 (1959).

[23] F. Ajzenberg-Selove, C. F. Osgood, and C. P. Baker, Phys. Rev. **116**, 1521 (1959).

[24] A. Galonsky and M. T. McEllistrem, Phys. Rev. **98**, 590 (1955).

[25] Y. C. Tang, K. Wildermuth, and L. D. Pearlstein, Nucl. Phys. **32**, 504 (1962).

a Hulthén like form.[26]

$$\varphi_d=\frac{1}{r}\{\exp(-b_1r)-b_2\exp(-b_3r)\}. \quad (19)$$

The coordinates applied in Eqs. (18) and (19) are related to the nucleon coordinates $\mathbf{r}_1, \cdots, \mathbf{r}_6$ and the relative distance between the mass centers of the clusters in the following way:

$$\begin{aligned}
\mathbf{r}_1&=\tfrac{1}{2}(-\mathbf{u}-\mathbf{v}-\mathbf{w})+\tfrac{1}{3}\mathbf{R},\\
\mathbf{r}_2&=\tfrac{1}{2}(\mathbf{u}-\mathbf{v}+\mathbf{w})+\tfrac{1}{3}\mathbf{R},\\
\mathbf{r}_3&=\tfrac{1}{2}(\mathbf{u}+\mathbf{v}-\mathbf{w})+\tfrac{1}{3}\mathbf{R},\\
\mathbf{r}_4&=\tfrac{1}{2}(-\mathbf{u}+\mathbf{v}+\mathbf{w})+\tfrac{1}{3}\mathbf{R},\\
\mathbf{r}_5&=\tfrac{1}{2}\mathbf{r}_{56}-\tfrac{2}{3}\mathbf{R},\\
\mathbf{r}_6&=-\tfrac{1}{2}\mathbf{r}_{56}-\tfrac{2}{3}\mathbf{R},\\
\mathbf{r}&\equiv\mathbf{r}_{56}=\mathbf{r}_5-\mathbf{r}_6.
\end{aligned} \quad (20)$$

Of special interest for the present work is the part of the wave function describing the relative motion of the clusters:

$$\chi_{\mathrm{rel}}(\mathbf{R})=R^2\{\exp(-c_1R^2)+c_2\exp(-c_3R^2)\}Y_{LM}(\hat{R}), \quad (21)$$

with $L=0$ for the ground state of ^{6}Li.

A minimum value for the expectation value of the six-body Hamiltonian operator was found for the following set of parameters:

$$\begin{aligned}
a_1&=1.17\ \mathrm{F}^{-2}, \quad a_2=0.25, \quad a_3=0.52\ \mathrm{F}^{-2},\\
b_1&=0.34\ \mathrm{F}^{-1}, \quad b_2=1.31, \quad b_3=1.34\ \mathrm{F}^{-1},\\
c_1&=0.18\ \mathrm{F}^{-2}, \quad c_2=0.25, \quad c_3=0.065\ \mathrm{F}^{-2}.
\end{aligned} \quad (22)$$

The small value of c_3 implies a long-range relative motion part. It should be noted that the refined cluster wave functions do not have the property of being reduced to harmonic oscillator shell-model wave functions for special values of the parameters. With this wave function the root-mean-square radius of the charge distribution is found to be 2.73 ± 0.15 F. (In obtaining this value, a root-mean-square radius of 0.72 F for the proton is included.) The root-mean-square radius found for ^{6}Li compares favorably with the value extracted from electron scattering data[27] which is 2.72 ± 0.15 F. Also a reasonable charge distribution and a good value for the total binding energy are obtained from the refined cluster wave functions.

The satisfactory value of the root-mean-square radius obtained in this case should be considered in relation to the present investigation. The integrals involving nuclear wave functions in the expression for the total Coulomb disintegration cross section [see Eqs. (8) and (9)] are very similar to the ones appearing in the evaluation of the root-mean-square radius.[4] This indicates that the refined cluster wave functions have the right behavior in the radial region involved in the calculation of the Coulomb disintegration cross section. The refined cluster wave functions do not have the correct asymptotic behavior required for an alpha-deuteron system. This, however, is not serious in the present context, because the asymptotic region does not contribute to the integrals involved. Stating this in a different manner, at least in the contributing region the tail of the ^{6}Li wave function seems to be well simulated by the chosen expression for the relative motion function. In the Coulomb disintegration case the main contributing region can be shown to be situated at radial distances around 6 F.

There is, in principle, no reason to assume the same representation for the free deuteron resulting from the Coulomb breakup of ^{6}Li as for the bound deuteron cluster. This simplifying assumption was made in Ref. 12. As an improvement we have chosen an Hulthén wave function[26] for the free deuteron,

$$\varphi_d^{(H)}=\exp(-\alpha r)(1-\exp(-\mu r))/r \quad (23)$$

with

$$\alpha=0.23\ \mathrm{F}^{-1} \quad \text{and} \quad \mu=0.72\ \mathrm{F}^{-1}.$$

The outcome of the calculations shows, however, that the form of the final-state deuteron function and also that of the deuteron cluster is of minor importance for the magnitude of the theoretical Coulomb disintegration cross section. The free alpha-particle wave function is taken to be identical with the one for the alpha cluster. Since both these four nucleon aggregates are tightly bound, this is probably a reasonable assumption. For further details, see Sec. III below.

F. Choice of Intermediate State Wave Functions

The refined cluster model wave functions for the ground state of ^{6}Li are given. With the assumed reaction mechanism the wave functions for the $L=2$ triplet states are needed.

From a cluster model point of view the nucleus ^{6}Li is pictured as a deuteron attached to an alpha particle, the relative motion being in an S state. For the excited states we have used the same wave function as for the ground state, except for the angular part of the relative motion function. This is assumed to be a D state. The intrinsic spins of the clusters couple to the orbital angular momentum, yielding the total angular momentum values $I=3$, 2, and 1. This idea is supported by the empirical level scheme[22] as well as supermultiplet considerations using the spectrum of ^{6}He (see Ref. 28 and Chap. VI of Ref. 29).

[26] L. Hulthén and M. Sugawara, in *Handbuch der Physik*, edited by S. Flügge (Springer Verlag, Berlin, 1957), Vol. 39, p. 33.
[27] U. Meyer-Berkhout, K. W. Ford, and A. E. S. Green, Ann. Phys. (N. Y.) 8, 119 (1959).
[28] E. P. Wigner, Phys. Rev. **51**, 106 (1937).
[29] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952).

The assumption of more or less unchanged internal cluster wave functions and unchanged radial part of the relative motion function necessitates some justification. For this purpose we consider the Coulomb energy differences of the $T=1$ levels in ^{6}He and ^{6}Li (and in ^{6}Be). The excited level of ^{6}He at 1.71 MeV is known to be a 2^+ level.[22] There are $T=1$ levels in ^{6}Li at 3.56 and 5.35 MeV. These levels correspond to the ground state and the 2^+ state in ^{6}He. The change in the Coulomb energy differences between corresponding levels in ^{6}He and ^{6}Li is therefore 0.08 MeV, which is quite negligible and an indication of equal cluster structure for the levels involved.[30,31] One might apply this result also for the $T=0$ levels in ^{6}Li. The supermultiplet theory may be utilized to support this view. The ground state of ^{6}He and ^{6}Be, the 3.56-MeV level and the ground state of ^{6}Li form a supermultiplet. Another supermultiplet is formed by the 1.71 MeV level in ^{6}He, the corresponding level in ^{6}Be, together with the mentioned D triplet and the 5.35 MeV, $T=1$ level in ^{6}Li. According to the supermultiplet theory, the spin and isospin parts of the wave functions change within a supermultiplet, whereas the space parts remain essentially unaltered. Consequently, for the $T=0$ levels in ^{6}Li neither the cluster structure nor the radial parts of the wave functions are subject to any appreciable change.[32] One might doubt that this reasoning is equally good for the somewhat broader 2^+ and 1^+ states in the triplet. However, in the present context this is not important. The reason for this is the strong depressive effect of the orbital Alder-Winther function on the Coulomb excitation cross section. This effect increases with increasing values of the excitation energy, cf. Subsec. IIC and Ref. 13. It is shown later that contributions to the total cross section from transitions to the two higher states in the triplet are small. It should be noted that the wave functions chosen yield maximum overlap. A small change in the wave functions of the excited states results only in a slight reduction of the matrix element. Hence, some uncertainty in the ascribed radial wave functions for these states is only of minor importance for the calculations of the total cross section, see also Subsec. IIIA.

G. Possibility of Other Intermediate States

In Subsecs. IID and IIF we have considered transitions only to excited states with $T=0$.

The Coulomb forces do not commute with the total isospin operator.[33] Thus transitions through other intermediate states can, in principle, occur. However, the $T=1$ state at 3.56 MeV in ^{6}Li is ruled out by angular momentum considerations. Contributions from higher lying states are reduced for several reasons. As already mentioned the Alder-Winther functions $f_{E2}(\xi)$ reduce the cross sections for higher excitation energies. Further, the total wave function being antisymmetric, changes in the isospin part must be accompanied by changes in the space or spin part of the wave function. Therefore, the corresponding matrix elements will be very small. Also, isospin impurities are rather small, less than 0.25%, as estimated in Ref. 33.

In conclusion, the contributions from intermediate states with other isospin values than $T=0$ are negligible for the process under consideration.

III. CALCULATIONS

A. Matrix Element

With the aid of formula (8) and (13) and the chosen wave functions (cf. subsections IIC, IIE, and IIF), we are able to write down the complete expressions for the quantities $g_{\text{int,rel}}$ and $g_{\text{int,rel}}'$.

We introduce the following notations, cf. Ref. 12,.

$$B_m=\frac{\int(\psi_{\text{int}}^{(f)}\chi_{\text{rel}}^{(f)})_m^*R^\lambda Y_{\lambda\mu}(\hat{R})(\psi_{\text{int}}^{(i)}\chi_{\text{rel}}^{(i)})_0 d\tau_{\text{int, rel}}}{\int Y_{l_fm_f}^*(\hat{R})Y_{\lambda u}(\hat{R})Y_{l_im_i}(\hat{R})d\Omega_R}, \tag{24}$$

$$A_m^{(k)}=\int(\psi_{\text{int}}^{(k)}\chi_{\text{rel}}^{(k)})_m^*(\psi_{\text{int}}^{(k)}\chi_{\text{rel}}^{(k)})_0 d\tau_{\text{int, rel}}, \quad k=i \text{ or } f, \quad m=0, 1 \text{ or } 2. \tag{25}$$

The index numbers m indicate the number of permutations. For the transitions to the discrete states we then have

$$g_{\text{int, rel}}=\frac{|B_0-2B_1+B_2|^2}{(A_0^{(i)}-2A_1^{(i)}+A_2^{(i)})(A_0^{(f)}-2A_1^{(f)}+A_2^{(f)})} \tag{26}$$

where the various terms result from the antisymmetrization procedure.[12,34] The factors in the denominator are

[30] K. Wildermuth and Y. C. Tang, Phys. Rev. Letters **6**, 17 (1961).

[31] K. Wildermuth, University of Maryland, Physics Department Technical Report No. 281, 1962 (unpublished).

[32] Further confirmation of this idea comes from the ^{6}Li(p,p') reaction recently studied by G. Tibell *et al.* They find angular distributions which are very similar for the members of the $L=2$ triplet, but essentially different for other states. We are very grateful to Dr. G. Tibell for sending us these results prior to publication.

[33] L. A. Radicati, Proc. Phys. Soc. (London) **A66**, 136 (1953).

[34] K. Wildermuth and Th. Kanellopoulos, Nucl. Phys. **9**, 449 (1958).

the normalization factors in the initial and final states, respectively. A completely analogous expression can be written for $\mathcal{J}_{\text{int,rel}}'$, i.e., for transitions to the alpha-deuteron continuum, the final-state relative motion now being given by a regular Coulomb wave function.

The Coulomb interaction is a long-range force. As already mentioned the main contributions to the integrals in the numerator in Eq. (26) come from radial distances rather big, i.e., ≈ 6 F. This implies that the terms B_1 and B_2 resulting from the antisymmetrization between the clusters internal to ^{6}Li must be small in comparison with the direct term B_0. Based on results from earlier calculatuons[12] it can be shown that the mentioned exchange terms contribute only fractions of a percent to the values of $\mathcal{J}_{\text{int,rel}}$ and $\mathcal{J}_{\text{int,rel}}'$. These exchange terms are therefore omitted in the present calculations.

However, in the normalization terms in the denominator in Eq. (26) for the transitions to the discrete states, the exchange terms can not be neglected off-hand. Especially in the ground state with a large degree of cluster overlap in the wave functions, some contributions from the exchange terms may be expected. Details concerning the computation of these terms are given below.

In the case of transitions to the alpha-deuteron continuum the exchange terms do not contribute to the final-state normalization. The reason for this is the required normalization condition for the final-continuum state wave function given at the end of Subsec. IIC, i.e., a normalization such that the final-state relative motion function has the asymptotic amplitude 1.

The short-range correlation factors in the refined cluster wave functions make all computations very difficult. However, long-range one-particle operators are almost insensitive to short-range many-particle correlations. This is well illustrated by the recent work on the refined cluster-model wave functions.[10] In this work also the influence of the short-range correlations on the proton distributions in ^{6}Li was investigated. The cutoff functions were shown to be of minor importance, cf. Fig. 2 in Ref. 10. This is the justification for leaving out the hard-core correlation functions in the present calculations.

B. Numerical Work

In this section we make some remarks on the numerical calculations, which were performed on an IBM 709 electronic computing machine using FORTRAN programs.

The numerical problems encountered were of numerical integrations and approximations of given functions, and standard procedures could be used. (cf. for instance Ref. 35).

The most elaborate part of our calculation is the continuum contribution of the cross section [cf. Eq. (13)] where the squared matrix element, weighted by the function $f_{E2}(\xi)$ has to be integrated over k. The orbital integral $f_{E2}(\xi)$ is tabulated in Ref. 17. It can be approximated by an expression of the form

$$f_{E2}(\xi)=e^{-\gamma\xi}\sum_{n=1}^{N} a_n\xi^{n-1}. \tag{27}$$

A polynomial of fifth order gives a reasonable fit, the constants a_n being

$$\left.\begin{aligned} &a_1=0.28458, \quad a_2=9.1695, \quad a_3=-1.0967,\\ &a_4=-1.7296, \quad a_5=0.66336, \quad a_6=-0.071721,\\ &\gamma=5.53285. \end{aligned}\right\}. \tag{28}$$

The matrix element B_0 depending upon k through the regular Coulomb function cannot be expressed simply in terms of elementary functions. Therefore, the evaluation was done numerically, generating the Coulomb functions by solving the corresponding differential equation. The integration over k also had to be done numerically.

The resonance contributions to $\sigma_C^{(\lambda=2)}$, on the other hand [cf. Eq. (8)], can be worked out analytically, and the evaluation of the resulting algebraic expressions is easily done.

Another problem involved the estimate of the importance of the exchange terms, $A_1^{(i)}$, $A_2^{(i)}$, for instance. In order to avoid an elaborate numerical integration, we approximate the internal wave function of the deuteron by a sum of Gaussians,

$$\rho_d(r)\approx\sum_{n=1}^{N} h_n\exp(-\epsilon_n r^2). \tag{29}$$

The constants $N=5$,

$$\begin{aligned} &h_1=0.0204, \quad h_2=0.1170, \quad h_3=0.2534,\\ &h_4=3.6258, \quad h_5=-4.0955, \quad \epsilon_1=0.0183,\\ &\epsilon_2=0.0653, \quad \epsilon_3=0.2352, \quad \epsilon_4=2.2485,\\ &\epsilon_5=2.5092, \end{aligned} \tag{30}$$

gave a reasonable fit. Using Eq. (29) all integrals can be worked out analytically, and again one is left with an evaluation of some algebraic expressions.

It turned out that the exchange terms reduce the normalization integral by about 11% for the ground state and increases the normalization by 1.2% in the excited states considered here.

Finally, we should mention that all numerical calculations were arranged in such a way that an accuracy of 0.5% or better in the final result is achieved.

C. Results

The results of the computations are shown in Fig. 1. The agreement with the experimental data in the energy region below the Coulomb barrier is very satisfactory. No fitting of the nuclear model parameters has been made.

[35] D. R. Hartree, *Numerical Analysis* (Oxford University Press, London, 1958), 2nd ed.

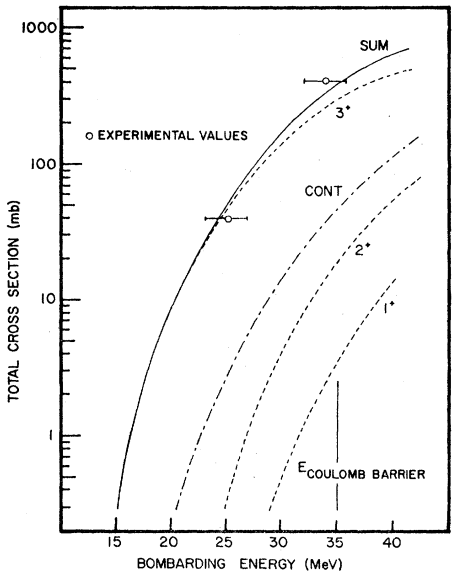

FIG. 1. Total Coulomb disintegration cross section for breakup of ^{6}Li projectiles into alpha particles and deuterons by impact on a gold target as a function of the bombarding energy (solid line). The other curves give the contributions from the different transitions. The experimental values are taken from the work of Anderson, Knox and Quinton (Ref. 2).

We note from Fig. 1 that the transitions to the 3^+, $T=0$ state in ^{6}Li are by far the dominating ones, and the more so with decreasing bombarding energies. The 2^+ and 1^+ transitions are much suppressed by the Alder-Winther function. This originates from the large excitation energies involved. These give rise to large ξ values and correspondingly small $f_{E2}(\xi)$ functions, cf. Subsec. IIC and Ref. 17. The continuum transition contribution increases somewhat more steeply with the bombarding energy than the discrete-discrete transitions, but nowhere does it compete seriously with the transitions from the ground state to the 3^+ state. An implication of this is that for bombarding energies of about 25 MeV and below, the measured alpha-deuteron breakup cross section is almost exclusively due to $1^+ \rightarrow 3^+$ transitions. One can thus from the measurements in Ref. 2 and application of Eq. (10) extract an empirical value for the reduced matrix elements $B(E2; 1 \rightarrow 3)$ which can be directly compared to the theoretically predicted value of this quantity. The significance of this is further discussed in the following section.

For bombarding energies above the Coulomb barrier at 35 MeV the bombarding nucleus will penetrate into the target nucleus. Phenomena involving nuclear forces will take place. Hence, in this energy region our simple picture of the process is no longer valid. Further, for bombarding energies below, say, 20 MeV the semiclassical method gradually turns bad, cf. Eq. (2). For this region of the bombarding energy, a calculation based on a purely quantum-mechanical treatment[16] is required. Thus, the method applied in this work is reliable only in a limited region of bombarding energies below the Coulomb barrier. The good fit obtained in this semiclassical energy region with the measurements in Ref. 2, lends support to the validity of the method itself as well as to the nuclear model applied.

IV. DISCUSSIONS

An excellent agreement is found between theory and experiment in the case of Coulomb disintegration of ^{6}Li.

From various points of view this agreement is instructive and suggests several conclusions to be drawn.

It seems that when the ^{6}Li dissociation is induced by a Coulomb field only, i.e., for bombarding energies well below the Coulomb barrier, the interesting situation occurs where the reaction mechanism appears to be well understood, and where further seemingly good wave functions for the projectile nucleus in question are available.

It should be stressed that in performing the calculations, no fitting of the parameters has been made. A few simplifying assumptions are introduced. These are the omission of the correlation factors in the wave functions and partial omission of the antisymmetrization. Because of the long-range character of the Coulomb force, this is a permissible procedure. The corrections to be made in this connection are completely negligible. The assumption of point-charge clusters in the disintegrating projectile should preferably have been avoided. Nevertheless, we do not consider this assumption to be a serious shortcoming.

The comparison between theory and experiment is based on measured absolute values of the total Coulomb breakup cross section[2] of ^{6}Li. The experimental values are believed to be very reliable.[36] From knowledge of these experimental values and of the fact that quadrupole Coulomb excitations are dominating for the lowest incident energy studied (25 MeV), an empirical value of the reduced matrix element $B(E2; 1 \rightarrow 3)$ can be estimated. Within the experimental errors this estimate is completely consistent with the present theoretical value of $B(E2; 1 \rightarrow 3) = 85 \times 10^{-52} e^2$ cm^4. This value is drastically different from the single-particle quadrupole estimate[18] which is $B(E2)_{sp} = 3.3$ in the same units as above.

Values of several reduced transition probabilities for transitions observed in ^{6}Li have recently been given by Bernheim and Bishop.[37] The ratios of the relevant measured reduced transition probabilities fit fairly well with the ratios for the $L=2$ triplet given by the present model, cf. Eq. (12). However, this is not the case with the absolute values of these quantities. For the $1^+ \rightarrow 3^+$

[36] We are very grateful to Professor D. A. Bromley for valuable comments concerning this point.

[37] M. Bernheim and G. R. Bishop, Phys. Letters 5, 270 (1963).

transition Bernheim and Bishop get $B(E2; 1 \rightarrow 3) = 30 \pm 3$, i.e., about ten times the single-particle value.

One might argue that the large theoretical $B(E2; I_i \rightarrow I_f)$ values are a consequence of the large diffuseness of the ^{6}Li nucleus. The rather large deviation from the single-particle estimate is perhaps not so surprising since the tail of the ^{6}Li charge distribution extends considerably farther out than the usual $1.2A^{1/3} \times 10^{-13}$ cm dependence.[27] The agreement with one set of experiments supports our belief in the quoted number. However, we would like to ask for further experimental tests on this point.

It has been stated earlier that the Pauli principle tends to reduce the difference between the various nuclear models.[38] Apparently different descriptions of nuclei seem to provide equally satisfactory predictions of nuclear behavior. Also connected with the indistinguishability of the nucleons is the important fact that nuclear states seem to behave differently for different nuclear reactions.[38] Because of the minor importance of the antisymmetrization for the present Coulomb breakup process, the pure alpha-deuteron picture is obviously good for the description of this reaction. Thus, this is a phenomenon where the cluster nature of ^{6}Li clearly stands out. However, this can only be expected to be true under the very special aspects of this process. One can easily imagine nuclear processes where the antisymmetrization of the wave function can be expected to come strongly into play. For ^{6}Li we have for instance the quasifree $(p,2p)$ reactions which for this nucleus have been much discussed recently.[7–9,39] Here the short-range nuclear forces are involved, and there is reason to expect the Pauli principle to be of importance. Under such circumstances the difference between nuclear models are not so strikingly shown. We want to stress that this by no means implies that the cluster model is unable to give a good description of these processes. On the contrary, it is a definite possibility that the cluster description of ^{6}Li can give better agreement with experiments than other nuclear models hitherto applied.[38] Nevertheless, the alpha-deuteron cluster character of the ^{6}Li nucleus seems to be most directly demonstrated by the present Coulomb breakup process.

ACKNOWLEDGMENTS

We would like to express our thanks to Professor Karl Wildermuth for his interest in this work and for numerous enlightening discussions during various stages of the investigation. Our best thanks are due to The Florida State University, especially the Physics Department, for their hospitality. We should also gratefully acknowledge free computing time at The FSU Computing Center and kind advice from its staff members on various occasions.

[38] K. Wildermuth, Nucl. Phys. **31**, 478 (1962).
[39] T. Berggren and G. Jacob, Nucl. Phys. **47**, 481 (1963).

APPENDIX: EVALUATION OF CROSS SECTION FORMULAS

In Eq. (8) we gave an expression for the total cross section for electric excitations of multipole order λ from the ground state to a discrete level. The evaluation is based on a two-cluster picture. One of the clusters is assumed to have spin zero. Let the symbols l_i and l_f be the relative angular momenta of the clusters in the initial and final states, m_i and m_f their magnetic quantum numbers, and $S_2^{(i)}$, $S_2^{(f)}$ and $m_2^{(i)}$, $m_2^{(f)}$ the corresponding intrinsic spin quantities of the second cluster. The resulting total angular momenta and total magnetic quantum numbers are called I_i, I_f and M_i, M_f, respectively. The differential cross section is then given by

$$d\sigma_C = \frac{1}{2I_i+1} \sum_{M_i,M_f} |b_{if}|^2 d\sigma_R . \tag{A1}$$

Here b_{if} denotes the probability amplitude for a transition of the system caused by the Coulomb interaction between each of the assumed clusters and a point-charge target nucleus. Further

$$d\sigma_R = \tfrac{1}{4} d^2 \sin^{-4}(\theta/2) d\Omega$$

is the differential cross section give by the Rutherford scattering law, with θ being the scattering angle in the center-of-mass system of the target and the projectile and with d denoting half the distance of closet approach in a head-on collision, cf. Subsec. IIC above. In Ref. 12 b_{if} is evaluated from first-order time-dependent perturbation theory in combination with a multipole expansion of the Coulomb interaction as given by Eq. (6) in this work or Eq. (10) in Ref. 12. Thus

$$b_{if} = \frac{4\pi Z_2 e^2}{i\hbar} \sum_{\lambda,\mu} q_\lambda \frac{1}{2\lambda+1} S_{E,\lambda\mu} \langle f | R^\lambda Y_{\lambda\mu}(\hat{R}) | i \rangle \tag{A2}$$

with λ being the multipole order.

The geometrical factor q_λ is a function of the masses and charge numbers of the clusters. With M_1 and Z_1 being the mass and charge numbers of the projectile, one has

$$M_1 = M^{(1)} + M^{(2)}, \quad Z_1 = Z^{(1)} + Z^{(2)} \tag{A3}$$

where the upper indices in brackets refer to the clusters. Defining

$$a_1 = (M^{(2)}/M_1), \quad a_2 = (M^{(1)}/M_1) \tag{A4}$$

the factor q_λ is given by

$$q_\lambda = \sum_{n=1}^{2} (-1)^{(n-1)\lambda} (a_n)^\lambda Z^{(n)} . \tag{A5}$$

The quantities $S_{E,\lambda\mu}$ in Eq. (A2) are orbital integrals introduced in the treatment of Coulomb excitation.[16,17] In general, the angular parts of the initial- and final-state wave functions in Eq. (A2) can be expressed by spin eigenfunctions, angular momentum eigenfunctions

and Wigner coefficients.

$$|i\rangle_{\text{ang}}=\sum_{m_i,m_2^{(i)}}\langle l_i S_2^{(i)},m_i m_2^{(i)}|I_i M_i\rangle \times|l_i m_i\rangle|S_2^{(i)} m_2^{(i)}\rangle$$
$$|f\rangle_{\text{ang}}=\sum_{m_f,m_2^{(f)}}\langle l_f S_2^{(f)} m_f m_2^{(f)}|I_f M_f\rangle \times|l_f m_f\rangle|S_2^{(f)} m_2^{(f)}\rangle. \quad \text{(A6)}$$

By inserting into (A1) Eqs. (A2) and (A6), a complicated expression arises. By repeated application of orthogonality relations for the Wigner coefficients (see Ref. 29, Appendix A) and the formula for the angular integral of a product of three spherical harmonics,[40] this expression is much simplified. After integration over the Rutherford scattering angle and exploiting relations given in standard works on Coulomb excitation,[16,17] the formula for the total electric Coulomb excitation cross section is arrived at. Thus,

$$\sigma_C^{(\lambda)}=\left(\frac{Z_2 e^2}{\hbar v}\right)^2\frac{1}{d^{2\lambda-2}}\frac{q_\lambda^2}{4\pi}(2I_f+1) \times\langle l_i\lambda 00|l_f 0\rangle^2 f_{E\lambda}(\xi)\vartheta_{\text{int, rel}}. \quad \text{(A7)}$$

The argument ξ and the nuclear part of the cross section $\vartheta_{\text{int,rel}}$ are defined in Subsec. IIC. The Alder and Winther functions $f_{E\lambda}(\xi)$ are tabulated in Ref. 17.

For transitions to free fragments in the continuum, the cross-section formula is given in Eq. (13). Assuming a plane-wave approximation of the final-state relative motion of the clusters and performing a standard expansion of the plane wave in spherical harmonics,[41] the cross section for continuum transitions of the quadrupole type is given by

$$\sigma_{C,\text{ cont}}^{(\lambda=2)}=\frac{15}{2}\frac{Z_2^2 e^4}{\pi^2\hbar^2 v^2 d^2}q_2^2 \times\int dk\, k^2 f_{E2}(\xi)\vartheta_{\text{int, rel}}'', \quad \text{(A8,a)}$$

where

$$\vartheta_{\text{int, rel}}''=\left|\int(\psi_{\text{int}}^{(f)} j_2(kR))^* \times R^\lambda\psi_{\text{int}}^{(i)}\chi_{\text{rel}}^{(i)}(R)d\tau_{\text{int, }R}\right|^2. \quad \text{(A8,b)}$$

Here $j_2(kR)$ is a spherical Bessel function. This expression was used for the calculations in Ref. 12. Owing to a writing error in Eq. (38a) in Ref. 12, the quantity A_{0R} in the denominator should be omitted because it is already absorbed in other terms in that equation. In the present calculation the spherical Bessel function was replaced by the regular Coulomb function times kR,[42]

$$j_l(kR)\rightarrow kRF_l(kR,\eta). \quad \text{(A9)}$$

Here η is a parameter characterizing the two fragments, $\eta=Z^{(1)}Z^{(2)}e^2/\hbar v_k$, with v_k denoting their relative velocity.[42]

[40] A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, N. J., 1957).

[41] W. Magnus and F. Oberhettinger, *Formulas and Theorems for the Functions of Mathematical Physics* (Chelsea Publishing Company, New York, 1954).

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