Collective Nuclear Vibrations Induced by High-Energy Neutrinos*

H. ÜBERALL
Department of Physics, The Catholic University of America, Washington, D. C.

(Received 10 September 1964)

Neutrino absorption in nuclei leading to states of collective vibration ("giant dipole") is investigated. The collective states are described schematically by using the Goldhaber-Teller model and its generalization to spin-isobaric spin vibrations.

I. INTRODUCTION

EXCITATION of nuclear giant dipole states by neutrinos was considered for the first time by Belyaev,[1] who related the probability of this reaction to the photonuclear giant dipole cross section. This work can, however, be considered only a rather crude estimate, since the axial vector matrix element was neglected, and no account was taken of the variation of the recoil with the angle of the emerging lepton. In the following, we shall evaluate the transition probability in greater detail and also obtain the lepton angular distribution, using the Goldhaber-Teller model[2] of the giant dipole transition in order to find the vector matrix element,[3] and a generalization of this model to collective spin-isobaric spin oscillations for a calculation of the axial vector matrix element.[4] Both matrix elements can be expressed in terms of the nuclear ground-state electromagnetic form factor, which causes the cross section to level off above $\sim$300 MeV neutrino energy. Numerical results were obtained for a C^{12} target, giving a cross section of $\sim 2\times 10^{-39}$ cm^2, large enough to make the process measurable.

II. EVALUATION

The reaction we consider is

$$\nu_l + A_Z \rightarrow A_{Z+1} + l^-, \tag{1}$$

where l = electron or muon, and A_{Z+1} may be in an excited state; its spin and magnetic quantum number will be called J', M', and those of A_Z will be called J, M. The corresponding reaction with antineutrinos is

$$\bar{\nu}_l + A_Z \rightarrow A_{Z-1} + l^+. \tag{1'}$$

* Supported in part by the U. S. Air Force Office of Scientific Research. Work begun under support by the Office of Naval Research, U. S. Navy, on a summer contract at the Harrison M. Randall Laboratory of Physics, University of Michigan, Ann Arbor, Michigan.

[1] W. B. Belyaev, Report No. 926, Dubna, 1962 (unpublished). The possibility of this process has independently been mentioned also by Professor R. R. Lewis.

[2] M. Goldhaber and E. Teller, Phys. Rev. **74**, 1046 (1948).

[3] This procedure was used by J. Goldemberg, Y. Torizuka, W. C. Barber, and J. D. Walecka [Nucl. Phys. **43**, 242 (1963)] for their treatment of the giant dipole excitation in inelastic electron scattering.

[4] For the connection of these vibrational modes with Wigner supermultiplet states, and their contribution to the axial vector matrix element in nuclear muon capture, see L. L. Foldy and J. D. Walecka, CERN report No. 8837/TH 424, 1964 (unpublished).

Kinematics gives for the nuclear recoil $\mathbf{q} = \boldsymbol{\nu} - \mathbf{l}$ (three-momenta of ν_l and l), and for the lepton energy $E_l = \nu - \delta$ (neglecting the recoil energy), with

$$\delta = \Delta + M(A_{Z\pm 1}) - M(A_Z), \tag{2}$$

where Δ is the excitation energy of $A_{Z\pm1}$. For the photonuclear effect in A_Z, one has $|\mathbf{q}| \cong \delta'$, the (fixed) excitation energy in A_Z, which is $\sim$20 MeV for the giant dipole transition (going down to $\sim$15 MeV in heavy nuclei), and which may differ from δ owing to the Coulomb effect, i.e., deviations from charge independence; but since we shall want to relate weak and electromagnetic matrix elements by the isobaric spin formalism, we shall disregard this difference. In reaction (1), $|\mathbf{q}|$ varies between $\nu - l$ at $\vartheta = 0°$ and $\nu + l$ at $\vartheta = 180°$, where $\vartheta = \sphericalangle(\boldsymbol{\nu}, \mathbf{l})$. With lepton mass μ, the threshold of (1) is $\nu \geqslant \mu + \delta$. The square of the four-momentum transfer becomes $q^2 = \mathbf{q}^2 - \delta^2$.

As the weak interaction Hamiltonian, we take the general form given by Fujii and Primakoff,[5] and obtain from it the nonrelativistic expression

$$H = (\phi_p{}^\dagger \psi_l{}^\dagger \mathcal{O} \psi_\nu \phi_n), \tag{3}$$

$$\mathcal{O} = [G_V + G_A \boldsymbol{\sigma}\cdot\boldsymbol{\sigma}^N + G_P(\mathbf{q}/2m)\cdot\boldsymbol{\sigma}^N\gamma_4 + iG_M(\mathbf{q}/2m)\cdot(\boldsymbol{\sigma}\times\boldsymbol{\sigma}^N)](1+\gamma_5)/\sqrt{2}, \tag{3'}$$

where ϕ are Pauli spinors and $\boldsymbol{\sigma}^N$ are Pauli matrices. The relativistic pseudoscalar and weak magnetic terms have been kept since their coefficients are large. The nucleon mass is m; we have $G_i = F_i(q^2)C_i$, and

$$C_V = 10^{-5}m^{-2}, \quad C_A = -XC_V, \quad C_P = \epsilon C_A, \quad C_M = \zeta C_V,$$

where

$$X = 1.20; \quad \epsilon \cong 8 (l=\mu), \cong 0 (l=e); \quad \zeta = \mu_p - \mu_n = 3.71.$$

Conserved vector current hypothesis suggests $F_V(q^2) = F_p(q^2)$, the proton charge form factor, for which we shall take

$$F_p(q^2) = (1 + (q^2/M_1{}^2))^{-2}, \tag{4}$$

$$M_1 = 0.90m = 840 \text{ MeV}, \tag{4'}$$

corresponding to the proton radius 0.8×10^{-13} cm. The other form factors may be assumed the same, and F_P may have to get an additional factor $F(q^2)$ = ratio of $(m_\pi{}^2 + \mu^2)\times(m_\pi{}^2 + q^2)^{-1}$ and Eq. (4), if it comes mainly

[5] A. Fujii and H. Primakoff, Nuovo Cimento **12**, 327 (1959).

from a single virtual exchanged pion.[6] The cross section of reactions (1) and (1′) is given by

$$d\sigma=\frac{2\pi}{2J+1}\sum_{MM'}\sum_{s_l s_\nu}(2\pi)^3\delta(\mathbf{v}-\mathbf{l}-\mathbf{q})|\mathcal{H}|^2 \times\frac{d^3q}{(2\pi)^3}\frac{d^3l}{(2\pi)^3}\delta(E_l+\delta-\nu)\,, \quad (5)$$

$$\mathcal{H}=(\Phi_{Z\pm1}{}^\dagger u_l{}^\dagger\sum_{i=1}^{A}\mathcal{O}_i e^{i\mathbf{q}\cdot\mathbf{r}_i}\tau_\pm{}^{(i)}u_\nu\Phi_Z)\,, \quad (5')$$

where $\tau_+p=0$, $\tau_+n=p$, etc. The use of the Born approximation is implied, treating the outgoing lepton as a free particle; this will be a good approximation for $Z/137\ll1$. In terms of the vector and axial vector matrix elements

$$\mathfrak{M}=(\Phi_{Z\pm1}{}^\dagger\sum_{i=1}^{A}e^{i\mathbf{q}\cdot\mathbf{r}_i}\tau_\pm{}^{(i)}\Phi_Z)\,, \quad (6)$$

$$\boldsymbol{\mathfrak{M}}=(\Phi_{Z\pm1}{}^\dagger\sum_{i=1}^{A}e^{i\mathbf{q}\cdot\mathbf{r}_i}\boldsymbol{\sigma}^{(i)}\tau_\pm{}^{(i)}\Phi_Z)\,, \quad (6')$$

(nuclear magnetic quantum numbers not written explicitly), we find

$$\begin{aligned}\sum_{s_l s_\nu}|\mathcal{H}|^2=&G_V{}^2|\mathfrak{M}|^2\left(1+\frac{\hat{\nu}\cdot\mathbf{l}}{E_l}\right)-2G_VG_A\left[\mathrm{Re}\mathfrak{M}^*\boldsymbol{\mathfrak{M}}\cdot\left(\hat{\nu}+\frac{\mathbf{l}}{E_l}\right)\pm\mathrm{Im}\mathfrak{M}^*\boldsymbol{\mathfrak{M}}\cdot\left(\hat{\nu}\times\frac{\mathbf{l}}{E_l}\right)\right]\\&+G_A{}^2\left[2\,\mathrm{Re}\boldsymbol{\mathfrak{M}}^*\cdot\hat{\nu}\boldsymbol{\mathfrak{M}}\cdot\frac{\mathbf{l}}{E_l}+|\boldsymbol{\mathfrak{M}}|^2\left(1-\hat{\nu}\cdot\frac{\mathbf{l}}{E_l}\right)\mp i\left(\hat{\nu}-\frac{\mathbf{l}}{E_l}\right)\cdot(\boldsymbol{\mathfrak{M}}^*\times\boldsymbol{\mathfrak{M}})\right]+2G_VG_P\frac{\mu}{E_l}\mathrm{Re}\mathfrak{M}^*\boldsymbol{\mathfrak{M}}\cdot\frac{\mathbf{q}}{2m}\\&-2G_AG_P\frac{\mu}{E_l}\mathrm{Re}\boldsymbol{\mathfrak{M}}^*\cdot\hat{\nu}\boldsymbol{\mathfrak{M}}\cdot\frac{\mathbf{q}}{2m}+G_P{}^2\left|\frac{\mathbf{q}}{2m}\cdot\boldsymbol{\mathfrak{M}}\right|^2\left(1-\frac{\hat{\nu}\cdot\mathbf{l}}{E_l}\right)\\&-2G_VG_M\left[\pm\left(\hat{\nu}+\frac{\mathbf{l}}{E_l}\right)\cdot\left(\frac{\mathbf{q}}{2m}\times\mathrm{Im}\mathfrak{M}^*\boldsymbol{\mathfrak{M}}\right)+\left(\frac{\mathbf{l}}{E_l}\times\hat{\nu}\right)\cdot\left(\frac{\mathbf{q}}{2m}\times\mathrm{Re}\mathfrak{M}^*\boldsymbol{\mathfrak{M}}\right)\right]\\&-2G_AG_M\left\{\mathrm{Re}\left[\boldsymbol{\mathfrak{M}}^*\times\left(\hat{\nu}-\frac{\mathbf{l}}{E_l}\right)\right]\cdot\left(\boldsymbol{\mathfrak{M}}\times\frac{\mathbf{q}}{2m}\right)\right.\\&\left.\pm\mathrm{Im}\left[\boldsymbol{\mathfrak{M}}^*\cdot\hat{\nu}\boldsymbol{\mathfrak{M}}\cdot\left(\frac{\mathbf{q}}{2m}\times\frac{\mathbf{l}}{E_l}\right)+\boldsymbol{\mathfrak{M}}^*\cdot\frac{\mathbf{l}}{E_l}\boldsymbol{\mathfrak{M}}\cdot\left(\frac{\mathbf{q}}{2m}\times\hat{\nu}\right)+\left(1-\frac{\hat{\nu}\cdot\mathbf{l}}{E_l}\right)\frac{\mathbf{q}}{2m}\cdot(\boldsymbol{\mathfrak{M}}^*\times\boldsymbol{\mathfrak{M}})\right]\right\}\,, \quad (7)\end{aligned}$$

where $\hat{\nu}=\mathbf{v}/\nu$. The upper sign refers to reaction (1), the lower sign to reaction (1′). With respect to the isobaric spin dependence (assuming charge independence), we may express $\mathfrak{M}_{(\mu)}$ in terms of Clebsch-Gordan coefficients and reduced matrix elements by virtue of the Wigner-Eckart theorem,

$$\mathfrak{M}_{(\mu)}=\mp\frac{1}{\sqrt{2}}\frac{(T'\|\mathfrak{M}_{(\mu)}{}^{(1)}\|T)}{(2T'+1)^{1/2}}C_{T1}(T'T_3';T_3,\pm1)\,, \quad (8)$$

where T, T_3 describe the initial and T', T_3' the final nucleus; clearly $T_3'=T_3\pm1$. Similarly, the "electromagnetic" matrix element

$$\mathfrak{M}'=\tfrac{1}{2}(\Phi_{Z'}{}^{*\dagger}\sum_{i=1}^{A}e^{i\mathbf{q}\cdot\mathbf{r}_i}\tau_3{}^{(i)}\Phi_{Z'}) \quad (9)$$

and its "axial vector" analog

$$\boldsymbol{\mathfrak{M}}'=\tfrac{1}{2}(\Phi_{Z'}{}^{*\dagger}\sum_{i=1}^{A}e^{i\mathbf{q}\cdot\mathbf{r}_i}\boldsymbol{\sigma}^{(i)}\tau_3{}^{(i)}\Phi_{Z'}) \quad (9')$$

may be expressed by

$$\mathfrak{M}_{(\mu)}{}'=\frac{1}{2}\frac{(T'\|\mathfrak{M}_{(\mu)}{}^{(1)}\|T)}{(2T'+1)^{1/2}}C_{T1}(T'\bar{T}_3;\bar{T}_3,0)\,, \quad (10)$$

with $\bar{T}_3$ the third component of isobaric spin of initial and final nucleus. $\mathfrak{M}'$ represents the actual matrix element of electromagnetic transitions provided that $|T'-T|=1$. This is a good assumption for self-conjugate nuclei $\bar{T}_3=0$ since in this case, Gell-Mann and Telegdi[7] have shown that the electric dipole transitions imply $|T'-T|=1$, to a very good accuracy. Thus, reaction (1) leads to the same final states as the giant dipole photonuclear transitions and will also have a large probability. Note that, from nuclear stability, we shall mostly deal with initial states $T_3=\bar{T}_3$. Assuming this, i.e., comparing reactions (1) and (1′) to "electromagnetic" transitions in the same nucleus A_Z, Eqs. (8) and (10) give us

$$\mathfrak{M}_{(\mu)}=\mp\sqrt{2}g(T',TT_3)\mathfrak{M}_{(\mu)}{}'\,, \quad (11)$$

[6] L. Wolfenstein, Nuovo Cimento **8**, 882 (1958).

[7] M. Gell-Mann and V. Telegdi, Phys. Rev. **91**, 169 (1953).

where

$$g(T',TT_3) = C_{T1}(T'T_3\pm 1;\, T_3,\, \pm 1)/C_{T1}(T'T_3;\, T_3,0); \quad (11')$$

g is unity for a nucleus with $T=0$, e.g., C^{12} or O^{16}.

The matrix elements $\mathfrak{M}'$, $\mathbf{\mathfrak{M}}'$ shall now be evaluated using a schematic Goldhaber-Teller type model. They may be written in the form

$$\mathfrak{M}' = \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}} \rho(\mathbf{r}), \quad (12)$$

$$\boldsymbol{\mathfrak{M}}' = \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}} \boldsymbol{\varrho}(\mathbf{r}), \quad (12')$$

with local transition densities and spin densities

$$\rho(\mathbf{r}) = \tfrac{1}{2}(\Phi_Z^{*\dagger} \sum_{i=1}^{A} \delta(\mathbf{r}-\mathbf{r}_i)\tau_3^{(i)}\Phi_Z), \quad (13)$$

$$\boldsymbol{\varrho}(\mathbf{r}) = \tfrac{1}{2}(\Phi_Z^{*\dagger} \sum_{i=1}^{A} \delta(\mathbf{r}-\mathbf{r}_i)\boldsymbol{\sigma}^{(i)}\tau_3^{(i)}\Phi_Z). \quad (13')$$

In this model, we consider nuclei with even numbers $Z=N=\frac{1}{2}A$, i.e., with $J=0^+$, $T=0$ ground states, and assume that the proton Fermi sphere oscillates as a whole against the neutron Fermi sphere (the Goldhaber-Teller model proper) with energy

$$H = (1/2m^*)\mathbf{p}_n^2 + \tfrac{1}{2}m^*\omega^2\mathbf{q}_n^2, \quad (14)$$

where the reduced mass is $m^* = \frac{1}{4}Am$; the first excited state of the oscillator, 1^- (giant dipole) has energy $\hbar\omega = \delta$. (We shall later set $\hbar = 1$.) The coordinate may be written in terms of spherical creation and annihilation operators[3]

$$(q_n)_{lm} = (\hbar/2m^*\omega)^{1/2}[a_{l,m}^{\dagger} + (-1)^m a_{l,-m}]. \quad (15)$$

There are, however, other collective modes of oscillation, degenerate with the Goldhaber-Teller ones to the extent that the nuclear forces are approximately spin independent[4] (so that Wigner's supermultiplet theory[8] is applicable) that may be described as the oscillation of a sphere of protons with spin in a given direction and neutrons with spin in the opposite direction, against a sphere of protons with spin in the opposite direction, and neutrons with spin in the given direction (spin-isobaric spin oscillations), and furthermore oscillations of a sphere of protons and neutrons with spin in a given direction against a sphere of protons and neutrons with spin in the opposite direction (spin waves). These latter two types of collective vibrations were first obtained by Wild,[9] and later independently by Glassgold *et al.*[10] Only the Goldhaber-Teller mode will contribute to the "vector" electromagnetic matrix element (9), and hence to giant dipole cross sections. The spin-isobaric spin mode contributes only to the "axial vector" matrix element (9′), the spin wave mode to neither, and they cannot be excited electromagnetically (to the extent that the approximations implied in the use of this model hold). It may be considered an interesting problem of nuclear physics to study excitations of these collective modes involving spin; inelastic electron scattering, muon capture and neutrino absorption offer a direct method of investigation of this phenomenon.

Assuming further the oscillations to be described by a rigid displacement of the ground-state densities $\rho_0(\mathbf{r})$ of protons and neutrons of either spin, we may write the nucleon transition density matrix $\phi(\mathbf{r})$ of the three modes of oscillation discussed above:

$$\phi_{\mathrm{GT}}(\mathbf{r}) = \frac{1+\tau_3}{2}\tfrac{1}{2}\rho_0(\mathbf{r}-\tfrac{1}{2}\mathbf{q}_n) + \frac{1-\tau_3}{2}\tfrac{1}{2}\rho_0(\mathbf{r}+\tfrac{1}{2}\mathbf{q}_n), \quad (16)$$

$$\phi_{s-is}(\mathbf{r}) = \frac{1+\tau_3\sigma_{m'}}{2}\tfrac{1}{2}\rho_0(\mathbf{r}-\tfrac{1}{2}\mathbf{q}_n) + \frac{1-\tau_3\sigma_{m'}}{2}\tfrac{1}{2}\rho_0(\mathbf{r}+\tfrac{1}{2}\mathbf{q}_n), \quad (16')$$

$$\phi_{sw}(\mathbf{r}) = \frac{1+\sigma_{m'}}{2}\tfrac{1}{2}\rho_0(\mathbf{r}-\tfrac{1}{2}\mathbf{q}_n) + \frac{1-\sigma_{m'}}{2}\tfrac{1}{2}\rho_0(\mathbf{r}+\tfrac{1}{2}\mathbf{q}_n), \quad (16'')$$

where

$$\int d^3r\, \rho_0(\mathbf{r}) = Z.$$

Here $m'=1, 0, -1$ expresses the fact that the collective modes involving spin correspond to triplet states of the final total spin S' (which is uncoupled from the spatial motion in our oscillator model, and generally in the supermultiplet theory,[4,8]) and the cross section contains a final state sum over m' for the *axial vector* matrix elements besides the sum over final oscillator orientations M'. Expanding in a Taylor series and keeping terms of first order in $\mathbf{q}_n$ only, which describe the $0^+ \to 1^-$ transition to the oscillator dipole states, we find

$$\phi_{\mathrm{GT}}(\mathbf{r}) = -\tfrac{1}{4}\tau_3\mathbf{q}_n\cdot\nabla\rho_0(\mathbf{r}), \quad (17)$$

$$\phi_{s-is}(\mathbf{r}) = -\tfrac{1}{4}\tau_3\sigma_{m'}\mathbf{q}_n\cdot\nabla\rho_0(\mathbf{r}), \quad (17')$$

$$\phi_{sw}(\mathbf{r}) = -\tfrac{1}{4}\sigma_{m'}\mathbf{q}_n\cdot\nabla\rho_0(\mathbf{r}). \quad (17'')$$

The densities (13) are obtained from the density matrix $\phi(\mathbf{r})$ by

$$\rho(\mathbf{r}) = \tfrac{1}{2}\,\mathrm{Tr}\tau_3\phi(\mathbf{r}), \quad (18)$$

$$\boldsymbol{\varrho}(\mathbf{r}) = \tfrac{1}{2}\,\mathrm{Tr}\boldsymbol{\sigma}\tau_3\phi(\mathbf{r}), \quad (18')$$

and we find, using Eqs. (17)

$$\rho(\mathbf{r}) = -\tfrac{1}{2}\mathbf{q}_n\cdot\nabla\rho_0(\mathbf{r}), \quad (19)$$

which gets its contribution from the Goldhaber-Teller

[8] E. P. Wigner, Phys. Rev. **51**, 106 (1937).
[9] W. Wild, Bayer. Akad. Wiss. Math.-Naturw. Klasse **18**, 371 (1956).
[10] A. E. Glassgold, W. Heckrotte, and K. M. Watson, Ann. Phys. (N. Y.) **6**, 1 (1959).

mode (17) only, and

$$\rho_i(\mathbf{r}) = -\tfrac{1}{2}\delta_{im'}^{(s)}\mathbf{q}_n\cdot\nabla\rho_0(\mathbf{r}), \qquad (19')$$

arising from the spin-isobaric spin mode (17′) only. We used the spherical Kronecker tensor

$$\delta_{im'}^{(s)} = \begin{cases} -2^{-1/2}(\delta_{i1}+i\delta_{i2}), & m'=1 \\ \delta_{i3}, & m'=0 \\ 2^{-1/2}(\delta_{i1}-i\delta_{i2}), & m'=-1. \end{cases} \qquad (20)$$

With the densities (19), we obtain the matrix elements for the $0^+\to 1^-(M')$ transition from Eqs. (12) using partial integration

$$\langle 1^- M'|\mathfrak{M}'|0^+\rangle = i|\mathbf{q}|F(\mathbf{q})\left(\frac{2\pi}{3Am\delta}\right)^{1/2} Y_{1M'}{}^*(\hat{q}), \qquad (21)$$

and further

$$\langle 1^- M'm'|\mathfrak{M}_i'|0^+\rangle = \delta_{im'}^{(s)}\langle 1^- M'|\mathfrak{M}'|0^+\rangle, \qquad (21')$$

so that the axial vector matrix element satisfies the relation found by Tolhoek[11,4]:

$$\sum_{m'}|\langle\mathfrak{M}'\rangle|^2 = 3|\langle\mathfrak{M}'\rangle|^2. \qquad (22)$$

The matrix elements come out proportional to the ground-state charge form factor

$$F(\mathbf{q}) = \int d^3r\, e^{i\mathbf{q}\cdot\mathbf{r}}\rho_0(\mathbf{r}). \qquad (23)$$

We take for it[12]

$$F(\mathbf{q}) = Z\left(1+\frac{\mathbf{q}^2}{M_0{}^2}\right)^{-2}, \qquad (24)$$

$$M_0 = 0.725A^{-1/3}m = 680A^{-1/3}\ \text{MeV}. \qquad (24')$$

One should note that for $q^2R^2 \ll 1$ (R = nuclear radius), i.e., using $|F(0)|^2 \equiv Z^2$, the Goldhaber-Teller matrix element $\langle 1M'|\mathfrak{M}'|00\rangle$ completely exhausts the dipole sum rule[13]

$$2m\sum_\delta \frac{\delta}{\mathbf{q}^2}|\langle 10|\mathfrak{M}'|00\rangle|^2 = \frac{NZ}{A}. \qquad (25)$$

Since, in practice, the giant dipole is not the only dipole transition (and indeed, experimentally does not completely exhaust the sum rule), we expect our results to provide an upper limit to the actual cross section.

With the matrix elements (21), the cross section is obtained from Eqs. (5) and (7). Contributions from the various collective modes enter incoherently; thus cross terms between vector and axial vector matrix elements vanish. The sums over the orientations m' of the $S'=1$ spin-isobaric spin collective mode give these results in expressions of the form

$$\sum_{m'}\mathbf{a}\cdot\langle\mathfrak{M}'^*\rangle\mathbf{b}\cdot\langle\mathfrak{M}'\rangle = |\mathfrak{M}'|^2\mathbf{a}\cdot\mathbf{b}, \qquad (26)$$

$$\sum_{m'}\langle\mathfrak{M}'^*\rangle\times\langle\mathfrak{M}'\rangle = 0, \qquad (26')$$

as well as Eq. (22). We find in this way the cross section of reactions (1) and (1′)

$$\frac{d\sigma}{d\Omega_l} = \frac{lE_l\mathbf{q}^2|F(\mathbf{q})|^2}{4\pi^2Am\delta}G_V{}^2\left\{1+\frac{\hat{p}\cdot\mathbf{l}}{E_l}+3X^2\left(1-\frac{1}{3}\frac{\hat{p}\cdot\mathbf{l}}{E_l}\right)\right.$$
$$\left.-\epsilon F(\mathbf{q}^2-\delta^2)X^2\frac{\mu}{E_l}\frac{\hat{p}\cdot\mathbf{q}}{m}+\tfrac{1}{4}\epsilon^2F^2(\mathbf{q}^2-\delta^2)X^2\frac{\mathbf{q}^2}{m^2}\left(1-\frac{\hat{p}\cdot\mathbf{l}}{E_l}\right)+2\zeta X\frac{\mathbf{q}}{m}\cdot\left(\hat{p}-\frac{\mathbf{l}}{E_l}\right)\right\}. \qquad (27)$$

As an example, we evaluated Eq. (27) for the reaction

$$\nu + {}_6C^{12} \to \underset{\text{giant dip.}}{{}_7N^{12}} + l^-. \qquad (28)$$

Note that the cross section of Eq. (27) will be roughly proportional to Z since $F(0)=Z$. We have used $\delta \cong 22.5$ MeV,[14] and have integrated Eq. (27) over the angles of the emitted lepton (e or μ). In the total cross section

$$\sigma = \frac{1.96\times10^{-34}\ \text{cm}^2}{(\nu^2)_{\text{MeV}}}\frac{1}{M_0{}^6}\int_{(\nu-l)^2}^{(\nu+l)^2} x\left\{2\nu E_l+\nu^2+l^2-x\right.$$
$$+3X^2[2\nu E_l-\tfrac{1}{3}(\nu^2+l^2-x)]-\epsilon F(x-\delta^2)X^2\frac{\mu}{m}(\nu^2-l^2+x)+\tfrac{1}{4}\epsilon^2F^2(x-\delta^2)X^2\frac{x}{m^2}(2\nu E_l-\nu^2-l^2+x)$$
$$\left.+2\zeta X\frac{1}{m}[\delta(l^2-\nu^2)+x(E_l+\nu)]\right\}\left(1+\frac{x}{M_0{}^2}\right)^{-4}\left(1-\frac{\delta^2}{M_1{}^2}+\frac{x}{M_1{}^2}\right)^{-4}dx, \qquad (29)$$

[11] J. R. Luyten, H. P. C. Rood, and H. A. Tolhoek, Nucl. Phys. **41**, 236 (1963).
[12] T. D. Lee, P. Markstein, and C. N. Yang, Phys. Rev. Letters **7**, 429 (1961).
[13] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), p. 641.
[14] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 1 (1959).

we dropped the terms containing ϵ and ζ in the numerical evaluation, since they can be seen to contribute very little for $\gtrsim 300$ MeV, where the cross section is large (although their contribution may be relatively large below this energy, where the cross section is small). Results are plotted in Fig. 1 versus the neutrino energy. (The same figure shows the cross section of

$$\nu + {}_6C^{12} \rightarrow {}_7N^{12}_{\text{ground}} + e^-, \quad (30)$$

as obtained earlier.[15]) The cross section, which at threshold behaves like l^2 (for electrons), levels off above 300 MeV at a value of $\sim 2.10^{-39}$ cm^2. This plateau is caused by the form factors F, Eq. (24), and F_p, Eq. (4), contained in $d\sigma$; without these, σ would increase indefinitely $\sim \nu^4$. With F alone, and $F_p \equiv 1$, σ would again level off at a value $\sim 5.10^{-39}$ cm^2. (This may indicate that in the nonrelativistic expansion of the Hamiltonian, higher terms $\sim (q/m)^2$ are to be kept, although they may be estimated from the same result to be of relative order 15–20% only.) For $l = \mu$, the high energy behavior of σ is similar, and only the threshold is higher by ~ 100 MeV.

Figure 2 presents the angular distributions of reaction (28), again dropping the ϵ, ζ terms, at $\nu = 400$, 600, and 800 MeV for $l = e$ (they are practically identical for $l = \mu$). They are mostly forward, again because of the form factors.

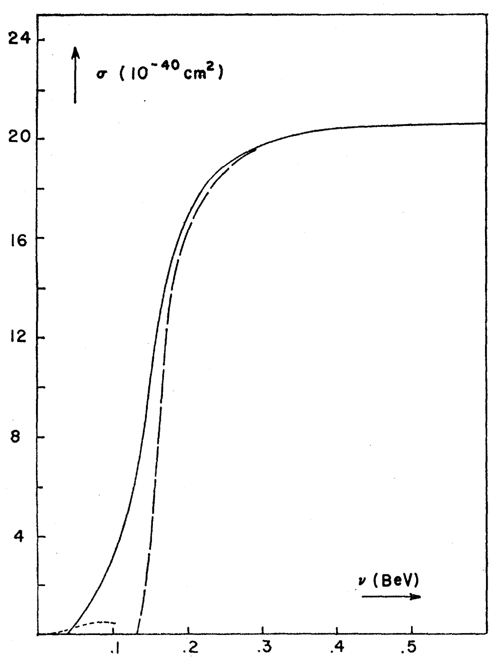

FIG. 1. Total cross section for the excitation of collective vibrations in C^{12} by neutrinos. Solid line: $\nu + {}_6C^{12} \rightarrow {}_7N^{12}_{\text{g.dip.}} + e^-$; dotted line: $\nu + {}_6C^{12} \rightarrow {}_7N^{12}_{\text{grd.}} + e^-$; broken line: $\nu + {}_6C^{12} \rightarrow {}_7N^{12}_{\text{g.dip.}} + \mu^-$.

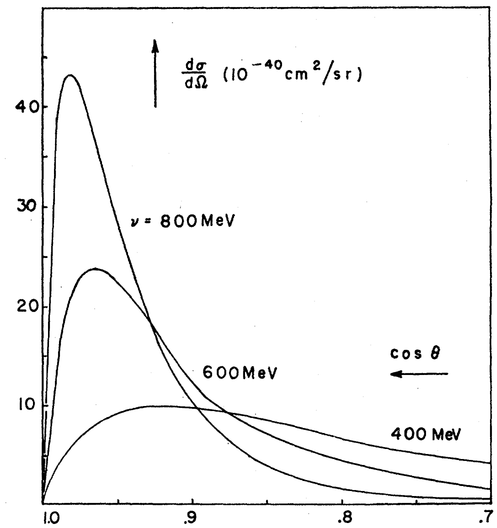

FIG. 2. Lepton angular distribution from the excitation of collective vibrations in C^{12} by neutrinos of 400, 600, and 800 MeV energy.

III. DISCUSSION

The cross section for the collective excitations turns out larger by a factor ~ 50 than the corresponding ground-state–ground-state transition. The absolute value may however not be too reliable, since it depends sensitively on the form factors, as noted above; also, the Goldhaber-Teller mode tends to overestimate the dipole matrix element, as remarked in Ref. 3; further, anisotropy of particles emitted in the photonuclear effect indicates some participation of direct interactions.[16] If the cross section of reaction (28) per neutron is compared with the direct reaction[17–19]

$$\nu + n \rightarrow p + l^-, \quad (31)$$

we find a ratio of about 5%. However, the reaction (31) may be inhibited somewhat by the exclusion principle.[20] The CERN experiment[21] so far obtained 2000 events of reaction (31) with $l = \mu$ in a brass-plated spark chamber, which would let one expect ~ 100 giant dipole excitation events.

The CERN and Brookhaven experiments used a neutrino flux of roughly 10^3–10^4/cm^2 sec below 1 BeV. With synchrocyclotrons of ~ 500 MeV energy, e.g., the CERN–SC, one may expect[22,23] perhaps $\sim 10^6$

[15] H. Überall, Phys. Rev. **126**, 876 (1962).

[16] Reference 14, p. 128.

[17] T. D. Lee and C. N. Yang, Phys. Rev. Letters **4**, 307 (1960).

[18] N. Cabibbo and R. Gatto, Nuovo Cimento **15**, 304 (1960).

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neutrinos/cm^2 sec; unfortunately, the energies of the neutrinos produced are small,[23] $\lesssim 250$ MeV. Nevertheless, perhaps several events per day of reaction (1) may be expected.

The question is how to identify these. Characteristic features of reaction (1) are the following:

(a) The lepton energy is given by $E_l=\nu-\delta$, i.e., it is independent of emission angle. The initial neutrinos are not monochromatic; their spectrum is known from calculation, however, and the leptons in Eq. (1) would have a spectrum equal to the neutrino spectrum, for $\nu \gtrsim 300$ MeV; below that, their spectrum is known from folding.

(b) The lepton angular distribution is more forward than in reaction (31), where angles below 20° are largely depressed owing to the exclusion principle.[19,20,24]

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(c) The excited dipole state will decay in a characteristic fashion,[14] i.e., $\rightarrow {}_7N^{12}+\gamma$, ${}_7N^{12*}+\gamma$, ${}_6C^{11}+p$, ${}_6C^{11*}+p$. The emitted particles have characteristic energies, given by the dipole state excitation energy Δ less the sum of the rest energies of the pair of final particles above the ${}_7N^{12}$ rest energy, with the excitation energy of the residual nucleus subtracted if it is formed in an excited state.

In total, there seems to be a good chance to observe the excitation of the giant dipole and related collective modes by neutrinos, even though the cross section is smaller than that of the direct (elastic) reactions. After the now-contemplated construction of high-intensity accelerators or "pion factories," the observation should be rather easy.

ACKNOWLEDGMENT

Helpful discussions with Dr. W. Danos are greatly appreciated. I also wish to thank Dr. P. Szydlik for aid with the numerical computations.

PHYSICAL REVIEW VOLUME 137, NUMBER 3B 8 FEBRUARY 1965

Quadrupole-Dipole Mixture of the N^{14} 3.95 → 0 Gamma-Ray Transition

F. RIESS AND W. TROST
Physikalisches Institut der Universität Freiburg, Freiburg, Germany

AND

H. J. ROSE AND E. K. WARBURTON*
Nuclear Physics Laboratory, Oxford, England

(Received 18 September 1964)

The quadrupole-dipole mixing ratio of the 4% gamma-ray branch from the N^{14} second excited state at 3.95 MeV to the ground state has been determined using the $C^{12}(He^3,p)N^{14}$ reaction and an He^3 beam of 5 MeV. Gamma rays due to the decay of the 3.95-MeV level were detected in coincidence with protons of the right energy to populate the 3.95-MeV level. The protons were detected in a surface-barrier counter at 0° with respect to the He^3 beam, and the populations of the magnetic substates were fixed from a simultaneous observation of the transition to the 0^+, 2.31-MeV level. Two regions of values for the mixing ratio x consistent with the experiment were found, namely, $-0.5\leq x\leq -0.2$ and $-5\leq x\leq -2$.

I. INTRODUCTION

THE extremely long lifetime of C^{14} is an experimental fact which has not received an unambiguous explanation despite several extensive efforts.[1–5] The crucial point is that the cancellation between the various contributions to this matrix element which must take place in order to reach the long lifetime of C^{14} cannot be achieved with a pure s^4p^{10} configuration and a conventional shell-model interaction, i.e., a central interaction between the particles together with a single-particle spin-orbit force. Two possible explanations for this cancellation have been advanced.[1] In one the s^4p^{10} configuration is assumed to be pure and the necessary modification of the s^4p^{10} wave function is achieved by introducing a small (but in this case non-negligible) tensor interaction between the nucleons.[2–4] In the other the cancellation is attributed to destructive interference between the contribution to the matrix element from s^4p^{10} and the contribution from admixtures of the doubly excited configurations generated by raising two p-shell nucleons into the $2s$ and $1d$ shells.[5] At the pres-

* National Science Foundation senior postdoctoral fellow, 1963–4. Permanent address: Brookhaven National Laboratory Upton, New York.
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