

THE PHYSICAL REVIEW

A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, Vol. 137, No. 3B

8 FEBRUARY 1965

Hyperfine Structure of Lu^{176m} by the Method of Atomic Beams*

M. B. WHITE† AND S. S. ALPERT‡

Physics Department, University of California, Berkeley, California

S. PENSELIN,§ T. I. MORAN, AND V. W. COHEN

Brookhaven National Laboratory, Upton, New York

AND

E. LIPWORTH||

Brandeis University, Waltham, Massachusetts

(Received 18 September 1964)

The hyperfine structure interaction constants of 3.7-h Lu^{176m} in the ${}^2D_{3/2}$ and ${}^2D_{5/2}$ electronic states have been measured using the atomic beam magnetic resonance technique. The nuclear spin was found to be 1. After correcting for the mutual perturbations between the two electronic states, we obtain for the ${}^2D_{3/2}$ state, $A = 97\,196\,440(300)$ cps and $B = -635\,193\,140(700)$ cps, while for the ${}^2D_{5/2}$ state, $A = 73\,172\,850(300)$ cps and $B = -781\,974\,690(700)$ cps. These lead to values of $\mu = 0.318(3)$ nm and $Q = -2.39(4)$ b for the nuclear moments. The values of g_j are 0.79931(5) for the ${}^2D_{3/2}$, and 1.20040(16) for the ${}^2D_{5/2}$ state. Comparing the ratio $A_{5/2}/A_{3/2}$ in Lu^{175} and in Lu^{176m} , one finds a difference in the hyperfine structure anomalies. The ratio $B_{5/2}/B_{3/2}$ is the same for the two isotopes within the precision of the presently available data in Lu^{175} .

I. INTRODUCTION

PREVIOUS investigations of the hyperfine structure of stable¹⁻⁵ Lu^{175} and radioactive^{6,7} Lu^{177} by atomic beam and spectroscopic methods have determined the nuclear spins of these isotopes and values of the magnetic dipole and electric quadrupole interaction constants. The nuclear magnetic dipole moment of Lu^{175} has been measured directly by nuclear magnetic resonance⁸ and atomic beams.⁵

Lutetium isotopes with their large electric nuclear quadrupole moments should be particularly interesting objects for study by high-precision resonance methods. Since both the ${}^2D_{3/2}$ and ${}^2D_{5/2}$ electronic states are well populated at the temperature required to produce an atomic beam, it is possible to measure both the magnetic dipole and electric quadrupole interaction constants A and B in two states of several isotopes with large nuclear deformation. An intercomparison of the values of the interaction constants for the two atomic states between different isotopes might be expected to show up small nuclear structure effects on the hyperfine interaction.

According to simple hyperfine structure theory, the relationship

$$B_{5/2}/B_{3/2} = eq_{5/2}/eq_{3/2}$$

should hold, for Q should be the same in both the $\frac{3}{2}$ and $\frac{5}{2}$ states. Furthermore, the ratio of the B 's is expected to be the same for all isotopes of a given element because the electric field gradient at the nucleus is determined by the electronic and not the nuclear structure. Deviations from this rule, which can only be studied experimentally when several isotopes of an element are investigated in more than one electronic state, might conceivably be due to (1)

* Work was performed under the auspices of the U. S. Atomic Energy Commission.

† Present address: Philco Research Laboratories, Newport Beach, California.

‡ Present address: Physics Department, Columbia University, New York, New York.

§ Work carried out at Brookhaven National Laboratory while on leave from the University of Heidelberg, Heidelberg, Germany.

|| Work carried out while at the Lawrence Radiation Laboratory, University of California, Berkeley, California and Brookhaven National Laboratory, Upton, New York.

¹ H. Schuler and T. H. Schmidt, *Z. Physik* **95**, 265 (1935).

² P. F. A. Klinkenberg, *Physica* **21**, 53 (1955).

³ H. Gollnow, *Z. Physik* **103**, 443 (1936).

⁴ A. Stuedel, *Z. Physik* **152**, 599 (1958).

⁵ G. Ritter, *Phys. Rev.* **126**, 252 (1962).

⁶ F. R. Petersen and H. Shugart, *Phys. Rev.* **126**, 252 (1962).

⁷ A brief report of the nuclear spin of Lu^{176m} was given by M. B. White, S. S. Alpert, and E. Lipworth, *Bull. Am. Phys. Soc.* **5**, 273 (1960).

⁸ A. H. Reddoch and G. J. Ritter, *Phys. Rev.* **126**, 1493 (1962).

higher order effects of finite nuclear volume (analogous to the magnetic hyperfine anomaly),⁹ (2) a nonlinearity in the atom core polarization (Sternheimer effect),¹⁰ or (3) a nuclear electric polarization which differs in the two electronic states.¹¹ Rough theoretical estimates indicate that these effects might conceivably show up if the B 's are measured to an accuracy of the order of 1 part in 10^7 or 10^8 .

As a first step towards detecting such effects, we studied the hfs of the two 2D states of the radioactive isotope Lu^{176m} ($\tau=3.7$ h) applying high precision methods. The results can be compared with measurements of the same interaction constants of the stable isotope Lu^{175} . Unfortunately, the accuracy of the B factors for Lu^{175} is smaller by a factor of about 10 than the accuracy we obtained for Lu^{176m} . The comparison is therefore limited at present by the Lu^{175} data (relative accuracy of the B factors about 3 to 5 parts in 10^6), and no effect of the type mentioned above shows up. Experiments are in progress to refine the data on Lu^{175} .

The isotopes Lu^{176m} and Lu^{175} are of particular interest, because their nuclear quadrupole moments differ by a factor of about two.

II. EXPERIMENTAL METHOD

The behavior¹²⁻¹⁴ of the $^2D_{5/2}$ energy levels of Lu^{176m} as a function of magnetic field is shown schematically in Fig. 1. The transitions labeled α and β are observable in a "flop-in" type apparatus. The α transitions are first examined at weak magnetic fields and followed to progressively higher fields until approximate values of A and B can be obtained. From these values, one can calculate the hyperfine intervals at low field and search there for the ($\Delta F = \pm 1$) or β transitions.

Observation of the direct transitions near zero field suffices to determine A and B to within a few kc/sec. These transitions, being field dependent, are broadened by C -field inhomogeneities. The transitions labeled β , however, pass through frequency minima at certain values of H where consequently the field dependence is small enough to permit the "Ramsey" separated oscillatory field method to be used. It is from the extremely sharp resonances obtained in this way that the final A 's and B 's are evaluated.

The frequencies of the observable $\Delta F=0$ transitions are highly field-dependent. At low fields they depend strongly on I , J , and g_I but are almost independent of g_I . Therefore, all these quantities except g_I can be

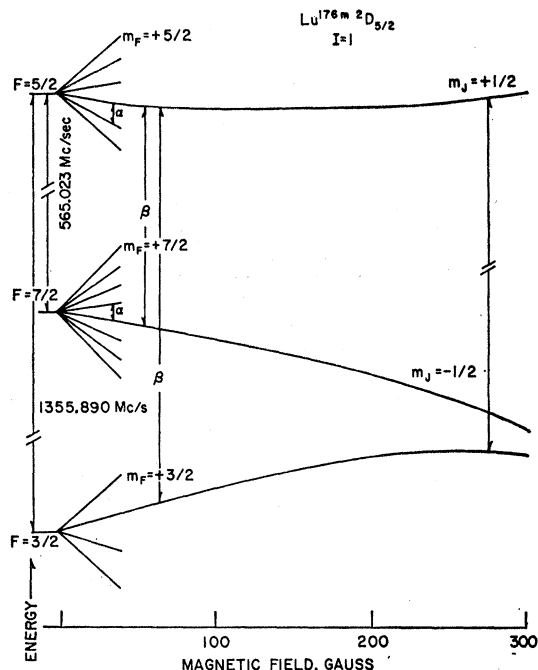


FIG. 1. Schematic diagram of the magnetic energy levels of the $^2D_{5/2}$ state of Lu^{176m} as a function of magnetic field (not to scale). The $^2D_{3/2}$ state is somewhat similar.

obtained from measurements of low-field α -transition frequencies.

III. EXPERIMENTAL PROCEDURE AND DATA

The initial experimental work, as well as a large portion of the data analysis, was done at the Lawrence Radiation Laboratory. The experimental procedure used in this early work (which was for the most part identical to that utilized at Brookhaven National Laboratory) and the atomic-beam apparatus employed have been described in detail elsewhere.¹⁵⁻¹⁷ During this early stage of the work, the nuclear spin ($I=1$) was measured and, after the inverted hyperfine level ordering of Lu^{176m} had been established, rough values of the interaction constants were obtained for both low-lying fine structure states. The more precise measurements reported in this paper were made on the apparatus used by Penselin, Moran, Cohen, and Winkler¹⁸ for their work with Rb^{85} and Rb^{87} .

The Lu^{176m} was produced by irradiating a 150-mg sample of Lu metal in a reactor for 3 h. An atomic beam was produced by heating the metal in a tantalum

⁹ A. Bohr and V. F. Weisskopf, Phys. Rev. **77**, 94 (1950) (Bohr-Weisskopf effect).

¹⁰ R. M. Sternheimer, Phys. Rev. **105**, 158 (1957); **95**, 736 (1954); **84**, 244 (1951).

¹¹ C. Townes and A. Schawlow, *Microwave Spectroscopy* (McGraw-Hill Book Company, Inc., New York, 1955), p. 138.

¹² *Recent Researches in Molecular Beams*, edited by E. Esterman, (Academic Press Inc., New York, 1959), p. 134.

¹³ K. F. Smith, Prog. Nucl. Phys. **6**, 52 (1957).

¹⁴ W. A. Nierenberg, Ann. Rev. Nucl. Sci. **7**, 349 (1957).

¹⁵ Matthew B. White, thesis, University of California, Lawrence Radiation Laboratory Report No. 10321, 1962 (unpublished).

¹⁶ H. L. Garvin, T. M. Green, and E. Lipworth, Phys. Rev. **111**, 534 (1958).

¹⁷ Hugh L. Garvin, thesis, University of California, Lawrence Radiation Laboratory Report No. 8860, 1959 (unpublished).

¹⁸ S. Penselin, T. I. Moran, V. W. Cohen, and G. Winkler, Phys. Rev. **127**, 524 (1962).

TABLE I. Lu^{176m}, $J = \frac{3}{2}$ resonance data and final hyperfine output. $A = 97.195 \pm 0.004$ Mc/sec, $B = -635.191 \pm 0.005$ Mc/sec, $\chi^2 = 1.4$, $g_J = 0.799311 \pm 0.000023$, $g_I < 0$.^a

Run No.	ν_K (Mc/sec)	H (G)	ΔH (G)	ν_{Lu} (Mc/sec)	$\Delta \nu_{Lu}$ (Mc/sec)	Residual (Mc/sec)	F_1	M_1	F_2	M_2	Weight factor
1	500.0	278.794	0.008	271.57	0.020	0.009	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	2120.9
2	500.0	278.794	0.008	191.645	0.020	-0.003	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	2313.8
3	1100.0	504.321	0.007	517.895	0.030	-0.007	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	1036.2
4	0.28	0.40	0.03	1574.450	0.030	0.014	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	424.5
5	0.28	0.40	0.03	1575.500	0.030	-0.007	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	423.4
6	1110.0	507.986	0.007	1270.500	0.015	-0.001	$\frac{3}{2}$	$-\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	4444.4
7	0.20	0.29	0.03	550.980	0.020	0.000	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	2473.0
8	6.0	8.25	0.02	5.530	0.015	-0.006	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	2482.1
9	6.0	8.25	0.02	6.820	0.015	0.005	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	2006.7
10	2.0	2.81	0.02	1.890	0.015	-0.002	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	2373.5
11	40.0	46.08	0.08	30.95	0.10	0.003	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	76.9
12	100.0	93.04	0.14	62.650	0.07	0.027	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	72.4
13	200.0	149.71	0.16	101.25	0.10	0.046	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	43.9
14	300.0	196.28	0.18	133.40	0.14	0.059	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	28.9
15	1100.0	504.32	0.25	365.40	0.50	-0.217	$\frac{5}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	3.4
16	12.0	15.92	0.06	13.20	0.10	-0.040	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	78.7
17	24.0	29.84	0.07	25.10	0.04	-0.005	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	187.5
18	40.0	46.08	0.08	39.30	0.05	0.016	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	130.1
19	100.0	93.04	0.14	82.20	0.10	0.014	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	36.7
20	200.0	149.71	0.16	137.10	0.10	-0.116	$\frac{3}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	26.9

^a The sign convention used for g_j and g_i is the one such that the g -factor for the free electron is positive, or $g_j I = -\mu_j/\mu_0$ and $g_i I = -\mu_i/\mu_0$.

oven to a temperature of about 2000°K by electron bombardment.

The oscillatory field power for the Ramsey loops was obtained by phase-locking a triode oscillator to a

precise variable reference signal obtained from a synthesizer system described by Cohen, Moran, and Penselin.¹⁹ The main crystal reference oscillator was compared with the standard 18-kc/sec signal transmitted by U. S. Naval Radio Station NBA in Panama. The General Radio Oscillators Nos. 1209A and 1218A were coupled to a varactor as shown in Fig. 2. The change in capacity of the varactor with the correction voltage applied by the syncriminator "pulls" the oscillator to lock it at the desired frequency.

The value of the C field was monitored frequently by observing field dependent resonances in K³⁹.

The $\Delta F = 0$ transitions in Lu^{176m} were observed at fields up to 500 G, corresponding to ν_K ³⁹ of 1100 Mc/sec. Tables I and II summarize the observed transitions together with the A and B values derived from them. The four direct transitions were observed at very weak field with a single rf loop. Linewidths were of the order of 25 kc/sec. Table III lists the zero field extrapolations of the transition frequencies thus obtained.

In the vicinity of the β transition frequency minima, the "Ramsey" type resonances with the maximum loop separation of 25 cm were about 2400 cps wide between adjacent valleys. A typical Ramsey resonance is shown in Fig. 3. The frequencies of the four observed "field-independent" transitions, together with the resulting A 's and B 's, are given in Table IV. To confirm that a

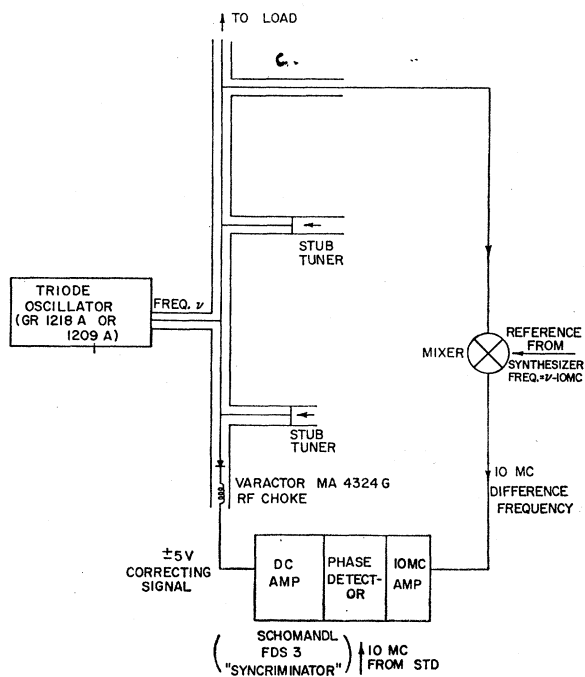


FIG. 2. Schematic circuit diagram of the phase locking circuit for the triode oscillator.

¹⁹ V. W. Cohen, T. I. Moran, and S. Penselin, Phys. Rev. **127**, 517 (1962).

TABLE II. Lu^{176m}, $J = \frac{5}{2}$ resonance data and final hyperfine output; $A = 77.6 \pm 2.4$ Mc/sec, $B = -782.3 \pm 4.8$ Mc/sec, $\chi^2 = 2.2$, $g_I < 0$.

Run No.	ν_K (Mc/sec)	H (G)	ΔH (G)	ν_{Lu} (Mc/sec)	$\Delta \nu_{Lu}$ (Mc/sec)	Residual (Mc/sec)	F_1	M_1	F_2	M_2	Weight factor
1	6.0	8.25	0.04	9.90	0.03	0.001	$\frac{7}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$-\frac{1}{2}$	315.1
2	40.0	46.08	0.06	55.40	0.07	-0.001	$\frac{7}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$-\frac{1}{2}$	96.0
3	100.0	93.04	0.14	112.50	0.10	0.015	$\frac{7}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$-\frac{1}{2}$	25.5
4	200.0	149.71	0.15	182.90	0.13	0.044	$\frac{7}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$-\frac{1}{2}$	19.0
5	500.0	278.80	0.28	352.80	0.50	0.291	$\frac{7}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$-\frac{1}{2}$	2.5
6	1100.0	504.33	0.29	694.50	0.70	0.011	$\frac{7}{2}$	$\frac{1}{2}$	$\frac{7}{2}$	$-\frac{1}{2}$	1.4
7	6.0	8.24	0.04	12.30	0.07	-0.024	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	$-\frac{3}{2}$	118.3
8	12.0	15.92	0.06	23.90	0.07	0.024	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	$-\frac{3}{2}$	73.5
9	24.0	29.84	0.07	45.00	0.10	-0.037	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	$-\frac{3}{2}$	46.0
10	40.0	46.08	0.06	70.00	0.07	-0.053	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	$-\frac{3}{2}$	71.4
11	100.0	93.04	0.14	143.90	0.10	-0.109	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	$-\frac{3}{2}$	16.8
12	200.0	149.71	0.14	235.00	0.22	-0.317	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	$-\frac{3}{2}$	10.0
13	500.0	278.80	0.28	446.40	0.60	0.425	$\frac{5}{2}$	$-\frac{1}{2}$	$\frac{5}{2}$	$-\frac{3}{2}$	1.8

frequency minimum for the $J = \frac{5}{2}$, $F = \frac{5}{2} \rightarrow F = \frac{3}{2}$ transition was being observed, the field was increased by 0.4 G

TABLE III. Experimental resonances in weak fields.

Electronic state	Transition	ν_K (kc/sec)	Observed frequency (kc/sec)	Extrapolated to zero field (kc/sec)
$^2D_{3/2}$	$(\frac{5}{2}, -\frac{1}{2}) \leftrightarrow (\frac{3}{2}, -\frac{1}{2})$	200	550 980(15)	551 023(20)
	$(\frac{5}{2}, -\frac{1}{2}) \leftrightarrow (\frac{3}{2}, +\frac{1}{2})$	180	1 574 675(20)	1 575 035(30)
$^2D_{5/2}$	$(\frac{7}{2}, -\frac{1}{2}) \leftrightarrow (\frac{5}{2}, -\frac{1}{2})$	200	564 942(10)	565 024(15)
	$(\frac{7}{2}, +\frac{1}{2}) \leftrightarrow (\frac{5}{2}, -\frac{1}{2})$	200	1 356 420(10)	
	$(\frac{7}{2}, -\frac{1}{2}) \leftrightarrow (\frac{5}{2}, +\frac{1}{2})$	200	1 355 330(10)	
	$(\frac{7}{2}, +\frac{1}{2}) \leftrightarrow (\frac{5}{2}, -\frac{1}{2})$	275	1 356 620(10)	

which led to a resonance frequency increase of approximately 400 cps. This is in good agreement with the theoretical relationship exhibited in Fig. 4, between the resonance frequency and field near a frequency minimum. For each transition, several resonance curves were taken. The limit of experimental error assigned to a transition frequency was based on the minimum error reasonably consistent with all these observations. The comparatively poor signal-to-noise ratio in lutetium resonances was due to the inability of the low-gradient A and B magnets to deflect any but the slowest atoms around the stop wire.

TABLE IV. Observed transition frequencies near the frequency minima and final A 's and B 's.

Electronic state	Transition ($F, m \leftrightarrow F', m'$)	Observed frequency (cps)	Field calibrating frequency ν_K^{39} (Mc/sec)	A (cps)	B (cps)
$^2D_{3/2}$	$(\frac{5}{2}, -\frac{1}{2}) \leftrightarrow (\frac{3}{2}, -\frac{1}{2})$	530 330 050(200)	13.670	97 196 440(300)	-635 193 140(700)
	$(\frac{5}{2}, -\frac{1}{2}) \leftrightarrow (\frac{3}{2}, +\frac{1}{2})$	1 270 504 300(200)	1116.00		
$^2D_{5/2}$	$(\frac{7}{2}, -\frac{1}{2}) \leftrightarrow (\frac{5}{2}, -\frac{1}{2})$	563 981 900(300)	10.240	73 172 850(300)	-781 974 690(700)
	$(\frac{7}{2}, -\frac{1}{2}) \leftrightarrow (\frac{5}{2}, +\frac{1}{2})$	1 119 200 000(200)	490.700		

³⁹ H. L. Garvin, T. M. Green, E. Lipworth, and W. A. Nierenberg, Phys. Rev. 116, 393 (1959).

IV. EVALUATION OF THE DATA

Data pertaining to the field dependent transitions observed are listed in Tables I and II. An IBM program —HYPERFINE²⁰ was used to reduce these data to obtain

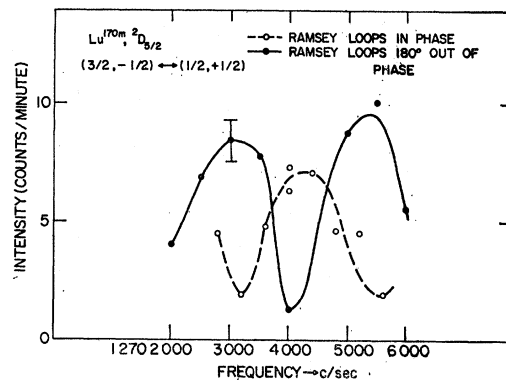


FIG. 3. A typical Ramsey type resonance with maximum loop spacing for the $(\frac{3}{2}, -\frac{1}{2}) - (\frac{1}{2}, +\frac{1}{2})$ transition.

the preliminary values of A 's, B 's, g_J 's, and g_I also listed in these tables.

For the final determination of the A 's and B 's, the transitions listed in Table IV were traced out around the frequency minima. The values of A and B obtained from HYPERFINE were corrected for (1) effects of the

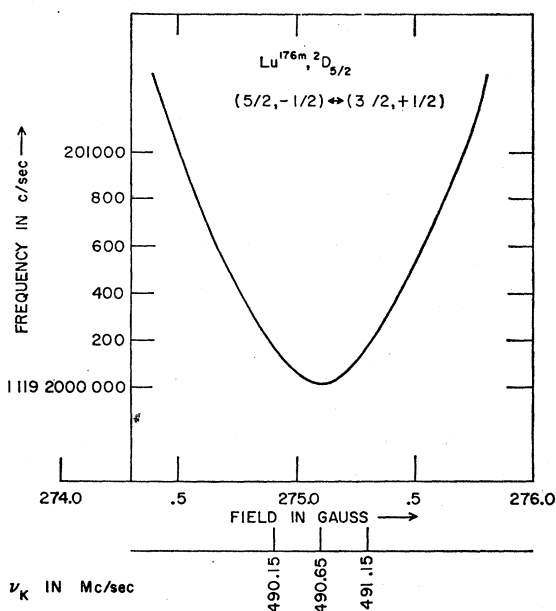


FIG. 4. The calculated frequency of the ${}^2D_{5/2}$ state transition $(\frac{5}{2}, -\frac{1}{2}) - (\frac{3}{2}, +\frac{1}{2})$ as a function of field in the vicinity of the minimum.

perturbation between $D_{5/2}$ and $D_{3/2}$ states and (2) mixing of the $6s^25d - 6s5d$ configurations.

In the notation of Schwartz,²¹ the Hamiltonian of the fine structure and hyperfine structure interaction of the $5d6s^2$ configuration in Lu^{176m} may be written

$$\mathcal{H} = \sum_{k=1,2} \tilde{T}_e^k \cdot \tilde{T}_n^k + \xi \mathbf{L} \cdot \mathbf{S} - \mathbf{u}_L \cdot \mathbf{H} - \mathbf{u}_S \cdot \mathbf{H} - \mathbf{u}_I \cdot \mathbf{H},$$

where \tilde{T}_e^k and \tilde{T}_n^k are tensor operators of rank k acting respectively on the electron and nuclear variables only, where²² $\xi(r) = (1/2m^2c^2)[(1/r)dV(r)/dr]$ and for Lu has the value 2.393×10^7 Mc/sec. Since the nuclear spin of Lu^{176m} is 1, tensors of higher rank than 2 (corresponding to interactions higher than quadrupole) are not present. In evaluating the appropriate matrix elements of the above Hamiltonian, the relativistic correction factors tabulated by Kopfermann²³ were used. Furthermore, the absolute value of the ratio of the normalization constants C''/C' for the $5d$ electron in the $D_{3/2}$ and $D_{5/2}$ states was assumed to be 1.²⁴ Diagonalization of the full matrix representation of H for the two 2D states including the off-diagonal matrix elements led, in the usual way, to a final A and B values of Table IV.

While the errors in measurements of the transitions were of the order of 200 cycles, an additional uncertainty is contributed by the term $\mathbf{u}_I \cdot \mathbf{H}$. The contribution of

this term, at 500 G, amounts to 80 KC; and since the precision with which \mathbf{u}_I is known is of the order of 1%, the uncertainty in the calculation of A and B will be of the order of 300 and 700 cycles respectively due to the uncertainty in \mathbf{u}_I alone.

The value of the nuclear magnetic dipole moment of Lu^{176m} may be calculated from the moment of Lu¹⁷⁵, $\mu = 2.230(11)$ as measured by Reddoch and Ritter using nuclear resonance,⁸ and the ratio of the A values of⁵ Lu¹⁷⁵ and Lu^{176m}. Due to a difference in the hfs anomalies (see Sec. 5) in the $D_{5/2}$ and $D_{3/2}$ states, the value of μ_{176m} calculated from the relation

$$\mu_{176m} = \frac{\mu_{175} A_{176m}}{\frac{1}{2} A_{175}}$$

is slightly different in the two states. Taking Ritter's values for the $\frac{3}{2}$ and $\frac{5}{2}$ states in Lu¹⁷⁵; namely, $A_{3/2} = 194.3316(4)$ and $A_{5/2} = 146.7790(8)$, we get

$$\mu_{176m} = 0.3187(15) \text{ using the } \frac{3}{2} \text{ data}$$

and

$$\mu_{176m} = 0.3176(15) \text{ using the } \frac{5}{2} \text{ data.}$$

The limit of error indicated is due only to the moment of Lu¹⁷⁵. One might choose a "best value" of 0.318(3) with the error including uncertainties both in μ_{175} and the hfs anomaly difference.

To obtain the nuclear electric quadrupole moment from the interaction constant B , we may use the expressions given by Ritter.⁵ The value of $1/\langle r^3 \rangle_{5d}$ involved in the calculation can be evaluated from the known fine structure splitting using the relationship

$$\delta = 2.911H(l, Z_i)Z_i(2l+1)a_0/\langle r^3 \rangle,$$

where $H(l, Z_i)$ is a relativistic factor, a_0 the Bohr radius, and Z_i is taken as 57.8. We obtain $Q_{(176m)} = -2.39(4)$ b, uncorrected for core polarization effects. These nuclear moments are not easy to explain on the basis of any existing nuclear model. The collective model, which might explain the large quadrupole moment, does not offer any simple assignment of neutron and proton states which can explain the observed nuclear magnetic moment.

V. DISCUSSION

As in the case of Lu¹⁷⁵, the measured ratios $A_{5/2}/A_{3/2}$ and $B_{5/2}/B_{3/2}$ for Lu^{176m} differ appreciably from those calculated on the basis of a pure $6s^25d$ electronic configuration. In fact

$$(A_{5/2}/A_{3/2})_{\text{meas}} = 0.752831, \quad (B_{5/2}/B_{3/2})_{\text{meas}} = 1.231083,$$

while

$$(A_{5/2}/A_{3/2})_{\text{calc}} = 0.41142, \quad (B_{5/2}/B_{3/2})_{\text{calc}} = 1.2538.$$

²¹ C. Schwartz, Phys. Rev. **97**, 380 (1955).

²² E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, New York, 1953), p. 193.

²³ H. Kopfermann, *Nuclear Moments*, translated by E. E. Schneider (Academic Press Inc., New York, N. Y., 1958), 2nd ed.

²⁴ H. B. G. Casimir, *On the Interaction Between Atomic Nuclei and Electrons* (Teyler's Tweede Genootschap, Haarlem, 1936).

Now s electrons, which have a large probability density at the nucleus, are most effective in producing magnetic hyperfine interactions. We therefore infer, since the discrepancy is so much greater for the ratio of the A 's than for the B 's that it is probably due to an admixture of excited $6sns5d$ configurations ($n \geq 7$) into the ground-state configuration. This point is strongly supported by the following experimental evidence. Comparing the A factors for Lu^{175} and Lu^{176m} , we find

$$\frac{(A_{175}/A_{176m})_{5/2}}{(A_{175}/A_{176m})_{3/2}} = \frac{(1 + {}_{175}\Delta_{176m})_{5/2}}{(1 + {}_{175}\Delta_{176m})_{3/2}} = 1 + [{}_{175}\Delta_{176m}]_{5/2} - [{}_{175}\Delta_{176m}]_{3/2} = 1.003773(6),$$

where Δ is the hyperfine anomaly for the appropriate electronic state. The fact that this ratio differs from 1 by as much as 0.003 implies that the Δ 's must be appreciable and that there must be a significant admixture of higher s states into the ground state.

In analogy to the hfs anomaly of the magnetic dipole interaction, one can define for the quadrupole interaction a quantity δ by the relation

$$B = eqQ(1 + \delta).$$

Here δ represents a small correction due to the effects mentioned in Sec. I. Considering all four B 's that have been measured for Lu^{176} and Lu^{175} , we may form the

ratio

$$\begin{aligned} & \frac{[B_{176m}/B_{175}]_{5/2}}{[B_{176m}/B_{175}]_{3/2}} \\ &= \left[\frac{eq_{5/2}Q_{176m}(1 + \delta_{176m})}{eq_{5/2}Q_{175}(1 + \delta_{175})} \right]_{5/2} / \left[\frac{eq_{3/2}Q_{176m}(1 + \delta_{176m})}{eq_{3/2}Q_{175}(1 + \delta_{175})} \right]_{3/2} \\ & \approx 1 + (\delta_{176m} - \delta_{175})_{5/2} - (\delta_{176m} - \delta_{175})_{3/2}. \end{aligned}$$

It is apparent from these relations that if the measured B ratio differs from 1, there must be some nonzero value for the δ 's. If the ratio is exactly 1, then no conclusion can be drawn since it could be consistent with $(\delta_{176m} - \delta_{175})_{5/2} = (\delta_{176m} - \delta_{175})_{3/2}$. Our measured value of 1.000005(5) therefore permits no definite conclusion to be drawn as to the presence or absence of the effects mentioned in Sec. I.

The limitation of our comparison is due principally to the experimental errors on the measurements of the B 's in Lu^{175} .

ACKNOWLEDGMENTS

We should like to acknowledge assistance from Mats Olsmats and Max Lipsicas during certain phases of this research and to Alfred W. Kane and John Ciperano for their technical assistance. We wish to thank Dr. Joseph Weneser, Dr. R. M. Sternheimer, Dr. M. Sternheim, and Dr. M. Blume for many helpful discussions; and in particular, for their pointing out to us that the configuration mixing in lutetium would enhance a quadrupole Bohr-Weisskopf effect.