discrepancy is due to the Irving-Gunn wave function becoming too singular near the origin.

In summary, our analysis of electron-trinucleon elastic scattering shows that (i) present theory is inadequate to use electron-trinucleon scattering as a reliable means of finding the electric form factor of the neutron; (ii) the scattering can be interpreted to give an S' state probability of either zero, or preferably of about 1%, in contrast to the original[3] interpretation of 4% probability and in agreement with later estimates[9]; (iii) the slope $F_{xV}'(0)$ is not in serious disagreement with the preliminary result[12] of Padgett *et al.*; (iv) nuclear wave functions for the trinucleon chosen with plausible shape, and with parameters adjusted to fit the Coulomb energy, give good fits to the form factor for the dominant S state.

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Unitary Symmetry and Weak Interactions. III. Nonleptonic Hyperon Decay*

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It is shown that B. W. Lee's relation for nonleptonic hyperon decay can be derived from T-L invariance and $\Delta T=\frac{1}{2}$, and that the vanishing of $\alpha(\Sigma^+\to n\pi^+)$ requires the additional assumption of R invariance. The vanishing of $\alpha(\Sigma^-\to n\pi^-)$ cannot be derived from these symmetries, and since there are no others applicable to weak interactions in $SU(3)$, it must result from weak-interaction dynamics. A comparison is made between this theory and those of Cabibbo and Coleman, Glashow and Lee. Mathematical aspects of T-L invariance are discussed in an appendix.

1. INTRODUCTION

In the first paper[1] of this series, a theory of weak interactions was proposed within the framework of unitary symmetry.[2] Several properties of nonleptonic hyperon decay, including Lee's relation[3,4]

$$\sqrt{3}\langle\Sigma^+|p\pi^0\rangle-\langle\Lambda|p\pi^-\rangle=2\langle\Xi^-|\Lambda\pi^-\rangle \tag{1}$$

and

$$\alpha(\Sigma^+\to n\pi^+)\approx 0\,, \tag{2}$$

were derived from the $\Delta T=\frac{1}{2}$ rule[5], T-L invariance and R conjugation. Despite its empirical success,[4] this derivation can be criticized on the grounds that R conjugation is not a valid symmetry of strong interactions and should not be applied to weak ones.[6] It is therefore desirable to ask whether the results in (1) and (2) can be derived without R conjugation.

Associated with this question is another, more general one. In any theory of elementary particles, the symmetries of weak interactions are limited by electric charge conservation.[7] Their number varies from one theory to another, and so does our ability to derive the properties of weak decays from them. In global symmetry,[8] for example, three weak symmetries are available, and when combined with $\Delta T=\frac{1}{2}$, they predict all the properties of nonleptonic hyperon decay.[9] By contrast, unitary symmetry contains only two weak symmetries, namely T-L invariance and R conjugation.[10] Since they are not as rich in predictions as the global symmetry ones, we must ask to what extent do they account for nonleptonic-hyperon decay.

The usefulness of this question arises in the following way. If a given property can be derived from symmetry principles, it may not cast much light on the dynamics of weak interactions. If, however, it is not derivable from symmetry principles, it must be a consequence of dynamics, and hence it provides a definitive test for dynamical models. A case in point is the vanishing of $\alpha(\Sigma^-\to n\pi^-)$. As shown below, this result does not

* Work supported in part by NSF and U. S. Air Force.
[1] S. P. Rosen, Phys. Rev. Letters **12**, 408 (1964).
[2] M. Gell-Mann, California Institute of Technology Report No. CTSL-20, 1961 (unpublished); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[3] B. W. Lee, Phys. Rev. Letters **12**, 83 (1964).
[4] M. L. Stevenson *et al.*, Phys. Letters **9**, 349 (1964). For a summary of other data on nonleptonic hyperon decay, see F. S. Crawford, *Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962* (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 827.
[5] For a discussion of the $\Delta T=\frac{1}{2}$ rule in nonleptonic decay, see R. H. Dalitz, *International Conference on Fundamental Aspects of Weak Interactions* (Brookhaven National Laboratory, Upton, New York, 1964).
[6] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964).
[7] S. P. Rosen, Phys. Rev. **135**, B1041 (1964).
[8] A. Pais, Phys. Rev. **110**, 574, 1480 (1958); **112**, 624 (1958).
[9] S. P. Rosen, Phys. Rev. Letters **9**, 186 (1962).
[10] The conservation of electric charge can be expressed in the form (Ref. 7), $\Delta Q\equiv\Delta Y_T+\Delta Y_L$. This relation is invariant only under $(Y_T\leftrightarrow Y_L,\ Q\to Q)$, and $(Q,Y_TY_L)\to(-Q,\ -Y_T,\ -Y_L)$. The former corresponds to T-L transformations and the latter to R conjugation.

follow either from T-L invariance or from R conjugation. Therefore, the prediction[4]

$$\alpha(\Sigma^- \to n\pi^-) \approx 0 \tag{3}$$

is an essential criterion for the success of a dynamical model of weak decays.

To a lesser degree, the same may be said about $\alpha(\Sigma^+ \to n\pi^+)$. We shall see later that Eq. (2) is derivable only with the aid of R conjugation. This derivation is subject to the criticism made in the first paragraph, and so its validity is rather doubtful. It is still possible, however, that the dynamics are such as to make R conjugation an *effective* symmetry of weak interactions.[3]

As indicated in our opening remarks, T-L invariance may be the only valid symmetry of weak interactions. The T-L transformation was originally defined as an interchange of isotopic spin and L-spin quantum numbers,[1] and as such it can be generated by any one of a large number of $SU(3)$ transformations.[11] These transformations all permute a unitary multiplet in the same way, but each one associates a different set of phases with the permutation. Since weak interactions are not invariant under the full $SU(3)$ group, differences of phase may lead to different physical consequences. Therefore, when we make statements about the consequences of T-L invariance, we must state precisely which form is being used.

Throughout the analysis we shall assume time-reversal invariance. When final state interactions are neglected, this assumption leads to "crossing" relations between the matrix elements $\langle Y|B\pi\rangle$ and $\langle B|Y\pi^*\rangle$. They are very useful in deriving the consequences of weak symmetries, but must be used with some care. It turns out that the crossing relations engendered by derivative coupling in the effective Hamiltonian are different from those engendered by nonderivative coupling. As a result, a given symmetry will not have the same consequences in one coupling scheme as it does in the other; however, this difficulty can be overcome by appropriate use of parity.

Our discussion is divided into five parts. We first analyze the crossing relations implied by time reversal invariance (Sec. 2) and then derive Lee's relation from T-L invariance and $\Delta T = \frac{1}{2}$ (Sec. 3). To show that this symmetry alone has no other observable consequences, we analyze in Sec. 4 the structure of the effective Hamiltonian. Next we determine the consequences of R conjugation (Sec. 5). In the final section, we compare our theory with those of other writers. Mathematical aspects of the T-L transformation are examined in an appendix.

2. TIME-REVERSAL INVARIANCE AND CROSSING RELATIONS

The most general, relativistically covariant Hamiltonian for

$$Y \to B + \pi\,, \tag{4}$$

includes both derivative and nonderivative coupling of the pion field. If Y and B are approximated as free particles on the mass shell, the derivative coupling can be reduced to a nonderivative form by means of the Dirac equation. It follows that the *effective* Hamiltonian need include only one type of coupling.

In the nonderivative coupling scheme, the effective Hamiltonian is

$$H = i[\bar{Y}(g_s + g_p\gamma_5)B]\pi + i[\bar{B}(-g_s^* + g_p^*\gamma_5)Y]\pi^*\,, \tag{5}$$

where g_s and g_p are the scalar and pseudoscalar coupling constants respectively. Time reversal invariance and the neglect of final-state interactions imply that g_s and g_p are real:

$$g_s = g_s^*\,,\ \ g_p = g_p^*. \tag{6}$$

Because the pion is pseudoscalar, Eqs. (5) and (6) lead to "crossing" relations

$$\begin{aligned}\langle Y|B\pi\rangle_{\rm pc} &= +\langle B|Y\pi^*\rangle_{\rm pc}\\ \langle Y|B\pi\rangle_{\rm pv} &= -\langle B|Y\pi^*\rangle_{\rm pv}\,,\end{aligned} \tag{7}$$

where pc denotes the parity conserving (P-wave pion) amplitude and pv the parity violating (S-wave pion) amplitude. Notice that (7) must be treated not as a precise equality, but rather as a "functional" equality[1]: that is, the left-hand side is a function of the dynamical variables (mass, momentum and spin) of Y, B, and π, and apart from the indicated sign, the right-hand side is the same function with the variables of Y, B, π, replaced by those of B, Y, π^*, respectively.

A similar analysis of the derivative coupling scheme leads to different crossing relations[12]:

$$\begin{aligned}\langle Y|B\pi\rangle_{\rm pc} &= +\langle B|Y\pi^*\rangle_{\rm pc}\\ \langle Y|B\pi\rangle_{\rm pv} &= +\langle B|Y\pi^*\rangle_{\rm pv}.\end{aligned} \tag{8}$$

To understand the difference between (7) and (8), we first note that the four coupling constants, g_s, g_p, g_v, g_a (the last two refer to vector and axial vector interactions, respectively), are taken to be independent of the masses of Y and B, and are therefore symmetric under the interchange $m_Y \leftrightarrow m_B$. Next the scalar and pseudoscalar coupling constants which arise when derivative coupling is reduced to nonderivative form, are given by:

$$\begin{aligned} g_s' &= (m_Y - m_B)g_v\,,\\ g_p' &= (m_Y + m_B)g_a.\end{aligned} \tag{9}$$

g_s' is antisymmetric under $m_Y \leftrightarrow m_B$, while g_p' remains symmetric. It is precisely this antisymmetry that removes the negative sign in (7) and yields the crossing relations of (8).

Now consider the decay modes $\Xi^- \to \Lambda\pi^-$ and $\Lambda \to p\pi^-$. If R conjugation is a valid symmetry, then

$$\langle \Xi^-|\Lambda\pi^-\rangle = \langle p|\Lambda\pi^+\rangle. \tag{10}$$

[11] S. P. Rosen (unpublished). See Appendix below.

[12] A. Pais, Phys. Rev. 122, 317 (1961).

From (7) and (10), we find[13]

$$\begin{aligned}\langle\Xi^-|\Lambda\pi^-\rangle_{\rm pc}&=-\langle\Lambda|p\pi^-\rangle_{\rm pc},\\ \langle\Xi^-|\Lambda\pi^-\rangle_{\rm pv}&=+\langle\Lambda|p\pi^-\rangle_{\rm pv},\end{aligned}\tag{11}$$

which implies[4]

$$\alpha_\Lambda\approx-\alpha_\Xi.\tag{12}$$

If we use (8) instead of (7), then[13]

$$\langle\Xi^-|\Lambda\pi^-\rangle=-\langle\Lambda|p\pi^-\rangle\tag{13}$$

for both pc and pv amplitudes, and hence

$$\alpha_\Lambda\approx+\alpha_\Xi.\tag{14}$$

Thus, the two types of coupling have different physical consequences, even though the same symmetries are used.

It also follows from this argument that if the observed relation[4,14] $\alpha_\Lambda\approx-\alpha_\Xi$ is to be derived from R invariance, we must use nonderivative coupling in the effective Hamiltonian. If, however, R invariance were replaced by invariance under the product RP of R conjugation and parity, then derivative coupling [Eq. (8)], would yield the desired result. In other words, by modifying the symmetry principle, we can derive the same result from one coupling scheme as we derived from the other.

This example illustrates a general rule:

If a given result follows from an invariance principle I and explicit use of nonderivative coupling [or equivalently, the crossing relation in (7)], then it also follows from IP and derivative coupling [Eq. (8)].

Consequently, it does not matter which coupling scheme we use; for convenience, however, we choose the derivative form, and hence the crossing relations of (8).

3. *T-L* INVARIANCE AND LEE'S RELATION

In general, the $SU(3)$ transformation properties of nonleptonic decays are very complicated, even when they satisfy the $\Delta T=\frac{1}{2}$ rule. They can, however, be greatly simplified by means of the transformation[1]:

$$\begin{aligned}(T,T_3,Y_T)&\leftrightarrow(L,-L_3,Y_L),\\ (K,K_3,Y_K)&\leftrightarrow(K,-K_3,Y_K).\end{aligned}\tag{15}$$

It has been shown elsewhere[7] that if the nonleptonic decay Hamiltonian H_{NL} satisfies $\Delta T=\frac{1}{2}$ and is symmetric with respect to (15), then it must transform according to the eight-dimensional representation of $SU(3)$. Since by definition, all T-L transformations[11] give rise to (15), this result holds for all forms of T-L invariance.

[13] We use a sign convention such that $(\pi^\pm)^*=-\pi^\mp$, $(\pi^0)^*=\pi^0$; see Ref. (11).

[14] L. Bertanza, V. Brisson, P. Connolly, E. Hart, I. Mittra, G. Moneti, R. Rau, N. Samios, I. Skillicorn *et al.*, Phys. Rev. Letters **9**, 229 (1962).

The most general form of an octet-type Hamiltonian is

$$H_{NL}=\alpha D_2{}^3+\beta D_3{}^2,\tag{16}$$

where, from time-reversal invariance, the coefficients α and β are real. Now, under any T-L transformation,[11]

$$D_2{}^3\leftrightarrow e^{i\varphi}D_3{}^2,\tag{17}$$

where the phase factor φ varies from one transformation to another. Because of the reality of α and β, there are only two choices of φ, α, β, such that H_{NL} is symmetric with respect to (15), (17):

$$\varphi=0,\quad\alpha=\beta,\tag{18a}$$

$$\varphi=\pi,\quad\alpha=-\beta.\tag{18b}$$

Accordingly, we need consider only two forms of T-L invariance, one corresponding to (18a) and the other to (18b).

Before the consequences of (18a) and (18b) are compared, it will be convenient to show how Lee's relation [Eq. (1)] may be derived. Our method depends upon three relations,

$$\langle\Sigma^+|n\pi^+\rangle=\langle\Sigma^-|\Xi^0\pi^-\rangle,\tag{19}$$

$$\begin{aligned}2\langle\Sigma^-|\Xi^-\pi^0\rangle+\sqrt{2}\langle\Sigma^0|n\pi^0\rangle+\sqrt{3}\langle\Lambda|p\pi^-\rangle&\\ =\sqrt{2}\langle\Sigma^-|\Xi^0\pi^-\rangle,&\end{aligned}\tag{20}$$

$$\langle\Sigma^-|\Xi^-\pi^0\rangle-\sqrt{3}\langle\Lambda|\Xi^-\pi^+\rangle=\langle\Sigma^+|p\pi^0\rangle-\sqrt{3}\langle\Lambda|p\pi^-\rangle,\tag{21}$$

for both pc and pv amplitudes. We eliminate the unobservable matrix elements[15] $\langle\Sigma|\Xi\pi\rangle$ and reduce (19)–(21) to one equation. Lee's relation then follows from (8) and the predictions of $\Delta T=\frac{1}{2}$:

$$\begin{aligned}\sqrt{2}\langle\Sigma^+|p\pi^0\rangle&=\langle\Sigma^-|n\pi^-\rangle-\langle\Sigma^+|n\pi^+\rangle,\\ 2\langle\Sigma^0|n\pi^0\rangle&=\langle\Sigma^-|n\pi^-\rangle+\langle\Sigma^+|n\pi^+\rangle.\end{aligned}\tag{22}$$

Our problem now is to relate (19)–(21) to the possibilities for H_{NL} in (18). For the T-L transformation corresponding to (18a), we choose T-$L(1)$, where[11]

$$\begin{aligned}T\text{-}L(1):\quad(B)\to(&p,\tfrac{1}{2}(\Sigma^0-\sqrt{3}\Lambda^0),-\Xi^-,\\ &-\tfrac{1}{2}(\sqrt{3}\Sigma^0+\Lambda^0);\Sigma^+,-\Xi^0,-n,-\Sigma^-),\\ (B)\equiv(&\Sigma^+,\Sigma^0,\Sigma^-,\Lambda;p,n,\Xi^0,\Xi^-),\end{aligned}\tag{23}$$

and for (18b) we choose T-$L(2)$[11]:

$$\begin{aligned}T\text{-}L(2):\quad(B)\to(&-p,\tfrac{1}{2}(\Sigma^0-\sqrt{3}\Lambda),\Xi^-,\\ &-\tfrac{1}{2}(\sqrt{3}\Sigma^0+\Lambda);\Sigma^+,\Xi^0,n,-\Sigma^-).\end{aligned}\tag{24}$$

The corresponding transformations for pseudoscalar mesons are obtained by substituting

$$(B)\to(\pi^+,\pi^0,\pi^-,\eta;K^+,K^0,-\bar{K}^0,K^-)\tag{25}$$

in (23) and (24).

The first point to notice is that (19) follows from T-$L(2)$ invariance but not from T-$L(1)$. To show this,

[15] The occurrence of unobservable matrix elements cannot be avoided in any theory constructed from baryon, antibaryon, and meson octets.

we consider the matrix element for $\Xi^0\to n\bar{K}^0$. The combination of T-$L(1)$ and (8) leads to a trivial identity for this matrix element; however, T-$L(2)$ and (8) imply that

$$\langle\Xi^0|n\bar{K}^0\rangle=0. \tag{26}$$

From (26) and the $\Delta T=\frac{1}{2}$ rule, we obtain

$$\langle\Xi^0|pK^-\rangle=\langle\Xi^-|nK^-\rangle. \tag{27}$$

Equation (19) now follows by applying T-$L(2)$ and (8) to (27).

Similarly, Eq. (20) is a consequence of T-$L(2)$ rather than T-$L(1)$. According to the $\Delta T=\frac{1}{2}$ rule

$$\begin{aligned}&\langle\Sigma^-|\Xi^-\pi^0\rangle-\sqrt{2}\langle\Sigma^0|\Xi^0\pi^0\rangle=-\sqrt{2}\langle\Sigma^-|\Xi^0\pi^-\rangle,\\&\langle\Sigma^-|\Xi^-\eta\rangle-\sqrt{2}\langle\Sigma^0|\Xi^0\eta\rangle=0.\end{aligned} \tag{28}$$

If we apply T-$L(1)$ to the left-hand side of (28), eliminate the η meson, and use

$$\langle\Lambda|p\pi^-\rangle=-\sqrt{2}\langle\Lambda|n\pi^0\rangle \tag{29}$$

we obtain an equation which differs from (20) only in the sign of $\langle\Sigma^-|\Xi^0\pi^-\rangle$. T-$L(2)$, however, yields (20) precisely.

By contrast, Eq. (21) can be derived from T-$L(1)$ and from T-$L(2)$. We apply either of these symmetries to

$$\langle\Sigma^+|\Sigma^0K^+\rangle=\langle\Sigma^0|\Sigma^-K^+\rangle \tag{30}$$

and then use (8)[13] together with

$$\begin{aligned}&\langle\Sigma^+|p\pi^0\rangle=\langle\Sigma^0|p\pi^-\rangle,\\&\langle\Sigma^-|\Xi^-\pi^0\rangle=\langle\Sigma^0|\Xi^-\pi^+\rangle.\end{aligned} \tag{31}$$

Equations (29)–(31) are consequences of the $\Delta T=\frac{1}{2}$ rule.

It is now clear that if we wish to derive Lee's relation (1) for pc and pv amplitudes simultaneously, we must assume T-$L(2)$ invariance and not T-$L(1)$. If, however, we consider the two types of amplitude separately, then there exists an alternative derivation. To see why, we note that T-$L(2)$ invariance requires H_{NL} to be odd under the permutation of indices[16] 2 and 3 [see (16), (18b)]. Another way of making H_{NL} odd is to leave the pc part T-$L(2)$ invariant but to make the pv part invariant under the product T-$L(1)\times P$, where P denotes the parity transformation. The arguments leading to (19)–(21) in this alternative scheme are essentially the same as above.

One other modification can be made in the arguments above, namely the use of the crossing relations (7) instead of those in (8). Lee's relation is then derived either from invariance under T-$L(2)\times P$, or by making the pc part of H_{NL} T-$L(2)$ invariant and the pv part T-$L(1)$ invariant. Equations (19)–(21) are still valid for pc amplitudes, but the corresponding equations for pv amplitudes involve some differences of sign.

[16] Use of this permutation has also been made by B. Sakita, Phys. Rev. Letters **12**, 379 (1964); and by Y. Hara, *ibid.* **12**, 378 (1964).

4. THE EFFECTIVE HAMILTONIAN

To show that T-L invariance alone has no further consequences, we must examine the structure of H_{NL}.

As pointed out in Sec. 3, H_{NL} transforms according to the eight-dimensional representation of $SU(3)$. A convenient way of constructing it is to combine baryons and antibaryons into various multiplets and then to combine each of these with the meson octet $\pi_\nu{}^\mu$. Altogether there will be eight independent terms. In three of them, the π-field always occurs with an index 3, and since this index refers to a K-meson, these terms do not contribute to observable decays. The five remaining terms are

$$\begin{aligned}&(D\pi)_2{}^3+(\pi D)_3{}^2,\\&(F\pi)_2{}^3+(\pi F)_3{}^2,\\&[10]_2{}^3\pi-[10^*]_3{}^2\pi,\\&[10]_3{}^2\pi-[10^*]_2{}^3\pi,\\&[27]_2{}^3\pi+[27]_3{}^2\pi,\end{aligned} \tag{32}$$

where the [10], [10*], and [27] terms are defined by Okubo,[17] and

$$\begin{aligned}&(AB)_\beta{}^\alpha=A_\lambda{}^\alpha B_\beta{}^\lambda,\\&[X]_\beta{}^\alpha\pi=[X]_{\beta\mu}^{\alpha\lambda}\pi_\lambda^\mu.\end{aligned} \tag{33}$$

D and F refer to the usual R symmetric and R-antisymmetric octet couplings respectively.[2]

Each term in (32) is an admixture of parity conserving and parity violating interactions. Because we assume derivative coupling and time reversal invariance, the pc and pv parts of H_{NL} have the same general form and their space-time dependence can be suppressed. Notice also that the second half of each term in (32) is the Hermitian conjugate of the first.

According to $\Delta T=\frac{1}{2}$, there are four independent amplitudes for observable decays (one each for Ξ and Λ decay, and two for Σ decay; each amplitude is, of course, an admixture of pc and pv parts). Since there are five terms in (32), the octet-transformation property of H_{NL} leads, by itself, to no observable consequences other than those of $\Delta T=\frac{1}{2}$.[18] The only effect of T-$L(1)$ invariance is to equate the coefficients of the two [10] terms, and hence to reduce the number of independent terms to four: again there are no observable consequences. On the other hand, T-$L(2)$ removes the [27] term and makes the coefficient of the [10] terms equal and opposite. Thus there will be three independent terms and hence, only one new relation, namely Lee's [see Eq. (1)], for observable decays. It is easy to see that the same conclusion holds for the alternative derivation of (1) (see Sec. 3).

The number of independent terms is further reduced if we assume RP invariance in addition to T-$L(2)$. The surviving ones are two pc terms (one from D and the other from [10]) and one pv term, which arises from

[17] S. Okubo, Progr. Theoret. Phys. (Kyoto) **28**, 24 (1962).
[18] B. W. Lee and S. L. Glashow (unpublished, 1964).

F-type coupling. We therefore expect one more relation among pc amplitudes and two more among pv amplitudes. They are investigated in the next section.

5. R INVARIANCE AND THE PROPERTIES OF $\Sigma^\pm \to n\pi^\pm$

We now consider the effects of adding RP invariance to the symmetries used in Sec. 3. Under R conjugation, baryons transform according to the rule[19]:

$$R: \quad (B) \to (\Sigma^-, -\Sigma^0, \Sigma^+, -\Lambda^0; -\Xi^-, \Xi^0, n, -p). \quad (34)$$

The corresponding rule for pseudoscalar mesons is given by (25) and (34). It is easy to see that if RP is applied to (19), then[13]

$$\langle \Sigma^+ | n\pi^+ \rangle_{\text{pv}} = 0. \quad (35)$$

In other words, the asymmetry parameter for $\Sigma^+ \to n\pi^+$ must vanish [see Eq. (2)].

As already shown in Sec. 2, RP invariance implies $\alpha_\Lambda \approx -\alpha_\Xi$ via Eq. (11). Combining (11) and (1), we find

$$\begin{aligned} \sqrt{3}\langle \Sigma^+ | p\pi^0 \rangle_{\text{pc}} &= -\langle \Lambda | p\pi^- \rangle_{\text{pc}}, \\ \langle \Sigma^+ | p\pi^0 \rangle_{\text{pv}} &= \sqrt{3}\langle \Lambda | p\pi^- \rangle_{\text{pv}}, \end{aligned} \quad (36)$$

which implies that α_0 and α_Λ are roughly equal in magnitude, but opposite in sign.[1,4]

Notice that the roles of pc and pv amplitudes in (11), (35), and (36) are not unique. They can be interchanged merely by introducing an overall change of sign[19] in (34). Hence we cannot draw a firm conclusion about the angular momentum of the pion in $\Sigma^+ \to n\pi^+$.

Equations (35) and (36) comprise the additional relations referred to at the end of the previous section. Since we have now exhausted the predictions of T-$L(2)$ and RP invariance, and since there are no more weak symmetries available in $SU(3)$, we cannot derive[4]

$$\alpha(\Sigma^- \to n\pi^-) \approx 0 \quad (3)$$

from symmetry principles. It must therefore be a dynamical effect.

6. SUMMARY AND DISCUSSION

We have shown that B. W. Lee's relation among the amplitudes for nonleptonic hyperon decay [see Eq. (1)] can be derived without using R conjugation. It follows from (i) time reversal invariance, (ii) the $\Delta T = \frac{1}{2}$ rule, (iii) T-$L(2)$ invariance [see (24), (25)], (iv) derivative-type crossing relations [see (8)]. On the other hand, the vanishing of $\alpha(\Sigma^+ \to n\pi^+)$ can be derived only if another condition, (v) RP invariance is added to the list.

An alternative derivation is obtained by replacing (iii) and (iv) with (iii)′ T-$L(2)$ invariance for pc interactions, T-$L(1)$ invariance for pv interactions [see (23)], (iv)′ nonderivative crossing relations [see Eq. (7)]. Again, $\alpha(\Sigma^+ \to n\pi^+)$ will vanish only if R conjugation is used. The assignment of crossing relations in (iv) and (iv)′ is not essential; if it is changed, however, the conditions (iii) and (iii)′ must be modified (see the final paragraphs of Secs. 2 and 3).

The only other consequences of (i)–(v) are contained in Eqs. (11) and (35). Since there are no more weak symmetries in $SU(3)$, the vanishing of $\alpha(\Sigma^- \to n\pi^-)$ must result from the dynamics of weak interactions.

We may compare our method for deriving Lee's relation with that of Coleman, Glashow, and Lee.[20] Theirs also does not require R invariance, but depends instead upon octets of scalar and pseudoscalar spurions with definite-charge conjugation properties. Now the component of the spurion octet which enters into nonleptonic-decay transforms like a K^0 meson, and its charge conjugate like a $\bar{K}^0$. Since K^0 and $\bar{K}^0$ are the $D_2{}^3$ and $D_3{}^2$ components of an octet, and since T-$L(1)$ invariance requires the Hamiltonian to be even under $2 \leftrightarrow 3$ [see (16) and (18a)], it follows that the use of a spurion with even charge conjugation parity[21] is equivalent to assuming T-$L(1)$ invariance. Similarly a spurion with odd charge conjugation parity is equivalent to T-$L(2)$ invariance. Thus the method of Coleman, Glashow, and Lee[20] is completely equivalent to ours.

There is one difference of approach that is worth noting. By using spurion octets, Coleman, Glashow, and Lee[20] assume that the nonleptonic decay Hamiltonian itself transforms as a member of an octet; from this they can deduce the $\Delta T = \frac{1}{2}$ rule. In our approach we assume $\Delta T = \frac{1}{2}$ and combine it with T-L invariance to deduce the octet transformation property of the Hamiltonian.[7]

Our last point concerns the Cabibbo theory.[22] As noted in a previous paper,[11] it leads to a nonleptonic decay Hamiltonian that is T-$L(1)$ invariant. Consequently, it predicts Lee's relation for parity-violating amplitudes, but not for parity-conserving ones.[6,21]

APPENDIX: THE T-L TRANSFORMATION

In this appendix, we examine those $SU(3)$ transformations that interchange isotopic spin and L-spin [see Eq. (15)]. We show that they are distinguished from one another not by the way in which they permute a unitary multiplet, but rather by the phases they associate with the permutation. As special cases, we determine the T-$L(1)$ and T-$L(2)$ transformations of Eqs. (23), (24).

We begin our analysis by classifying an arbitrary octet with respect to T, K, and L spins. The isospin

[19] For operators, $R^{-1}D_\nu{}^\mu R = -D_\mu{}^\nu$ (see Ref. 1, footnote 12); for state vectors, $R|D_\nu{}^\mu\rangle = \lambda|D_\mu{}^\nu\rangle$, where λ is an undertermined phase factor. In (34), we take $\lambda = -1$.

[20] S. Coleman, S. L. Glashow, and B. W. Lee, Ann. Phys. (N. Y.) (to be published); see also S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

[21] Our assignment of charge conjugation parity may differ from that of other writers (Gell-Mann, Refs. 2 and 6; Coleman, Glashow, and Lee, Ref. 20) because of the way in which we identify mesons with components of an octet (see Ref. 11).

[22] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

TABLE I. Isotopic spin classification of an octet.

Y_T	T	$T_3 \rightarrow$ increasing
0	1	$-D_2{}^1,\ \frac{1}{\sqrt{2}}(D_2{}^2-D_1{}^1),\ D_1{}^2$
0	0	$-\frac{\sqrt{3}}{\sqrt{2}}D_3{}^3$
1	$\frac{1}{2}$	$D_2{}^3,\quad D_1{}^3$
-1	$\frac{1}{2}$	$D_3{}^1,\quad -D_3{}^2$

classification enables us to identify each baryon (and each meson) with a definite component of the octet, phase included: we find, for example, that $\Sigma^+\sim D_1{}^2$ and $\Sigma^-\sim -D_2{}^1$. Using this identification, we can then determine their K-spin and L-spin properties, again with the correct phases.

Our method is based upon certain sets of commutation relations. In unitary spin space, the components $D_\nu{}^\mu$ of a traceless tensor operator satisfy[23]

$$[B_\beta{}^\alpha,D_\nu{}^\mu]=\delta_\beta{}^\mu D_\nu{}^\alpha-\delta_\nu{}^\alpha D_\beta{}^\mu\,, \tag{A1}$$

where $B_\beta{}^\alpha$ is an infinitesimal generator of $SU(3)$. In isospace the components of a spherical tensor $S(k,q)$ satisfy[24]

$$\begin{aligned}[][T_\pm,S(k,q)]&=[k(k+1)-q(q\pm1)]^{1/2}S(k,\,q\pm1)\,,\\ [T_3,S(k,q)]&=qS(k,q)\,,\\ [Y_T,S(k,q)]&=Y_{T'}S(k,q)\,,\end{aligned} \tag{A2}$$

where $Y_{T'}$ is the hypercharge quantum number. By identifying isospin and hypercharge with certain generators of $SU(3)$,[25] we can use (A1) and (A2) to identify each $D_\nu{}^\mu$ with an $S(k,q)$. The results are displayed in Table I. Notice that the numerical factors in the $T_3=0$ terms arise from the normalization of the $D_\nu{}^\mu$.[26]

The K- and L-spin classifications are determined in exactly the same way,[25] and are displayed in Tables II and III. It should be noted that these classifications are not entirely independent of one another. In particular, they must be consistent with such commutation relations as[27]

$$[T_-,L_-]=K_+\,. \tag{A3}$$

To show that they are, we take the commutator of both sides of (A3) with $D_\nu{}^\mu$, and use the Jacobi identity

$$[[A,B],C]=[A,[B,C]]-[B,[A,C]]\,. \tag{A4}$$

The required result follows from Tables I–III.

We now use Table I to identify baryons with the components of the octet:

$$\begin{aligned}&\Sigma^+\sim D_1{}^2,\ \Sigma^0\sim(1/\sqrt{2})(D_2{}^2-D_1{}^1),\ \Sigma^-\sim -D_2{}^1\,,\\ &\Lambda^0\sim-(\sqrt{3}/\sqrt{2})D_3{}^3\,,\\ &p\sim D_1{}^3,\ n\sim D_2{}^3;\ \Xi^0\sim -D_3{}^2,\ \Xi^-\sim D_3{}^1\,.\end{aligned} \tag{A5}$$

Their K- and L-spin properties now follow from (A5) and Tables II and III. To obtain the corresponding properties of pseudoscalar mesons, we use the substitution of Eq. (25).

Equation (A2) demands that the spherical components of an isovector be related to its Cartesian components by[24]

$$S(1,\pm1)=\mp\frac{1}{\sqrt{2}}(S_1\pm iS_2)\,. \tag{A6}$$

For π mesons, this implies

$$\pi^\pm=\mp\frac{1}{\sqrt{2}}(\pi_1\pm i\pi_2)$$

and hence[13]

$$(\pi^\pm)^*=-\pi^\mp\,. \tag{A7}$$

Another point about (A2) is its consistency with the Condon and Shortley[28] phase convention for spherical harmonics. It allows us to use standard Clebsch-Gordan coefficients[24] for the addition of isospin. The same is true for K spin and L spin.

A comparison of Tables I and III indicates that transformations which interchange isospin and L spin must interchange the indices 2 and 3. To analyze such transformations we work in the three-dimensional space from which $SU(3)$ is generated.

In this space, cogredient vectors

$$X\equiv\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

TABLE II. K-spin classification of an octet.

Y_K	K	$K_3 \rightarrow$ increasing
0	1	$-D_3{}^2,\ \frac{1}{\sqrt{2}}(D_3{}^3-D_2{}^2),\ D_2{}^3$
0	0	$-\frac{\sqrt{3}}{\sqrt{2}}D_1{}^1$
1	$\frac{1}{2}$	$D_3{}^1\quad D_2{}^1$
-1	$\frac{1}{2}$	$D_1{}^2\quad -D_1{}^3$

[23] S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

[24] D. M. Brink and G. R. Satchler, *Angular Momentum* (Clarendon Press, Oxford, England, 1962).

[25] See Ref. 7, Eq. (11).

[26] The normalization of $D_\nu{}^\mu$,

$$\begin{aligned}(D_\nu{}^\mu,D_\beta{}^\alpha)&=\delta^{\mu\alpha}\delta_{\nu\beta}\quad(\mu\neq\nu),\\ &=\tfrac{2}{3}\delta^{\mu\alpha}\delta_{\nu\beta}\quad(\mu=\nu),\\ &=-\tfrac{1}{3}\quad(\mu=\nu,\ \alpha=\beta,\ \mu\neq\alpha),\end{aligned}$$

is determined by the traceless condition $D_\lambda{}^\lambda=0$.

[27] See Ref. 7, Eqs. (9) and (11). This point has also been discussed by P. Jassallette, Nuovo Cimento **32**, 136 (1964). J. J. de-Swart, Rev. Mod. Phys. **35**, 916 (1963).

[28] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, New York, 1959), p. 52.

TABLE III. L-spin classification of an octet.

Y_L	L	$L_3 \to$ increasing
0	1	$-D_1^3,\ \frac{1}{\sqrt{2}}(D_1^1-D_3^3),\ D_3^1$
0	0	$-\frac{\sqrt{3}}{\sqrt{2}}D_2^2$
1	$\frac{1}{2}$	$D_1^2 \quad D_3^2$
-1	$\frac{1}{2}$	$D_2^3 \quad -D_2^1$

transform like[29]

$$X \to UX \tag{A8}$$

and contragredient ones

$$\tilde{X} \equiv (x^1, x^2, x^3)$$

like[29]

$$\tilde{X} \to \tilde{X} U^\dagger. \tag{A9}$$

The matrix U is unitary and unimodular. One way of extending a given transformation to higher representations of $SU(3)$ is to note that the basis of any representation can be constructed from products of X's and $\tilde{X}$'s: for example[29]

$$D_\nu^\mu = x^\mu x_\nu - \tfrac{1}{3}\delta_\nu^\mu x^\alpha x_\alpha. \tag{A10}$$

Another is to express U in terms of the generators of $SU(3)$ and then use commutators such as those in (A1).

The most general transformation that interchanges the indices 2 and 3 is

$$U = \begin{bmatrix} e^{i\alpha} & 0 & 0 \\ 0 & 0 & i\beta \\ 0 & e^{i\gamma} & 0 \end{bmatrix}, \tag{A11}$$

where, from the unimodular condition,

$$\alpha+\beta+\gamma = (2n+1)\pi \tag{A12}$$

and n is a positive integer. There are infinitely many choices of α, β, γ consistent with (A12), and hence there are infinitely many transformations which engender Eq. (15). It is also obvious that they differ from one another only in the matter of phase.

In general, the phase angle φ of Eq. (17) is given by

$$\varphi = \beta - \gamma. \tag{A13}$$

The simplest choices of α, β, γ consistent with $\varphi = 0$, π [see Eq. (18)] are:

$$\begin{aligned} &\text{(i)} \quad \varphi = 0: \quad \alpha=\beta=\gamma=\pi, \\ &\text{(ii)} \quad \varphi=\pi: \quad \alpha=\gamma=0,\ \beta=\pi, \end{aligned} \tag{A14}$$

and the corresponding transformation matrices are

$$U_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}. \tag{A15}$$

U_1 engenders the T-$L(1)$ transformation [see Eq. (23)] via (A5), (A8)–(A10), and U_2 engenders T-$L(2)$ [see Eq. (24)].

With an appropriate representation of the generators of $SU(3)$,[11] it can be shown that

$$\begin{aligned} U_1 &= \exp[-i\pi(\tfrac{3}{2}Y_K + K_1)], \\ U_2 &= \exp[-i\pi K_2], \\ K_1 \pm iK_2 &= K_\pm. \end{aligned} \tag{A16}$$

We note that U_1 is precisely the Weyl reflection W_2 as defined by Macfarlane, Sudarshan, and Dullemond.[30] The other two Weyl reflections interchange 1, 2, and 3, 1, respectively, and by analogy with (A16) they can be expressed as

$$\begin{aligned} W_1 &= \exp[-i\pi(\tfrac{3}{2}Y_T + T_1)], \\ W_2 &= \exp[-i\pi(\tfrac{3}{2}Y_L + L_1)]. \end{aligned} \tag{A17}$$

It also appears that U_2 is a special case of a more general transformation discussed by d'Espagnat and Prentki.[31]

Note added in proof. In Cabibbo's theory,[22] weak interactions are engendered by a Hamiltonian:

$$\begin{aligned} H &= J^\dagger J, \\ J &= \cos\theta D_1^2 + \sin\theta D_1^3. \end{aligned} \tag{A18}$$

The terms giving rise to nonleptonic decays, namely $\sin\theta\ \cos\theta(D_1^2D_3^1 + D_1^3D_2^1)$ are invariant under T-$L(1)$, but not under T-$L(2)$.

Finally, we note that Eq. (A5) can be used to derive the rule for R conjugation given in Eq. (34).

[29] R. E. Behrends, J. Dreitlein, C. Fronsdal, and B. W. Lee, Rev. Mod. Phys. **34**, 1 (1962).

[30] A. J. Macfarlane, E. C. G. Sudarshan, and C. Dullemond, Nuovo Cimento **30**, 845 (1964).

[31] B. d'Espagnat and J. Prentki, Nuovo Cimento **24**, 497 (1962).