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Form Factors in Electron-Trinucleon Scattering*†

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We analyze the four form factors in electron-trinucleon (e-H^3 or e-He^3) elastic scattering using two alternative sets of four unknowns: (i) the neutron electric form factor, the form factor F_1 for the dominant S state, and the isovector- and isoscalar-exchange magnetic form factors—here we neglect the mixed symmetry S' state; (ii) the form factors F_O and F_L for odd and like nucleons, and the isovector- and isoscalar-exchange magnetic form factors—here we assume knowledge of the neutron electric form factor, essentially 0.02 q^2 in our region. The first alternative gives positive neutron electric form factors that seem somewhat high; the second alternative gives a plausible value of 1% for the probability of the mixed-symmetry S' state, and also gives a reasonable shape for the isovector-exchange form factor. The isoscalar-exchange form factor stays less than 0.06 nm. We find good agreement between our results for F_1 and calculations based on three different wave functions adjusted to give the experimental Coulomb energy.

INTRODUCTION

ELASTIC electron-trinucleon scattering can in the one-photon approximation be interpreted in terms of four form factors[1,2] since each isospin state of the trinucleon (H^3 and He^3) has an electric and a magnetic form factor. One might hope that these four independent measurements at each squared momentum transfer q^2 would make it possible to cancel out effects of nuclear structure, so that the electric form factor of the neutron G_{En} could be determined. For instance, Schiff[3] has analyzed the four experimental form factors in terms of the following four unknowns: G_{En}, the nuclear form factors for like nucleons (F_L) and for the odd nucleon (F_O), and the isovector magnetic-exchange form factor[4] F_{xV}. On the other hand, Levinger and Chow[5] have argued that another choice of 4 unknowns is not clearly less plausible than Schiff's choice. They choose the set G_{En}, $F_O=F_L$, G_{xV}, and the isoscalar magnetic-exchange form factor G_{xS}. The same experimental data[1] give markedly different values for G_{En} when analyzed according to these two different recipes. Thus Schiff[3] finds that the neutron form factor stays near zero, while Levinger and Chow find that G_{En} climbs to a value of 0.22 at $q^2=5\text{F}^{-2}$.

One purpose of the present paper is to apply the Levinger-Chow analysis to the latest measurements[2] which are both more extensive and more accurate than those analyzed earlier. We also analyze the recent data in a still different manner by choosing the set of 4 unknowns as F_O, F_L, G_{xV}, and G_{xS}. This analysis uses values of the neutron electric form factor based on interpolation of the Wilson-Levinger selective compilation[6] of results from electron-neutron scattering.

We then compare the nuclear form factors found by our two different analyses of the recent scattering data: namely $F_1=F_O=F_L$ from the second section with $F_1=(\frac{2}{3})F+(\frac{1}{3})F_O$ from the third section. We also compare these nuclear form factors F_1 found from scattering data with nuclear form factors found by taking the Fourier transform of the squared trinucleon wave function. We use exponential wave functions[7] with two and with three adjustable parameters.

In the final section we summarize the results of this paper, and we make brief comparisons with other trinucleon calculations. There have been several estimates[3,8–10] of the percentage of the mixed-symmetry S' state, which gives a difference between F_O and F_L. Also, other workers[3,11] have used assumed wave functions to find the function $F(q^2)$, while a preliminary estimate has been made[12] of the shape of the function $F_{xV}(q^2)$.

ANALYSIS ASSUMING PURELY SYMMETRIC S STATE

Our phenomenological analysis[4,5] is based on the following four equations relating the measured form factors F_{ET}, F_{MT}, F_{EG}, and F_{MG} to the nuclear and nucleon form factors. (Here the subscripts E and M stand for electric and magnetic, respectively; the subscripts T and G stand for the triton H^3 and for He^3, respectively.)

$$F_{ET}=F_OF_{Ep}+2F_LG_{En}, \tag{1}$$

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12 D. W. Padgett, J. G. Brennan, W. M. Frank, and T. Spriggs, Bull. Am. Phys. Soc. 9, 467 (1964), and (private communications).

$$2.98F_{MT}=2.71F_OF_{Mp}+G_{xS}+0.27F_{xV}\,, \qquad (2)$$

$$2F_{EG}=F_OG_{En}+2F_LF_{Ep}\,, \qquad (3)$$

$$-2.13F_{MG}=-1.86F_OF_{Mn}+G_{xS}-0.27F_{xV}\,. \qquad (4)$$

Here F_O and F_L are the nuclear form factors for odd and like nucleons introduced by Schiff[3]; the subscripts p and n refer to proton and neutron, respectively; x refers to exchange moment effects, with S and V to isoscalar and isovector, respectively. We normalize at the static limit by making all form factors unity except that $G_{En}=G_{xS}=0$. (The statement $G_{xS}=0$ is the "mirror theorem"[13] that the exchange effects are equal and opposite for the two members of the isospin doublet.) Of course, there may also be meson-exchange effects[14] for electric form factors F_{ET} and F_{EG} for $q^2\neq 0$. For the time being we suppress explicit consideration of these exchange effects by including them implicitly with F_O and F_L. To the extent that these (or other relativistic effects) are of importance, we cannot use the standard interpretation of F_O and F_L as Fourier transforms of squared wave functions.

Equations (1) to (4) are those of Schiff[3] or Levinger[4] with some modifications. In particular, we follow the former in distinguishing between F_O and F_L, while we follow the latter in using 0.27 nm as the static isovector exchange moment. The D state contributes[13] about -0.08 nm to the static magnetic moment of the triton, so we decrease the coefficient of F_OF_{Mp} in Eq. (2) from 2.79 to 2.71, and increase the value of the static isovector exchange moment from 0.19 to 0.27 nm. Of course, our procedure is not exact, since now F_O has slightly different meanings in Eqs. (1) and (2), respectively. These D-state effects have been considered by Krueger and Goldberg.[15] We make the further approximation of treating the trinucleon system as a pure isospin $\frac{1}{2}$ state.[16] It is clear that some approximations are needed, since we already have five unknowns to be determined by four measurements, and we could easily[14,15] have seven or even more unknown quantities.

TABLE I. Calculations assuming pure S state.

q^2 (F^{-2})	G_{En}	Error[a]	F_1	Error[a]	F_{xV}	Error[a]	G_{xS}	Error[a]
0	0.00	0.00	1.00	0.00	1.00	0.00	0.00	0.00
1	0.06	0.01	0.625	0.008	1.76	0.26	0.02	0.08
$1\frac{1}{2}$	0.10	0.01	0.494	0.007	1.11	0.18	0.01	0.05
2	0.10	0.015	0.398	0.007	1.05	0.15	0.02	0.03
$2\frac{1}{2}$	0.13	0.015	0.324	0.006	0.90	0.07	0.06	0.02
3	0.15	0.015	0.267	0.004	0.61	0.07	0.06	0.02
$3\frac{1}{2}$	0.16	0.015	0.217	0.004	0.50	0.06	0.06	0.015
4	0.15	0.015	0.184	0.003	0.48	0.04	0.07	0.015
$4\frac{1}{2}$	0.16	0.015	0.145	0.003	0.53	0.03	0.05	0.007
5	0.22	0.02	0.115	0.003	0.45	0.04	0.03	0.01
6	0.16	0.03	0.088	0.003	0.30	0.03	0.04	0.007
8	0.25	0.08	0.029	0.004	0.25	0.02	0.025	0.005

[a] The statistical errors given include *only* errors due to errors of measurement of the four trinucleon form factors. G_{En} is the electric form factor of the neutron; F_1 is the form factor of the dominant S state; F_{xV} and G_{xS} are the isovector- and isoscalar-exchange magnetic form factors, respectively. See Eqs. (1) to (4) with $F_0=F_L=F_1$.

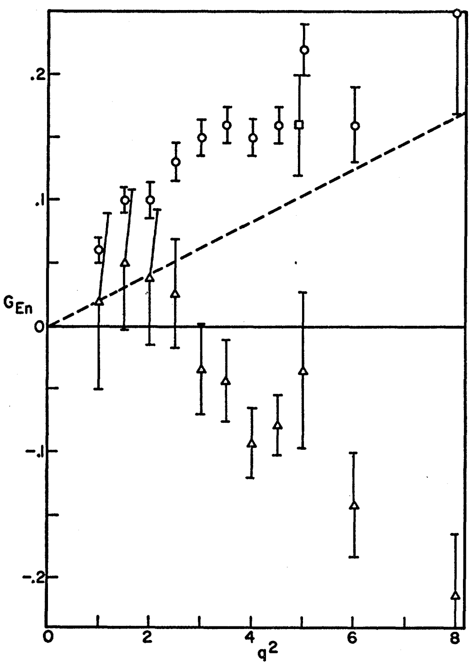

FIG. 1. The neutron electric form factor G_{En} found neglecting the S' state (Table I) shown as circles; the triangles show the results of Hofstadter's analysis of the same data, Ref. 2; the one square shows the measurement of Stein *et al.*, Ref. 17; the dashed line shows the slope from scattering of thermal neutrons. The squared four-momentum transfer q^2 is given in F^{-2}.

In this section we assume $F_O=F_L=F_1$. We obtain F_{Ep}, F_{Mp}, and F_{Mn} from measurements[6] on electron-proton and inelastic electron-deuteron scattering (see Table II). We use the latest Hofstadter results[2] for F_{ET}, F_{MT}, F_{EG}, and F_{MG}, and solve for the four unknowns G_{En}, F_1, F_{xV}, and G_{xS}. Our results are given in Table I, along with statistical errors based on the quoted errors of the measured form factors $F_{ET}\cdots F_{MG}$. The errors in the table are approximate for three reasons: They do not include correlated experimental errors among the trinucleon form factors; they do not include effects of errors in the three nucleon form factors; and they omit the effects of serious theoretical approximations.

Table I and Fig. 1 show values for the neutron electric form factor G_{En} similar to those of our earlier analysis[5] using the same assumptions, and earlier[1] data. The neutron electric form factor rises initially quite rapidly with increasing q^2, so that the slope $G_{En}'(0)$ seems high when compared with the value of 0.021 F^2 from scattering of thermal neutrons by electrons. Our value for G_{En} at $q^2=5$ F^{-2} is consistent within statistical errors with

[13] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1953).

[14] A. Q. Sarker, Phys. Rev. Letters 13, 375 (1964).

[15] D. A. Krueger and A. Goldberg, Phys. Rev. 135, B934 (1964).

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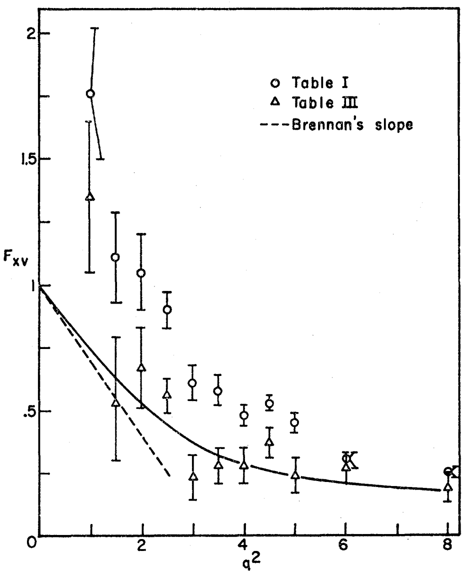

FIG. 2. F_{xV} is the isovector-exchange magnetic form factor from Tables I and III (circles and triangles respectively). The statistical errors shown are based only on the (assumed uncorrelated) statistical errors of Ref. 2. The curve for the triangles is drawn merely to aid the reader see a smooth curve. The dashed line shows the slope of preliminary calculations by Padgett *et al.*, Ref. 12.

that of Stein *et al.*,[17] but it is higher than the Stein value: Fig. 1 also shows Hofstadter's[2] results of G_{En} using current data analyzed according to Schiff.

As in our earlier paper[4] based on electron-He^3 scattering data, the isovector magnetic-exchange form factor F_{xV} shown in Table I and Fig. 2 falls rather slowly from its static value of unity, reaching a half at about 5 F^{-2} and a quarter at $q^2=8$ F^{-2}. The isoscalar magnetic-exchange form factor G_{xS} starts by construction with a static value of zero and rises only to about 0.07 nm, thereafter falling slowly with increasing q^2.

ANALYSIS ASSUMING KNOWLEDGE OF NEUTRON FORM FACTOR

In this section we choose another set of four unknowns, namely F_O, F_L, F_{xV}, and G_{xS}. We use the same three nucleon form factors F_{Ep}, F_{Mp}, and F_{Mn} used in the previous section, and we also use the *assumed* neutron electric form factors G_{En} given in Table II. These values for the neutron form factor are based on an interpolation of the Wilson-Levinger[6] selection of data. We are placing considerable weight on the slope from thermal-neutron scattering, $G_{En}'(0)=0.021$ F^2, and on the point of Stein *et al.*[17] at $q^2=5$ F^{-2} based on a measurement of the ratio of electron-neutron coincidences to electron-proton coincidences in inelastic electron-deuteron scattering. We interpret this measurement assuming that G_{En} is positive; an assumption as to sign is needed since G_{En}^2 is measured.

[17] P. Stein, R. W. McAllister, B. D. McDaniel, and W. M. Woodward, Phys. Rev. Letters 9, 403 (1962).

TABLE II. Values assumed for nucleon form factors. F_{Ep} and F_{Mp} are the proton electric and magnetic form factors, respectively, while G_{En} and F_{Mn} are the neutron form factors. See Ref. 6.

q^2 (F^{-2})	F_{Ep}	$F_{Mp}=F_{Mn}$	G_{En}
0	1.000	1.000	0.000
1	0.886	0.881	0.02
$1\frac{1}{2}$	0.835	0.845	0.03
2	0.794	0.795	0.04
$2\frac{1}{2}$	0.756	0.759	0.05
3	0.724	0.723	0.06
$3\frac{1}{2}$	0.692	0.680	0.07
4	0.663	0.652	0.08
$4\frac{1}{2}$	0.636	0.623	0.09
5	0.610	0.587	0.10
6	0.563	0.537	0.11
8	0.491	0.473	0.11

Our results for the four unknowns, F_O, F_L, F_{xV}, and G_{xS} are given in Table III. The statistical errors given represent minimum values since, as in Table I, we have included only the effects of (assumed uncorrelated) experimental errors in the four measured form factors.

This analysis gives values for F_O larger than corresponding values for F_L by about three standard errors of the difference. The difference between F_O and F_L gives us an estimate of the percentage probability P of the mixed symmetry S' state, following Schiff's interpretation.[3] For Gaussian wave functions, P is proportional to $(F_2/q^2F_1)^2$, where $F_2=F_O-F_L$, and F_1 is defined by Eq. (5) below. Our values of F_2^2 are, on the average, about one quarter those found by Schiff *et al.*[3] for a 4% probability, so the numbers in Table III could be interpreted as suggesting roughly a 1% probability of the S' state. For an Irving wave function, with the adjustable parameter adjusted by Schiff to give the experimental Coulomb energy, we again find an S' state probability of about 1%; see Fig. 3.

Our values for F_{xV} decrease somewhat more rapidly with increasing q^2 than do those found by the analysis of the previous section, as is illustrated in Fig. 2. Our

TABLE III. Calculation assuming knowledge of neutron electric form factor.

q^2 (F^{-2})	F_O	Error[a]	F_L	Error[a]	F_{xV}	Error[a]	G_{xS}	Error[a]
0	1.000	0.000	1.000	0.000	1.000	0.000	0.00	0.00
1	0.679	0.009	0.638	0.006	1.35	0.30	0.00	0.07
$1\frac{1}{2}$	0.575	0.010	0.513	0.006	0.53	0.23	0.00	0.05
2	0.454	0.011	0.411	0.006	0.67	0.16	0.00	0.03
$2\frac{1}{2}$	0.376	0.011	0.336	0.005	0.56	0.07	0.04	0.02
3	0.329	0.010	0.281	0.004	0.23	0.09	0.05	0.02
$3\frac{1}{2}$	0.270	0.008	0.228	0.004	0.28	0.07	0.04	0.01
4	0.222	0.009	0.192	0.002	0.28	0.07	0.06	0.01
$4\frac{1}{2}$	0.177	0.008	0.152	0.003	0.37	0.06	0.04	0.01
5	0.157	0.011	0.124	0.003	0.24	0.07	0.03	0.01
6	0.095	0.013	0.092	0.003	0.27	0.06	0.04	0.01
8	0.044	0.017	0.031	0.004	0.19	0.06	0.02	0.01

[a] The statistical errors given include *only* errors due to (assumed uncorrelated) errors of measurement of the four trinucleon form factors. The numbers are based on Eqs. (1) to (4) using nucleon form factors from Table II. F_O and F_L are the form factors for odd and like nucleons respectively; F_{xV} and G_{xS} are the isovector- and isoscalar-exchange moment form factors, respectively.

TABLE IV. Calculations of form factor F_1 for dominant S state. F_1 is the Fourier transform of the squared wave function. See Eqs. (6) to (8) for ψ_1 to ψ_3. Schiff (Ref. 3) uses Irving's wave function. All wave functions except ψ_3 have parameters adjusted to give the experimental Coulomb energy, assuming point protons, while the parameters in ψ_3 are found by a variational calculation (Ref. 18).

q^2 (F^{-2})	Using ψ_1	Using ψ_2	Using ψ_3	Schiff-Irving
0	1.000	1.000	1.000	1.000
1	0.618	0.640	0.563	0.637
$1\frac{1}{2}$	0.499	0.520	0.433	...
2	0.407	0.427	0.337	0.426
$2\frac{1}{2}$	0.337	0.352	0.265	...
3	0.282	0.295	0.211	0.297
$3\frac{1}{2}$	0.238	0.247	0.170	...
4	0.203	0.208	0.138	0.215
$4\frac{1}{2}$	0.174	0.177	0.111	...
5	0.151	0.151	0.091	0.159
6	0.115	0.111	0.063	0.122
8	0.070	0.064	0.029	0.073

present values for G_{xS} are similar to those of the previous section, but a bit smaller.

THE TRINUCLEON FORM FACTOR F

In the preceding two sections we have interpreted the four measured form factors in electron-trinucleon scattering in two alternative ways. The first method, by construction, gives just the trinucleon form factor $F_1(q^2)$ which, in a nonrelativistic approximation, is just the Fourier transform of the squared trinucleon wave function. The second method gives two different form factors F_O and F_L which we now combine to give an average form factor[3]

$$F_1=\tfrac{2}{3}F_L+\tfrac{1}{3}F_O. \tag{5}$$

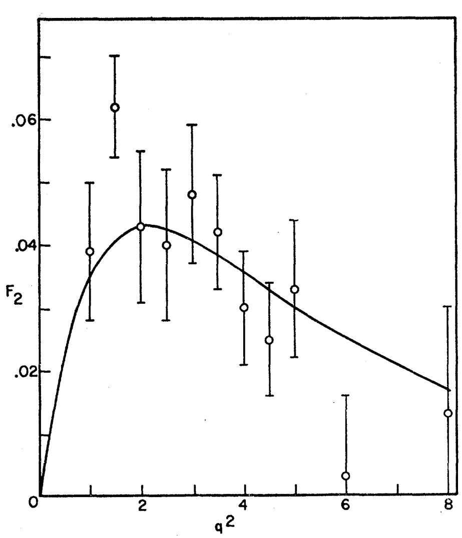

FIG. 3. $F_2=F_O-F_L$, the difference of form factors of odd and like nucleons, using the results of Table III, based on assumed knowledge of the neutron electric form factor. The statistical errors shown are based only on the (assumed uncorrelated) statistical errors of Ref. 2. The curve shows Schiff's calculations for an Irving wave function, with 1% probability for the S' state.

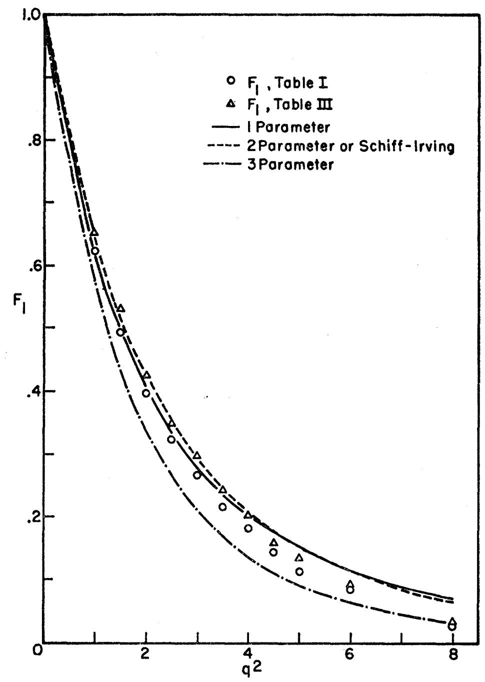

FIG. 4. The circles and triangles show the S state form factor F_1 from Tables I and III, respectively. The solid curve, dashed curve and dash-dot curve show the calculations in Table IV based on wave functions ψ_1, ψ_2, and ψ_3, respectively. Schiff's calculations for an Irving wave function (Ref. 3) are indistinguishable from the dashed curve.

F_1 is the form factor corresponding to the dominant completely symmetric S state. In Fig. 1 and Table IV we compare these two values of $F_1(q^2)$, and we also give values of F_1 calculated from four different choices for the trinucleon wave function.

Our first calculation[7] uses a wave function

$$\psi_1=N\exp[-(\tfrac{1}{2})\kappa(r_{12}+r_{13}+r_{23})]. \tag{6}$$

The value of the single parameter is chosen as $\kappa=0.74$ F^{-1}, to obtain agreement with the observed Coulomb energy difference between He^3 and H^3. We have presented earlier[7] the methods of obtaining the Fourier transform and its numerical results.

Our second calculation[18] uses a two-parameter wave function

$$\psi_2=N'\{\exp[-\tfrac{1}{2}\kappa(r_{12}+r_{13}+r_{23})] -\exp[-\tfrac{1}{2}\lambda(r_{12}+r_{13}+r_{23})]\}. \tag{7}$$

The two parameters κ and λ are chosen to fit the coulomb energy difference, allowing one degree of freedom remaining which we exploit to improve the agreement with the experimental values of F_1. We use $\kappa=0.83$ F^{-1}, and $\lambda=1.51$ F^{-1}. The calculation uses the function $F_B(x)$ already published.[7]

Our third calculation uses the three-parameter wave function

$$\psi_3=N''\{\exp[-\tfrac{1}{2}\kappa(r_{12}+r_{13}+r_{23})] +A\exp[-\tfrac{1}{2}\lambda(r_{12}+r_{13}+r_{23})]\}. \tag{8}$$

The parameters κ, λ, and A are determined[18] by a variational calculation assuming a central, spin-independent but velocity-dependent nucleon-nucleon potential. The values found are $\kappa=0.732$ F^{-1}, $\lambda=1.415$ F^{-1}, and $A=-1.305$; these are used with our function $F_B(x)$ to give the form factors in the last column of Table IV.

Table IV also gives Schiff's calculation[3] of F_1 for an Irving wave function, adjusted to give the Coulomb energy. We see from Table IV and Fig. 4 that the two-parameter wave function of Eq. (7) and the Schiff-Irving values each gives quite a good fit to our analyses of the measured form factors except at the highest values of q^2, where the calculated form factor does not decrease quite rapidly enough. The one-parameter wave function ψ_1 is moderately successful in fitting F_1. The three-parameter wave function has a satisfactory shape but the wrong scale, since it decreases too rapidly at the origin. As might be expected, ψ_3 gives too low a value for the Coulomb energy difference.

DISCUSSION

We shall discuss four different questions rather briefly: (i) What information on the neutron electric form factor can be extracted from electron-trinucleon elastic scattering? (ii) How much S' state of mixed symmetry is present in the trinucleon? (iii) Is the shape of the isovector-exchange magnetic form factor F_{xV} in agreement with theoretical estimates? (iv) What information on the trinucleon wave function can be extracted from the values of the form factor F_1? We will not discuss the values we have found for the isoscalar-exchange magnetic form factor G_{xS} since we do not know of theoretical estimates of this quantity for the trinucleon,[19] nor will we discuss Sarker's suggestion of the importance of exhange contributions to electric form factors.

Figure 1 illustrates our belief that our present knowledge of the trinucleon system is insufficient to use electron-trinucleon scattering for a reliable determination of the neutron electric form factor. Two major uncertainties at present are the size of the difference F_O-F_L, and the size of the isoscalar-magnetic-exchange form factor G_{xS}. Different assumptions concerning these poorly known quantities allow the divergent results illustrated. [Note that the straight line representing the known slope $G_{En}'(0)$ is essentially the input data of our third section, which gives not implausible results for F_O-F_L, and for G_{xS}.]

The analysis of Schiff *et al.*[1,3] gave a probability of 4% for the mixed symmetry S' state in the trinucleon wave function. Schiff *et al.*[9] have recently summarized several independent estimates of the probability of the S' state. The cross section for capture of thermal neutrons by deuterium is a magnetic dipole transition involving both spin moments (via the S' state) and also exchange moments.[20] Schiff now[9] estimates an S' probability of 2% or less. Further, Blatt and Delves[21] estimate this same probability by treating it as a variational parameter in a trinucleon calculation using realistic static potentials with hard cores. They find a probability of about 2%. Thirdly, Schiff *et al.*[9] find that inelastic electron-He^3 scattering suggests an S' probability of 1% or less. Finally, Blin-Stoyle[10] analyzes beta decay of H^3 and finds a serious discrepancy for 4% probability of the S' state.

These four estimates are individually weak, but together they represent a strong heuristic argument that the S' probability is less than 2%. We find it satisfactory that we are able to obtain not completely unreasonable values for the neutron electric form factor, and for the isoscalar magnetic-exchange form factor assuming no S' state; and that alternatively by assuming reasonable values for the neutron electric form factor, we find an S' probability of only 1%.

Recently Padgett *et al.*[12] have returned to the old problem of calculating the isovector-exchange magnetic moment using meson theory. At present they give preliminary results both for the magnitude of the static exchange moment, and for the slope of F_{xV} at $q^2=0$. Their static value of about 0.2 nm is in satisfactory agreement with the value 0.27 nm which we have used above. They find a slope $F_{xV}'(0)$ of -0.3 F^2. In Fig. 2 we compare our two sets of values of $F_{xV}(q^2)$, extracted from the scattering data on two different assumptions, with a straight line showing Padgett's slope. At low q^2 our values from Tables I and III have large statistical errors, and in some cases (e.g., $q^2=1$) are implausible since they are appreciably above the static value of unity. But by and large the triangles, showing the results of Table III, are not in sharp disagreement with Padgett's slope.

Finally, we examine to what extent our determination of the form factor F_1 selects the form of the nuclear wave function of the trinucleon, for the dominant S state. We find in Table IV and Fig. 3 that our two different analyses of the data give reasonably consistent values for F_1, and that several different calculations give values for F_1 in good agreement with each other and with the values we have derived from experiment. In particular, our two-parameter wave function ψ_2, and Schiff's use of Irving's wave function, with its single parameter adjusted to give the observed Coulomb energy, give values of F_1 that are well represented by a single curve in Fig. 3. However, Koester's use[11] of an Irving-Gunn wave function gives too large values of F_1 at large q^2. One of us (BKS) has remarked[7] that this

[18] B. K. Srivastava, Ph.D. thesis, Cornell University, 1964 (unpublished).

[19] For a calculation of the isoscalar-exchange form factor for the deuteron see, for example, D. R. Harrington, Phys. Rev. **133**, B142 (1964).

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discrepancy is due to the Irving-Gunn wave function becoming too singular near the origin.

In summary, our analysis of electron-trinucleon elastic scattering shows that (i) present theory is inadequate to use electron-trinucleon scattering as a reliable means of finding the electric form factor of the neutron; (ii) the scattering can be interpreted to give an S' state probability of either zero, or preferably of about 1%, in contrast to the original[3] interpretation of 4% probability and in agreement with later estimates[9]; (iii) the slope $F_{xV}'(0)$ is not in serious disagreement with the preliminary result[12] of Padgett *et al.*; (iv) nuclear wave functions for the trinucleon chosen with plausible shape, and with parameters adjusted to fit the Coulomb energy, give good fits to the form factor for the dominant S state.

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Unitary Symmetry and Weak Interactions. III. Nonleptonic Hyperon Decay*

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It is shown that B. W. Lee's relation for nonleptonic hyperon decay can be derived from T-L invariance and $\Delta T=\frac{1}{2}$, and that the vanishing of $\alpha(\Sigma^+\to n\pi^+)$ requires the additional assumption of R invariance. The vanishing of $\alpha(\Sigma^-\to n\pi^-)$ cannot be derived from these symmetries, and since there are no others applicable to weak interactions in $SU(3)$, it must result from weak-interaction dynamics. A comparison is made between this theory and those of Cabibbo and Coleman, Glashow and Lee. Mathematical aspects of T-L invariance are discussed in an appendix.

1. INTRODUCTION

In the first paper[1] of this series, a theory of weak interactions was proposed within the framework of unitary symmetry.[2] Several properties of nonleptonic hyperon decay, including Lee's relation[3,4]

$$\sqrt{3}\langle\Sigma^+|p\pi^0\rangle-\langle\Lambda|p\pi^-\rangle=2\langle\Xi^-|\Lambda\pi^-\rangle \quad (1)$$

and

$$\alpha(\Sigma^+\to n\pi^+)\approx 0\,, \quad (2)$$

were derived from the $\Delta T=\frac{1}{2}$ rule[5], T-L invariance and R conjugation. Despite its empirical success,[4] this derivation can be criticized on the grounds that R conjugation is not a valid symmetry of strong interactions and should not be applied to weak ones.[6] It is therefore desirable to ask whether the results in (1) and (2) can be derived without R conjugation.

Associated with this question is another, more general one. In any theory of elementary particles, the symmetries of weak interactions are limited by electric charge conservation.[7] Their number varies from one theory to another, and so does our ability to derive the properties of weak decays from them. In global symmetry,[8] for example, three weak symmetries are available, and when combined with $\Delta T=\frac{1}{2}$, they predict all the properties of nonleptonic hyperon decay.[9] By contrast, unitary symmetry contains only two weak symmetries, namely T-L invariance and R conjugation.[10] Since they are not as rich in predictions as the global symmetry ones, we must ask to what extent do they account for nonleptonic-hyperon decay.

The usefulness of this question arises in the following way. If a given property can be derived from symmetry principles, it may not cast much light on the dynamics of weak interactions. If, however, it is not derivable from symmetry principles, it must be a consequence of dynamics, and hence it provides a definitive test for dynamical models. A case in point is the vanishing of $\alpha(\Sigma^-\to n\pi^-)$. As shown below, this result does not

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