Remark on the Balázs-Type Bootstraps

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The dependence of the Balázs-type "bootstrap" calculations on the subtraction as well as the matching point is numerically checked for two typical examples, namely, the ρ parameters and the deuteron pole parameters. In the case of the calculation of the ρ parameters, the results are very sensitive to the choice of the matching point. The matching point was varied from $s=0$ to $s=-8m_\pi^2$. In this range m_ρ^2 varied from ~ 10 to $\sim 32m_\pi^2$. The input and output widths were equal only when the matching point was between $s=0$ and $s=-1.6m_\pi^2$. In this region m_ρ^2 is a very sensitive function of the matching point. The results are almost independent of the subtraction point. In the second example of the calculation of the deuteron parameters, there exists no self-consistent solution in the expected region.

MUCH enthusiasm has recently been witnessed for getting bound states and resonances from a bootstrap mechanism.[1] We have scrutinized the typical example of getting the ρ-meson parameters from the π-π scattering amplitude. Assuming that only the ρ exchange dominates in the crossed channel, and approximating the left-hand cut in the $\nu(=\frac{1}{4}s-1)$ plane[2] by two well-placed poles, Balázs[3] applies the N/D method to get the pion-pion scattering amplitude. It has been claimed that the theory does not contain any free parameters, with the tacit assumption that the calculation is essentially independent of the choice of the matching point ν_F and the subtraction point ν_0. This last assumption, accepted as a matter of faith, was never put to a test. We have just done this and our results are as follows:

As far as the subtraction point ν_0 is concerned, no appreciable variation of self-consistent mass ν_ρ was observed when ν_0 varies between -3 and -1 (in units of the pion mass squared, m_π^2). On the contrary, the variation with respect to the matching point ν_F is considerable. Figure 1 shows a plot of ν_ρ as a function of ν_F. In the region where ν_ρ has a maximum, one sees from Fig. 2 that the quantity $2(\Gamma_{\text{out}}-\Gamma_{\text{in}})/(\Gamma_{\text{out}}+\Gamma_{\text{in}})$ does not pass through zero. Thus in this region there is no self-consistent solution. On Fig. 1, we have also shown the curve on which the denominator function $D_\nu(\nu)$ becomes infinite. For ν_ρ below -3, this curve approaches the curve on which $D_\nu(\nu)$ vanishes; the approach depends on the input width Γ_{in}, and the Balázs solution gradually disappears.

As a second example, we scrutinized the Bose-Dersarkissian work[4] which is apparently a self-consistent calculation of the deuteron parameters. In this work, as far as one can see, the deuteron does not appear in any of the crossed channels and hence the self-consistency requirements are themselves not clear. We tried a numerical solution of Eqs. (23)–(25), and (29) of Ref. 3, and apparently the results showed a strong dependence on the choice of the subtraction point. However, it was pointed out to us, very correctly, that the subtraction point can be entirely eliminated (see the Appendix) while everything else remains finite. The preceding solutions, including that of Bose and Desarkissian, must therefore be in error. We repeated our calculations without the subtraction point and found that in the relevant region $-0.5\lesssim\nu_D\lesssim 0$, $2\lesssim\gamma_D\lesssim 4$ there exists no solution. We think that the numbers which have previously been believed to be solutions must have been the accumulated pure noise arising from the restricted accuracy of the calculations.

We may add a few concluding remarks.

(1) It may be argued that since only a few partial waves are kept in such calculations, the results can be quite sensitive to the variations in the matching point. The dependence on the matching point will weaken as more and more partial waves are taken into account. If a theory is meaningful only when one keeps all the partial waves, then it is useless for all practical purposes, since it is almost impossible to work with a large number of terms. On the other hand, if an approximation is very sensitive to the values of certain quantities then these quantities are effective parameters in the theory and must be supplied from outside.

(2) Some people might think that the matching point is not an arbitrary parameter, since it has to be as far away as possible from any singular region—in other words, the middle point of the singularity-free gap—since a partial wave expansion is most valid there. This view is also not tenable, since an arbitrary point in the singularity-free gap, unless it be infinitely close to one of the singular regions, can, by a simple transformation, be made to correspond to the middle point of the singularity-free gap. For example, in the case of the ρ, if $z=4a\nu/[4b+(b-a)\nu]$, $a>0$, $b>0$, then $\nu=-4$ and $\nu=0$ correspond respectively to $z=-4$ and $z=0$, while $\nu=-2$ corresponds to $z=-4a/(b+a)$, an arbitrary number lying between -4 and 0.

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[1] F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962). L. A. P. Balázs, Phys. Rev. **125**, 2179 (1962); **128**, 1935 (1962). V. Singh and B. M. Udgaonkar, Phys. Rev. **128**, 1820 (1962); **123**, 1487 (1961). S. R. Choudhury and L. K. Pande, Phys. Rev. **135**, B1027 (1964).

[2] Our notations are the same as in the corresponding papers, Refs. 3 and 4.

[3] L. A. P. Balázs, Phys. Rev. **128**, 1935 (1962).

[4] S. K. Bose and M. Dersarkissian, Nuovo Cimento **30**, 878 (1963).

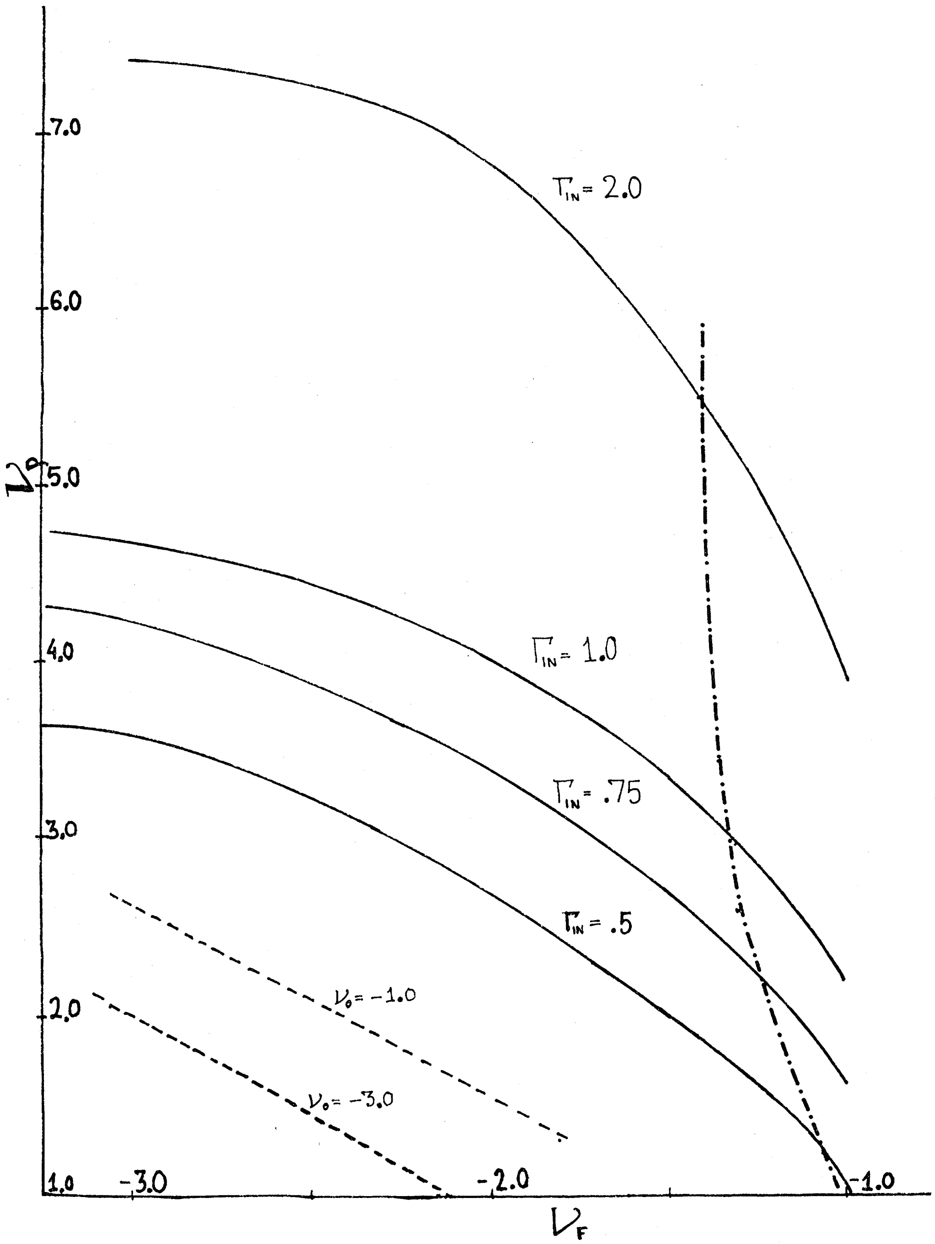

FIG. 1. Variations of ν_ρ with respect to the variations in the matching point, i.e., the curve on which $D_\nu(\nu)=0$. The dashed lines — — — show the value of ν for which $D_\nu(\nu)$ is infinite, as a function of the matching point ν_F. The dashed and dotted lines —·—·—·— show those self-consistent values of ν_p for which $\Gamma_{\text{in}}=\Gamma_{\text{out}}$.

(3) One might wonder why we do not favor a plot of self-consistent values of ν_ρ and Γ as functions of ν_F. Self-consistent ν_ρ and Γ exist only for a small range of values of the ν_F, which is shown as the dashed and dotted curve in Fig. 1. For $\nu_F \lesssim -1.5$ no self-consistent values of both ν_ρ and Γ exist. Such a situation is perhaps better illustrated by plots of the kind we have used.

(4) If for a given Γ_{in}, the ν_ρ has an extremum as a function of ν_F, then for that particular value of Γ_{in} the ν_ρ does not vary as ν_F is varied around the extremum. If one can get $\Gamma_{\text{in}}=\Gamma_{\text{out}}$ around such a region, then at least the self-consistent ν_ρ will not be sensitive to small changes in ν_F. Thus the extremum values of ν_ρ are to be preferred.

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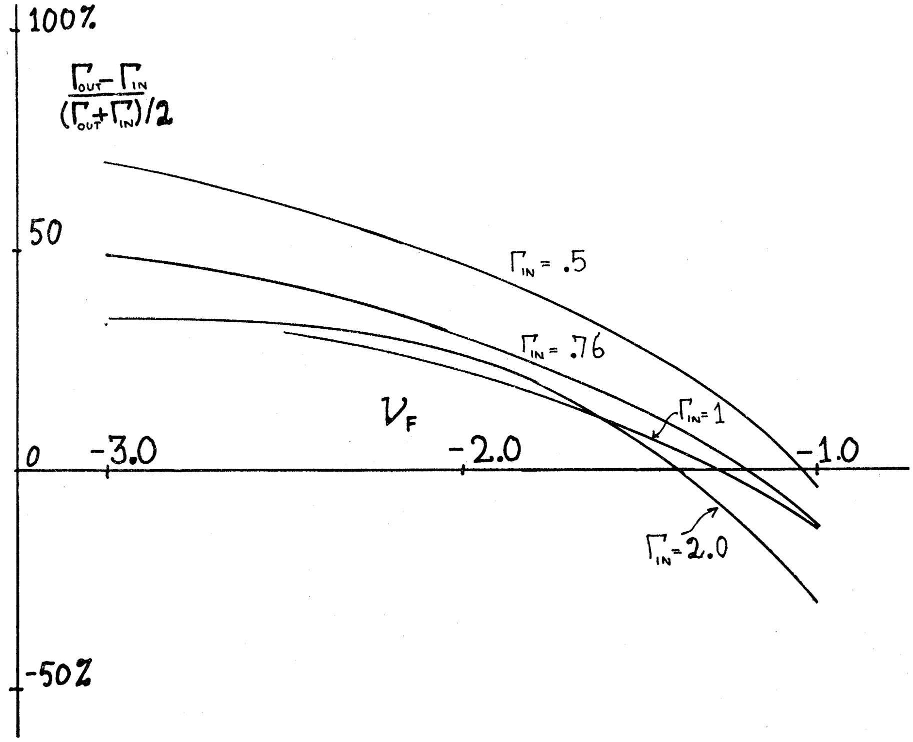

FIG. 2. Variation of the output width as a function of the matching point evaluated at $D_{\nu_p}(\nu_p)=0$.

APPENDIX

The dependence of the result on the subtraction point is a general feature and is true no matter what ansatz is made for the left-hand singularities. More specificially, Bose and Dersarkissian took

$$N(\nu)=\sum_{i=1}^{2}\frac{a_i}{\nu+\alpha_i},$$

and

$$\begin{aligned}D(\nu)&=1-\frac{\nu-\nu_0}{\pi}\int_0^\infty d\nu'\frac{\rho(\nu')N(\nu')}{(\nu'-\nu_0)(\nu'-\nu)}\\&=1+\frac{1}{\pi}\int_0^\infty d\nu'\frac{\rho(\nu')N(\nu')}{\nu'-\nu_0}-\frac{1}{\pi}\int_0^\infty d\nu'\frac{\rho(\nu')N(\nu')}{\nu'-\nu}\\&=c-\frac{1}{\pi}\int_0^\infty d\nu'\frac{\rho(\nu')N(\nu')}{\nu'-\nu},\end{aligned}$$

where c is a constant depending on ν_0. Therefore

$$h_0(\nu)=\frac{N}{D}=n(\nu)\bigg/\left(1-\frac{1}{\pi}\int_0^\infty d\nu'\frac{\rho(\nu')n(\nu')}{\nu'-\nu}\right),$$

where

$$n(\nu)=\frac{1}{c}N(\nu)=\sum_{i=1}^{2}\frac{a_i/c}{\nu+\alpha_i}.$$

Assuming that the α_i are known, this function depends on only two quantities (a_1/c) and (a_2/c). The same is therefore true of $h_0(\nu)$ as can be seen from Eqs. (3) and (4). Thus, if we require a certain value and derivative at a certain matching point, we *uniquely* determine (a_1/c) and (a_2/c). This in turn *uniquely* determines $h_0(\nu)$ *everywhere* through Eqs. (3) and (4) *no matter what* c (and hence ν_0) is.