# Some Considerations on Nonleptonic Decays

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A mechanism, based on the hypothesis that there is a direct interaction of the mesons with the charged vector field  $Z_{\mu}$ , has recently been proposed by Schwinger for the pure  $\Delta T = \frac{3}{2} \operatorname{decay} K^+ \to \pi^+ + \pi^0$ . An extension of this mechanism to other predominantly  $\Delta T = \frac{1}{2}$  nonleptonic decays is considered in this paper. The three branching ratios  $\Gamma(K_1^0 \to \pi^+\pi^-)/\Gamma(K_1^0 \to \pi^0\pi^0)$ ,  $\Gamma(\Lambda \to p\pi^-)/\Gamma(\Lambda \to m\pi^0)$ , and  $\Gamma(\Xi \to \Lambda\pi^-)/\Gamma(\Xi^0 \to \Lambda\pi^0)$  are seen to be closely related. The isotopic-spin change is seen to be  $\Delta T = \frac{1}{2}$  for all  $3\pi$  modes of the K-meson decays. Relations are derived among the parameters describing the energy spectra of  $K_{\pi3}$ decays.

### I. INTRODUCTION

A<sup>S</sup> a phenomenological transcription of his field theory of fundamental weak interactions, Schwinger<sup>1</sup> has suggested a model in which the mesons are directly coupled to the charged vector field  $Z_{\mu}$ . The socalled Goldberger-Treiman relations are immediate consequences of this point of view. Using this model, Schwinger<sup>2</sup> has computed the absolute rate for the  $\Delta T = \frac{3}{2}$  process  $K^+ \rightarrow \pi^+ + \pi^0$ . In order to account for the observed  $K^+ \rightarrow \pi^+ + \pi^0$  to  $K_{1^0} \rightarrow \pi^+ + \pi^-$  amplitude ratio of 1/23, the  $K_{\mu3}$ -decay parameter  $\xi$  is required to be around -6. However, recent experimental determinations<sup>3</sup> indicate that this parameter is in the neighborhood of zero. Hence a simple application of the model would give an amplitude ratio  $\sim 1/10$  instead of the observed 1/23. This probably means that higher order effects (in strong interactions) would have to be included in order to obtain quantitative agreement.

In this work, we will extend the considerations to the predominantly  $\Delta T = \frac{1}{2}$  nonleptonic decays. In Sec. II we briefly describe the model. In Sec. III we consider the two-particle nonleptonic decays of  $K_1^0$  and hyperons, for which the quantitative results obtained are believed to be reasonably reliable. These results are compared with the still conflicting experimental data. In Sec. IV we consider the  $K_{\pi3}$  decays. Among other things, a relation is derived connecting the energy spectrum parameters  $\sigma_{\tau}$  and  $\sigma_{\tau'}$  for  $\tau$  and  $\tau'$  decays, respectively. Since higher order corrections are not attempted, the result may again be off by a factor of 2 or so.

#### **II. THE MODEL**

In Schwinger's field theory of matter,<sup>1</sup> the fundamental weak interactions consist of the interaction between the charged vector field  $Z_{\mu}$  and the leptonic field  $X_a(a=1, 2, 3)$ 

$$e'\bar{Z}_{\mu}[\bar{\chi}_{1}\gamma^{\mu}(1+i\gamma_{5})\chi_{2}+\bar{\chi}_{3}\gamma^{\mu}(1-i\gamma_{5})\chi_{1}]+\mathrm{adj.}, \quad (1)$$

and the interaction between the Z field and the funda-

mental nucleonic charge-bearing fermion field  $\psi_a$  and the boson field  $V_a(a=1, 2, 3)$ 

$$\begin{aligned} e' Z_{\mu} [ \bar{\psi}_{1} \gamma^{\mu} (1 + i\gamma_{5}) \psi_{2} + \bar{\psi}_{1} \gamma^{\mu} (1 - i\gamma_{5}) \psi_{3} \\ + i (\bar{V}_{1}^{\mu\nu} V_{\nu 2} - \bar{V}_{\nu 1} V_{2}^{\mu\nu}) + i (\bar{V}_{1}^{\mu\nu} V_{\nu 3} - \bar{V}_{1} V_{3}^{\mu\nu}) ] \\ + \mathrm{adj.}, \quad (2) \end{aligned}$$

where the leptonic fields  $\chi_1$ ,  $\chi_2$ , and  $\chi_3$  are identified with  $\nu$ ,  $\mu^+$ , and  $e^-$ , respectively. If the strongly interacting particles are identified as the low-lying excitations produced by combinations of fields with opposite nucleonic charges, the vector and pseudovector currents that are coupled to the Z field in (2) can generate meson states of spin-parity  $1^{\pm}$  and therefore can be approximately represented by the phenomenological fields of known 1<sup>-</sup> and 0<sup>-</sup> mesons, i.e.,

$$e'\bar{Z}^{\mu}\left[\gamma_{Z\rho}\rho_{\mu}^{+}+\frac{\gamma_{Z\pi}}{m_{\pi}}\pi_{\mu}^{+}+\gamma_{ZK}*K_{\mu}^{*+}+\frac{\gamma_{ZK}}{m_{K}}K_{\mu}^{+}\right]+\mathrm{adj.},\,(3)$$

where  $\gamma$ 's are the phenomenological coupling constants. Both the muon decay and the two-body leptonic decays of  $\pi^{\pm}$  and  $K^{\pm}$  proceed through the Z field. From the known muon lifetime, the coupling constant e' is related to the mass of the Z field by

$$e'^2/m_z^2 = 1.60 \times 10^{-7} m_\pi^{-2}.$$
 (4)

The mesons in (3) are strongly coupled to the baryons and meson pairs. As a result, there arise the three-body leptonic decays and the two-body nonleptonic decays  $(\Delta T = \frac{1}{2} \text{ and } \frac{3}{2})$  of the mesons and baryons. One immediate consequence of this model is the emergence of the Goldberger-Treiman type relations for these processes.

#### **III. TWO-BODY NONLEPTONIC DECAYS**

Since the mixed  $\Delta T = \frac{1}{2}$  and  $\frac{3}{2}$  transitions are mediated by the *charged* vector field  $Z_{\mu}$ , our mechanism contributes only to those two-body nonleptonic decays that have a *charged* pion in the final state. Insofar as the decay rate is concerned, the s-wave (parity-violating) transition is important. The dynamical mechanism for the dominant  $\Delta T = \frac{1}{2}$  s-wave transitions is also comparatively simple. We will therefore restrict ourselves to the consideration of only the s-wave transitions.

<sup>&</sup>lt;sup>1</sup> J. Schwinger, Phys. Rev. 135, 816 (1964); 136, B1821 (1964).

 <sup>&</sup>lt;sup>2</sup> J. Schwinger, Phys. Rev. Letters 12, 630 (1964).
 <sup>3</sup> G. Gidal, W. M. Powell, R. March, and S. Natali, Phys. Rev. Letters 13, 95 (1964); and T. H. Groves, P. R. Klein, and V. VanderBurg, Phys. Rev. 135, B1269 (1964).

As has been noted by Schwinger,<sup>2,4</sup> all s-wave  $\Delta T = \frac{1}{2}$ nonleptonic decays can be accounted for by the socalled pseudoscalar-vacuon mechanism, in which  $K^*$ and  $\pi$  are coupled. On the other hand, through the intermediary of the charged vector Z field, s-wave  $\Delta T = \frac{3}{2}$  transitions can occur from an effective coupling of charged  $K^*$  with  $\pi$ . These considerations lead to a phenomenological model in which all the parity-violating (s-wave), nonleptonic decays proceed through an intermediate K\*. Further, the same  $\Delta T = \frac{3}{2}$  admixture will be present in all such decays which have a charged pion in the final state. As a first result, this implies that the sum rule<sup>2,4,5</sup> for the s-wave  $\Delta T = \frac{1}{2}$  coupling constants is not affected even with the inclusion of the  $\Delta T = \frac{3}{2}$  admixtures.

The amount of  $\Delta T = \frac{3}{2}$  admixture in the s-wave channel is characterized by the  $K^+ \rightarrow \pi^+ + \pi^0$  to  $K_1^0 \rightarrow \pi^+ + \pi^-$  amplitude ratio, which is empirically known to be<sup>6</sup>

$$A (K^+ \to \pi^+ + \pi^0) / A (K_1^0 \to \pi^+ + \pi^-) = A_3 / (A_1 + 2A_3) = 0.045 \pm 0.0005, \quad (5)$$

where  $A_1$  and  $A_3$  represent the  $\Delta T = \frac{1}{2}$  and  $\Delta T = \frac{3}{2}$ amplitudes, respectively. Since there is no charged pion in the final state of  $K_1^0 \rightarrow \pi^0 + \pi^0$ , there is  $no \Delta T = \frac{3}{2}$ admixture in this transition. By elementary isotopic-spin considerations, we infer from (5) that the amplitude ratio is

$$A(K_1^0 \to \pi^+ + \pi^-) / A(K_1^0 \to \pi^0 + \pi^0) = (A_1 + 2A_3) / A_1 = 0.917 \text{ or } 1.10, \quad (6)$$

depending on the relative phase<sup>7</sup> of the  $\Delta T = \frac{1}{2}$  and  $\Delta T = \frac{3}{2}$  channels. It follows from (6) that the branching ratio is

$$\frac{\Gamma(K_1^0 \to \pi^+ + \pi^-)}{\Gamma(K_1^0 \to \pi^0 + \pi^0)} = 1.97 \times \left(\frac{A_1 + 2A_3}{A_1}\right)^2 = \frac{62.4}{37.6} \text{ or } \frac{70.4}{29.6}.$$
 (7)

The latter ratio, 70.4/29.6, is consistent with the experimental result.<sup>6</sup> This implies that the two channels,  $\Delta T = \frac{1}{2}$  and  $\Delta T = \frac{3}{2}$ , interfere constructively.

The decays  $\Lambda \to p + \pi^-$  and  $\Xi^- \to \Lambda + \pi^-$  are known to be predominantly s wave, in the sense that it constitutes 88.5% and 93.0% of the decay rates, respectively.<sup>8,9</sup> To the extent that the  $\Delta T = \frac{3}{2}$  admixture in

<sup>6</sup> B. W. Lee, Phys. Rev. Letters 12, 83 (1964).
<sup>6</sup> R. H. Dalitz, in Brookhaven National Laboratory Report No. BNL 837, 1963, p. 383 (unpublished).
<sup>7</sup> Time-reversal invariance is assumed.
<sup>8</sup> The percentage 88.5% for the Λ decay rate is based on the *p*- to *s*-wave amplitude ratio 0.36, given by J. M. Cronin and O. E. Overseth, Phys. Rev. 129, 1795 (1963).
<sup>9</sup> H. K. Ticho, in Brookhaven National Laboratory Report No. BNL 837, 1963, p. 140 (unpublished). Our percentage 93.0% for the Ξ<sup>-</sup> decay rate is based on the *p*- to *s*-wave amplitude ratio (0.08)<sup>1/2</sup> given in this reference.

the p-wave (parity-conserving) channel is neglected, we obtain the branching ratios

$$B_{\Lambda} = \frac{\Gamma(\Lambda \to p + \pi^{-})}{\Gamma(\Lambda \to n + \pi^{0})} = 2 \times \frac{88.5 \times (1.10)^{2} + 11.5}{100} = \frac{70.3}{29.7},$$

$$B_{\Xi} = \frac{\Gamma(\Xi^{-} \to \Lambda + \pi^{-})}{\Gamma(\Xi^{0} \to \Lambda + \pi^{0})} = 2 \times \frac{93.0 \times (1.10)^{2} + 7}{100} = \frac{70.5}{29.5},$$
(8)

where small corrections due to small mass differences have been neglected. Experimental results concerning these branching ratios are conflicting. The experimental determinations available for the branching ratios  $B_{\Lambda}$ have been summarized by Chretien et al.<sup>10</sup> Their own value  $(70.9\pm3.4)/(29.1\mp3.4)$  is in good agreement with our predicted value. The  $\Xi^- \rightarrow \Lambda + \pi^-$  decay rate is experimentally better known than that for  $\Xi^0 \rightarrow \Lambda + \pi^0$ . The  $\Xi^-$  lifetime<sup>9</sup> is  $(1.76 \pm 0.05) \times 10^{-10}$  sec. According to (8), the  $\Xi^0$  lifetime is predicted to be around 4.2  $\times 10^{-10}$  sec. The experimental situation concerning this lifetime has been reviewed by Ticho,<sup>9</sup> and is still not definite. Of the three measurements quoted by Ticho, the results of the UCLA group and the École Polytechnique group, which are, respectively,  $(3.5_{-0.7}^{+0.9}) \times 10^{-10}$ sec and  $(3.8_{-0.65}^{+1.0}) \times 10^{-10}$  sec, are consistent with our predicted value. A reliable determination of the branching ratios  $B_{\Lambda}$  and  $B_{\Xi}$  is therefore of particular interest.

#### IV. $K_{\pi^3}$ DECAYS

As a result of the direct coupling of the mesons with the charged vector field  $Z_{\mu}$ , there is the weak transition  $K^{*+} \rightarrow \rho^+$ . The isotopic spin change can be  $\frac{1}{2}$  and  $\frac{3}{2}$ . This gives rise to an effective coupling for  $K_{\pi 3}$  decays, via the successive strong, weak, and strong couplings  $(K\pi)^+ \to K^{*+} \to \rho^+ \to (\pi\pi)^+$ . The effective coupling constant can be obtained from the known leptonic decay rates. The relevant strong couplings are

$$g_{K^*K\pi}(K^{*+})^{\lambda} [(1/\sqrt{2})(\bar{K}^-\pi_{\lambda}^0 - \bar{K}_{\lambda}^-\pi^0) + (\bar{K}^0\pi_{\lambda}^- - \bar{K}_{\lambda}^0\pi^-)] + \text{adj.} \quad (9)$$
  
and

$$g_{\rho\pi\pi}(\rho^+)^{\lambda}(\pi^-\pi_{\lambda}^0-\pi_{\lambda}^-\pi^0) + \mathrm{adj.}$$
(10)

As a combined result of (1), (3), (9), and (10), there arise the following effective couplings for  $\pi^+ \rightarrow \pi^0$  $+e^++\nu$ ,  $K^+ \rightarrow \pi^0 + \mu^+ + \bar{\nu}$ , and  $K \rightarrow 3\pi$  ( $\Delta T = \frac{1}{2}$  and  $\frac{3}{2}$ ), respectively,

$$(g_{\pi\pi l}/m_{\pi}^{2})(\pi^{+}\pi_{\lambda}^{0}-\pi_{\lambda}^{+}\pi^{0})\bar{\nu}\gamma^{\lambda}(1-i\gamma_{5})e^{-}, \qquad (11)$$

$$(g_{K\pi l}/m_{\pi}m_{K})(K_{\lambda}+\pi^{0}-K+\pi_{\lambda}^{0})\bar{\mu}-\gamma^{\lambda}(1+i\gamma_{5})\nu, \quad (12)$$

and

$$(g_{K\pi\pi\pi}/m_{\pi}m_{K})[(K^{+}\pi^{0\lambda}-K^{+\lambda}\pi^{0})(\pi^{-}\pi_{\lambda}^{0}-\pi_{\lambda}^{-}\pi^{0}) + (\pi^{+}K_{2}^{\lambda}-\pi^{+\lambda}K_{2})(\pi^{-}\pi_{\lambda}^{0}-\pi_{\lambda}^{-}\pi^{0}) - (\pi^{-}K_{2}^{\lambda}-\pi^{-\lambda}K_{2})(\pi^{0}\pi_{\lambda}^{+}-\pi_{\lambda}^{0}\pi^{+})], \quad (13)$$

<sup>10</sup> M. Chretien, V. K. Fischer, H. R. Crouch, Jr., R. E. Lanou, Jr., J. T. Massimo et al., Phys. Rev. 131, 2208 (1963).

<sup>&</sup>lt;sup>4</sup> J. Schwinger, Phys. Rev. Letters **13**, 355 (1964). <sup>5</sup> B. W. Lee, Phys. Rev. Letters **12**, 83 (1964).

where

$$\frac{g_{\pi\pi l}}{m_{\pi}^2} = \frac{e^{\prime 2}}{m_Z^2} \gamma_{Z\rho} g_{\rho\pi\pi} \frac{1}{m_{\rho}^2}, \qquad (14)$$

$$\frac{g_{K\pi l}}{m_{\pi}m_{K}} = \frac{e^{\prime 2}}{m_{Z}^{2}} \gamma_{ZK*} \frac{g_{K*K\pi}}{\sqrt{2}} \frac{1}{m_{K*}^{2}},$$
(15)

and

$$\frac{g_{K\pi\pi\pi}}{m_{\pi}m_{K}} \approx \frac{e^{\prime 2}}{m_{Z}^{2}} \gamma_{Z\rho} \gamma_{ZK*} g_{\rho\pi\pi} \frac{g_{K*K\pi}}{\sqrt{2}} \frac{1}{m_{\rho}^{2}} \frac{1}{m_{K*}^{2}}.$$
 (16)

It is to be noted that the  $K^*K\pi$ -coupling (9) has been written down in anticipation of the recent experimental result that the form factor ratio  $\xi$  in  $K_{\mu3}$  decays is consistent with zero.<sup>3</sup> Even if a coupling term of the form  $(K^{*+})^{\lambda}(\bar{K}^{-}\pi_{\lambda}^{0}+\bar{K}_{\lambda}^{-}\pi^{0})$  is included in (9), it will not show up in (13), since the current coupled to  $\rho_{\mu}^{+}$ is conserved. Combining (4), (14), (15), and (16), one obtains

$$g_{K\pi\pi\pi} = g_{\pi\pi l} g_{K\pi l} / g_{ll}. \tag{17}$$

The coupling constants  $g_{\pi\pi l}$  and  $g_{K\pi l}$  are empirically known<sup>11</sup> to be

$$g_{\pi\pi l}^{2}/4\pi = 4.38 \times 10^{-15}$$
$$g_{K\pi l}^{2}/4\pi = 0.747 \times 10^{-15}.$$

Thus, we obtain

$$g_{K\pi\pi\pi^2}/4\pi = 1.61 \times 10^{-15}$$
. (18)

The dominant  $\Delta T = \frac{1}{2} K_{\pi 3}$  decays can be regarded as a consequence of the scalar-vacuon mechanism,<sup>4</sup> within the framework of a broken  $SU_3$  symmetry. Denoting by  $A_0$  and  $A_{\tau'}$  the dominant  $\Delta T = \frac{1}{2}$  amplitudes for  $K_{2^{0}} \rightarrow \pi^{+} + \pi^{-} + \pi^{0}$  and  $K^{+} \rightarrow \pi^{0} + \pi^{0} + \pi^{+}$ , respectively, we have

$$X_{0}(\omega_{+},\omega_{-},\omega_{0}) + (g_{K\pi\pi\pi}/m_{\pi}m_{K}) \\ \times [(P+p_{+})^{\lambda}(p_{0}-p_{-})_{\lambda} + (P+p_{-})^{\lambda}(p_{0}-p_{+})_{\lambda}]$$
(19)

and

$$\begin{array}{l} A_{\tau'}(\omega_{0},\omega_{0}',\omega_{+}) + (g_{K\pi\pi\pi}/m_{\pi}m_{K}) \\ \times \left[ (P+p_{0})^{\lambda}(p_{+}-p_{0}')_{\lambda} + (P+p_{0}')^{\lambda}(p_{+}-p_{0})_{\lambda} \right] \end{array} (20)$$

as the total amplitudes for these decays, where the  $\omega$ 's refer to the energies and the p's to the four momenta of the pions, and P to the four momentum of the K; the subscripts attached to the  $\omega$ 's and p's are selfexplanatory. It is known<sup>12</sup> on general grounds that the  $\Delta T = \frac{1}{2}$  amplitudes  $A_0(\omega_+, \omega_-, \omega_0)$  and  $A_{\tau'}(\omega_0, \omega_0', \omega_+)$  are identical functions of the respective arguments and that they are symmetrical with respect to the first two variables. These two properties insure some exact predictions of the  $\Delta T = \frac{1}{2}$  rule (to the extent that small

corrections due to small mass differences are neglected); the branching ratio  $\Gamma(K_2^0 \rightarrow \pi^+ \pi^- \pi^0) / \Gamma(K^+ \rightarrow \pi^0 \pi^0 \pi^+)$ is 2; the energy spectra of the corresponding pions in the two decays are identical. As is explicit from (19) and (20), these two general properties of the  $\Delta T = \frac{1}{2}$ amplitudes are preserved even with the inclusion of the  $g_{K\pi\pi\pi}$  terms. This means that only the  $\Delta T = \frac{1}{2}$  component of the weak transition  $K^{*+} \rightarrow \rho^+$  is effective in  $K_{\pi 3}$  decays. It follows that we still have the same prediction of a strict  $\Delta T = \frac{1}{2}$  rule for the two decays  $K_{2^{0}} \rightarrow \pi^{+} + \pi^{-} + \pi^{0}$  and  $K^{+} \rightarrow \pi^{0} + \pi^{0} + \pi^{+}$ . Experimentally, the value 2 for the branching ratio has recently been confirmed.<sup>13</sup> The experimental findings concerning the pion spectra of these two decays are also consistent with the prediction.14,15

Contrary to the case of  $\tau'$  decay, our mechanism does not contribute to the  $\tau$  decay  $K^+\!\rightarrow \pi^+\!+\pi^+\!+\!\pi^-$  (and  $K_{2^{0}} \rightarrow \pi^{0} + \pi^{0} + \pi^{0}$ . The branching ratio is insensitive to the presence of the additional p-wave amplitude in the  $\tau'$  decay, but a comparison of the pion energy spectra of the  $\tau$  and  $\tau'$  decays can provide a practicable test of its presence.<sup>16</sup> To proceed, we shall decompose the dominant  $\Delta T = \frac{1}{2} \tau$  and  $\tau'$  decay amplitudes  $A_{\tau}(\omega_{+},\omega_{+}',\omega_{-})$  and  $A_{\tau'}(\omega_{0},\omega_{0}',\omega_{+})$  into s waves and p waves. In terms of the Dalitz variables x and y defined by

$$x_{\tau} = \sqrt{3} (T_{+} - T_{+}')/Q_{\tau}, \quad y_{\tau} = (3T_{-} - Q_{\tau})/Q_{\tau}, \text{ etc.},$$

where T's refer to the kinetic energies of the pions and Q the total kinetic energy available for the process, we obtain

 $a_{\tau} + b_{\tau} (Q_{\tau}/m_K) y_{\tau}$ 

and

$$a_{\tau'}+b_{\tau'}(Q_{\tau'}/m_K)y_{\tau'}-2g_{K\pi\pi\pi}(Q_{\tau'}/m_K)y_{\tau'}$$

as the amplitudes for  $\tau$  and  $\tau'$  decays, respectively. The absence of terms linear in x is a result of the Bose statistics for the pions. The dominant  $\Delta T = \frac{1}{2}$  terms satisfy the relations<sup>16</sup>

$$a_{\tau} = -2a_{\tau'}, \qquad (21)$$
$$b_{\tau} = b_{\tau'}.$$

The differential spectrum for the unlike pion in both  $\tau$  and  $\tau'$  decays is of the form

with 
$$\begin{bmatrix} 1 - 2\sigma m_K (2Q/3)y \end{bmatrix} x(y) dy,$$
$$\sigma_z = -(3/2m_K^2) b_z/a_z.$$

$$\sigma_{\tau'} = -\frac{3}{2m_K^2} \left[ \frac{b_{\tau'}}{a_{\tau'}} - \frac{m_K}{m_{\pi}} \frac{g_{K\pi\pi\pi}}{a_{\tau'}} \right], \tag{22}$$

<sup>13</sup> D. Stern, T. O. Binford, V. G. Lind, J. A. Anderson, F. S. Crawford, Jr., and R. L. Golden, Phys. Rev. Letters **12**, 459

<sup>&</sup>lt;sup>11</sup> While the value for  $g_{K\pi t^2}/4\pi$  is quoted from Ref. 2, that for  $g_{\pi\pi t^2}/4\pi$  is inferred from the decay rate given by P. Depommier, J. Heintze, C. Rubbia, and V. Zoergel, Phys. Letters 5, 61 (1963). We remark that the conserved vector-current theory predicts the value  $g_{\pi\pi t^2}/4\pi = 2(g_{1t}^2/4\pi) = 4.06 \times 10^{-16}$ . <sup>12</sup> R. F. Sawyer and K. C. Wali, Nuovo Cimento 17, 938 (1960).

<sup>(1964).</sup> <sup>14</sup> G. Barton, C. Kacser, and S. P. Rosen, Phys. Rev. 130, 783

<sup>(1963).
&</sup>lt;sup>15</sup> G. E. Kalmus, A. Kernan, R. T. Pu, W. M. Powell, and R. Dowd, Phys. Rev. Letters 13, 99 (1964).
<sup>16</sup> S. Weinberg, Phys. Rev. Letters 4, 87 and 585(E) (1960).

where x(y) is the maximum value of x for a given y. The two parameters  $\sigma_r$  and  $\sigma_{r'}$  are then related by the relation

$$_{\tau'} - (3/m_{\pi}m_K)(g_{K\pi\pi\pi}/a_{\tau'}) = -2\sigma_{\tau}.$$
 (23)

The value of the parameter  $a_{\tau'}$  is fixed by the  $\tau'$  decay rate<sup>17</sup> to be

$$a_{\tau'} = [64(2\pi)^2 3^{3/2} m_K(\Gamma(\tau')/Q_{\tau'}^2)]^{1/2} = 9.12 \times 10^{-7}.$$

Hence, we obtain the relation (in the unit of  $m_{\pi}^{-2}$ )

$$\sigma_{\tau'} = -2\sigma_{\tau} \pm 0.13, \qquad (24)$$

where the  $\pm$  signs allow for the two possible relative phases of the two channels. The latest experimental information<sup>15</sup> consists of

and

$$\sigma_{\tau'} = (0.24 \pm 0.02) m_{\pi}^{-2}$$
  
 $\sigma_{\tau'} / \sigma_{\tau} = -2.3 \pm 0.4$ .

<sup>17</sup> G. Alexander, S. P. Almeida, and F. S. Crawford, Jr., Phys. Rev. Letters 9, 68 (1962).

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Although these data are not well correlated by our relation (24), the emergence of the ratio -2.3 is encouraging. As we have already remarked in the introduction, the value of the coupling constant  $g_{K\pi\pi\pi}$  as given by (18) may be off by a factor of 2 or so. Within this limit, the relation (24) can indeed be consistent with the experimental result. Of course, more experimental effort to determine the  $\sigma$  parameters more accurately is desirable.

To make explicit the result already stated, we also record the relation

$$\sigma_{\tau'} = \sigma(+-0)$$

where  $\sigma(+-0)$  refers to the energy spectrum of  $\pi^0$  in the decay  $K_{2^0} \rightarrow \pi^+ + \pi^- + \pi^0$ .

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## Strange-Particle Production by $3-\text{BeV}/c \pi^-$ Mesons in Hydrogen\*

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Over 600 strange-particle events were identified on the basis of both kinematical fitting and bubble-density information. The center-of-mass angular distributions generally show the familiar backward tendency for the baryons and forward tendency for the K mesons. Invariant mass distributions show the production of several resonant states including the  $Y_1^*(1385 \text{ MeV})$ ,  $K^*(885 \text{ MeV})$ , and  $Y_0^*(1405 \text{ MeV})$ . The two-body reactions  $\pi^- + p \rightarrow K^* + Y_1^*$  and  $\pi^- + p \rightarrow K^* + Y_0^*$  are also reported and studied. The two-, three-, and four-particle reactions are discussed and an attempt is made to put limits on the  $SU_3$  multiplet of the  $f^0$  from the number of  $K_1^0K_1^0$  events with a mass near the  $f^0$ . The cross sections are calculated and the total strange-particle cross section at this energy is estimated to be  $\sigma = (1.68 \pm 0.20)$  mb.

#### INTRODUCTION

**F**ROM an analysis of 60 000 pictures of  $\pi^{-}p$  interactions at a pion momentum of 3 BeV/c in the B.N.L. 20-in. hydrogen bubble chamber, over 600 measured events were identified as strange-particle events. A complete scan of all the film was done for charged decays and neutral V's and two-thirds of the film was rescanned. The film was measured on digitized projectors and IBM-704 computer programs performed the tasks of geometrical reconstruction and kinematical fitting to the various mass hypotheses. A revised version of GUTS<sup>1</sup> formed the basis of the kinematical fitting program. Visual estimates of the bubble density of each track were also used in the analysis. All events found in the scanning were measured and analyzed except for single V events with zero prongs.

After the analysis was completed there were 437 events believed to be unambiguous fits to strange particle hypotheses, 130 events ambiguous between two categories, and 98 events with two or more neutral particles. Of the 130 ambiguous events 62 were ambiguous between  $\pi^-K^+\Lambda^0$  and  $\pi^-K^+\Sigma^0$ . The remaining 68 events were distributed among the various categories according to the lowest  $\chi^2$ . The  $\chi^2$  distributions for the one-constraint and four-constraint fits seen in Figs. 1(a) and 1(b), show good agreement with the theoretical distributions.

#### CROSS SECTIONS

The cross sections were determined by normalizing to the total cross section of 32.5 mb measured by a

<sup>\*</sup> Work done under the auspices of the U. S. Atomic Energy Commission. <sup>1</sup> J. P. Berge, F. T. Solmitz, H. D. Taft, Rev. Sci. Instr. **32**, 538

<sup>(1961).</sup>