Neutron-Deuteron Scattering at Low Energies*

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Using the exact three-body formalism with separable potentials proposed sometime ago by us, the scattering lengths and low-energy phase shifts in n-d scattering have been evaluated. The nature of the contribution of the so-called "potential scattering" to the scattering lengths has also been examined within this formalism and found not to exceed 10% of the "nucleon exchange" contribution.

SOME time ago one of the authors (A.N.M.) proposed an exact formalism for the three-body problem[1] as a means of studying the properties of the triton. Subsequently this formalism was extended by two of us (ANM and VSB) to the problem of neutron-deuteron scattering.[2] The preliminary results on the n-d scattering lengths in the $S=\frac{3}{2}$ and $S=\frac{1}{2}$ states were given in Ref. (2), which we refer to here as A. The formalism of the three-body problem as given in these two papers can be cast almost without modification in the language of Faddeev's three-particle theory.[3] In the case $S=\frac{3}{2}$, for example, the Faddeev wave function $\Psi^{(1)}$ for the scattering of particle 1 by the composite of 2 and 3 is expressible in terms of the wave function $F(\mathbf{P}_1)$ of A [see Eqs. (3.5)] by the relation

$$\Psi^{(1)}(\mathbf{p}_{23},\mathbf{P}_1)=Ng(p_{23})F(\mathbf{P}_1)[p_{23}^2+\tfrac{3}{4}P_1^2-ME]^{-1}. \quad (1)$$

Very recently a paper by Sitenko and Kharchenko[4] on the three-body problem using separable potentials with a formulation closely resembling that of A, has come to our notice.

We would like to present here the results of a more accurate evaluation of the n-d scattering lengths as well as the low energy n-d phase shifts. Since the formalism already exists in the papers mentioned above, we shall omit the algebraic details completely and make free use of the notation and results of A. However, in addition to the usual "exchange scattering" (represented by the graphs in Fig. 1), we have considered here the effects of the so-called "potential scattering" as represented by the graphs in Fig. 2. It may be remarked that the results of A or of Ref. (4) can be interpreted in terms of the contribution of exchange scattering only.

Following Faddeev,[3] the exact $S=\frac{3}{2}$ n-d scattering amplitude works out to be

$$\langle\Phi_f^{(3/2)}|V_{12}+V_{13}|\Psi_{3/2}\rangle=-(2\pi)^3N^2K_1(\mathbf{k}_i,\mathbf{k}_f)+(2\pi)^3N^2\lambda_{31}\int d\mathbf{p}\,K_1(\mathbf{p},\mathbf{k}_i)K_1(\mathbf{p},\mathbf{k}_f)+4\pi N^2\lambda_{31}\int d\mathbf{p}\,h_{31}(p)K_1(\mathbf{p},\mathbf{k}_f)$$
$$\times a_{3/2}(\mathbf{p})/(p^2-k_i^2-i\epsilon)-4\pi N^2\lambda_{31}\int\int d\mathbf{p}d\mathbf{q}\,K_1(\mathbf{p},\mathbf{k}_f)K_2(\mathbf{q},\mathbf{p})a_{3/2}(\mathbf{q})/(q^2-k_i^2-i\epsilon), \quad (2)$$

where

$$\Phi_f^{(3/2)}=Ng(p_{23})(p_{23}^2+\alpha^2)^{-1}\delta(\mathbf{P}_1-\mathbf{k}_f)\chi^s\zeta', \quad (3)$$

$$N^{-2}=\int d\mathbf{q}\,g^2(q)(q^2+\alpha^2)^{-2}, \quad (4)$$

$$K_1(\mathbf{p},\mathbf{q})=g(\mathbf{p}+\tfrac{1}{2}\mathbf{q})g(\mathbf{q}+\tfrac{1}{2}\mathbf{p})[p^2+\tfrac{1}{4}q^2+\mathbf{p}\cdot\mathbf{q}+\alpha^2]^{-1}, \quad (5)$$

$$K_2(\mathbf{p},\mathbf{q})=g(\mathbf{p}+\tfrac{1}{2}\mathbf{q})g(\mathbf{q}+\tfrac{1}{2}\mathbf{p})[p^2+q^2+\mathbf{p}\cdot\mathbf{q}-ME]^{-1}, \quad (6)$$

and $a_{3/2}(\mathbf{p})$ satisfies the equation [cf. A, Eq. (3.28)]

$$4\pi k_{31}(p,k_i)a_{3/2}(\mathbf{p})=+(2\pi)^3K_2(\mathbf{p},\mathbf{k}_i)-4\pi\int d\mathbf{q}\,K_2(\mathbf{p},\mathbf{q})a_{3/2}(\mathbf{q})/(q^2-k_i^2-i\epsilon). \quad (7)$$

$\mathbf{k}_i$ and $\mathbf{k}_f$ are, respectively, the initial and final momenta of the neutron in the center-of-mass system.

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[3] L. D. Faddeev, Zh. Eksperim. i Teor. Fiz. 39, 1459 (1960) [English transl.: Soviet Phys.—JETP 12, 1014 (1961)].
[4] A. G. Sitenko and V. F. Kharchenko, Nucl. Phys. 49, 15 (1963).

In Eq. (2), the first two terms on the right-hand side represent, respectively, the lowest-order contributions to the graphs of Fig. (1) and Fig. (2). Iterated solutions for $a_{3/2}(\mathbf{p})$ substituted from (7) in (2) would lead to the contributions to successive higher order graphs shown in Figs. (1) and (2). The effect of the virtual deuteron dissociation and recombination (the so-called self-energy "bubbles") is incorporated in this formalism through the function

$$[1-\lambda_{31}h_{31}(p)]^{-1} \tag{8}$$

which appears in Eq. (7).

For the $S=\frac{1}{2}$ case the corresponding expression for the scattering amplitude is

$$\begin{aligned}\langle\Phi_f^{(1/2)}|V_{12}+V_{13}|\Psi_{1/2}\rangle &= \tfrac{1}{2}(2\pi)^3N^2K_1(\mathbf{k}_i,\mathbf{k}_f)+\left(\frac{1}{4}\lambda_{31}+\frac{9}{4}\lambda_{13}\right)N^2(2\pi)^3\int d\mathbf{p}\;K_1(\mathbf{p},\mathbf{k}_i)K_1(\mathbf{p},\mathbf{k}_f)+4\pi N^2\int d\mathbf{p}\;K_1(\mathbf{p},\mathbf{k}_i)(p^2-k_i^2-i\epsilon)^{-1}\\ &\times[\tfrac{3}{2}\lambda_{13}h_{13}(p)a_2(\mathbf{p})-\tfrac{1}{2}\lambda_{31}h_{31}(p)a_1(\mathbf{p})]+\pi N^2\int\int d\mathbf{p}d\mathbf{q}\;K_1(\mathbf{p},\mathbf{k}_f)(q^2-k_f^2-i\epsilon)^{-1}\\ &\times[3(\lambda_{31}+\lambda_{13})K_3(\mathbf{q},\mathbf{p})a_2(\mathbf{q})-(\lambda_{31}+9\lambda_{13})K_2(\mathbf{q},\mathbf{p})a_1(\mathbf{q})],\end{aligned} \tag{9}$$

where

$$\begin{pmatrix}4\pi k_{31}(p)a_1(\mathbf{p})\\4\pi k_{13}(p)a_2(\mathbf{p})\end{pmatrix}=\tfrac{1}{2}(2\pi)^3\begin{pmatrix}-K_2(\mathbf{k}_i,\mathbf{p})\\3K_3(\mathbf{p},\mathbf{k}_i)\end{pmatrix}+2\pi\int\frac{d\mathbf{q}}{q^2-k_i^2-i\epsilon}\begin{pmatrix}K_2(\mathbf{q},\mathbf{p}) & -3K_3(\mathbf{q},\mathbf{p})\\-3K_3(\mathbf{p},\mathbf{q}) & K_4(\mathbf{p},\mathbf{q})\end{pmatrix}\begin{pmatrix}a_1(\mathbf{q})\\a_2(\mathbf{q})\end{pmatrix}, \tag{10}$$

$$\Phi_f^{(1/2)}=Ng(p_{23})(p_{23}^2+\alpha^2)^{-1}\delta(\mathbf{P}_1-\mathbf{k}_f)\chi''\zeta', \tag{10a}$$

and

$$\begin{aligned}K_3(\mathbf{q},\mathbf{p})&=g(\mathbf{q}+\tfrac{1}{2}\mathbf{p})f(\mathbf{p}+\tfrac{1}{2}\mathbf{q})[p^2+q^2+\mathbf{p}\cdot\mathbf{q}-ME]^{-1},\\ K_4(\mathbf{q},\mathbf{p})&=f(\mathbf{q}+\tfrac{1}{2}\mathbf{p})f(\mathbf{p}+\tfrac{1}{2}\mathbf{q})[p^2+q^2+\mathbf{p}\cdot\mathbf{q}-ME]^{-1}.\end{aligned} \tag{11}$$

The unexplained notations are the same as given in A.

The scattering lengths and low-energy phase shifts have been evaluated for the case of central s-wave forces discussed in A. Two sets of parameters have been considered (see also A):

(I): $\beta_1=\beta_2=6.255\alpha$, (Yamaguchi[5])

(II): $\beta_1=5.8\alpha$, $\beta_2=5.5\alpha$, (Naqvi[6]).

The numerical computations were made on the IBM 709 situated at the Indiana University Computing Center. The results for the scattering lengths (doublet and quartet) are shown in Table I, and those for low-energy phase shifts (below break-up threshold) appear in Table II.

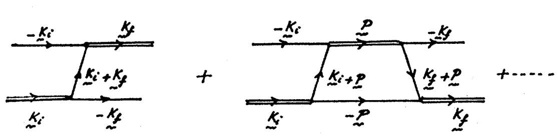

FIG. 1. Sum of the graphs representing "nucleon exchange" in n-d scattering. Double lines denote the deuterons, while nucleons are represented by single lines.

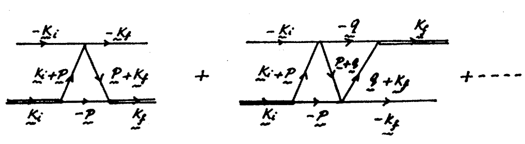

FIG. 2. Sum of the graphs corresponding to the so-called "potential scattering" in n-d scattering. Double lines indicate the deuterons and single lines the nucleons.

[5] Y. Yamaguchi, Phys. Rev. **95**, 1628 (1954).

[6] J. H. Naqvi, Nucl. Phys. **36**, 578 (1962).

TABLE I. Quartet and doublet scattering lengths.

Spin state	Set No.	Values of the scattering lengths: With potential scattering (70×70 matrix)	Without potential scattering (70×70 matrix)	Without potential scattering (30×30 matrix)
$S=\frac{3}{2}$	I	6.19 F	6.28 F	6.22 F
	II	6.27 F	6.38 F	6.23 F
$S=\frac{1}{2}$	I	−2.41 F	−2.62 F	−2.03 F
	II	−1.87 F	−1.57 F	−1.31 F

TABLE II. Low-energy phase shifts.

Spin state	Set No.	Energy (MeV) in lab. system	Phase shifts (in degrees): Present calculations	C & G Results[a]
$S=\frac{3}{2}$	I	0.60	−40	−28
		3.0	−66.2	−66.0
	II	0.60	−41	−28
		3.0	−67	−66
$S=\frac{1}{2}$	I	0.60	4	...
		3.0	−15	−19
	II	0.60	−0.5	...
		3.0	−22.0	−19

[a] See Ref. 7.

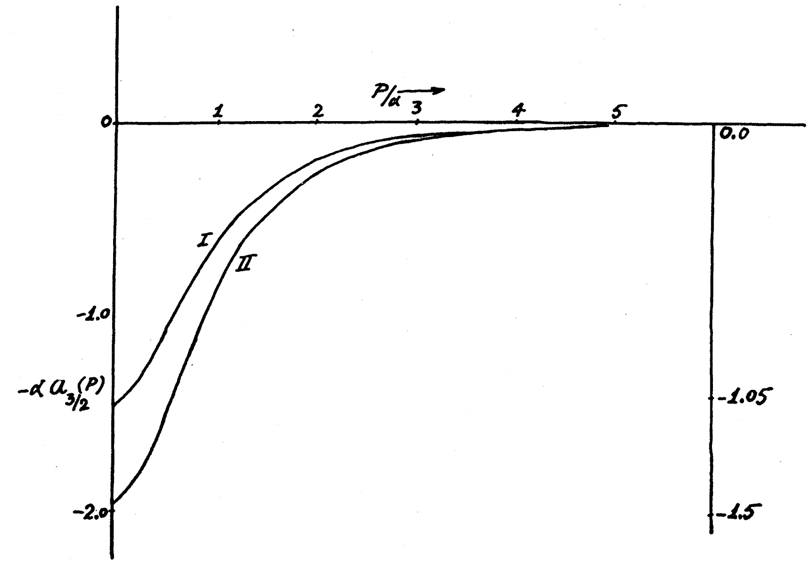

FIG. 3. Amplitude $-\alpha a_{3/2}(P)$ as a function of P/α for $k_i=0$. Curve I refers to the case $\beta_1=\beta_2=6.255\alpha$ (Yamaguchi, Ref. 5) and curve II corresponds to the parameters $\beta_1=5.8\alpha$, $\beta_2=5.5\alpha$ (Naqvi, Ref. 6). Right-hand scale refers to curve II.

The last column in Table I shows the results of evaluation of the parameters using a 30×30 matrix representation for the integral equations, to be compared with 70×70 matrix representations used for determining the various quantities shown in all the other columns. It is seen that there is very little difference for the case of $S=\frac{3}{2}$ but the correction is substantial ($\sim20\%$) in the $S=\frac{1}{2}$ case. This fact might explain the large discrepancy between the cruder solution obtained in A for the doublet scattering length and the present one which is much more accurate, though there has been little change in the results for the quartet scattering length.

The graphs of the amplitudes $a_{3/2}(\mathbf{p})$ and $a_{1/2}(\mathbf{p})$ for $k^2=0$ are shown in Figs. 3 and 4, respectively.

Table I shows that the quartet scattering length is in excellent agreement with the experimental value of 6.4 F, corresponding to set I,[2] in agreement with A as well as Ref. (4). Potential scattering yields only a small correction to this value. Further, as was already pointed out in A, the Naqvi set (set II) of parameters[6] β_1 and β_2 gives a slightly better agreement for $a_{3/2}$ with experiment than the corresponding set (set I) of Yamaguchi's effective s-wave potential.[5] As for the doublet scattering length, we find from Table I that it has the wrong sign compared with its experimental value of $+0.7$ F which goes with $a_{3/2}=6.4$ F.[2] This result is in general agreement with that of Ref. (4). Potential scattering again gives a small effect on the scattering length. In this connection we note that the Naqvi set (set II) of β_1 and β_2 helps in substantially reducing the magnitude of $a_{1/2}$, a requirement badly needed to give at least the

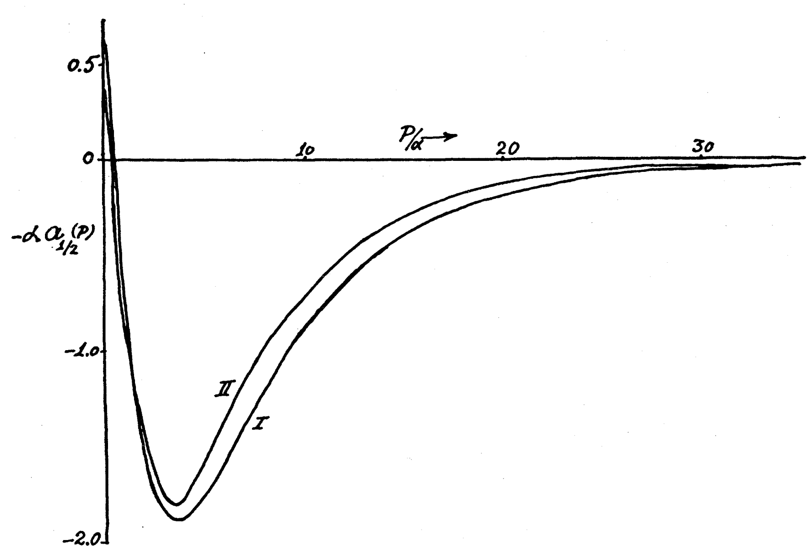

FIG. 4. Amplitude $-\alpha a_{1/2}(P)$ as a function of P/α for $k_i=0$. Curve I corresponds to the case $\beta_1=\beta_2=6.255\alpha$ (Yamaguchi, Ref. 5) and curve II refers to the parameters $\beta_1=5.8\alpha$, $\beta_2=5.5\alpha$ (Naqvi, Ref. 6).

right order of magnitude to the *n-d* cross section at zero energy. In terms of Fig. (4) the point at which $a_{1/2}(P)$ intersects the ordinate, while still remaining above the P axis, is substantially lower for set II than for set I. However, it is not low enough to intersect the ordinate slightly below the P axis, as needed by the experimental value of $a_{1/2}$.

The *n-d* phase shifts are given in Table II, for the energies 0.60 and 3.0 MeV (laboratory system). The results seem to be in general agreement with the corresponding results of Christian and Gammel[7] which are shown in the last column for comparison.

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The effect of tensor forces on the scattering parameters will be the subject of a separate and more detailed communication.

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Elastic Scattering of 3.3-MeV Polarized Neutrons*

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Measurements of azimuthal asymmetry in the scattering of partially polarized neutrons from the $D(d,n)He^3$ reaction have been carried out for Zn, Se, Sr, Y, and Ag, at a laboratory scattering angle of 90°. The results, combined with values obtained from previously measured elements, are compared with predictions based on a nonlocal nuclear potential. No distinct departure from optical-model behavior is observed.

I. INTRODUCTION

THE relation of optical-model parameters to the strength and radial dependence of spin-orbit forces has been described recently in a variety of comparisons between scattering data and nuclear potentials.[1–5] Any finally acceptable set of parameters is expected to predict successfully, among various aspects of nuclear interaction cross sections, the azimuthal asymmetries in elastic scattering of fast neutrons from nuclei. A requisite feature of asymmetry measurements is an incident particle beam which is partially polarized. Although the reaction dynamics and cross section of the $D(d,n)He^3$ reaction are advantageous in this respect, currently existing disagreement[6–9] in the measured magnitude of the neutron polarization injects an additional uncertainty when comparing asymmetries with theoretical prediction. Nevertheless, useful features can be observed; for example, both theory and experiment indicate an abrupt change in polarization as the target mass is increased from $A=50$ to $A=100$, for a neutron energy of 3.3 MeV and a scattering angle near 90°. Investigation of the region in which this change occurs was undertaken partly in order to search for marked departure from the predicted optical-model behavior. Thus, the results reported here are an extension of earlier work[10] to include scattering from Ca, Zn, Se, Sr, Y, Ag, In, and Sb, with additional data obtained for some of the elements measured previously.

* Supported by the U. S. Atomic Energy Commission.

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II. EXPERIMENTAL

Details of the experimental technique have been described in an earlier paper.[10] The left-right asymmetry of neutrons scattered from the cylindrical samples (1 in. diam×1 in.) was obtained simultaneously from pulse-height analysis of the response of two identical stilbene crystals. Contributions from γ rays were removed by pulse-shape discrimination, and the component from inelastically scattered neutrons was suppressed by restricting attention to the upper portion of

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