PHYSICAL REVIEW VOLUME 137, NUMBER 2B 25 JANUARY 1965

Λ-Nucleon Interaction from Analysis of S-Shell Hypernuclei*

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(Received 9 September 1964)

From the binding-energy data of the S-shell hypernuclei, the triplet and singlet depth of a central Λ-nucleon potential, which has a hard core of radius 0.4 F and an attractive well of exponential shape with a range suggested by the mechanism of two-pion exchange, are determined. The results show that the singlet interaction is stronger than the triplet interaction. The well-depth parameters are equal to 0.865 and 0.675, indicating that the strength of the Λ-nucleon potential is not sufficient to bind a hyperdeuteron, although it is sufficient to allow the existence of a bound excited state in the hypernuclei $_\Lambda H^4$ and $_\Lambda He^4$ with a binding energy B_Λ in the neighborhood of 0.35 MeV. Using the Λ-nucleon interaction found in this analysis, the binding energy of a Λ particle in nuclear matter and the Λ-nucleon scattering cross sections have also been computed. In both these cases, fair agreement with experimental results has been obtained.

I. INTRODUCTION

At the present time, the best way to gain information about the Λ-nucleon interaction is from the binding-energy data of the light hypernuclei. Although there are some experimental data on the scattering of Λ particles by nucleons,[1–7] the number of events so far recorded are too few to allow any quantitative deduction about the characteristics of this interaction. In this investigation, we shall attempt to establish some gross features of the Λ-nucleon interaction by analyzing the S-shell hypernuclei with $A=3$ to 5 using a spin-dependent central potential with a hard core, which is considered to contain also the effect of a possible tensor component.

Our interest in the hypernuclear systems with $A=3$ to 5 has been stimulated by our recent finding that using a central nucleon-nucleon potential with a hard core of radius equal to 0.4 F which fits the two-nucleon low-energy scattering data, it is possible to get good agreement with the experimental values of the binding energies and rms radii of H^3 and He^4 and the Coulomb energy of He^3.[8] Since these nuclei form the cores of the hypernuclei $_\Lambda H^4$, $_\Lambda He^4$, and $_\Lambda He^5$, this finding gives us confidence that with this nucleon-nucleon potential, the results which we will obtain on the hypernuclear systems will certainly be meaningful.

Until now, most of the efforts have been directed toward analyses of the hypertriton, which is the lightest hypernucleus known to date.[9–18] The most elaborate calculations are those of Downs and Dalitz[10] with a potential without a hard core, and those of Smith and Downs[12] with a hard-core potential. The results they obtained are probably quite good, since the variational functions used in these analyses have a great deal of flexibility.[19] On the other hand, owing to mathematical complexity, the analyses of the hypernuclei $_\Lambda H^4$ and $_\Lambda He^5$ have not been nearly so complete. In the calculation of Dalitz

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and Downs,[20] the core nuclei have been considered in a rather crude manner; only compressive distortion of the core has been approximately included. It seems to us, therefore, that for these hypernuclei, further calculations with a variational function as flexible as that used in the calculation of ${}_\Lambda H^3$ should be performed.

Using the independent-pair method proposed by Mang and Wild for light nuclei,[21] Dietrich *et al.* have also considered the hypernuclear system with $A=3$ to 5.[22] With this method, the two-body correlations in these light hypernuclei are taken into account quite accurately, but the square-well shape for the Λ-nucleon interaction assumed in their calculation is not too realistic. Also, we feel that this method may not be too accurate when the binding energy of the Λ particle is small. In a calculation on the triton with a simple Gaussian potential, Folk has found a value of -9.2 MeV for the ground-state energy,[23] which is already larger than the upper bound of -9.74 MeV obtained by a variational calculation.[24]

In the next section, the form of the nucleon-nucleon potential and the Λ-nucleon potential is given, together with a description of the trial wave function. The nucleon-nucleon potential used is that of Kikuta *et al.*,[25] which has a hard core of radius 0.4 F. For the Λ-nucleon potential, a core radius of the same magnitude will be assumed. Only the case with an intrinsic range of 1.5 F, corresponding to the mechanism of two-pion exchange, will be considered. For the trial wave function, we adopt a form originally proposed by Austern and Iano.[26] In this form, the trial function is written as a product of functions, each depending individually on the interparticle distance. For each of these functions, the solution of the two-body Schrödinger equation is used up to a certain interparticle separation, which is then connected to a variational function for larger distances.

In Sec. III, the depths of the average Λ-nucleon potentials in the hypernuclei ${}_\Lambda H^3$, ${}_\Lambda H^4$ and ${}_\Lambda He^4$, and ${}_\Lambda He^5$ are determined from the binding-energy data. From these depths, we can find the strength of the Λ-nucleon interaction in the triplet and singlet state.

The properties of the Λ-nucleon interaction are discussed in Sec. IV. Specifically, what we consider is the binding energy, D, of a Λ particle in nuclear matter and the Λ-nucleon scattering cross sections. Finally, in Sec. V, we give a summary of the results of this investigation.

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II. TWO-BODY POTENTIALS AND TRIAL WAVE FUNCTION

A. Nucleon-Nucleon and Λ-Nucleon Potential

The nucleon-nucleon potential is assumed to be of the form

$$V_{ik}=[(1+P_{ik}^{\sigma})/2]V_t(r_{ik})+[(1-P_{ik}^{\sigma})/2]V_s(r_{ik})+V_c(r_{ik})\epsilon_{ik}\,, \qquad (1)$$

where P_{ik}^{σ} denotes the spin-exchange operator and the last term represents the Coulomb interaction, with ϵ_{ik} equal to 1 if i and k are protons, and 0 otherwise. The quantities $V_t(r)$ and $V_s(r)$ are the triplet and singlet potential in the even states and are chosen to be of the following exponential type[25]:

$$\begin{aligned} V_t(r)&=\infty\,, && (r<r_c)\\ &=-V_{0t}\exp[-\kappa_t(r-r_c)]\,, && (r>r_c)\\ V_s(r)&=\infty\,, && (r<r_c)\\ &=-V_{0s}\exp[-\kappa_s(r-r_c)]\,, && (r>r_c) \end{aligned} \qquad (2)$$

with $r_c=0.4$ F, $V_{0t}=475.044$ MeV, $V_{0s}=235.414$ MeV, $\kappa_t=2.5214\ \mathrm{F}^{-1}$, and $\kappa_s=2.0344\ \mathrm{F}^{-1}$. The potential in the odd states does not need to be specified, since, in this investigation, the trial function is taken to be symmetric with respect to the space exchange of all the nucleons.

The fit of this potential to the low-energy nucleon-nucleon data is shown in Table I, where the experimental values are taken from the compilation of Gammel and Thaler,[27] and of MacGregor *et al.*[28]

With this nucleon-nucleon potential, we have also examined the nuclear three- and four-body problems.[8] The results obtained are listed in Table II. In this table, E_t and E_α are the upper bounds of the ground-state energies of the triton and the alpha particle, respectively.[29] The quantities $(\mathrm{rms})_t$ and $(\mathrm{rms})_\alpha$ represent the rms radii of the nucleon distribution in these two nuclei, and E_c is the Coulomb energy of He^3 calculated with

TABLE I. Fit to low-energy nucleon-nucleon data.

		Calculated[a]	Experimental
	E_d(MeV)	−2.253	−2.225
n-p triplet	Scattering length (F)	5.35	5.39
	Effective range (F)	1.73	1.704
n-p singlet	Scattering length (F)	−23.17	−23.74
	Effective range (F)	2.72	2.67
p-p singlet	Scattering length (F)	−8.80	−7.68
	Effective range (F)	2.59	2.65

[a] Our calculated values are slightly different from those tabulated by Kikuta *et al.* (Ref. 25).

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TABLE II. Ground-state energies and rms radii of H^3 and He^4.

		Calculated	Experimental
E_t	(MeV)	-7.78 ± 0.05	-8.48
$(\text{rms})_t$	(F)	1.65	1.70 ± 0.10[a]
E_c	(MeV)	0.70	0.76
E_α	(MeV)	-29.75 ± 0.18	-28.3
$(\text{rms})_\alpha$	(F)	1.40	1.44 ± 0.07

[a] See Ref. 8.

the extended charge distribution of the proton taken into account. As is seen, the agreement between the calculated and experimental values is fairly good. This is important, since we feel that to obtain reliable values for the triplet and singlet depths of the Λ-nucleon potential from the binding-energy data of the hypernuclei, it is necessary that the nucleon-nucleon potential should adequately predict the binding energies and rms radii of the core nuclei.

For the Λ-nucleon potential, we use a spin-dependent central potential which has a hard core of the same size as that in the nucleon-nucleon potential. It has the form

$$U_{i\Lambda}=[(1+P_{i\Lambda}{}^{\sigma})/2]U_t(r_{i\Lambda})+[(1-P_{i\Lambda}{}^{\sigma})/2]U_s(r_{i\Lambda}) \quad (3)$$

with

$$\begin{aligned} U_t(r)&=\infty\,, & (r<r_c)\\ &=-U_{0t}\exp[-\lambda(r-r_c)]\,, & (r>r_c)\\ U_s(r)&=\infty\,, & (r<r_c)\\ &=-U_{0s}\exp[-\lambda(r-r_c)]\,. & (r>r_c) \end{aligned} \quad (4)$$

The value of λ is chosen in such a way as to yield an intrinsic range of 1.5 F, which corresponds to a range of $\hbar/2m_\pi c$ for a Yukawa potential without a hard core.[11] Such a choice gives λ equal to 5.059 F^{-1}.

With a trial function which is symmetric with respect to the space-exchange of all the nucleons, the depths of the spin-averaged Λ-nucleon potentials in the S-shell hypernuclei can be expressed in terms of the triplet depth U_{0t} and the singlet depth U_{0s}. Depending upon whether $U_{0s}>U_{0t}$ or $U_{0s}<U_{0t}$, we have the following relations:

$$U_{0s}>U_{0t}:\quad \begin{aligned} U_{03}&=\tfrac{1}{4}U_{0t}+\tfrac{3}{4}U_{0s}\,,\\ U_{04}&=\tfrac{1}{2}U_{0t}+\tfrac{1}{2}U_{0s}\,,\\ U_{05}&=\tfrac{3}{4}U_{0t}+\tfrac{1}{4}U_{0s}\,; \end{aligned} \quad (5)$$

$$U_{0t}>U_{0s}:\quad \begin{aligned} U_{03}&=U_{0t}\,,\\ U_{04}&=\tfrac{5}{6}U_{0t}+\tfrac{1}{6}U_{0s}\,,\\ U_{05}&=\tfrac{3}{4}U_{0t}+\tfrac{1}{4}U_{0s}\,, \end{aligned} \quad (6)$$

where the symbol U_{0A} denotes the depth of the spin-averaged Λ-nucleon potential in the hypernucleus ${}_\Lambda Z^A$. From the binding-energy data, we will obtain the values of U_{03}, U_{04}, and U_{05}. With these values determined, the next step is to see which of the above two equations yields a combination of U_{0s} and U_{0t} which is most consistent with all three relations. In this way, we hope to get information about whether the triplet or the singlet interaction is the stronger one in the Λ-nucleon potential.

For the trial wave function, we use a function which is symmetric with respect to the space exchange of all the nucleons. It has the form

$$\Psi=\psi\chi\,, \quad (7)$$

with ψ and χ being the spatial and the appropriate spin function, respectively. The function ψ will be chosen as

$$\psi=\Big[\prod_{i<j=1}^{A-1} g(r_{ij})\Big]\Big[\prod_{i=1}^{A-1} f(r_{i\Lambda})\Big]\,, \quad (8)$$

with i and j representing the nucleons.

For the function $f(r)$, we adopt a form originally proposed by Austern and Iano,[26] i.e.,

$$\begin{aligned} f(r)&=u_f(r)/r\,, & (r<d_f)\\ &=A_f r^{n_f}[\exp(-\alpha_f r)+B_f\exp(-\beta_f r)]\,, & (r>d_f) \end{aligned} \quad (9)$$

where $u_f(r)$ is a solution of the equation

$$-\frac{\hbar^2}{2\mu_f}\frac{d^2}{dr^2}u_f(r)+[V_f(r)-e_f]u_f(r)=0\,, \quad (10)$$

with μ_f being the reduced mass of the nucleon and the Λ particle. The potential $V_f(r)$ is the spin-averaged Λ-nucleon potential effective in the hypernucleus ${}_\Lambda Z^A$; it is equal to $U_A(r)$ where

$$\begin{aligned} U_A(r)&=\infty\,, & (r<r_c)\\ &=-U_{0A}\exp[-\lambda(r-r_c)]\,. & (r>r_c) \end{aligned} \quad (11)$$

The constants A_f and B_f in Eq. (9) are adjusted such that the function $f(r)$ and its first derivative are continuous at the separation distance d_f. There are a total of five variational parameters in this function, namely, α_f, β_f, e_f, d_f, and n_f. The function $g(r)$ is defined in an analogous manner, except that μ_f is replaced by μ_g, the reduced mass of two nucleons, and the potential function in Eq. (10) is replaced by the potential $V_g(r)$ which is equal to $V_t(r)$ for ${}_\Lambda H^3$ and equal to $\frac{1}{2}[V_t(r)+V_s(r)]$ for ${}_\Lambda H^4$, ${}_\Lambda He^4$, and ${}_\Lambda He^5$. The variational parameters in this latter function are α_g, β_g, e_g, d_g, and n_g.

The trial function is constructed such that, for a certain choice of n_f and n_g, it will have a correct asymptotic form in the regions of large separations. For instance, when the Λ particle is far away from the c.m. of the nucleons, the wave function takes on the asymptotic form

$$\psi\sim\Big[\prod_{i<j=1}^{A-1} g(r_{ij})\Big]R_\Lambda{}^{(A-1)n_f}\exp[-(A-1)\alpha_f R_\Lambda]\,, \quad (12)$$

where R_Λ is the distance from the Λ particle to the c.m. of the nucleons. Similarly, when a nucleon is far away, the function ψ behaves as

$$\psi \sim \Big[\prod_{i<j=1}^{A-2} g(r_{ij})\Big]\Big[\prod_{i=1}^{A-2} f(r_{i\Lambda})\Big] R_N^{\,n_f+(A-2)n_g} \times \exp[-\alpha_f R_N-(A-2)\alpha_g R_N]\,, \quad (13)$$

with R_N being the distance from this nucleon to the c.m. of the rest of the system. Thus, we see that the best values of n_f and n_g are

$$n_f=n_g=-1/(A-1)\,. \quad (14)$$

In an actual calculation on $_\Lambda H^3$,[14] we have found, however, that it is not necessary to choose n_f and n_g strictly according to the above equation. As long as they have a value close to that given by Eq. (14), we can always vary α_g, β_g, α_f, and β_f to get very nearly the same value for the upper bound as that which can be obtained if n_f and n_g are given their best values. Thus, for convenience, we take $n_g=-\frac{1}{2}$, $-\frac{1}{2}$, and $-\frac{1}{3}$ for $_\Lambda H^3$, $_\Lambda H^4$ and $_\Lambda He^4$, and $_\Lambda He^5$ to take advantage of the fact that we have already found the optimum values of the variational parameters α_g, β_g, e_g, and d_g, of the core nuclei,[8,14] and these values can be used as a starting set for the variational process in this calculation.

For n_f, we use the values $-\frac{1}{2}$, $-\frac{1}{3}$, and $-\frac{1}{4}$ for $_\Lambda H^3$, $_\Lambda H^4$ and $_\Lambda He^4$, and $_\Lambda He^5$. With this choice, it is seen from Eq. (12) that if our trial function is a good approximation to the ground-state eigenfunction, the variationally determined value of α_f should be close to the value of α_f' given by

$$\alpha_f'=\left\{\frac{2M_N M_\Lambda B_\Lambda}{(A-1)[M_\Lambda+(A-1)M_N]\hbar^2}\right\}^{1/2}, \quad (15)$$

where B_Λ is the binding energy of the Λ particle. From the results to be presented in the next section, we find that this is indeed the case.

We would also like to mention that with the type of trial function used here, very good upper bounds have been obtained in the nuclear three- and four-body problems.[8] With the hard-core potential given by Eqs. (1) and (2), we have found that the upper bounds obtained are only a few percent larger than the gound-state eigenvalues.

For the evaluation of the various expectation values, we have used a Monte Carlo method. As this method has already been described in detail previously,[30] we shall not go into it further here.[31]

III. RESULTS

To find the depths U_{03}, U_{04}, and U_{05} of the average Λ-nucleon potentials, the following procedure will be adopted.[32] We take three suitably chosen values of U_{0A} and compute the corresponding values of the ground-state energy E_A. From the value of E_A, the energy of the core nucleus listed in Tables I and II is then subtracted off; the negative of the resultant is thus the binding energy B_Λ of the Λ particle. With these three sets of values for U_{0A} and B_Λ, we find the constants a_A, b_A, and c_A in the interpolation formula

$$U_{0A}=a_A+b_A B_\Lambda^{1/2}+c_A B_\Lambda\,, \quad (16)$$

from which the depths U_{0A} corresponding to the observed values of the binding energies can be determined.

The results of this calculation are given in Table III.[33] To achieve the statistical accuracy listed in this table, a rather large number of estimates (about 200 000) was needed in the Monte Carlo calculation.

TABLE III. Results of the variational calculation for the S-shell hypernuclei.

Hyper-nucleus $_\Lambda Z^A$	U_{0A} (MeV)	E_A (MeV)	B_Λ (MeV)	a_A (MeV)	b_A (MeV)$^{1/2}$	c_A
	1140	-2.43 ± 0.07	0.18 ± 0.07			
$_\Lambda H^3$	1180	-2.90 ± 0.05	0.65 ± 0.05	1090.4	123.2	-14.9
	1202	-3.32 ± 0.03	1.07 ± 0.03			
	1050	-8.96 ± 0.21	1.18 ± 0.22			
$_\Lambda H^4$, $_\Lambda He^4$	1090	-10.00 ± 0.18	2.22 ± 0.19	920.7	133.3	-13.2
	1130	-11.57 ± 0.13	3.79 ± 0.14			
	970	-31.21 ± 0.69	1.46 ± 0.71			
$_\Lambda He^5$	1010	-32.36 ± 0.49	2.61 ± 0.52	787.8	192.0	-33.8
	1050	-35.02 ± 0.33	5.27 ± 0.38			

The observed binding energies of the S-shell hypernuclei are

$$\begin{aligned} B_\Lambda(_\Lambda H^3)&=0.31\pm0.15\ \text{MeV}\,,\\ B_\Lambda(_\Lambda H^4,\,_\Lambda He^4)&=2.18\pm0.10\ \text{MeV}\,,\\ B_\Lambda(_\Lambda He^5)&=3.10\pm0.05\ \text{MeV}\,. \end{aligned} \quad (17)$$

In the above equation, the values for $_\Lambda H^3$ and $_\Lambda He^5$ are those listed by Levi-Setti.[34] For $_\Lambda H^4$ and $_\Lambda He^4$, the value taken is an average of the binding energies of these two hyperfragments given by Raymund.[35] These latter values are $B_\Lambda(_\Lambda H^4)=2.03\pm0.09$ MeV and $B_\Lambda(_\Lambda He^4)=2.33\pm0.10$ MeV.

Using Eqs. (16) and (17), we get, with the values of a_A, b_A, and c_A listed in Table III,

$$\begin{aligned} U_{03}&=1154.3\pm16.0\ \text{MeV}\,,\\ U_{04}&=1088.8\pm6.3\ \text{MeV}\,,\\ U_{05}&=1020.8\pm9.2\ \text{MeV}\,. \end{aligned} \quad (18)$$

With these values of the potential depth, the optimum parameters of the upper bound of the ground-state energy are given in Table IV.

[30] E. W. Schmid, Nucl. Phys. 32, 82 (1962); E. W. Schmid, Y. C. Tang, and R. C. Herndon, Nucl. Phys. 42, 95 (1963).

[31] The computation was done on the IBM 7030 and CDC 3600 computers at the Lawrence Radiation Laboratory, Livermore, California.

[32] This procedure is necessary, since, in our method, the average depth U_{0A} also appears indirectly in the trial function.

[33] The results for $_\Lambda H^3$ have been previously reported (Ref. 14). Here, we use a slightly different procedure of analysis.

[34] R. Levi-Setti, Proceedings of the International Conference on Hyperfragments, St. Cergue, Switzerland, 1963 (unpublished).

[35] R. Raymund, Nuovo Cimento 32, 555 (1964).

TABLE IV. Optimum values of the variational parameters.

Hypernucleus ${}_{\Lambda}Z^{A}$	α_f (F^{-1})	β_f (F^{-1})	e_f (MeV)	d_f (F)	α_g (F^{-1})	β_g (F^{-1})	e_g[a] (MeV)	d_g (F)	U_{0A} (MeV)
${}_{\Lambda}H^3$	0.065	4.5	4.0	1.0	0.38	3.0	−4.0	1.2	1154.3
${}_{\Lambda}H^4$, ${}_{\Lambda}He^4$	0.112	3.0	2.0	1.0	0.28	3.7	−28.0	1.2	1088.8
${}_{\Lambda}He^5$	0.10	4.0	−5.0	1.0	0.29	2.7	−3.0	1.2	1020.8

[a] Due to the insensitivity of the upper bound with the variation of e_g and the statistical uncertainty arising from the Monte Carlo calculation, the value of e_g can be determined only to within about ±15 MeV.

The self-consistent values α_f' given by Eq. (15) are equal to 0.053, 0.10, and 0.093 F^{-1} for ${}_{\Lambda}H^3$, ${}_{\Lambda}H^4$ and ${}_{\Lambda}He^4$, and ${}_{\Lambda}He^5$, which are quite close to the variationally determined values of α_f listed in Table IV. This indicates that our trial wave functions are a good approximation to the ground-state eigenfunctions of these hypernuclei.

The amount of compression sustained by the core nuclei in the hyperfragments ${}_{\Lambda}H^4$ and ${}_{\Lambda}He^5$ can be determined by calculating the rms value of the separation distance between two nucleons. Using the optimum wave functions, the values we obtained are 2.50 and 2.20 F for the cores in ${}_{\Lambda}H^4$ and ${}_{\Lambda}He^5$, respectively. Comparing these values with the corresponding values of 2.84 and 2.28 F in H^3 and He^4, we note that the amount of compression is 14% and 3% in these two hypernuclei, which is very nearly the same amount determined by Dalitz and Downs[20] in their calculation with a Λ-nucleon potential containing no hard core.

From U_{03} and U_{05}, we determine the depths U_{0s} and U_{0t} of the Λ-nucleon potential. Depending upon whether $U_{0s} > U_{0t}$ or $U_{0s} < U_{0t}$, their values are given in Table V, where the values of the well-depth parameters s_s and s_t are also listed. From this table, we note that when $U_{0s} > U_{0t}$, the value of U_{04} computed from the values of U_{03} and U_{05} is nearly the same as that determined variationally, which is, however, not the case when U_{0s} is less than U_{0t}. This indicates, therefore, that the singlet interaction is the stronger one in the Λ-nucleon potential and the spins of the S-shell hypernuclei are $\frac{1}{2}$, 0, 0, and $\frac{1}{2}$ for ${}_{\Lambda}H^3$, ${}_{\Lambda}H^4$, ${}_{\Lambda}He^4$, and ${}_{\Lambda}He^5$, respectively. This latter conclusion is, indeed, in agreement with the experimental finding on the spins of these hypernuclei.[36,37]

The ratio of the triplet to singlet strength is 0.78, which is larger than the value of 0.45 obtained by Downs and Dalitz with a Λ-nucleon potential containing no hard core,[10] and the value of 0.53 obtained by Dietrich *et al.*[22] with a hard-core potential of core radius 0.2 F. This shows that the triplet-to-singlet ratio increases with the radius of the hard core. Consequently, we believe that for a core radius equal to 0.6 F, this ratio will be at least as large as 0.8. Using the relation

$$s_s = s\left[\tfrac{3}{4} + \tfrac{1}{4}(s_t/s_s)\right]^{-1}, \tag{19}$$

where s is the average well-depth parameter in the hypertriton, we get

$$s_s \leq 0.974, \tag{20}$$

for the Λ-nucleon singlet potential with a core radius of 0.6 F by using s equal to 0.925 as determined by Smith and Downs.[12] Thus, even with such a large core radius in the Λ-nucleon potential, the existence of a bound hyperdeuteron can still be ruled out.[38]

The values of the low-energy parameters of the Λ-nucleon potential are listed in Table VI, where a comparison with the values corresponding to the potentials obtained by Downs and Dalitz[10] and Dietrich *et al.*[22] is also made. We note that the agreement between our values and the values of Downs and Dalitz is rather good. On the other hand, the agreement with the values of Dietrich *et al.* is somewhat poorer. The reason for this is probably as follows: The method of calculation employed by Dietrich *et al.* is not too accurate when the binding energy of the Λ particle is small.[39] As a consequence, the average well depth obtained in their calculation may have a value which is somewhat too large. This shows up quite clearly in the calculation on the hypertriton. With a hard-core radius of 0.2 F, Dietrich *et al.* need a potential with an average well-depth parameter s equal to 0.754 to get a binding energy of 0.2 MeV for the Λ particle, while Smith and Downs[12] need only a potential with s equal to 0.701.

With U_{0s} and U_{0t} determined, we can decide if there exists a particle-stable excited state of $J=1$ for the hypernuclei ${}_{\Lambda}H^4$ and ${}_{\Lambda}He^4$. For this state, the spin-

TABLE V. Depth of triplet and singlet Λ-nucleon potentials.

	U_{0s} (MeV)	U_{0t} (MeV)	s_s	s_t	U_{04} (MeV) (Variationally determined)	U_{04} (MeV) (From U_{03} and U_{05})
$U_{0s} > U_{0t}$	1221.1±24.4	954.1±16.0	0.865±0.017	0.675±0.011	1088.8±6.3	1087.6±9.2
$U_{0s} < U_{0t}$	620.4±60.5	1154.3±16.0	0.440±0.043	0.818±0.011	1088.8±6.3	1065.3±8.2

[36] M. M. Block, R. Gessaroli, J. Kopelman, S. Ratti, M. Schnuberger *et al.*, Proceedings of the International Conference on Hyperfragments, St. Cergue, Switzerland, 1963 (unpublished).
[37] R. H. Dalitz and L. Liu, Phys. Rev. **116**, 1312 (1959).
[38] This statement may have to be modified if there is a strongly repulsive three-body potential which makes a significant contribution to the binding of the hypertriton. However, there is at present little evidence for the existence of such a three-body potential [see Ref. 13, and J. D. Chalk, III, and B. W. Downs, Phys. Rev. **132**, 2727 (1963)].
[39] See, also, Ref. 18.

TABLE VI. Low-energy scattering parameters of the Λ-nucleon potentials.

	Singlet Scattering length (F)	Singlet Effective range (F)	Triplet Scattering length (F)	Triplet Effective range (F)
This analysis	$-(2.89_{-0.41}^{+0.59})$	1.94 ± 0.08	-0.71 ± 0.06	3.75 ± 0.22
Downs and Dalitz[a]	$-(2.25_{-0.54}^{+0.80})$	1.97 ± 0.14	-0.51 ± 0.09	3.62 ± 0.35
Dietrich *et al.*[b]	$-(4.6_{-1.7}^{+3.8})$	$1.7\ \pm0.1$	-0.53 ± 0.11	3.88 ± 0.65

[a] Ref. 10.
[b] Ref. 22.

averaged well depth is

$$U_{04}^*=\tfrac{5}{6}U_{0t}+\tfrac{1}{6}U_{0s}=998.6 \text{ MeV}, \qquad (21)$$

which, according to Eq. (16), would indicate that there is a bound excited state with B_Λ equal to about 0.35 MeV. The fact that this binding energy is so small is a rather fortunate situation. It has been shown by Block *et al.*[36] that the existence of a bound excited state with a small binding energy does not upset the argument, based on the observed creation of the hyperfragments $_\Lambda H^4$ and $_\Lambda He^4$ following K^--He^4 capture events, which leads to the prediction that the relative K-Λ parity is negative.

IV. BINDING ENERGY OF Λ PARTICLE IN NUCLEAR MATTER AND Λ-NUCLEON SCATTERING

A. Binding Energy of Λ Particle in Nuclear Matter

The binding energy D of a Λ particle in nuclear matter has been calculated by Downs and Ware with the independent-pair approximation.[40] The spin-averaged Λ-nucleon potential used in their calculation is of the form

$$\begin{aligned} U_N&=\infty, && (r<0.4\text{ F})\\ &=-U_{0N}\exp[-5.059(r-0.4)], && (r>0.4\text{ F}) \end{aligned} \qquad (22)$$

where U_{0N} is easily seen to be just equal to U_{05} appropriate in the hypernucleus $_\Lambda He^5$. With a nucleon density $\rho=0.172$ nucleons/F^3 and $U_{05}=969.2$ MeV, they obtained a value of D equal to 32.7 MeV.

The value of U_{05} used by Downs and Ware is, however, an underestimated one. From our analysis of $_\Lambda He^5$, we have instead arrived at a larger value for U_{05} equal to 1020.8 MeV. Using this latter value, the procedure of calculating D, fortunately, does not have to be repeated. Rather, we can simply use Eq. (17a) and Table II in the paper of Downs and Ware to conclude that D is equal to 37.6 MeV in our case.

Recent measurements of the binding energies of hypernuclei with mass numbers in the range $60\lesssim A\lesssim 100$ have led to the estimate that D is likely to be in the neighborhood of 30 MeV,[41] which is in essential agreement with the value arrived at with the Λ-nucleon potential found in this investigation.

B. Λ-Nucleon Scattering[42]

With a central potential of the form given by Eq. (4), the scattering phase shifts can be easily calculated.[43] In Table VII, we list the phase shifts and the total cross sections in the energy range from 2 to 140 MeV in the c.m. system. To obtain these phase shifts, we have used an ordinary (nonexchange) interaction in accordance with the assumption of a two-pion exchange mechanism for the Λ-nucleon potential.

The comparison of the total cross sections with the experimental values[2,4,5,7] is shown in Fig. 1. In this figure, we note that the agreement is quite adequate for energies up to around 40 MeV.[44] At higher energies, our cross sections seem to be somewhat too small. But this is more or less to be expected, since, in our calculation, we have not taken into account the presence of the Σ-production channel. It has been shown by de Swart and Dullemond[45] that in the neighborhood of the Σ-production threshold (about 76 MeV), the Λ-nucleon scattering cross sections may be rather strongly affected by the presence of this latter channel.

TABLE VII. Λ-nucleon scattering phase shifts and total cross sections.

E (MeV)	Triplet phase shifts (deg) δ_0	δ_1	δ_2	δ_3	Singlet phase shifts (deg) δ_0	δ_1	δ_2	δ_3	σ (mb)
2	9.03	0.12	0	0	30.06	0.17	0	0	194.93
5	11.76	0.44	0.01	0	37.10	0.64	0.01	0	117.43
10	14.45	1.01	0.03	0	39.68	1.66	0.04	0	71.95
20	15.28	2.71	0.14	0.01	38.25	4.10	0.18	0.01	37.63
40	12.38	6.33	0.41	0.07	31.62	10.22	0.58	0.08	18.54
60	8.22	9.74	0.94	0.11	25.13	16.17	1.24	0.12	14.77
80	3.75	12.61	1.63	0.20	19.02	20.86	2.19	0.23	14.21
100	−0.41	14.68	2.43	0.28	13.69	24.16	3.38	0.35	14.18
120	−4.33	16.05	3.39	0.46	8.86	26.28	4.69	0.56	14.05
140	−8.07	16.81	4.38	0.59	4.38	27.42	6.18	0.78	13.74

[40] B. W. Downs and W. E. Ware, Phys. Rev. **133**, B133 (1964).
[41] D. H. Davis, R. Levi-Setti, M. Raymund, O. Skjeggestad, G. Tomasina, J. Lemonne, P. Renard, and J. Sacton, Phys. Rev. Letters **9**, 464 (1962).

[42] Similar calculations to find the Λ-nucleon scattering cross sections have also been made by B. Ram and B. W. Downs, Phys. Rev. **133**, B420 (1964), and by J. S. Kovacs and D. Lichtenberg, Nuovo Cimento **13**, 371 (1959). The hard-core Λ-nucleon potentials used by these authors were, however, not the result of a direct analysis of the hypernuclear data.
[43] The calculation of the phase shifts was done on the IBM 7094 computer at the Brookhaven National Laboratory.
[44] It is interesting to note that the potential used here is suggested by the hypernuclear binding-energy data, which are determined by the Λ-nucleon interaction for energies up to about 40 MeV.
[45] J. J. de Swart and C. Dullemond, Ann. Phys. (N. Y.) **16**, 263 (1961); Nuovo Cimento **25**, 1072 (1962).

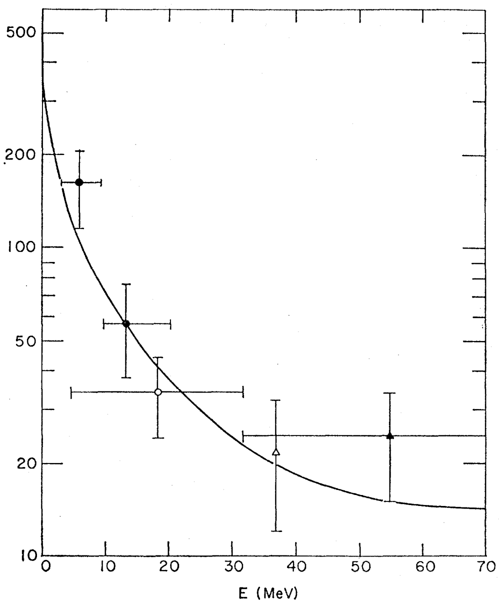

FIG. 1. Total Λ-p elastic scattering cross section as a function of the c.m. energy. The experimental points are from Ref. 7 (closed circle), Ref. 5 (open circle), Ref. 4 (open triangle), and Ref. 2 (closed triangle).

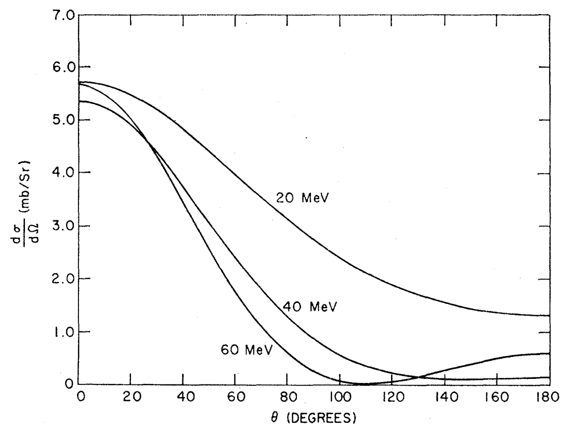

FIG. 2. Angular distributions of Λ-p elastic scattering at c.m. energies of 20, 40, and 60 MeV.

The differential cross sections at $E=20$, 40, and 60 MeV are shown in Fig. 2. Here, it is seen that at higher energies the angular distribution is prominently peaked in the forward direction. The forward-to-backward ratios (F/B) are equal to 1.3, 2.1, 8.1, and 12.2 at E equal to 10, 20, 40, and 60 MeV, respectively. As a comparison, we note that with 22 events in the energy range 3 to 21 MeV, Alexander *et al.*[7] obtained $F/B=1.1^{+1.0}_{-0.6}$, and with 11 events in the energy range 5 to 32 MeV, Piekenbrock and Oppenheimer[5] obtained $F/B\approx 3$. For both of these experimental results, the agreement with our computed values is fairly good. This is satisfying, since we expect F/B to be a rather sensitive function of the extension of the Λ-nucleon potential; hence, the agreement with experiments constitutes some additional support for the use of a deep and narrow attractive potential in our calculation.

V. CONCLUSION

In this investigation, we determine from the binding-energy data of the S-shell hypernuclei the triplet and singlet depth of a central Λ-nucleon potential which has a hard core of radius 0.4 F and an attractive well of exponential shape. The intrinsic range is taken as 1.5 F, corresponding to the mechanism of two-pion exchange. The results indicate that the singlet interaction is stronger than the triplet interaction, with the well-depth parameters being 0.865 and 0.675, respectively. The ratio of the triplet-to-singlet depth is equal to 0.78, which is larger than the values obtained by other investigators.[10,22]

The strength of the Λ-nucleon potential found in this calculation clearly suggests the nonexistence of a hyperdeuteron. It is, however, still large enough to cause the existence of a bound excited state in the hypernuclei $_\Lambda H^4$ and $_\Lambda He^4$, with a binding energy B_Λ in the neighborhood of 0.35 MeV.

With this Λ-nucleon potential, we have also calculated the binding energy, D, of a Λ particle in nuclear matter and the Λ-nucleon scattering cross sections. In both cases, fair agreement with experimental data has been obtained. Together with the fact that this potential yields also the observed S-shell hypernuclear binding energies, these results indicate that from a purely phenomenological point of view, there is no need to use potentials with more complicated features, such as three-body potentials, to explain the existing experimental phenomena. At the present time, the use of a two-body, spin-dependent central potential with a hard core seems to be entirely sufficient.

As a next step, it seems desirable to calculate also the binding energies of a Λ particle in the P-shell hypernuclei. In this respect, the hypernucleus $_\Lambda Li^7$ is a good candidate, since the core nucleus Li^6 has already been examined with a hard-core nucleon-nucleon potential similar in nature to that used in this investigation and very satisfactory results about its binding energy and rms radius have been obtained.[46] The calculation on such a large system is, of course, a rather difficult mathematical problem; however, with the computers presently available, it can still be handled with the Monte Carlo technique employed in this calculation. In any case, we feel that it is a worthwhile project, since the results should give us more information about the characteristics of the Λ-nucleon interaction and the structure of the P-shell hypernuclei.

ACKNOWLEDGMENTS

The authors wish to thank Dr. C. E. Porter and Dr. K. W. Lai for their interest in this work.

[46] E. W. Schmid, Y. C. Tang, and K. Wildermuth, Phys. Letters 7, 263 (1963).