where

$$[B,A]_n=[B,[B,A]_{n-1}]$$

and

$$[B,A]_0=A\,,$$

readily yields Eq. (A2). Q. E. D.

For the case that A and B each commute with the commutator $[A,B]$, Eq. (A2) simplifies to

$$\partial\Gamma/\partial\alpha=-\alpha[B,A]\Gamma$$

or

$$\Gamma=\exp\{-\tfrac{1}{2}\alpha^2[B,A]\}\Gamma(\alpha=0)=\exp\{-\tfrac{1}{2}\alpha^2[B,A]\}\,,$$

i.e.,

$$e^{(A+B)}=e^Be^A\exp\{\tfrac{1}{2}[A,B]\}\,. \qquad \text{(A6)}$$

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Polarization of Protons Elastically Scattered by Oxygen*

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Measurements of the polarization of protons elastically scattered by oxygen have been made between 2 and 12 MeV by double scattering using gas targets. Scattering by helium at 46° was used as the polarization analyzer for most of the work. In addition, four angular distributions of the cross section were measured between 2.5 and 3.8 MeV. The experimental results show that away from the known sharp resonances at 2.66 and 3.47 MeV the polarization changes slowly with energy between 2 and 5 MeV. At 3 MeV the polarization is negative at forward angles and positive at back angles, the observed extrema being −0.14±0.02 at 45° and 0.43±0.03 at 115°. At higher energies the pronounced resonance structure causes rapid fluctuations of the polarization with energy. Angular distributions of the polarization were measured at eleven energies away from sharp resonances. At 10.7 MeV the observed extrema in the polarization are −0.88±0.04 at 50°, 0.92±0.04 at 65°. and −0.83±0.03 at 133°. A phase shift analysis of the polarization and cross section was made between 2 and 5 MeV and at four energies between 5 and 7 MeV. The phase shifts obtained differ from those of Salisbury and Richards by less than 10 deg. The measured angular distributions of the polarization between 8 and 12 MeV are also compared with the predictions of the optical model. New results of the polarization for *p*-α scattering and *p*-C scattering between 2 and 4 MeV are also reported.

1. INTRODUCTION

PREVIOUS studies of the elastic scattering of protons by oxygen consisted primarily of measurements of the differential cross section as a function of energy.[1–5] At Wisconsin excitation curves of the cross section have been measured for several angles. Eppling[2] made measurements up to 4.6 MeV, Salisbury *et al.*[3] from 4.2 to 8.6 MeV, and Hardie *et al.*[4] from 8.5 to 13 MeV. In addition, Hardie measured angular distributions at thirteen energies between 4.8 and 13 MeV. Phase-shift analyses of cross-section data have been reported between 2.5 and 5.2 MeV by Harris *et al.*[5] and between 2 and 7.6 MeV by Salisbury and Richards.[6]

The polarization can, in principle, be calculated from the phase shifts, but these calculations are seldom reliable since the polarization and the cross section depend on the phase shifts in different ways. For this reason measurements of the polarization provide an independent test of the validity of the phase shifts obtained by fitting cross-section data and can be used in conjunction with cross-section data for a more accurate determination of the phase shifts. Indeed, at higher energies where there are more parameters to be determined, cross-section data alone is not sufficient to determine the phase shifts unambiguously.

Previous measurements of the polarization for proton-oxygen scattering are not very extensive. Early measurements by Sorokin *et al.*[7] near 2.7 MeV and Al-Jeboori

* Work supported by the U. S. Atomic Energy Commission.

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[2] F. J. Eppling, PhD thesis, University of Wisconsin, 1953 (unpublished); and private communication.

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[4] G. Hardie, R. L. Dangle, and L. D. Oppliger, Phys. Rev. 129, 353 (1963).

[5] R. W. Harris, G. C. Phillips, and C. Miller Jones, Nucl. Phys. 38, 259 (1962).

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[7] P. V. Sorokin, A. K. Val'ter, B. V. Gavrilovskii, K. V. Karadzhev, V. I. Man'ko, and A. Ya. Taranov, Zh. Eksperim. i Teor. Fiz. 33, 606 (1957) [English transl.: Soviet Phys.—JETP 6, 466 (1958)].

et al.[8] at 8.7 MeV were made with poor resolution. The more recent measurements by Rosen *et al.* at 8 MeV[9] and 10 MeV[10] and by Gorodetsky *et al.*[11,12] near 4 MeV are compared with the present results in Sec. 8.

In the present experiment excitation curves of the polarization were measured for laboratory angles of 46 and 115° between 2 and 5 MeV and at 65° between 5 and 11 MeV. The polarization was measured between 45 and 115° at 3.0, 3.4, and 3.8 MeV, and more complete angular distributions of the polarization were measured at eight energies between 4.8 and 12 MeV.

The polarization measurements were made by double scattering from gas targets. The Wisconsin tandem electrostatic accelerator provided the incident proton beam. Scattering by helium at $\theta_{lab}=46°$ was used as the polarization analyzer for most of the measurements. The analyzing power of helium is known to an accuracy of about 0.01 from measurements by Brown *et al.*[13] at several energies between 4 and 12 MeV. These measurements were made by double scattering from helium and, therefore, do not depend on other polarization results.

Below 4 MeV the analyzing power for scattering by helium at 46° is too small for accurate polarization measurements. Measurements of the *p*-oxygen polarization at lower energies were made using other analyzers for which the analyzing power was first measured relative to that of helium at higher energies. The result of these supplementary polarization measurements are given in Sec. 3.2.

Since it was found that the polarization calculated from the phase shifts[5,6] did not agree with the measured polarization below 3.8 MeV, angular distributions of the cross section were measured at 2.4, 3.0, 3.4, and 3.8 MeV over the angular range 18 to 167°. A new phase-shift analysis was made between 2 and 5 MeV and at four energies between 5 and 7 MeV, using the present polarization and cross-section results and the cross section measured by Eppling,[2] by Salisbury,[3] and by Hardie.[4] The results of this analysis are described in Sec. 9.

At the highest energies for which the polarization was measured in the present experiment, two optical-model analyses of proton-oxygen scattering had previously been made.[4,14] The polarization values[15,16] calculated from the optical-model parameters obtained in these analyses are compared with the present results in Sec. 10.

[8] M. A. Al-Jeboori, M. S. Bokhari, B. Hird, and A. Strzalkowski, Proc. Phys. Soc. (London) **74**, 705 (1959).
[9] L. Rosen, J. E. Brolley, Jr., M. L. Gursky, and L. Stewart, Phys. Rev. **124**, 199 (1961).
[10] L. Rosen, J. E. Brolley, Jr., and L. Stewart, Phys. Rev. **121**, 1423 (1961).
[11] S. Gorodetsky, J. Ullman, G. Bergdolt, and A. Gallman, Nucl. Phys. **38**, 177 (1962).
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[15] C. B. Duke (private communication).
[16] G. Hardie, PhD thesis, University of Wisconsin, 1962 (unpublished) (available through University Microfilms, Ann Arbor, Michigan).

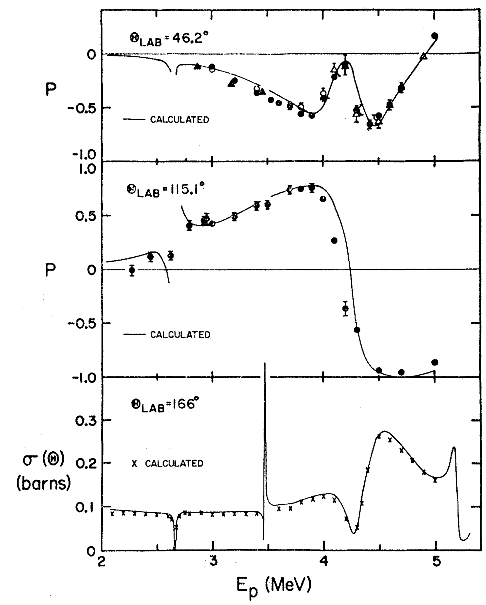

FIG. 1. Excitation curves of the polarization and cross section for proton-oxygen scattering between 2 and 5 MeV. Different symbols are used for the polarization values, depending on the method of measurement: ● oxygen as the second target after scattering by helium at 46°, ○ oxygen as the first target, scattering by helium at 90° as the analyzer; ▲ oxygen as the second target after scattering by oxygen at 115°; △ oxygen as the first target, scattering by oxygen at 46° as the analyzer. The error bars indicate the total uncertainty of the measured polarization. Where no error bar is shown the uncertainty is comparable to the size of the dot. The cross-section curve is a smooth line through the measured points of Eppling (Ref. 2) and of Salisbury *et al.* (Ref. 3) The calculated polarization curves and cross-section points are based on the phase shifts obtained in the present analysis.

2. APPARATUS

The double scattering apparatus has been described previously.[13] The target gases were confined in small cells by foil windows. Doubly scattered protons were detected in identical counter telescopes at symmetric second scattering angles. For most of the present work each telescope consisted of a proportional counter and a CsI(Tl) scintillation counter operated in coincidence. The coincident scintillation counter pulses from each telescope were recorded in separate sections of a multichannel pulse-height analyzer.

For some measurements at low bombarding energies scattering by helium at $\theta_{lab}=90°$ was used as the analyzer. Because of the low proton energy (as low as 0.8 MeV at the entrance of the telescope), the scintillation counters of the telescopes were replaced by proportional counters. To reduce energy losses, the window foils on the counters were eliminated, and the entire volume of the telescope was filled with argon at a pressure ranging from 2 to 16 cm Hg.

3. EXPERIMENTAL RESULTS

3.1. Proton-Oxygen Scattering

Excitation curves of the polarization between 2 and 5 MeV are shown in Fig. 1, together with the differential

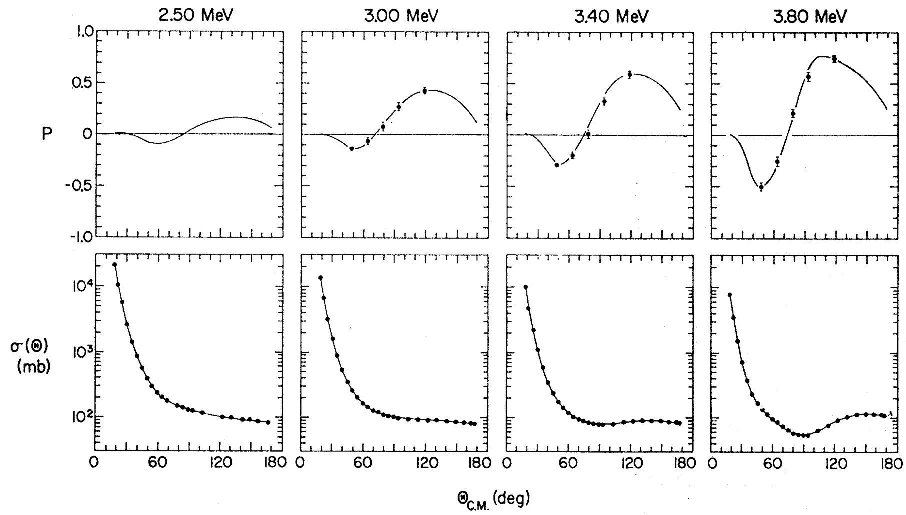

FIG. 2. Angular distributions of the polarization and cross section for proton-oxygen scattering at laboratory proton energies of 2.50, 3.00, 3.40, and 3.80 MeV. The smooth curves were calculated from the phase shifts of Table III.

cross section at a back angle as measured by Eppling[2] up to 4.2 MeV and by Salisbury *et al.*[3] above 4.2 MeV. The cross section reveals narrow resonances at 2.66 MeV and at 3.47 MeV and a broad anomaly between 3.8 and 5 MeV. The energy resolution with which the polarization could be measured did not allow study of the two sharp resonances, but was sufficient to resolve the structure related to the broad anomaly in the cross section.

The measurements at $\theta_{\rm lab}=115.1°$ (Fig. 1) were made with oxygen as the second target. Scattering by helium at 45.4° provided incident protons of known polarization. Initial measurements at $\theta_{\rm lab}=46.2°$ were made in the same way, but to study the structure near 4 MeV with better energy resolution, it was necessary to make measurements with oxygen as the first target.[17] Since the polarization in p-α scattering at 46° is small below 4 MeV, other analyzers were used as indicated in Fig. 1 and Table I. The present results for scattering by oxygen at 46° between 3.2 and 3.8 MeV provided the analyzer for measurements of the polarization between 4.1 and 4.9 MeV. To extend measurements made with oxygen as the first target to lower energies, scattering by helium at 90° was used as the analyzer. The analyzing power of helium between 2 and 4 MeV was known from auxiliary measurements which are described in Sec. 3.2.

The polarizations measured between 2 and 5 MeV indicate that p-oxygen scattering might be useful as a polarization analyzer over certain energy intervals. For this reason, the results of the present measurements are also given in Table I.[18]

The angular dependence of polarization was further investigated at 3.0, 3.4, and 3.8 MeV. The results of these measurements which made use of p-α scattering at 90° for analysis are plotted in Fig. 2. The points at $\theta_{\rm cm}=48.4°$ and at 118.1 are taken from Fig. 1. Angular distributions of the cross section were measured at 2.5, 3.0, 3.4, and 3.8 MeV to provide additional information

TABLE I. Polarization in proton-oxygen scattering at $\theta_{\rm lab}=46.2°$ and $\theta_{\rm lab}=115.1°$.

	$\theta_{\rm lab}=46.2°$		$\theta_{\rm lab}=115.1°$	
E_p (MeV)	P	Analyzer	E_p (MeV)	P[b]
2.87±0.10	−0.116±0.015	O_2, 115°[a]	2.27±0.10	−0.011±0.046
3.00±0.09	−0.116±0.026	He, 45°[a]	2.45±0.09	0.113±0.046
3.00±0.01	−0.140±0.019	He, 90°	2.63±0.09	0.123±0.041
3.18±0.09	−0.279±0.016	O_2, 115°[a]	2.79±0.10	0.410±0.042
3.20±0.09	−0.247±0.018	He, 45°[a]	2.93±0.09	0.457±0.037
3.40±0.09	−0.360±0.020	He, 45°[a]	2.95±0.10	0.471±0.050
3.40±0.02	−0.295±0.018	He, 90°	3.00±0.12	0.429±0.031
3.45±0.09	−0.352±0.023	O_2, 115°[a]	3.20±0.14	0.492±0.037
3.53±0.13	−0.422±0.019	He, 45°[a]	3.40±0.10	0.592±0.034
3.60±0.09	−0.453±0.021	He, 45°[a]	3.50±0.09	0.604±0.033
3.70±0.09	−0.481±0.032	He, 45°[a]	3.70±0.09	0.738±0.037
3.80±0.09	−0.566±0.029	He, 45°[a]	3.80±0.09	0.748±0.030
3.80±0.02	−0.496±0.038	He, 90°	3.90±0.09	0.757±0.036
3.90±0.09	−0.572±0.027	He, 45°[a]	4.00±0.09	0.654±0.033
4.00±0.09	−0.418±0.025	He, 45°[a]	4.10±0.09	0.264±0.021
4.00±0.01	−0.365±0.050	He, 90°	4.20±0.09	−0.366±0.070
4.10±0.09	−0.216±0.027	He, 45°[a]	4.30±0.09	−0.563±0.021
4.10±0.01	−0.126±0.065	O_2, 46°	4.50±0.09	−0.940±0.031
4.20±0.09	−0.102±0.100	He, 45°[a]	4.70±0.09	−0.957±0.027
4.20±0.01	−0.094±0.040	O_2, 46°	5.00±0.09	−0.863±0.027
4.30±0.09	−0.528±0.053	He, 45°[a]		
4.30±0.01	−0.563±0.078	O_2, 46°		
4.42±0.09	−0.655±0.041	He, 45°[a]		
4.50±0.09	−0.577±0.019	He, 45°[a]		
4.50±0.01	−0.630±0.059	O_2, 46°		
4.60±0.09	−0.460±0.019	He, 45°[a]		
4.60±0.01	−0.463±0.045	O_2, 46°		
4.70±0.09	−0.315±0.017	He, 45°[a]		
4.70±0.01	−0.301±0.042	O_2, 46°		
4.90±0.01	−0.007±0.029	O_2, 46°		
5.00±0.09	+0.167±0.022	He, 45°[a]		

[a] For this measurement, P_1 was the known polarization, P_2 the unknown, i.e., the "analyzer" in fact was the first scatterer.
[b] For all measurements at 115.1°, oxygen was the second target. The first scattering took place from helium at $\theta_{\rm lab}=45.4°$.

[17] For the equipment used, the mean angle of p-oxygen scattering was 45.4° when oxygen was the first target, and 46.2° when it was the second target. For consistency the results at 45.4° were corrected to correspond to scattering at $\theta_{\rm lab}=46.2°$. The correction was determined from angular distributions calculated from the p-O^{16} phase shifts (see Sec. 9).

[18] A complete tabulation of all experimental results is contained in: R. A. Blue, PhD thesis, University of Wisconsin, 1963 (unpublished) (available through University Microfilms, Ann Arbor, Michigan).

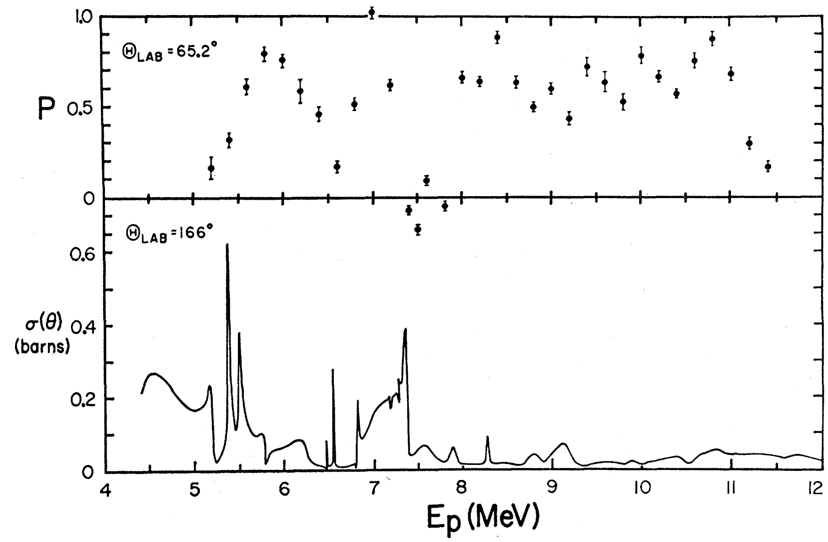

FIG. 3. Excitation curves of the polarization and cross section for proton-oxygen scattering between 5 and 12 MeV. The cross section shown is taken from measurements by Salisbury *et al.* (Ref. 3) and by Hardie *et al.* (Ref. 4).

for the phase-shift analysis. The measured cross sections are also plotted in Fig. 2. The cross sections were measured using a differentially pumped gas scattering chamber.[19] The uncertainty in the measured cross section is about $\pm 5\%$.

The results of the polarization measurements at $\theta_{lab}=65.2°$ between 5 and 12 MeV are shown in Fig. 3. The cross-section curve which is shown for comparison is taken from measurements by Salisbury *et al.*[3] and by Hardie *et al.*[4] The energy resolution of the polarization measurements is not sufficient to resolve the resonance structure which is seen in the cross section.

Because of the pronounced resonance structure for proton-oxygen scattering above 5 MeV, angular distributions of the polarization were measured only in regions where the cross section is not changing rapidly with energy. The energies were selected so that averaging over an energy interval of 0.1 MeV should not seriously distort the polarization.

The experimental angular distributions of the polarization at eight energies between 4.8 and 12 MeV are shown in Figs. 4 and 5. The angular distribution at 4.79 MeV was measured using scattering by oxygen at 46° as the analyzer. Scattering by helium at 46° was the analyzer for the higher energies. The angular distributions of the cross section shown in Figs. 4 and 5 were measured by Hardie *et al.*,[4] except at 5.66 and 6.00 MeV where the cross sections were taken from excitation curves measured by Salisbury *et al.*[3]

3.2. Supplementary Polarization Measurements

The polarization for p-α scattering at 90° was measured to provide known analyzing powers for proton

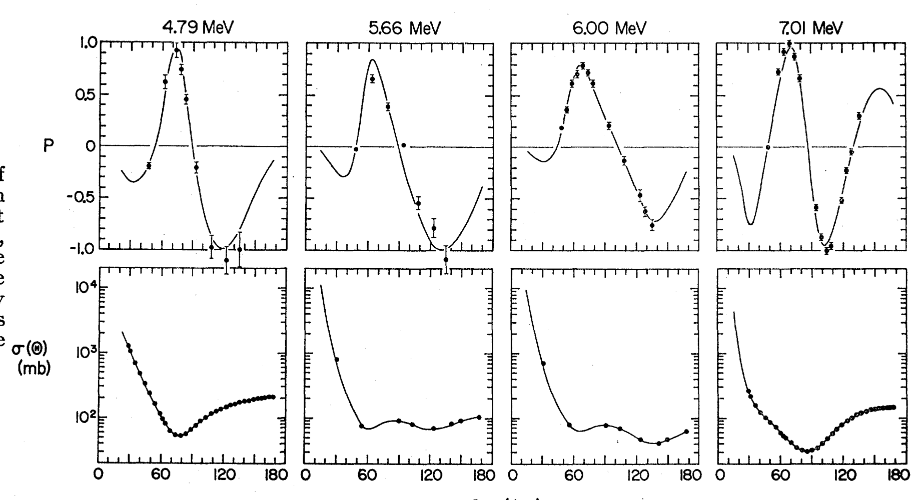

FIG. 4. Angular distributions of the polarization and cross section for proton-oxygen scattering at laboratory proton energies of 4.79, 5.66, 6.00, and 7.01 MeV. The cross sections are those of Hardie *et al.* (Ref. 4) and of Sailsbury *et al.* (Ref. 3). The smooth curves were calculated from the phase shifts of Table III.

[19] We should like to thank Professor H. T. Richards and his group for the use of this chamber. For a description of this equipment, see Ref. 3 and E. A. Silverstein, Phys. Rev. **124**, 868 (1961).

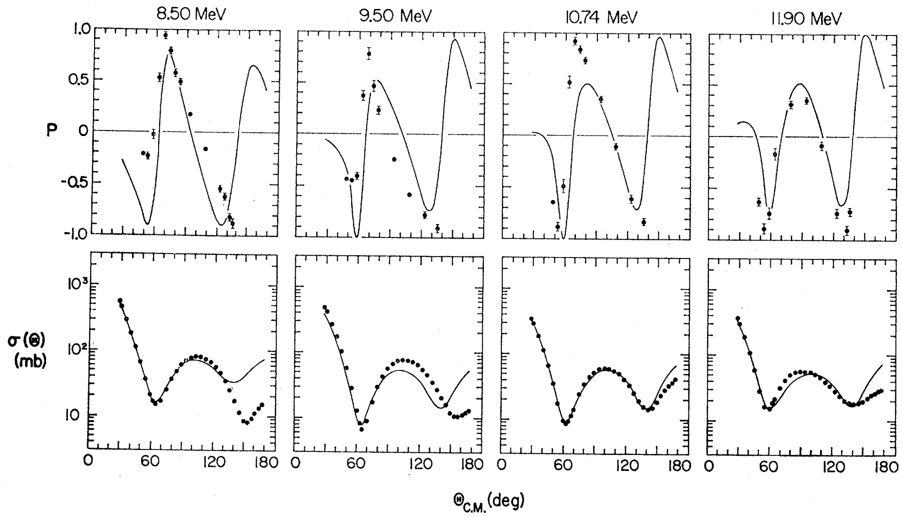

FIG. 5. Angular distributions of the polarization and cross section for laboratory proton energies of 8.50, 9.50, 10.74, and 11.90 MeV. The cross sections are those of Hardie *et al.* (Ref. 4). The smooth curves were calculated by Hardie *et al.* from optical model parameters obtained in an analysis of the cross section only.

energies below 4 MeV. Since these results may be useful for other polarization measurements, they are listed in Table II. Also included are new measurements of the polarization in *p*-carbon scattering below 4 MeV.

The polarization for scattering by helium at $\theta_{lab} = 90.4°$ is plotted in Fig. 6. The present results are given by the closed circles, a measurement by Scott[20] is shown as an open circle, and the solid line gives the polarization calculated from *p*-α phase shifts by Brockman.[21]

The polarization for scattering by carbon at $\theta_{lab} = 60.8°$ is plotted in Fig. 7. The present results are given by the closed circles while the open circles give the results by Evans and Grace.[22]

4. GAS PURITY AND BACKGROUNDS

The target gases used in this experiment were Bureau of Mines Grade A helium and electrolytic oxygen. The stated helium purity is 99.99%. An analysis of the oxygen indicated an impurity concentration of less than 0.35%. Together with the 0.2% natural abundance of O^{18} this gives a total contaminant concentration of less than 0.55%. This concentration of impurities could have a significant effect on the experimental results only if a resonance for one of the impurities occurred at an energy where the p-O^{16} cross section happens to be small.

The use of coincidence between the proportional counter and the scintillation counter eliminated background pulses except those due to charged particles passing through both counters. Accidental coincidences did not contribute significantly to the observed background. Background measurements were generally made by evacuating the second target cell. For measurements above 5 MeV the background was always less than 0.5% so that background subtraction was not necessary. Since the background tended to increase as the energy of the detected protons decreased, background subtractions were made below 5 MeV. The largest background was about 5%. The background at low energies is caused to a large extent by foil-to-foil scattering in the windows

TABLE II. Polarization of protons elastically scattered by helium at $\theta_{lab}=90.4°$ and by carbon at $\theta_{lab}=60.8°$.

p-He, $\theta_{lab}=90.4°$		*p*-C, $\theta_{lab}=60.8°$	
E_p (MeV)	P	E_p (MeV)	P
2.13±0.15	0.915±0.037	2.40±0.14	−0.230±0.017
2.32+0.15	0.929±0.035	2.57±0.13	−0.225±0.021
2.51±0.13	0.892±0.033	2.89±0.11	−0.260±0.022
3.01±0.11	0.799±0.029	3.12±0.16	−0.249±0.018
3.47±0.08	0.601±0.025	3.65±0.13	−0.314±0.017
3.91±0.10	0.470±0.021	3.89±0.11	−0.400±0.015
4.28±0.10	0.345±0.020		

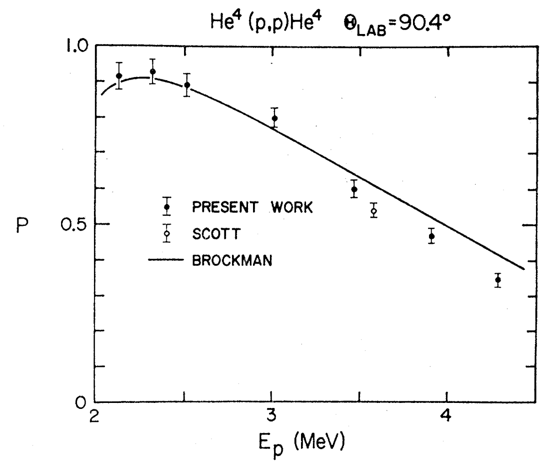

FIG. 6. Polarization in proton-helium scattering between 2 and 4 MeV. The present results at $\theta_{lab}=90.4°$ are shown as solid dots. The open circle is a measurement by Scott (Ref. 20). The solid line is the polarization calculated from *p*-α phase shifts by Brockman (Ref. 21).

[20] M. J. Scott, Phys. Rev. **110**, 1398 (1958).
[21] K. W. Brockman, Jr., Phys. Rev. **110**, 163 (1958).
[22] J. E. Evans and M. A. Grace, Nucl. Phys. **15**, 646 (1960).

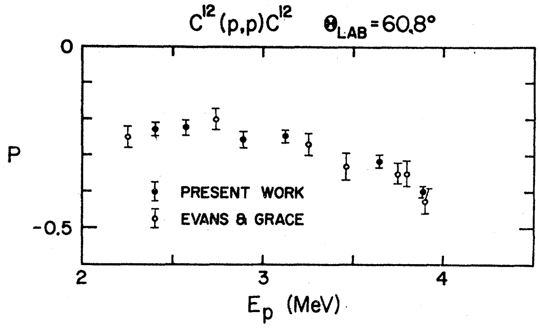

FIG. 7. Polarization in proton-carbon scattering between 2 and 4 MeV. The present results at $\theta_{lab}=60.8°$ are shown as solid dots. The open circles are measurements by Evans and Grace (Ref. 22).

of the second target cell; i.e., protons scattered by the entrance foil can strike the exit foil and can then be scattered into the detector. The effect of foil-foil scattering in the second target was eliminated by the background subtractions mentioned above. Foil-foil scattering in the first target cell is not eliminated by the background subtraction, which causes an estimated error of 0.005 in the polarization at 45° for 3.0 MeV. The effect decreases rapidly with increasing angle and energy since it depends on the product of two Rutherford cross sections.

5. RESOLUTION AND UNCERTAINTY IN ENERGY AND ANGLE

In the present geometry the scattering angles have an uncertainty of about 0.1°. All angles quoted are mean angles of scattering. They were evaluated as in Ref. 13. With the inclusion of multiple scattering in the target windows, the rms angular spread in the first scattering decreases from 2.0° at 3.0 MeV to the geometrically determined 1.4° above 5 MeV. The corresponding range of the spread in the second scattering angle is from 3.6° at 2.3 MeV to a minimum of 2.2°.

For angular distributions of the polarization measured above 4.0 MeV an oxygen target thickness of 0.10 MeV was used except at 5.66 MeV where the target thickness was about 0.18 MeV. Below 4.0 MeV, target thicknesses up to 0.20 MeV were used. For measurements made with oxygen as the second target the energy spread is somewhat larger, because the energy spread of the protons incident upon the second cell is comparable with the oxygen target thickness. The uncertainty in the mean energy results primarily from the uncertainty with which energy losses in the foil and the target gas can be determined. The first scattering energy has an uncertainty of about 0.01 MeV. Because of the larger energy losses the second scattering energy has an uncertainty of about 0.1 MeV.

6. CORRECTIONS FOR EXTENDED TARGETS AND DETECTORS

To obtain acceptable counting rates for a double scattering experiment measurements must be made with thicker targets and a larger angular spread than would be necessary for a single scattering. The effect of the increased spread in energy and angle upon the experimental results cannot, in general, be expressed as an averaging of the polarization for each scattering over energy and angle. Coupling between the first and second scatterings and between the cross section and the polarization prevents any such simple interpretation of the results.

The effects of the extent of the targets and of the detectors has been discussed for the present geometry by Brown *et al.*[13] The present results were corrected for the effect of averaging over the range in azimuthal angle allowed by the slit height following the method described there. Corrections due to the slit width and the resulting spreads in energy and angle can be calculated if the derivatives of the cross section and the polarization with respect to energy and angle are known. These derivatives were estimated from smooth curves drawn through the experimental data. The corrections resulting from the slit width were applied to all measurements below 5 MeV except at 3.4 and 3.5 MeV where the effect of averaging over the sharp resonance at 3.47 MeV could not be reliably estimated. The corrections to P_1P_2 were generally less than 0.002, although corrections as large as 0.02 for some data points near 3.9 MeV did result from the rapid variation of the polarization and cross section with energy.

7. UNCERTAINTIES

The uncertainty of P_1P_2 is largely the statistical uncertainty for the number of counts recorded. Because the corrections (Sec. 6) are rather uncertain, an additional uncertainty equal to one-half the correction was also included. The uncertainty of the final results, as indicated by the error bars on the plotted data, included further the uncertainty of the analyzing power as well as the effect of the energy uncertainty on the analyzing power. In most cases the uncertainty of the analyzing power is small compared to the statistical uncertainty of P_1P_2. The most notable exceptions are at 4.79 and 5.66 MeV where the large uncertainty in the polarization at back angles is mainly the result of the uncertainty in the analyzing power.

8. COMPARISON WITH OTHER MEASUREMENTS

Because the polarization is changing rapidly over most of the energy range covered by the present experiment, only qualitative comparisons can be made with the results of other measurements which are at slightly different energies and angles. Measurements made at Strasbourg between 3.8 and 4.6 MeV for laboratory angles of 51.5°[11] and 130°[12] show essentially the same structure as that observed in the present experiment at 46 and 115°, respectively. The Strasbourg results are in good agreement with the polarizations calculated from the phase shifts given in Sec. 9.

Angular distributions of the polarization have also been measured at 7.9 MeV[9] and 10 MeV[10] by Rosen *et al.* Due to the resonances in this energy region (see Fig. 3) it is not surprising that the angular distribution at 7.9 MeV is different from the present results at neighboring energies. The angular distribution at 10 MeV is quite similar to the present results at 9.5 MeV.

9. ANALYSIS

9.1. Proton-Oxygen Scattering Phase Shifts

The elastic scattering of protons by O^{16} can be described by the well-known partial-wave expansion for the scattering of protons by a nucleus with zero spin. The formulas for the differential cross section and the polarization have been given, for example, by Harris *et al.*[5] In the present analysis the polarization was taken to be positive in the direction $\mathbf{k}_{\text{in}}\times\mathbf{k}_{\text{out}}$ in accordance with the Basel convention.[23]

The proton wave number k and the Coulomb parameter η were evaluated relativistically. At 7 MeV relativistic effects change $1/k^2$ by 0.4% and η by 0.6%. Additional effects which result from the Coulomb spin-orbit coupling[24] are of comparable size but have been neglected in the present analysis.[25]

To determine the proton-oxygen phase shifts at a given energy, a least-squares fit of the calculated cross section and polarization was made to the experimental data with the phase shifts as adjustable parameters. Partial waves for $l\leq 3$ were included. The calculations were made using a digital computer. The computer program uses the gradient search method to find the best fit. Starting from an initial set of phase shifts, which are part of the input information, the program changes all of the phase shifts simultaneously in steps proportioned to give the maximum rate of decrease in the squared error for the fit.

The calculated polarization and cross section obtained in fitting the experimental data at eight energies are given by the solid curves in Figs. 2 and 4. The phase shifts are listed in Table III. At 2.5 MeV where no polarization was measured, initial fits to the cross section alone gave a calculated polarization of 0.28 at 115° which does not agree with a smooth curve drawn through the measured points at this angle (see Fig. 1). Therefore, a polarization of 0.15 at 115° was included in the input data. The quality of the resulting fit to the cross section was not changed. At 3.4 MeV the phase shifts were determined by the fit to the cross section alone since the energy spread with which the polarization was measured overlapped the sharp resonance of 3.47 MeV.

TABLE III. Phase shifts for p-O^{16} scattering (in degrees). The phase shift for $J^\pi=\frac{5}{2}^-$ was set equal zero.

E_p (MeV)	$\frac{1}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$	$\frac{7}{2}^-$
2.50	−36.8	2.0	3.2	2.2	−1.6	0.0[a]
3.00	−45.8	−3.7	4.6	5.1	−2.2	0.4
3.40	−52.1	−2.8	8.1	10.1	−3.1	0.8
3.80	−57.7	−4.3	12.9	19.6	−3.0	1.0
4.79	−69.1	−3.8	−6.2	−68.0	−5.3	2.5
5.66	−78.0	−2.0	−3.3	−44.0	−9.8	−6.4
5.89	−70.1	−14.3	−1.2	−31.6	−9.8	−2.3
6.00	−65.8	−11.7	1.3	−30.3	−10.6	−1.9
7.01	77.4	−0.4	26.3	72.3	−10.6	2.6

[a] At this energy, the 7/2− phase shift was held at zero.

The accuracy with which the phase shifts were determined was investigated at two energies by introducing forced variations of each phase shift in turn while the other phase shifts were left free to find a new fit. The results of these investigations indicate that the phase shifts have an uncertainty of one to two degrees, the S-wave phase shift being the least accurately determined. A search for alternate sets of phase shifts was also made at two energies by varying the starting phase shifts over a wide range. No alternate solutions were found which gave reasonable fits to the experimental data.

In order to determine to what extent the polarization measurements help in reducing the uncertainty of the phase shifts, the analysis of the 4.79-MeV data was repeated using the cross-section data only. It was found that in this particular case a good fit to the data could be obtained over a ten times larger range of values of the $\frac{1}{2}^+$ and $\frac{1}{2}^-$ phases than in the case that the polarization measurements were included also. To quote a specific example, it was possible to use any value of the $\frac{1}{2}^+$ phase shift between −46° and −76° and still fit the cross section very well. The rms deviation between calculated and measured cross sections for all angles measured[4] (28° to 167°) never exceeded 2%. As the $\frac{1}{2}^+$ phase was changed over this interval, the $\frac{1}{2}^-$ phase shift which gave the best fit varied from +32° to −5°. The accuracy of the cross section measurements used[4] was about 1.5% so that a large improvement in the accuracy of the phase shifts cannot be readily obtained without polarization data. In any case, the above example makes it fairly clear that at higher energies where the phase shifts become complex, i.e., where the number of parameters doubles, polarization measurements are essential.

The phase shifts between 2 and 5 MeV are plotted in Fig. 8. The closed symbols give the results of fits to the measured angular distributions. For several energies between 3.8 and 5 MeV additional calculations were made to determine the behavior of the two phase shifts

[23] The formula for the polarization given in Ref. 5 uses the opposite sign, in agreement with their stated sign convention. However, the sign of the calculated polarization shown in their Fig. 15 should be reversed to be consistent with their sign convention.

[24] L. Wolfenstein, Ann. Rev. Nucl. Sci. 6, 43 (1956).

[25] Neglect of the Coulomb spin-orbit coupling results in small errors in the nuclear phase shifts because of the incorrect separation of nuclear and Coulomb effects. In work done by L. E. Porter at our laboratory (unpublished) the size of these errors has been estimated at 10.5 MeV by comparing optical model calculations with and without the additional Coulomb spin-orbit potential. For this case differences in the phase shifts of about 0.5° result, with the largest changes occuring in the phase shifts which make the largest contributions to the scattering amplitudes.

that are changing rapidly with energy. The cross section from excitation curves measured by Eppling[2] at six angles and by Salisbury *et al.*[3] at eight angles and the polarization from the excitation curves shown in Fig. 1 were fit with only the $\frac{3}{2}^-$ and the $\frac{3}{2}^+$ phase shifts as free parameters. The other phase shifts were fixed at values given by smooth curves through the earlier points. The phase shifts obtained from these fits are plotted in Fig. 8 as open symbols.

In agreement with Salisbury and Richards[6] and with Harris *et al.*[5] the broad anomaly in the cross section between 3.8 and 5 MeV must be attributed to broad, overlapping $\frac{3}{2}^-$ and $\frac{3}{2}^+$ resonances. Even though Salisbury's phase shifts predicted polarizations that are an order of magnitude too small near 3 MeV, relatively minor changes in these phase shifts were sufficient to obtain agreement with the measured polarization. Below 5 MeV the $\frac{3}{2}^-$ and $\frac{3}{2}^+$ phase shifts obtained in the present analysis are 5 to 10° more positive than the previous results. For the other phase shifts, the present values differ by only 2° or 3° from those of Salisbury. A much larger difference exists between the present results and those of Harris *et al.*[5] The difference in the $\frac{1}{2}^+$ phase shift approaches 30° at 5 MeV. For the *P* waves the difference is generally less than 10°, but the discrepancy is as much as 18° for the $\frac{3}{2}^+$ phase shift.

The polarization and cross section as functions of energy between 2 and 5 MeV were calculated using phase shifts taken from smooth curves drawn through the points of Fig. 8. Near 2.66 MeV the effect of the $\frac{1}{2}^-$ resonance was included in the calculation, making use of the level parameters obtained by Salisbury and Richards[6] for this resonance. The effect of the very sharp ($\Gamma = 1.6$ keV) resonance at 3.47 MeV was not considered. The agreement of the calculated values with the experimental excitation curves is shown in Fig. 1. As previously mentioned, these phase shifts also are in

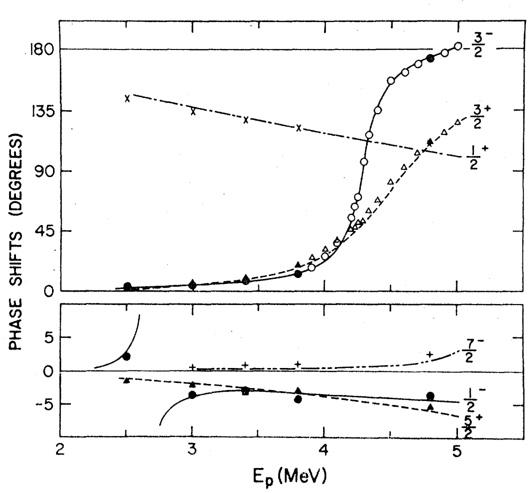

FIG. 8. Phase shifts for proton-oxygen scattering between 2 and 5 MeV. The curves were calculated from the F^{17} level parameters.

satisfactory agreement with other recently measured polarizations.[11,12]

Phase shifts were also determined at higher energies by fitting the angular distributions of the polarization and cross section at 5.66, 5.89, 6.00, and 7.01 MeV. Fits to the polarization and cross section at three of these energies are shown in Fig. 4. At 5.89 MeV no polarizations were measured, but a phase shift analysis was made of an angular distribution of the cross section measured by Hardie *et al.*[4] The phase shifts obtained are included in Table III although the fit is not shown. The measured and calculated cross sections and the calculated polarization at this energy are nearly identical to those at 6.00 MeV. The results of the analysis at these four energies confirm the essential features of the analysis by Salisbury and Richards[6] although again differences of 5 to 10° do occur for some phase shifts.

The present analysis, as well as the analysis by Salisbury and Richards, was made using the partial wave expansion with real phase shifts, which is strictly applicable only when there are no open channels other than elastic scattering. At a laboratory proton energy of 5.6 MeV the $O^{16}(p,\alpha)N^{13}$ channel opens, and above 6.3 MeV inelastic scattering leaving oxygen in its first excited state becomes possible. With the opening of these additional channels the phase shifts become complex so that the number of parameters to be determined is doubled. It is expected that neglect of these additional open channels does not seriously alter the results of the phase shift analysis up to 7 MeV because the reaction cross sections are probably still small. However, recent measurements of the $O^{16}(p,p')O^{16*}$ and the $O^{16}(p,\alpha)N^{13}$ cross sections by Dangle *et al.*[26] indicate that these channels become important at slightly higher energies.

The effects of the inelastic channels on the elastic p-O^{16} scattering has been included in a recent analysis between 7.0 and 8.5 MeV by Oppliger and Croley.[27] This analysis is discussed in Sec. 9.2.

9.2. Level Parameters for States in F^{17}

The energy dependence of the phase shifts for p-O^{16} scattering provides information about the energy levels of the compound nucleus F^{17}. Salisbury and Richards[6] had obtained level parameters for most resonances in the energy region covered by the present analysis through the application of nuclear dispersion theory to their phase shifts. The differences between the results of the present analysis and the phase shifts of Salisbury and Richards are not large enough to significantly affect the extracted level parameters except for the $\frac{3}{2}^-$ and $\frac{3}{2}^+$ phase shifts. In the following, the phase shifts determined from the present data will be discussed in relation to the excited states of F^{17}. As usual the phase

[26] R. L. Dangle, L. D. Oppliger and G. Hardie, Phys. Rev. **133**, B647 (1964).
[27] L. D. Oppliger and D. R. Croley, Jr., Bull. Am. Phys. Soc. **8**, 538 (1963); D. R. Croley, Jr., and L. D. Oppliger, *ibid.* **8**, 538 (1963); (private communication).

TABLE IV. Level parameters for the $\frac{3}{2}^-$ and $\frac{3}{2}^+$ states in F^{17} from four-level analyses of p-O^{16} phase shifts between 2.5 and 8.5 MeV.[a]

J^π	$E\lambda_{lab}$ (MeV)	γ^2 [b] (MeV cm)
	4.35	0.446×10^{-13}
$\frac{3}{2}^-$ [c]	5.23	0.102×10^{-13}
	6.84	0.286×10^{-15}
	8.30	0.111×10^{-12}
	4.61	0.573×10^{-12}
$\frac{3}{2}^+$ [d]	5.52	0.177×10^{-13}
	6.57	0.730×10^{-15}
	7.11	0.187×10^{-12}

[a] Taken from Ref. 27.
[b] The reduced width γ^2 was computed using an interaction radius of 4.15×10^{-13} cm for the $3/2^-$ levels and a radius of 5.10×10^{-13} cm for the $3/2^+$ levels.
[c] The $3/2^-$ potential phase shift could not be expressed as a hard-sphere phase shift. The potential phase shift was taken to proportional to energy, with a slope of -1.77 deg/MeV.
[d] The $3/2^+$ potential phase shift was taken to be $\phi=-\arctan(F_l/G_l)$ and was evaluated for a radius of $r=4.15\times10^{-13}$ cm.

shifts were considered as the sum of resonance and potential scattering phase shifts. In most cases good agreement with the experimentally determined phase shifts could be obtained by using hard-sphere phases as the potential scattering phase shifts, but for some of the partial waves it was necessary to use a hard sphere radius which differed substantially from a reasonable nuclear radius. However, all values used satisfy the condition that the potential scattering be slowly varying with energy. As pointed out by Teichmann and Wigner[28] this is the only real condition that can be imposed.

The $\frac{1}{2}^+$ phase shift. The $\frac{1}{2}^+$ phase shift between 2 and 5 MeV differs by no more than 5° from the calculated potential phase shift for a hard-sphere radius given by $r=R_0(A^{1/3}+1)$ with $R_0=1.55$ F. The smooth curve for the $\frac{1}{2}^+$ phase shift in Fig. 8 gives this potential phase shift. Better agreement with the point at 4.8 MeV would be obtained if the effect of the broad $\frac{1}{2}^+$ resonance at 6.33 MeV were included.

The $\frac{1}{2}^-$ phase shift. The existence of a $\frac{1}{2}^-$ level at a bombarding energy of 2.66 MeV is well established.[2,6] The only other $\frac{1}{2}^-$ resonance below 7 MeV is that at 5.78 MeV with a width of 30 keV. It has little effect on the phase shift below 5 MeV. The $\frac{1}{2}^-$ phase shift given by the smooth curve of Fig. 8 was calculated by adding the resonant phase shift to a hard-sphere potential phase shift. The hard-sphere radius parameter R_0 which gives the best fit between 2 and 5 MeV is 0.70 F. The resonance parameters were taken from Ref. 6.

The very small hard-sphere radius used here is consistent with the observation that the P-wave optical-model phase shifts[4] in this energy region are much smaller than the conventional hard-sphere phases.

The $\frac{3}{2}^-$ phase shift. The $\frac{3}{2}^-$ phase shifts extracted from the present measurements show a resonance near 4.3 MeV (Fig. 8). Below 7 MeV three further $\frac{3}{2}^-$ resonances have been reported.[6] Except for the one at 5.23 MeV, they are too narrow to be of importance here. However, a very broad $\frac{3}{2}^-$ resonance at 8.3-MeV bombarding energy has recently been discovered by Oppliger and Croley.[27] They made use of the phase-shift search program described in Sec. 9.1 to fit the present polarization measurements and the cross section[4] at 8.5 MeV. Since the phase shifts are also known accurately at 7 MeV (Table III), it was then possible to extend the analysis through the intermediate energy interval by fitting the cross sections measured at eight angles by Salisbury *et al.*[3] Use was also made of the results on the (p,p') and the (p,α) reaction by Dangle *et al.*[26] The $\frac{3}{2}^-$ phase shift as a function of energy between 2.5 and 8.5 MeV was then fit with a four-level formula. The parameters of the levels which Oppliger and Croley obtained in this way are given in Table IV. The curve shown in Fig. 8 is based on these level parameters. The calculated curve is in excellent agreement with the phase-shift points obtained from the experiment over the whole energy range.

The $\frac{3}{2}^+$ phase shift. The $\frac{3}{2}^+$ phase shift increases rapidly above 4 MeV (see Fig. 8) as expected from the known level[6] at a bombarding energy of about 4.8 MeV. Since again the phase shift in our energy region is affected by the levels at higher energies, the calculated dashed curve in Fig. 8 took into account all four known $\frac{3}{2}^+$ levels. The curve shown was obtained from the level parameters of Croley and Oppliger[28] listed in Table IV.

The $\frac{5}{2}^+$ phase shift. Since there are no $J^\pi=\frac{5}{2}^+$ resonances, below 8 MeV, the calculated curve of the $\frac{5}{2}^+$ phase shift in Fig. 8 is the potential scattering phase shift only. An interaction radius of $r=4.15$ F was used, which corresponds to a radius parameter $R_0=1.18$ F. This is the same radius as had been used for the $\frac{3}{2}^+$ potential phase shift.

The $\frac{5}{2}^-$ phase shift. In the initial stages of the phase shift analysis it was found that the $\frac{5}{2}^-$ phase shift which gave the best fit to the data did not differ significantly from zero. In the final analysis the $\frac{5}{2}^-$ phase shift was assumed to be zero for all proton bombarding energies. While it has recently been shown[29,30] that the resonance at 3.47 MeV has $J^\pi=\frac{5}{2}^-$, this resonance is so sharp ($\Gamma=1.6$ keV) that it does not affect the present analysis, except possibly at 3.40 MeV where the expected phase shift is about one degree. No other $\frac{5}{2}^-$ resonances have been reported for bombarding energies below 8.5 MeV.

The $\frac{7}{2}^-$ phase shift. The $\frac{7}{2}^-$ phase shift was found to be small and positive below 5 MeV. This is explained by the presence of a resonance at about 5.4 MeV. The curve (Fig. 8) was calculated from the resonance parameters of Ref. 6. It agrees with the phase shifts obtained from the measurements within a fraction of a degree.

[28] T. Teichmann and E. P. Wigner, Phys. Rev. 87, 123 (1952).

[29] R. E. Segel, P. P. Singh, R. G. Allas, and S. S. Hanna, Phys. Rev. Letters **10**, 345 (1963).

[30] E. A. Silverstein, L. D. Oppliger, and R. A. Blue, Bull. Am. Phys. Soc. 9, 68 (1964).

10. OPTICAL MODEL CALCULATIONS

Two optical model analyses[4,14] of the elastic scattering of protons by oxygen have been made in the energy range above 8 MeV. When these analyses were made, the only polarization data available in this energy region was the angular distribution at 10 MeV of Rosen *et al.*[10] It is, therefore, of interest to compare the results of the present measurements with the predictions of these calculations.

The solid curves in Fig. 5 give the polarization and the cross section calculated from the optical-model parameters which Hardie *et al.*[4,16] obtained from the analysis of the cross section along. Hardie did not make a complete search of parameter space but, after selecting parameters on the basis of the fit to the cross section at 13 MeV, allowed only the depths of the real and imaginary central potentials to vary at other energies. No attempt was made to fit the cross section back of 130°. Duke,[14] in a more detailed analysis, obtained better fits to the cross section. The resulting polarization[15] is quite similar to that of Hardie.

The application of the optical model to proton-oxygen scattering below 12 MeV is, perhaps, questionable. The level spacing is still larger than the experimental resolution so that the cross section and polarization are not averaged over many resonances. Under these conditions the optical model cannot be expected to reproduce the experimental results in detail. The optical model for proton-oxygen scattering, however, successfully fits the qualitative features of the cross section and the polarization and, as Hardie[4] has shown, provides useful information about the position of the single-particle states.

11. CONCLUSIONS

As is the case for other light nuclei, the scattering of protons by oxygen exhibits polarizations which often approach ± 1.0. Below 5 MeV two very broad resonances give rise to polarizations which change slowly with energy except near sharp resonances at 2.66 and 3.47 MeV. Except in the immediate vicinity of these resonances proton-oxygen scattering may prove useful as a polarization analyzer.

The present polarization measurements demonstrate again that small errors in the phase shifts determined from fits to the cross section can result in large errors in the calculated polarization. A phase shift analysis using both cross section and polarization data allowed a more accurate determination of the proton-oxygen scattering phase shifts between 2 and 5 MeV. Further use of polarization measurements at higher energies was limited by the energy resolution attainable in a double scattering experiment. Measurements made at energies selected to avoid the many pronounced resonances were useful for a phase-shift analysis. This analysis confirmed the results of an earlier analysis by Salisbury and Richards.[6] The phase-shift analysis has recently been extended to 8.50 MeV.[27] The success of this analysis depended, in part, on the polarization data obtained in the present experiment for an accurate determination of the phase shifts at 7 and 8.5 MeV. Further extension of the analysis of proton-oxygen scattering to higher energy may well require more polarization data since the number of parameters to be determined increases due to the increasing importance of other open channels and higher orbital angular momenta.

Comparison of the present polarization measurements with the predictions of an optical-model calculation shows qualitative agreement even though the conditions for the applicability of the optical model are not strictly satisfied in proton-oxygen scattering below 12 MeV.

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