Hyperon Production by Neutrinos in an SU_3 Model*

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An SU_3 model of weak interactions is used to discuss amplitudes and cross sections for hyperon production by neutrinos. Numerical results for the cross sections are given. The notion of first- and second-class currents is extended to currents transforming like multiplets under SU_3.

I. INTRODUCTION

THIS paper presents a study of the production of hyperons by neutrinos through the "elastic" mechanism[1]

$$\bar{\nu}+N\rightarrow B+\bar{l}.$$

Using an SU_3 model of weak interactions due to one of us,[2] we make detailed predictions of cross sections and polarizations for all the possible processes of "elastic" hyperon production.

A number of theoretical studies of the $\Delta S=0$ elastic reactions

$$\begin{aligned}\bar{\nu}+p&\rightarrow n+\mu^+,\\ \nu+n&\rightarrow p+\mu^-,\end{aligned}\tag{1}$$

have been performed.[3–5] Experimental work is still preliminary but already interesting discoveries such as the existence of two neutrinos have been made.[6,7]

Among the $\Delta S=1$ reactions, the $\Delta S=\Delta Q$ rule allows only three "elastic" reactions, all arising from antineutrinos:

$$\begin{aligned}\bar{\nu}+p&\rightarrow\Lambda+\mu^+,\\ \bar{\nu}+n&\rightarrow\Sigma^-+\mu^+,\\ \bar{\nu}+p&\rightarrow\Sigma^0+\mu^+.\end{aligned}\tag{2}$$

The third reaction is related to the second by the $\Delta I=\frac{1}{2}$ selection rule for the $\Delta S=1$ weak current

$$d\sigma(\Sigma^0)=\tfrac{1}{2}d\sigma(\Sigma^-).\tag{3}$$

Formulas for the differential cross section,[8–10] polarization,[10,11] and total cross section[9] have been given by one of us, among others.

Let us review briefly the relevant facts. The matrix element has the form

$$\langle Bl|T|N\bar{\nu}\rangle=(G/\sqrt{2})\langle B|J_\lambda^\dagger|N\rangle\bar{u}_l\gamma^\lambda(1+i\gamma_5)u_{\bar{\nu}}.\tag{4}$$

The matrix element of J_λ, the weak current of strongly interacting particles, can be expressed in a general form in terms of certain form factors $f(q^2)$ which are functions of the invariant momentum transfer to the baryons. For the $\Delta S=0$ reactions (1) the vector form factors are related through the conserved-vector-current (CVC) hypothesis to the electromagnetic form factors of the nucleon.[12] While the axial-vector form factors are not known they would still be expected to be decreasing functions of q^2, similar to the vector form factors. Qualitatively, for large neutrino energy E_ν the differential cross section is

$$\frac{d\sigma(E_\nu,q^2)}{dq^2}\sim G^2|f(q^2)|^2,$$

and the total cross section

$$\sigma(E_\nu)\sim G^2\int_{q^2_{\max}}^{q^2_{\min}}|f(q^2)|^2dq^2.$$

In the high-energy limit $q^2_{\min}\rightarrow 0$ and $q^2_{\max}\rightarrow -4E_\nu^2$. The behavior of the total cross section depends on the large q^2 behavior of $f(q^2)$. We now consider three cases depending on the power with which $f(q^2)$ vanishes at ∞:

(i) $f(q^2)\rightarrow$ constant as $q^2\rightarrow-\infty$,

here $\sigma\sim E_\nu^2$;

(ii) $q^2f(q^2)\rightarrow$ constant as $q^2\rightarrow-\infty$,

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[1] We call both nucleon and hyperon neutrino interactions "elastic" in the spirit of SU_3 symmetry.

[2] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

[3] T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960); Phys. Rev. **126**, 2239 (1962).

[4] N. Cabibbo and R. Gatto, Nuovo Cimento **15**, 304 (1960).

[5] Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) **23**, 1117 (1960).

[6] G. Danby, J. M. Gaillard, K. Goulianos, L. M. Lederman, N. Mistry, M. Schwartz, and J. Steinberger, Phys. Rev. Letters **9**, 36 (1962).

[7] G. Bernardini, *et al.*, *Proceedings of the Sienna International Conference on Elementary Particles* (Societa Italiana di Fisica, Bologna, 1964), pp. 555 and 571.

[8] J. S. Bell and S. M. Berman, Nuovo Cimento **25**, 404 (1962).

[9] F. Chilton, Nuovo Cimento **31**, 447 (1964).

[10] S. Adler, Nuovo Cimento **30**, 1020 (1963).

[11] L. Egardt, Nuovo Cimento **29**, 954 (1963).

[12] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

yields $\sigma\sim\ln E_\nu$, and

$$\text{(iii)}\quad q^4 f(q^2)\to\text{constant}\quad\text{as}\quad q^2\to-\infty,$$

yields $\sigma\sim$const.

The second and third cases are the more realistic if one examines the electromagnetic form factors of the nucleon. Because of the relatively low energy of present neutrino beams, cases (ii) and (iii) probably cannot be distinguished by present experiments.

Our study of hyperon production by neutrinos is based on an[13,14] SU_3 model of weak interactions developed by one of us.[2] The model assumes that the weak current of strongly interacting particles is a member of an SU_3 octet. This is the simplest hypothesis about the current which is compatible with the conserved vector-current hypothesis.[15] If one also assumes a certain degree of universality in weak interactions the current can be written as

$$J_\lambda=\cos\theta\, J_\lambda{}^{(0)}+\sin\theta\, J_\lambda{}^{(1)}. \tag{5}$$

$J_\lambda{}^{(0)}$ and $J_\lambda{}^{(1)}$ are the $\Delta S=0$, $\Delta Q=1$, and $\Delta S=\Delta Q=1$ members of an octet of currents which are partly vector, partly axial-vector. The octet transformation properties have as a direct consequence a $\Delta I=\frac{1}{2}$ (and no $\Delta S=-\Delta Q$) selection rule for strangeness nonconserving leptonic processes, and a $\Delta I=1$ rule for $\Delta S=0$ processes.

As a consequence of the CVC hypothesis, the vector part is in the same octet as the electromagnetic current, so that its matrix elements can be related to known electromagnetic properties of different particles.

The axial part of the current cannot be obtained in a similar way, but its different matrix elements can all be expressed in terms of two independent ones. The treatment of axial parts will be discussed in more detail in Sec. II.

The parameter θ can be determined by comparing say, K_{l3} and π_{l3} and also independently by comparing $K_{\mu2}$ and $\pi_{\mu2}$. It is essential that the same value $\theta=0.26$ is obtained from both determinations.

In Sec. II we will discuss the matrix elements for processes (2) and our determinations of the coupling constants and form factors. Section III is a discussion of an extension of the notion of classification of currents by G parity to explain why certain possible terms ("electric dipole," induced scalar) can presumably be ignored.[16] In Sec. IV the analytic expressions for differential cross section, hyperon spectrum, and total cross section are given. Section V contains a discussion of some numerical examples.

[13] M. Gell-Mann, California Institute of Technology Report, CTSL-20 (unpublished); Phys. Rev. **125**, 1067 (1962).

[14] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

[15] The conserved vector current has the isospin character of the $|1,1,1\rangle$ member of an octet.

[16] S. Weinberg, Phys. Rev. **112**, 1375 (1958).

II. STRUCTURE OF THE MATRIX ELEMENTS

The matrix elements for the processes in which we are interested can be written as

$$\langle|T|\rangle=[\bar v(\nu_\mu)\gamma^\lambda(1+i\gamma_5)v(\mu^+)]\times\begin{cases}\langle\Sigma^-|J_\lambda{}^\dagger|n\rangle\\ \langle\Lambda|J_\lambda{}^\dagger|p\rangle.\end{cases} \tag{6}$$

This expression is valid in the first order in weak interactions disregarding radiative corrections; $J_\lambda{}^\dagger$ is here the Hermitian conjugate of the weak current J_λ. It is well known that matrix elements of a mixed vector and axial-vector current among two baryon states B_1 and B_2 can be expressed in terms of six independent form factors, functions of the momentum transfer.

$$\langle B_2|J_\lambda{}^\dagger|B_1\rangle=\frac{1}{\sqrt2}\bar u_2(p_2)\biggl\{\gamma_\lambda(G_V+G_Ai\gamma_5)+\sigma_{\lambda\beta}q^\beta\biggl(\frac{F_V}{\Sigma}-\frac{F_A}{\Delta}i\gamma_5\biggr)+q_\lambda\biggl(-\frac{H_V}{\Sigma}+\frac{H_A}{\Delta}i\gamma_5\biggr)\biggr\}u_1(p_1), \tag{7}$$

where $\Sigma=M_1+M_2$ and $\Delta=M_2-M_1$. All form factors are presumed to be real, a consequence of T invariance. In the following we will, according to the SU_3 model, express the matrix elements in which we are interested in terms of two "reduced" matrix elements. We will then discuss the structure of these and obtain explicit formulas for the form factors [as defined in Eq. (7)] for Σ and Λ production. We would like to note the following points:

(a) The vector form factors G_V, F_V will be expressed in terms of the proton and neutron em form factors.

(b) The axial form factor G_A will be expressed in terms of the analogous one for $\Delta S=0$ processes [Eq. (1)], and one other parameter (also a function of q^2) whose value at zero momentum transfer can be measured from the leptonic decays of hyperons.

(c) The equations we derive are only true in the ideal situation of exact SU_3, but we also take into account some of the more clearly foreseeable consequences of symmetry breaking, as we will discuss at the end of the section.

(d) The form factors F_A, H_V would come from "second-class" currents. This classification, introduced by Weinberg for $\Delta S=0$ currents on the basis of g parity, is, in the next section, extended to octet currents. We will assume the existence of "first-class" currents only; thus we put $F_A=H_V=0$ in the following.

It is convenient to work with matrix elements of J_λ (instead of $J_\lambda{}^\dagger$) and write

$$\begin{aligned}\langle\Sigma^-|J_\lambda{}^\dagger|n\rangle&=\langle n|J_\lambda|\Sigma^-\rangle^\dagger,\\ \langle\Lambda|J_\lambda{}^\dagger|p\rangle&=\langle p|J_\lambda|\Lambda\rangle^\dagger.\end{aligned} \tag{8}$$

Since in our model the current is a member of an SU_3 octet, and the same is true of the initial and final states, the relevant matrix elements can be expressed in terms of two reduced matrix elements. In general if B^i and B^k are members of the baryon octet, and $J_\lambda{}^j$ is an arbitrary member of the octet to which J_λ belongs, we have

$$\langle B^i|J_\lambda{}^j|B^k\rangle = if_{ijk}O_\lambda + d_{ijk}E_\lambda\,, \tag{9}$$

where O_λ and E_λ are reduced matrix elements, f_{ijk} and d_{ijk} are numbers—the SU_3 equivalents of Clebsch-Gordan coefficients.[17] We have in particular

$$\begin{aligned}\langle n|J_\lambda|\Sigma^-\rangle &= \sin\theta\,(O_\lambda - E_\lambda)\,,\\ \langle p|J_\lambda|\Lambda\rangle &= -\sin\theta\,(\sqrt{\tfrac{3}{2}})(O_\lambda + \tfrac{1}{3}E_\lambda)\,.\end{aligned} \tag{10}$$

The problem is therefore reduced to that of knowing the structure of O_λ and E_λ.

$J_\lambda{}^V$, the vector part of J_λ, is assumed to belong to the same octet as the electromagnetic current j_λ. As a consequence, the vector parts of O_λ and E_λ can be obtained from matrix elements of the electromagnetic current[18]:

$$\begin{aligned}\langle p|J_\lambda|p\rangle &= O_\lambda{}^V + \tfrac{1}{3}E_\lambda{}^V\,,\\ \langle n|J_\lambda|n\rangle &= -\tfrac{2}{3}E_\lambda{}^V.\end{aligned} \tag{11}$$

From these, and the well-known expressions of these matrix elements in terms of form factors we get

$$\begin{aligned}O_\lambda{}^V &= [F_1{}^p(t) + \tfrac{1}{2}F_1{}^n(t)]\gamma_\lambda\\ &\quad + (1/2M_N)[F_2{}^p(t) + \tfrac{1}{2}F_2{}^n(t)]\sigma_{\lambda\beta}q^\beta\,,\\ E_\lambda{}^V &= -\tfrac{3}{2}F_1{}^n(t)\gamma_\lambda - (3/4M_N)F_2{}^n(t)\sigma_{\lambda\beta}q^\beta\,,\end{aligned} \tag{12}$$

where the spinors have been omitted; $t \equiv q^2$. The form factors are normalized so that at zero momentum transfer

$$\begin{aligned}F_1{}^p(0) &= 1\,, \quad F_1{}^n(0) = 0\\ F_2{}^p(0) &= \mu_p\,, \quad F_2{}^n(0) = \mu_n\,.\end{aligned} \tag{13}$$

These equations, together with Eq. (12) completely identify the vector part of the matrix elements in which we are interested. For the axial parts $O_\lambda{}^A$ and $E_\lambda{}^A$ we can write[19]

$$\begin{aligned}O_\lambda{}^A &= G_A{}^O(t)\gamma_\lambda i\gamma_5 + H_A{}^O(t)q_\lambda i\gamma_5\,,\\ E_\lambda{}^A &= G_A{}^E(t)\gamma_\lambda i\gamma_5 + H_A{}^E(t)q_\lambda i\gamma_5\,.\end{aligned} \tag{14}$$

The combination $G_A{}^O + G_A{}^E$ can be measured in $\Delta S = 0$ processes [Eq. (1)]; in fact we have [as another case of Eq. (10)]

$$\langle p|J_\lambda|n\rangle = \cos\theta\,(O_\lambda + E_\lambda) \tag{15}$$

so that

$$G_A{}^{n\to p} = \cos\theta\,[G_A{}^O(t) + G_A{}^E(t)] \equiv \cos\theta\,\tilde{G}_A(t)\,. \tag{16}$$

[17] Clebsch-Gordan coefficients for SU_3 can be found in P. McNamee and F. Chilton, Rev. Mod. Phys. **36**, 1005 (1964).

[18] N. Cabibbo and R. Gatto, Nuovo Cimento **21**, 872 (1961).

[19] Note the notation: H has a different meaning in this paper compared to Ref. 2.

It is therefore convenient to introduce a parametrization

$$\begin{aligned}G_A{}^O(t) &\equiv \tilde{G}_A(t)x(t)\,,\\ G_A{}^E(t) &\equiv \tilde{G}_A(t)[1 - x(t)].\end{aligned} \tag{17}$$

$x(t)$ measures the ratio of f to d coupling, which can in principle depend on q^2. For $t=0$, $\tilde{G}_A(t)$ is equal to the ratio G_A/G_V in beta decay so that

$$\tilde{G}_A(0) \approx 1.25\,. \tag{18}$$

Similarly the value of x for $q^2=0$ can be measured from the leptonic decays of hyperons. From the decay of Σ^- we find $x \approx 0.25$.[2]

Presuming that we regard the pseudoscalar coefficients as induced, they can be obtained by using Goldberger-Treiman relations. Then $H_A{}^O$ and $H_A{}^E$ would be proportional to $G_A{}^O$ and $G_A{}^E$. It is pleasing to note that our f/d ratio value of $x \approx 0.25$ is quite consistent with the values for the f/d ratio resulting from determinations of meson-baryon coupling constants.[20,21]

In practice, however, the coefficients H_A are not important since all their contributions to cross sections are proportional to the square of the lepton mass $m_l{}^2$, and thus contribute in ν_μ reactions only of the order of 1% compared to the largest terms. For this reason we will ignore the pseudoscalar terms in the following. In correlation terms such as polarizations, the pseudoscalar term can give contributions proportional to m_l, not just $m_l{}^2$, but we are not interested in polarizations in this paper.

To summarize, in the limit of exact SU_3 symmetry we would obtain from Eqs. (10), (12), and (14),

$$\langle p|J_\lambda|\Lambda\rangle = -\frac{G}{\sqrt{2}}\sin\theta\,(\sqrt{\tfrac{3}{2}})\left\{F_1{}^p(t)\gamma_\lambda + F_2{}^p(t)\frac{\sigma_{\lambda\beta}q^\beta}{2M_N} + \frac{1+2x(t)}{3}\tilde{G}_A(t)\gamma_\lambda i\gamma_5\right\} \tag{19}$$

and

$$\langle n|J_\lambda|\Sigma^-\rangle = \frac{G}{\sqrt{2}}\sin\theta\left\{[F_1{}^p(t) + 2F_1{}^n(t)]\gamma_\lambda + \frac{1}{2M_N}[F_2{}^p(t) + 2F_2{}^n(t)]\sigma_{\lambda\beta}q^\beta - (1-2x(t))\tilde{G}_A(t)\gamma_\lambda i\gamma_5\right\}.$$

Note that the Λ current has V/A negative while the Σ^- current has V/A positive.

Some ambiguities result from the broken SU_3 symmetry which corresponds more closely to nature than the exact SU_3 symmetry discussed in this section. For example, the weak magnetism term in Eq. (19) con-

[20] A. W. Martin and K. C. Wali, Phys. Rev. **130**, 2455 (1963); Nuovo Cimento **31**, 1324 (1964).

[21] R. E. Cutkosky, Ann. Phys. (N. Y.) **23**, 415 (1963).

tains a factor $(2M_N)^{-1}$ which can possibly be interpreted in a variety of ways since the eight baryons are not degenerate. We choose to replace $2M_N$ by the sum of the initial and final baryon masses $\Sigma = M_1 + M_2$, although such a choice cannot be made uniquely.

The effect of symmetry breaking upon the form factors is harder to assess. The following seems like the most reasonable alternative. We know that the isovector nucleon electromagnetic form factor is dominated by the ρ meson while the isoscalar form factor is dominated by the ϕ and ω mesons. Similarly the form factors for the hyperon-nucleon current should be dominated by K^* exchange. When normalized to unity for $t=0$, this choice of form factor yields

$$f(t) = M_{K^*}^2/(M_{K^*}^2 - t)\,. \qquad (20)$$

As for the axial-vector form factor, nothing is known about the detailed dependence of $G_A(t)$ even for the nucleon-nucleon current. A calculation of the axial-vector form factor has been made, but it necessarily depends on some scattering amplitudes that are unknown.[22] Presumably the axial-vector form factor falls off from its value at $t=0$ also, so that for the purpose of estimation in this study, it is reasonable to again choose the form factor $f(t)$ in Eq. (20) which implies

$$\tilde{G}_A(t) = 1.25 f(t)\,. \qquad (21)$$

Although $x(t)$ is, in principle, a function of t we will regard it as constant $x(t) \equiv x \simeq 0.25$. Any dependence of $x(t)$ on t is probably a much finer detail than experiments will be able to attack for some time.

In conclusion then, we have the following choices for the six form factors in the current Eq. (7). For $\langle p|J_\lambda|\Lambda\rangle$

$$\begin{aligned} G_V &= -G\sin\theta\,(\sqrt{\tfrac{3}{2}})f(t)\,,\\ G_A &= -1.25G\sin\theta\,(\sqrt{\tfrac{3}{2}})\frac{1+2x}{3}f(t)\,,\\ F_V &= -G\sin\theta\,(\sqrt{\tfrac{3}{2}})\mu_p f(t)\,, \end{aligned} \qquad (22)$$

while $F_A=0$, $H_V=0$ and H_A is ignorable. G is the coupling constant for muon decay. For $\langle n|J_\lambda|\Sigma^-\rangle$

$$\begin{aligned} G_V &= G\sin\theta\, f(t)\,,\\ G_A &= 1.25G\sin\theta\,(1-2x)f(t)\,,\\ F_V &= G\sin\theta\,(\mu_p+2\mu_n)f(t)\,, \end{aligned} \qquad (23)$$

and again F_A, H_V, and H_A are either absent or negligible.

III. FIRST- AND SECOND-CLASS CURRENTS

In this section we discuss the reasons for presuming $F_A = 0 = H_V$. For beta decay Weinberg has discussed the role of G parity in classifying $\Delta S=0$ currents.[16] He

[22] C. H. Woo, Phys. Rev. Letters **12**, 308 (1964).

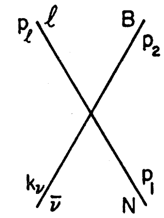

FIG. 1. Basic Feynman diagram for "elastic" neutrino reactions.

defines first-class currents as currents which obey

$$GJ_VG^{-1} = +J_V \qquad (24)$$

for vector currents and

$$GJ_AG^{-1} = -J_A \qquad (25)$$

for axial-vector currents. Second class currents have the opposite signs under the G transformation. For beta decay γ_λ, $\gamma_\lambda i\gamma_5$, $\sigma_{\lambda\beta}q^\beta$ and $q_\lambda i\gamma_5$ are all first class, while q_λ and $\sigma_{\lambda\beta}q^\beta i\gamma_5$ are second class.

The hypothesis is usually made that only first-class currents are coupled in beta decay. There is not much experimental verification of this idea, but the hypothesis is appealing because it points to a possible simplicity of the weak interaction currents. In our scheme the weak current belongs to an octet of currents $J_\lambda{}^i$. The hypothesis of definite G parity [Eqs. (24) and (25)] for the $\Delta S=0$ part of J_λ implies a well-defined behavior of the octet under charge conjugation. For example the neutral members of the octet have $C=-1$ for the vector part, $C=+1$ for the axial part.[23]

In the SU_3 limit these properties have well-defined consequences for $\Delta S=1$ transitions. In particular the form factors F_A and H_V vanish for all baryon-baryon transitions (including also such $\Delta S=0$ transitions as $\Sigma\to\Lambda e\nu$). This conclusion, however, is only true in the limit of exact SU_3.[24]

IV. CROSS SECTIONS

In this section we discuss kinematics, the differential cross section, the hyperon spectrum, and the total cross section.

First some clarification of kinematics will be useful. Our momenta are labeled as in Fig. 1. There is some utility in using Mandelstam variables in the expressions for basic quantities. For example,

$$s \equiv (p_1+k_\nu)^2 \equiv (p_2+p_l)^2 = M_1^2 + 2M_1E_\nu \qquad (26)$$

in terms of the lab neutrino energy E_ν, while

$$\begin{aligned} t = (k_\nu - p_l)^2 = (p_2-p_1)^2 &= m_l^2 - 2E_\nu(E_l - p_l\cos\theta_l)\\ &= M_1^2 + M_2^2 - 2M_1E_2 \end{aligned}$$

[23] The definition of C for an octet of operators has been discussed by N. Cabibbo, Phys. Rev. Letters **12**, 62 (1964), and M. Gell-Mann, *ibid.* **12**, 83 (1964). According to the notation introduced by Gell-Mann, the vector octet has $C=-1$, the axial octet has $C=+1$.

[24] A similar result has been obtained by L. Wolfenstein, Carnegie Institute of Technology (unpublished).

in terms of the lab lepton energy, momentum and angle, and alternatively, in terms of the lab energy of the hyperon E_2. To discuss the angle of production of the heavy particle θ_2 it is convenient to use the variable

$$u\equiv M_1^2+M_2^2+m_l^2-s-t\equiv(p_l-p_1)^2\equiv(k_\nu-p_2)^2$$
$$=M_2^2-2E_\nu(E_2-p_2\cos\theta_2). \quad (27)$$

The differential cross section is most conveniently expressed as $d\sigma/dt$ and then the particular $d\sigma/d\cos\theta_l$, $d\sigma/d\cos\theta_2$ and $d\sigma/dE_2$ derived from $d\sigma/dt$.

In the formula for $d\sigma/dt$ below not only have the axial magnetism-induced scalar and pseudoscalar terms been dropped for the reasons mentioned earlier but also those other V and A contributions which have an over-all factor of m_l^2. Again these contributions are about 1%. This abbreviated formula for $d\sigma/dt$ is then[9]

$$\frac{d\sigma}{dt}=\frac{1}{32\pi k_{Bi}^2 s}\Bigg\{\left[G_V^2+G_A^2-\frac{F_V^2 t}{\Sigma^2}\right]$$
$$\times\left((j\cdot j')^2-\Sigma^2\Delta^2-(m_l^2-t)(\Sigma^2+\Delta^2-t)\right)$$
$$+2\left[(G_V+F_V)^2(\Delta^2-t)+G_A^2(\Sigma^2-t)\right](m_l^2-t)\Big]$$
$$-4(G_V+F_V)G_A\left[j\cdot j' t+m_l^2\Sigma\Delta\right]\Bigg\}, \quad (28)$$

where the initial c.m. momentum $k_{Bi}=\frac{1}{2}(s-M_1^2)s^{-1/2}$ and $j\cdot j'=(k_\nu+p_l)\cdot(p_1+p_2)=2s-M_1^2-M_2^2-m_l^2+t$. The quantity $j\cdot j'$, which is like a spinless charge-current interaction, is the largest quantity present for a wide range of circumstances.

The various lab differential cross sections can be obtained in terms of $d\sigma/dt$ by taking the appropriate Jacobian. The results are

$$\frac{d\sigma}{d\cos\theta_l}=\left|\frac{4M_1E_\nu p_l^3}{E_l(2M_1E_\nu+M_1^2-M_2^2+m_l^2)-2(E_\nu+M_1)m_l^2}\right|\frac{d\sigma}{dt},$$
$$\frac{d\sigma}{d\cos\theta_2}=\left|\frac{4M_1E_\nu p_2^3}{2(E_\nu+M_1)M_2^2-E_2(2M_1E_\nu+M_1^2+M_2^2-m_l^2)}\right|\frac{d\sigma}{dt}, \quad (29)$$

and

$$d\sigma/dE_2=2M_1(d\sigma/dt). \quad (30)$$

A mixed notation has been used for convenience; E_ν, E_l, and E_2 could be expressed in terms of s and t if it were desirable.

For any particular model of the form factors, the total cross section can be obtained by integration of the $d\sigma/dt$. Since low-energy experiments cannot distinguish cases (ii) and (iii) above, we give asymptotic formulas for both models.

Model (ii) has $f(t)=(1-t/b^2)^{-1}$ and the result is

$$\sigma\rightarrow\frac{b^2k_{Bf}}{2\pi k_{Bi}}\left[G_V^2(0)+G_A^2(0)+\frac{F_V^2(0)b^2}{\Sigma^2}\times\left(\ln\frac{4k_{Bi}k_{Bf}}{b^2}-2\right)\right]. \quad (31)$$

Model (iii) has $F(t)=(1-t/a^2)^{-2}$. The result is

$$\sigma\rightarrow\frac{a^2k_{Bf}}{6\pi k_{Bi}}\left[G_V^2(0)+G_A^2(0)+\frac{1}{2}\frac{F_V^2(0)a^2}{\Sigma^2}\right]. \quad (32)$$

In each equation above

$$\Sigma=M_1+M_2,\quad k_{Bi}=\tfrac{1}{2}(s-M_1^2)s^{-1/2},$$

and

$$k_{Bf}=\tfrac{1}{2}(s-(M_2+m_l)^2)^{1/2}(s-(M_2-m_l)^2)^{1/2}s^{-1/2}.$$

The modification required by a^2 or b^2 being different for G_V, G_A, and F_V is evident. These asymptotic formulas are really the first terms in an expansion in the variables $a^2/t_{\max}\approx a^2/4k_{Bi}k_{Bf}$. The next term in the expansion contains the VA interference contribution. This contribution is positive for V/A negative and negative for V/A positive. Thus the total cross section rises faster with energy for V/A negative than for V/A positive and this fact may be useful in distinguishing between the two. Since our SU_3 model predicts a positive (though small) V/A for Σ^- instead of the negative V/A encountered in β decay, the question of V/A positive versus V/A negative is of considerable interest.

We see from the above asymptotic formulas that the total cross section is also sensitive to two features of

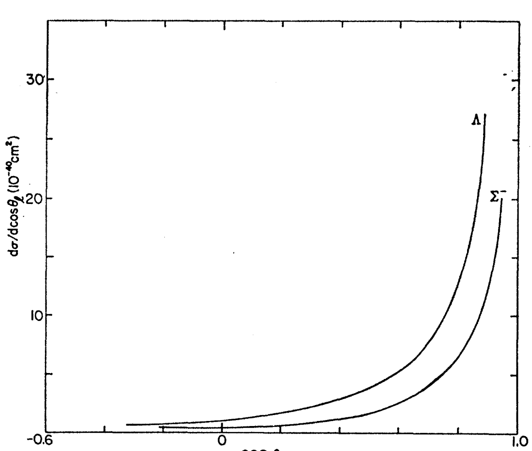

FIG. 2. Cross section differential in the lab lepton angle for $E_\nu=2$ BeV.

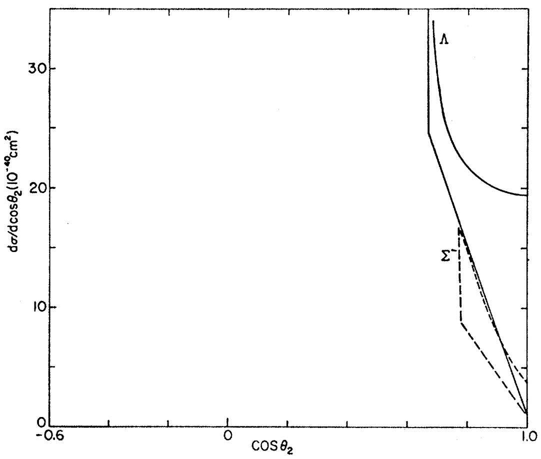

FIG. 3. Cross section differential in the lab baryon angle for $E_\nu=2$ BeV.

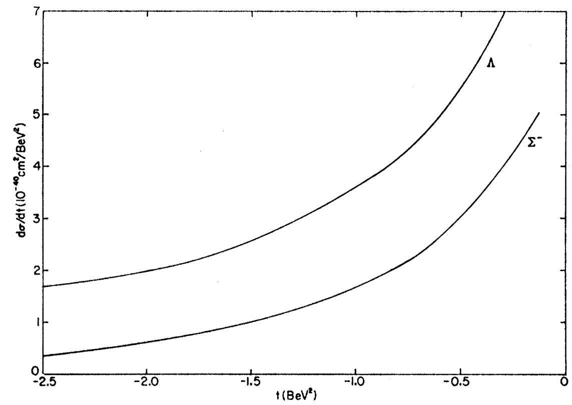

FIG. 4. $d\sigma/dt$, the recoil baryon spectrum, for $E_\nu=2$ BeV.

the form factors, the average mass a^2 or b^2 and the asymptotic behavior of the form factors for large t.

V. RESULTS

Numerical examples were computed using the Stanford 7090. The cross section, differential in lepton and baryon lab angles and differential in t (i.e., the baryon spectrum) were computed for five energies, $E_\nu=0.5$, 1.0, 2.0, 5.0, and 10.0 BeV. The total cross section was also computed. This was done for both Λ and Σ^- production. The sensitivity of the results to the f/d ratio was examined by redoing the calculation for several neighboring values of the f/d ratio. Qualitatively, the results were not sensitive to small changes in the f/d ratio.

Our principle motivation, beyond being able to make detailed quantitative predictions, was to see if there was a way to distinguish qualitatively or semiquantitatively the main features of the SU_3 model, such as the $V/A>0$ prediction for Σ^-.

In the figures below we present differential cross sections for $E_\nu=2$ BeV only. The qualitative features of the differential cross sections can all be clearly seen by looking at this set of examples.

Figure 2 presents the cross section differential in the lab lepton angle ($E_\nu=2$ BeV) for both Λ and Σ^- production; the general feature of the angular distribution is a substantial forward peaking. This forward peaking is primarily a consequence of the monotonic decrease of the form factors. We see no obvious qualitative difference in $d\sigma/d\cos\theta_l$ for Λ and Σ^- although the quantitative predictions are different. For increasing E_ν the forward peaking of $d\sigma/d\cos\theta_l$ becomes sharper.

Figure 3 shows the cross section differential in the baryon lab angle ($E_\nu=2$ BeV) for Λ and Σ^- production. It has been plotted in a double-valued fashion to distinguish the two different baryon momenta in the lab which occur at each baryon angle in the lab. If you do not distinguish momenta, then $d\sigma/d\cos\theta_l$ is the sum of the two curves. This doubling corresponds to the fact that the baryon goes forward in the lab for both forward and backward leptons. Again there is no obvious qualitative difference beyond that due to kinematics. With increasing E_ν the maximum angle slowly becomes larger.

These distributions in the heavy-particle angle have one feature that may be of some technical value. At any particular energy the maximum baryon angle is fairly sensitive to the mass of the baryon. Further, the cross sections increase rapidly in the vicinity of the maximum angle. This may permit one to discriminate between the masses of the recoil baryons and to construct an uncontaminated sample of neutrino events. For example, if $E_\nu=2$ BeV, then roughly 30% of the Λ events fall at larger angles than would be possible for Σ^0. The same idea could be applied to nucleon-N^* discrimination to obtain an uncontaminated sample of elastic events. In this case the mass difference is larger so the difference of maximum angles would also be larger.

Figure 4 shows the baryon spectra $d\sigma/dt$ for Λ and

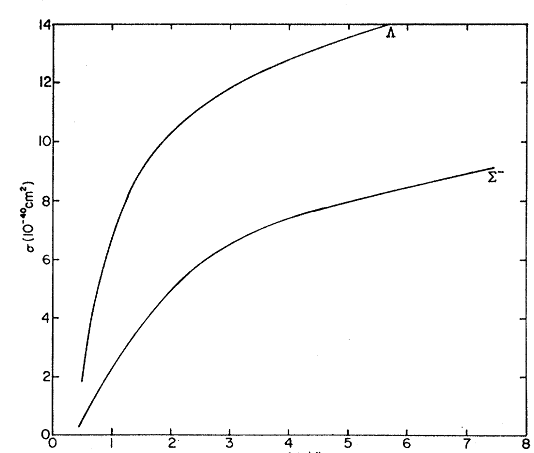

FIG. 5. The total cross section.

Σ^-. Qualitatively there is very little difference between the Λ and Σ spectra. At higher energies the spectra peak more sharply.

Figure 5 presents the total cross sections for Λ and Σ^-. The main qualitative difference, typical of $V-A$ versus $V+A$ is the much more rapid rise for Λ production. This is a possible way to verify that in the Σ^- interaction V/A is positive.

It is also of some interest to use our asymptotic formulas to make estimates of the hyperon to nucleon ratios. If we use Eq. (32) and choose $a^2/2=M_\rho^2$ or $M_{K^*}^2$, then the results are

$$\sigma_\Lambda/\sigma_N \to 0.078 \approx 1/13,$$

and

$$\sigma_\Sigma/\sigma_N \to 0.055 \approx 1/18.$$

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Rates of Nuclear Reactions in Solid-Like Stars*

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In stellar matter as cool and dense as the interior of a white dwarf, the Coulomb energies between neighboring nuclei are large compared to the kinetic energies of the nuclei. Each nucleus is constrained to vibrate about an equilibrium position, and the motion of the nuclei in the interior of a white dwarf is similar to the motion of the atoms in a solid or liquid. We propose a solid-state method for calculating the rate at which a nuclear reaction proceeds between two identical nuclei oscillating about adjacent lattice sites. An effective potential $U(\mathbf{r})$ derived by analyzing small lattice vibrations is used to represent the influence of the Coulomb fields of the lattice on the motion of the two reacting nuclei. The wave function describing the relative motion of the two reacting particles is obtained by solving the Schrödinger equation containing the effective potential $U(\mathbf{r})$. From this wave function, we derive an expression for the reaction rate. The rates of the $p+p$ and $C^{12}+C^{12}$ reactions calculated using this solid-state method are typically 1 to 10 orders of magnitude smaller than those calculated by the method previously suggested by Cameron.

I. INTRODUCTION

THE motions of nuclei in the interiors of cool, dense stars resemble the motions of atoms in solids or liquids. The mean free path between collisions suffered by a given nucleus is much smaller than the average distance between nuclei and may be comparable to the particle's quantum-mechanical wavelength. Each nucleus is therefore forced to oscillate about a fixed position in a lattice structure.[1]

Reactions between charged particles in stars are inhibited by the small probability of penetrating the Coulomb barrier between nuclei. However, the probability of penetrating the barrier increases rapidly with the energies of the colliding particles. In most stars, the effective energies are due primarily to thermal motions. In stars as cold as white dwarfs, the thermal energies alone are too small to allow charged particles to react at significant rates. However, the Coulomb potential of the lattice combined with the ground-state vibrational energy of the reacting nuclei can, at high densities, enable nuclei at adjacent lattice sites to react rapidly even at zero temperature.

It is important that one be able to calculate the rates of reactions occurring at high densities and low temperatures, reactions to which Cameron[2] has applied the name "pycnonuclear." Cameron has suggested that such reactions might be the source of energy for nova explosions. A knowledge of the rates of pycnonuclear reactions would also be useful in mathematical studies of white dwarfs. From the rates of reactions at high densities, one can infer certain limitations on the possible compositions of the interiors and envelopes of white-dwarf stars, compositions which would otherwise be completely unknown.[3] Any future attempts to evolve stellar models into the white-dwarf state from higher temperature configurations will also require detailed knowledge of pycnonuclear reaction rates.

In this paper we develop a method for finding the rate at which nuclear reactions proceed between particles vibrating about adjacent lattice sites. For reactions between particles with $Z \geq 2$, the solid-state approach applies to the temperatures and densities in region I of Fig. 1. Figure 1 also shows typical central temperatures and densities for various types of stars.

We consider primarily reactions in a lattice of identical nuclei, although we do suggest a rough model for

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