Hypercharge Conservation, CP Invariance and the Possible Existence of a Zero-Mass Zero-Spin Field*

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It is shown that by introducing a neutral zero-mass zero-spin field ϕ, the Lagrangian density of all interactions (including weak interactions) can be made invariant under the hypercharge gauge transformation. Consequently, there exists a hypercharge current density J_μ that is absolutely conserved. The current density J_μ is related to the usual hypercharge current density j_μ of all the presently known particles by

$$J_\mu = j_\mu - \lambda^{-1}(\partial\phi/\partial x_\mu),$$

where λ is a coupling parameter. The same Lagrangian density is invariant under CP transformation, time-reversal transformation, and Lorentz transformation. It turns out that if the conserved quantity $\int J_4 d^3r \neq 0$, then there exists an energy difference between any hypercharged particle and its antiparticle with the same momentum. Such an energy difference would induce decays such as $K_2^0 \to 2\pi$, and the decay rate is proportional to the square of the K-meson energy.

1. INTRODUCTION

It is well-known that the strong and electromagnetic interactions conserve the hypercharge Y and are invariant under the hypercharge gauge transformation

$$\psi_a(\mathbf{r},t) \to e^{iY_a\theta}\psi_a(\mathbf{r},t)\,, \qquad (1)$$

where θ is an arbitrary constant, $Y_a(=0, \text{ or } \pm 1, \cdots)$ is the hypercharge of the particle a, and ψ_a is its field operator. The weak interaction allows processes such as

$$\Lambda^0 \to p + \pi^-, \text{ etc.} \qquad (2)$$

Since $Y_\Lambda = Y_\pi = 0$ and $Y_p = 1$, it follows that reaction (2) violates Y conservation, and thus the weak interaction is usually regarded to be noninvariant under the hypercharge gauge transformation.

Recently, Christenson, Cronin, Fitch, and Turlay[1] observed that the long-lived component of the neutral K-meson, the K_2^0, decays into two π mesons

$$K_2^0 \to \pi^+ + \pi^- \qquad (3)$$

which apparently indicates that CP is also not conserved in the weak interaction. The CP invariance is connected with the transformation

$$\psi_a(\mathbf{r},t) \to \eta_a\bar{\psi}_a(-\mathbf{r},t)\,, \qquad (4)$$

where η_a is a phase factor and $\bar{\psi}_a$ is the field operator for the antiparticle of a. The experiment by Christenson *et al.* seems to imply that the weak interaction might not be invariant under the transformation (4).

The purpose of this paper is to point out that by introducing a neutral field, ϕ which is associated with a zero-rest mass and zero-spin particle, it is possible to preserve both the hypercharge gauge invariance and the CP invariance for the weak interaction, and at the same time allow all the observed weak reactions such as (2) and (3).

To see how this can be achieved we may consider, for example, reaction (2) and represent the relevant part of the usual weak-interaction Lagrangian density by (in the absence of ϕ)

$$G[\bar{p}\Lambda\pi] + \text{H.c.}, \qquad (5)$$

where G is the appropriate weak-coupling constant and

$$[\bar{p}\Lambda\pi] = \psi_p^\dagger\gamma_4\gamma_\mu(1+b\gamma_5)\psi_\Lambda\frac{\partial\psi_\pi}{\partial x_\mu};$$

the $\gamma_1, \gamma_2, \cdots \gamma_5$ are the usual Dirac matrices, b = constant, and the dagger denotes Hermitian conjugation. The expression (5) is not invariant under the hypercharge gauge transformation. In order to maintain the gauge invariance, we replace (5) by

$$G[\bar{p}\Lambda\pi]\exp(-i\lambda\phi) + \text{H.c.} \qquad (6)$$

and assume that under the hypercharge gauge transformation (1), the field ϕ transforms according to

$$\phi \to \phi - \lambda^{-1}\theta\,, \qquad (7)$$

where the parameter λ is real and has the dimension of a length. The gauge invariance requires that ϕ have zero mass. Under the CP transformation, we assume that

$$\phi(\mathbf{r},t) \to -\phi(-\mathbf{r},t)\,. \qquad (8)$$

The new Lagrangian density (6) is, then, invariant under the Lorentz transformation, the hypercharge gauge transformation, and the CP transformation.

As a consequence of the hypercharge gauge invariance, there exists a conserved hypercharge current density

$$J_\mu \equiv j_\mu - \lambda^{-1}(\partial\phi/\partial x_\mu) \qquad (9)$$

which satisfies the conservation law

$$(\partial J_\mu/\partial x_\mu) = 0\,, \qquad (10)$$

where j_μ is the usual hypercharge current density due to all the presently known particles. For example, in

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[1] J. H. Christenson, J. W. Cronin, V. L. Fitch, and R. Turlay, Phys. Rev. Letters 13, 138 (1964).

the case of fermions such as p, n, Ξ, etc., we have

$$j_\mu = i \sum_a Y_a \psi_a^\dagger \gamma_4 \gamma_\mu \psi_a. \tag{11}$$

Equation (10) implies that

$$\int J_4 d^3r \equiv iQ \tag{12}$$

is absolutely conserved, and its value is determined by the initial condition of the system. Decays such as $\Lambda^0 \to p + \pi^-$ do not violate the conservation of hypercharge current J_μ, provided we also include in the definition of J_μ the contribution of the ϕ field.

It will be shown that if the system is in any state with $Q \neq 0$, there must exist a difference of energy between any hypercharge particle and its antiparticle of the same momentum. For K^0 and $\bar{K}^0$, this energy difference is given by

$$V = -2\lambda^2(Q/\Omega), \tag{13}$$

where Ω is the volume of the system and is, presumably, the same as the volume of the "universe." Under CP transformation, J_4 changes sign. The CP invariance implies that, for every eigenstate (of the entire system) with $Q = Q_0$, there exists another eigenstate of the same energy but with $Q = -Q_0$. The existence of this energy difference V between K^0 and $\bar{K}^0$ in a given state of the system, therefore, does not contradict the requirement of CP invariance.

It has been pointed out[2] that, independent of the precise nature of the mechanism, if such an energy difference V exists between K^0 and $\bar{K}^0$ then the decay $K^0 \to 2\pi$ can occur, and its rate is proportional to

$$|\epsilon|^2, \tag{14}$$

where ($\hbar = c = 1$)

$$\epsilon \cong \tfrac{1}{2}\gamma V[(m_1 - m_2) - i\tfrac{1}{2}(\Gamma_1 - \Gamma_2)]^{-1}; \tag{15}$$

m_i, Γ_i are the rest mass and the decay width of K_i^0 ($i = 1, 2$),

$$\gamma = (1 - v^2)^{-1/2}, \tag{16}$$

and v is the velocity of K_2^0 with respect to the rest system of the "universe." Since in the present theory, the energy difference V is independent of γ, the observed rate for the 2π decay of K_2^0 must be proportional to γ^2.

Identical considerations can, of course, be applied to other quantities such as strangeness, or I_z. The hypercharge is used only as a representative of any such quantities that could be conserved by introducing a zero-mass and zero-spin field. Throughout this paper we follow the method of canonical formalism of quantum-field theory. The effect of general relativity is not discussed.

[2] J. Bernstein, N. Cabibbo, and T. D. Lee, Phys. Letters 12, 146 (1964).

2. LAGRANGIAN AND HAMILTONIAN

In general, we can write the usual weak-interaction Lagrangian density in the absence of the ϕ field as

$$L_{\text{weak}}(0) = G \sum L_{\Delta Y}, \tag{17}$$

where the subscript ΔY denotes the amount of hypercharge violation and the sum extends over $\Delta Y = 0$ and ± 1. The weak-interaction Lagrangian density in the presence of the ϕ field is given by

$$L_{\text{weak}} = G \sum (L_{\Delta Y}) \exp[-i\lambda(\Delta Y)\phi]. \tag{18}$$

The Lagrangian density of the entire system can be written as

$$L = L_\phi + L_\psi + L_{\text{int}}, \tag{19}$$

in which the free-field Lagrangian density is given by L_ϕ and L_ψ, where

$$L_\phi = -\tfrac{1}{2}(\partial\phi/\partial x_\mu)^2, \tag{20}$$

$$L_\psi = -\sum_a \psi_a^\dagger \gamma_4(\gamma_\mu(\partial/\partial x_\mu) + m_a)\psi_a, \tag{21}$$

and the interaction Lagrangian density is given by

$$L_{\text{int}} = L_{\text{strong}} + L_{\text{elect}} + L_{\text{weak}}, \tag{22}$$

where L_{strong} and L_{elect} are, respectively, the Lagrangian density for the strong interaction and the electromagnetic interaction, and L_{weak} is given by Eq. (18). For simplicity, we include only the expression for the free Fermions in Eq. (21). All repeated indices are to be summed over.

The Lagrangian density L is invariant under the Lorentz transformation, the time-reversal transformation, the CP transformation and the hypercharge gauge transformation. By using L, the equation of motion is found to be

$$(\partial^2\phi/\partial x_\mu^2) = -(\partial/\partial\phi)L_{\text{weak}} \tag{23}$$

and

$$\gamma_4\left(\gamma_\mu \frac{\partial}{\partial x_\mu} + m_a\right)\psi_a = \frac{\partial}{\partial \psi_a^\dagger} L_{\text{int}} - \frac{\partial}{\partial x_\mu}\left[\frac{\partial}{\partial \psi_{a,\mu}^\dagger} L_{\text{int}}\right], \tag{24}$$

where

$$\psi_{a,\mu} = (\partial/\partial x_\mu)\psi_a \quad \text{and} \quad \psi_{a,\mu}^\dagger = (\partial/\partial x_\mu)\psi_a^\dagger.$$

The hypercharge current density J_μ is related to the Lagrangian density by

$$J_\mu = -i \sum_a Y_a\left[\frac{\partial L}{\partial \psi_{a,\mu}}\psi_a - \psi_a^\dagger \frac{\partial L}{\partial \psi_{a,\mu}^\dagger}\right] + \lambda^{-1}\frac{\partial L}{\partial \phi_{,\mu}}, \tag{25}$$

where

$$\phi_{,\mu} = \partial\phi/\partial x_\mu.$$

From the equations of motion and the gauge-invariance property of L, the conservation law [Eq. (10)]

$$\partial J_\mu/\partial x_\mu = 0$$

follows. An alternative form of J_μ is given by Eq. (9)

$$J_\mu = j_\mu - \lambda^{-1}(\partial\phi/\partial x_\mu),$$

where

$$j_\mu = -i\sum_a Y_a\left[\frac{\partial L}{\partial\psi_{a,\mu}}\psi_a - \psi_a^\dagger\frac{\partial L}{\partial\psi_{a,\mu}^\dagger}\right]. \quad (26)$$

The usual canonical formalism can be directly applied to the present problem. The conjugate momentum of the ϕ field is given by

$$\pi = \partial\phi/\partial t. \quad (27)$$

The corresponding Hamiltonian density can be written as

$$H = H_\phi + H_\psi + H_{\text{int}}, \quad (28)$$

where

$$H_\phi = \tfrac{1}{2}\pi^2 + \tfrac{1}{2}(\nabla\phi)^2, \quad (29)$$

$$H_\psi = \sum_a \psi_a^\dagger\left[\boldsymbol{\alpha}\cdot\frac{1}{i}\nabla + \beta m_a\right]\psi_a, \quad (30)$$

and

$$H_{\text{int}} = H_{\text{strong}} + H_{\text{elect}} + H_{\text{weak}}. \quad (31)$$

The matrices $\boldsymbol{\alpha}$ and β are the usual Dirac matrices. The H_{strong}, H_{elect}, and H_{weak} describe, respectively, the strong, the electromagnetic, and the weak interaction which includes the modification due to the ϕ field.

For simplicity, we have assumed in Eq. (30) that except for ϕ all other fields are Fermion fields. To simplify further our discussions, we consider, in the following, only the case that L_{int} does not contain derivatives of ψ_a. (All our conclusions can, of course, be applied to any other case.) Thus,

$$H_{\text{weak}} = -L_{\text{weak}} \quad (32)$$

and j_μ is given by Eq. (11). Both π and ϕ are Hermitian operators which satisfy

$$[\pi(\mathbf{r},t),\phi(\mathbf{r}',t)] = -i\delta^3(\mathbf{r}-\mathbf{r}'), \quad (33)$$

and the ψ_a obeys the usual anticommutation relation.

3. A CANONICAL TRANSFORMATION

To exhibit more explicitly the constant of motion implied by the conservation of hypercharge current, we consider the canonical transformation[3,4] generated by the unitary operator

$$U \equiv \exp\left[i\lambda\int\phi\rho d^3r\right], \quad (34)$$

where

$$\rho = -ij_4 = \sum_a Y_a\psi_a^\dagger\psi_a \quad (35)$$

and is the hypercharge density of the ψ_a's. Under the canonical transformation,

$$U\psi_a U^\dagger = \psi_a \exp(-i\lambda Y_a\phi), \quad (36)$$

$$U\phi U^\dagger = \phi, \quad (37)$$

and

$$U\pi U^\dagger = \pi - \lambda\rho. \quad (38)$$

The Hamiltonian density is transformed into

$$H_U = UHU^\dagger = H_\phi + H_\psi + H_{\text{strong}} + H_{\text{elect}} + H_{\text{weak}}(0) - \lambda[\rho\pi + \mathbf{j}\cdot\nabla\phi] + \tfrac{1}{2}\lambda^2\rho^2, \quad (39)$$

where $\mathbf{j}$ is the spatial part of the 4-vector j_μ, the terms H_ϕ, H_ψ, H_{strong}, and H_{elect} are the same as that given by Eqs. (29)–(31), but $H_{\text{weak}}(0)$ is related to Eq. (17) by

$$H_{\text{weak}}(0) = -L_{\text{weak}}(0), \quad (40)$$

and is independent of ϕ. In H_U, the coupling between ϕ and ψ_a is of a derivative type with λ as the coupling constant.

To understand the meaning of conservation of hypercharge in this new representation, it is convenient to consider the Fourier expansions of ϕ and π in a volume Ω:

$$\phi(\mathbf{r}) = \Omega^{-1/2}[\phi_0 + \sum_{\mathbf{k}\neq 0}\phi_{\mathbf{k}}\exp(i\mathbf{k}\cdot\mathbf{r})] \quad (41)$$

and

$$\pi(\mathbf{r}) = \Omega^{-1/2}[\pi_0 + \sum_{\mathbf{k}\neq 0}\pi_{\mathbf{k}}\exp(-i\mathbf{k}\cdot\mathbf{r})], \quad (42)$$

where

$$\pi_{\mathbf{k}} = \pi_{-\mathbf{k}}^\dagger, \quad \phi_{\mathbf{k}} = \phi_{-\mathbf{k}}^\dagger,$$

and the commutation relations

$$[\pi_{\mathbf{k}},\phi_{\mathbf{k}'}] = -i\delta_{\mathbf{k}\mathbf{k}'}$$

are satisfied for all $\mathbf{k}$ and $\mathbf{k}'$. The transformed Hamiltonian H_U is independent of ϕ_0. Thus,

$$d\pi_0/dt = 0. \quad (43)$$

The conservation of hypercharge, after the canonical transformation, becomes simply

$$\pi_0 = \text{constant}, \quad (44)$$

which is a quantum number characterizing the particular state of the system. Under CP transformation, any state with $\pi_0 \neq 0$ is transformed into another state which has a π_0 of the opposite sign but with the same magnitude. The π_0 is related to the Q, introduced in Eq. (12), by $\pi_0 = \Omega^{-1/2}(\lambda Q)$.

It is important to notice that the hypercharge gauge transformation becomes a totally trivial operation after the canonical transformation. This may at first sight seem rather strange, but it is actually a general feature of such gauge invariance. In the Appendix, we give a simple example to illustrate further the same property.

4. APPARENT CP NONINVARIANCE

In this section we discuss the various consequences for the state which has a $\pi_0 \neq 0$. From Eq. (39), we

[3] Similar canonical transformations have been used by F. J. Dyson, Phys. Rev. **73**, 929 (1948), and R. J. Glauber, *ibid.* **84**, 395 (1951).

[4] The present considerations are, however, suggested by some results on the (physically unrelated) polaron problem by T. D. Lee, F. Low, and D. Pines, Phys. Rev. **90**, 297 (1953). See the discussions given in the Appendix.

notice that, after the canonical transformation, the Hamiltonian can be written as

$$\int H_U d^3r = -\Omega^{-1/2}\lambda\pi_0 \int \rho d^3r + \int H' d^3r, \qquad (45)$$

where

$$H' = H_\phi + H_\psi + H_{\text{strong}} + H_{\text{elect}} + H_{\text{weak}}(0) + H_1 \qquad (46)$$

and

$$H_1 = -\lambda[\rho\pi_1 + \mathbf{j}\cdot\nabla\phi_1] + \tfrac{1}{2}\lambda^2\rho^2. \qquad (47)$$

In Eq. (47), the operators ϕ_1 and π_1 are defined to be

$$\phi_1 = \sum_{\mathbf{k}\neq 0} \Omega^{-1/2}\phi_{\mathbf{k}} \exp(i\mathbf{k}\cdot\mathbf{r})$$

and

$$\pi_1 = \sum_{\mathbf{k}\neq 0} \Omega^{-1/2}\pi_{\mathbf{k}} \exp(-i\mathbf{k}\cdot\mathbf{r}). \qquad (48)$$

The first term on the right-hand side of Eq. (45) shows that there is an energy difference V between any hypercharge particle and its antiparticle of the same momentum. For particles with $Y=1$, this potential energy difference is given by

$$V = -2\Omega^{-1/2}\lambda\pi_0. \qquad (49)$$

As mentioned in the Introduction, the existence of such an energy difference implies that

$$K_2^0 \to 2\pi$$

can occur and the analysis made in Ref. 2 is applicable. In the following, we list the various consequences for the weak decays. Most of these results have already been stated in Ref. 2.

(i). In a vacuum (i.e., in the absence of any neighboring matter), the two states $|K_1^0\rangle$ and $|K_2^0\rangle$, each of which has a single lifetime, are related to $|K^0\rangle$ and its CP conjugate state $|\bar{K}^0\rangle$ by

$$|K_1^0\rangle = [2(1+|\epsilon|^2)]^{-1} \times[(1+\epsilon)|K^0\rangle + (1-\epsilon)|\bar{K}^0\rangle]e^{-i\lambda_1 t} \qquad (50)$$

and

$$|K_2^0\rangle = [2(1+|\epsilon|^2)]^{-1} \times[(1-\epsilon)|K^0\rangle - (1+\epsilon)|\bar{K}^0\rangle]e^{-i\lambda_2 t}, \qquad (51)$$

where $|K^0\rangle$ and $|\bar{K}^0\rangle$ are time-independent,

$$\lambda_1 = \eta + (\xi^2+\zeta^2)^{1/2}, \qquad (52)$$

$$\lambda_2 = \eta - (\xi^2+\zeta^2)^{1/2}, \qquad (53)$$

$$\epsilon = \zeta^{-1}[-\xi + (\xi^2+\zeta^2)^{1/2}], \qquad (54)$$

$$\eta = (2\gamma)^{-1}[(m_1^0+m_2^0) - i\tfrac{1}{2}(\Gamma_1^0+\Gamma_2^0)], \qquad (55)$$

$$\xi = (2\gamma)^{-1}[(m_1^0-m_2^0) - i\tfrac{1}{2}(\Gamma_1^0-\Gamma_2^0)], \qquad (56)$$

and

$$\zeta = \tfrac{1}{2}V. \qquad (57)$$

In Eqs. (55) and (56), the m_1^0, Γ_1^0 and m_2^0, Γ_2^0 are the mass and width of K_1^0 and K_2^0 in the absence of V, and γ is given by Eq. (16). Since $V\neq 0$, the actual mass and width of K_1^0 and K_2^0 are determined by λ_1 and λ_2. We may define

$$\lambda_j \equiv \gamma^{-1}[m_j - i\tfrac{1}{2}\Gamma_j], \qquad (58)$$

where $j=1, 2$. The "observed mass" m_1, m_2 and the "observed width" Γ_1 and Γ_2 depend on γ. If $|\epsilon|\ll 1$ (which corresponds to $\gamma \ll 10^3$), then

$$(m_1 - i\tfrac{1}{2}\Gamma_1) \cong (m_1^0 - i\tfrac{1}{2}\Gamma_1^0) + \tfrac{1}{4}\gamma^2 V^2[\Delta m^0 - i\tfrac{1}{2}\Delta\Gamma^0]^{-1} \qquad (59)$$

and

$$(m_2 - i\tfrac{1}{2}\Gamma_2) \cong (m_2^0 - i\tfrac{1}{2}\Gamma_2^0) - \tfrac{1}{4}\gamma^2 V^2[\Delta m^0 - i\tfrac{1}{2}\Delta\Gamma^0]^{-1}, \qquad (60)$$

where

$$\Delta m^0 = m_1^0 - m_2^0, \qquad (61)$$

and

$$\Delta\Gamma^0 = \Gamma_1^0 - \Gamma_2^0. \qquad (62)$$

For large values of γ ($\gg 10^3$),

$$\lambda_1 \cong \tfrac{1}{2}V + \frac{1}{2\gamma}[(m_1^0+m_2^0) - i\tfrac{1}{2}(\Gamma_1^0+\Gamma_2^0)] \qquad (63)$$

and

$$\lambda_2 \cong -\tfrac{1}{2}V + \frac{1}{2\gamma}[(m_1^0+m_2^0) - i\tfrac{1}{2}(\Gamma_1^0+\Gamma_2^0)]. \qquad (64)$$

Thus, the lifetimes of K_1^0 and K_2^0 become the same at the extremely high-energy limit.

(ii) From Eqs. (50), (51), and the CP invariance property of the theory, we find (in vacuum)

$$\frac{\text{Rate}(K_2^0 \to \pi^+ + \pi^-)}{\text{Rate}(K_1^0 \to \pi^+ + \pi^-)} = \frac{\text{Rate}(K_2^0 \to \pi^0 + \pi^0)}{\text{Rate}(K_1^0 \to \pi^0 + \pi^0)} = |\epsilon|^2. \qquad (65)$$

The result of Christenson *et al.*[1] shows that at $\gamma \cong 2$, $|\epsilon| \cong 2\times 10^{-3}$. This corresponds to a

$$|V| \cong 2\times 10^{-8} \text{ eV} \qquad (66)$$

For $\gamma \ll 10^3$, the parameter $|\epsilon|^2$ varies according to γ^2; for $\gamma \gg 10^3$, the parameter $|\epsilon|^2 \cong 1$.

For the 3π decay mode, if we neglect the high angular momentum states in the final $(\pi^- + \pi^+ + \pi^0)$ system, then

$$\frac{\text{Rate}(K_1^0 \to \pi^+ + \pi^- + \pi^0)}{\text{Rate}(K_2^0 \to \pi^+ + \pi^- + \pi^0)} = \frac{\text{Rate}(K_1^0 \to \pi^0 + \pi^0 + \pi^0)}{\text{Rate}(K_2^0 \to \pi^0 + \pi^0 + \pi^0)} = |\epsilon|^2. \qquad (67)$$

If we assume the $\Delta Q = \Delta S$ rule for the leptonic decay modes, then

$$\text{Rate}(K_1^0 \to \pi^- + l^+ + \nu_l) = \text{Rate}(K_2^0 \to \pi^+ + l^- + \bar{\nu}_l), \qquad (68)$$

$$\text{Rate}(K_1^0 \to \pi^+ + l^- + \bar{\nu}_l) = \text{Rate}(K_2^0 \to \pi^- + l^+ + \nu_l), \qquad (69)$$

and

$$\frac{\text{Rate}(K_1^0 \to \pi^- + l^+ + \nu_l)}{\text{Rate}(K_1^0 \to \pi^+ + l^- + \bar{\nu}_l)} = \left|\frac{1+\epsilon}{1-\epsilon}\right|^2, \tag{70}$$

where $l = e$ or μ.

(iii) All above results [Eqs. (50)–(54) and (65)–(70)] are applicable to K_1^0 and K_2^0 in a medium, provided we replace Eqs. (55)–(57) by

$$\eta = (2\gamma)^{-1}[(m_1^0 + m_2^0) - i\tfrac{1}{2}(\Gamma_1^0 + \Gamma_2^0)] - \tfrac{1}{2}(n + n' - 2)kv, \tag{71}$$

$$\xi = (2\gamma)^{-1}[(m_1^0 - m_2^0) - i\tfrac{1}{2}(\Gamma_1^0 - \Gamma_2^0)], \tag{72}$$

and

$$\zeta = \tfrac{1}{2}[V - (n - n')kv], \tag{73}$$

where k is the momentum of the K meson and n, n' are, respectively the complex index of refraction of K^0 and $\bar{K}^0$ in the medium.

The sign of V can be determined either by observing the interference term between the decay of K_1^0 with that of K_2^0 in vacuum, or by studying the decay rates of K_1^0 or K_2^0 in a medium.

In all above formulas, the γ is measured with respect to the rest system of the entire volume Ω. On the other hand, we can also measure the value of γ with respect to earth by direct means. Combining these two measurements, it would become possible to measure the "absolute" velocity of the earth.

(iv) In the decays of all other particles, no apparent CP noninvariant term can be observed if an energy difference of the order of 10^{-8} eV between the particle and its antiparticle can be neglected.

5. EMISSION AND ABSORPTION OF ϕ QUANTA

From Eq. (47) we find that the coupling between ϕ and ψ_a is of a derivative type and that the coupling parameter, λ has the dimension of a length. It is convenient to introduce a dimensionless coupling constant, f:

$$f = \lambda m_N, \tag{74}$$

where m_N = mass of the nucleon. To study the emission and absorption processes of ϕ quanta with $\mathbf{k} \neq 0$, we briefly describe the rules for Feynman graphs.

Let us start with H_U [Eq. (45)] and use the interaction representation. From Eq. (48), it follows that as $\Omega \to \infty$, the propagator of ϕ_1 is given by[5]

$$\phi_1^{\cdot}(x)\phi_1^{\cdot}(y) = (16\pi^4)^{-1} \int (ik^2)^{-1} \exp[ik_\lambda(x-y)_\lambda] d^4k \equiv \tfrac{1}{2} D_F(x-y), \tag{75}$$

where $k^2 = k_\lambda k_\lambda$ and d^4k is real. It is well known that the second time derivative of the right-hand side of Eq. (75) differs from the corresponding contraction of the time derivatives of ϕ_1 by a $\delta^4(x-y)$ function which, in the evaluation of the S matrix, contributes terms that are exactly canceled by the corresponding terms generated by the $\frac{1}{2}\lambda^2\rho^2$ in H_1 [Eq. (47)]. Therefore, in deriving the Feynman rules we can[6] ignore the $\delta^4(x-y)$ function, but regard the contraction of the derivatives of the ϕ_1 as given by

$$\left[\frac{\partial}{\partial x_\mu}\phi_1(x)\right]^{\cdot}\left[\frac{\partial}{\partial y_\nu}\phi_1(y)\right]^{\cdot} = -\frac{\partial^2}{\partial x_\mu \partial x_\nu}[\tfrac{1}{2}D_F(x-y)] \tag{76}$$

and, at the same time, replace Eq. (47) by

$$H_1 = -\frac{f}{m_N} j_\mu \frac{\partial}{\partial x_\mu}\phi_1(x). \tag{77}$$

In the limit that the weak-coupling constant $G \to 0$, the current j_μ is conserved. From Eq. (77), it follows that the emission and absorption rate of any ϕ quanta with $\mathbf{k} \neq 0$ must be zero if $G = 0$. Therefore, the actual emission and absorption probability for each ϕ quantum is proportional to $G^2 f^2$. The same conclusion can also be derived in a more transparent way by using the original Hamiltonian, H given by Eq. (28).

The ϕ quanta can be emitted whenever there is a violation of the conservation of j_μ such as, e.g.,

$$\Lambda^0 \to p + \pi^-. \tag{78}$$

The rate for the process

$$\Lambda^0 \to p + \pi^- + \phi \tag{79}$$

is (to lowest order in G^2 and f^2) proportional to

$$f^2 |M|^2 (16\pi^3 E m_N^2)^{-1} d^3k, \tag{80}$$

where M is the matrix element for the $\Lambda^0 \to p + \pi^-$ vertex in the ϕ emission, k is the momentum of ϕ and $E = |k|$. For the emission of soft ϕ quantum, we can regard M as independent of k. The branching ratio is given by

$$B_1 \equiv \frac{\text{Rate}(\Lambda^0 \to p + \pi^- + \phi)}{\text{Rate}(\Lambda^0 \to p + \pi^-)} = f^2 E_m^2 (8\pi^2 m_N^2)^{-1}, \tag{81}$$

where E_m is the maximum energy of the soft ϕ quantum emitted. Similarly, the rate for

$$\Lambda^0 \to p + \pi^- + \phi_1 + \phi_2 + \cdots + \phi_N \tag{82}$$

is (to lowest order in G^2 and f^2) proportional to

$$(n!)^{-1} f^{2N} |M|^2 \prod_i (16\pi^3 E_i m_N^2)^{-1} d^3k_i, \tag{83}$$

where k_i is the momentum of ϕ_i and $E_i = |k_i|$. For soft

[5] See G. C. Wick, Phys. Rev. **80**, 268 (1950) for the definition of $\phi^{\cdot}(x)\phi^{\cdot}(y)$.

[6] For a detailed proof of this well-known result see, for example, the Appendix in T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962).

quanta emission, the corresponding branching ratio

$$B_n \equiv \frac{\text{Rate}(\Lambda^0 \to p+\pi^- + n\phi)}{\text{Rate}(\Lambda^0 \to p+\pi^-)} \tag{84}$$

is given by

$$B_n = [(2n)!n!]^{-1}[f^2 E_m{}^2 (4\pi^2 m_N{}^2)^{-1}]^n, \tag{85}$$

where E_m is the maximum value of the *total* energy given to the n ϕ-quanta system.

Identical formulas can be applied to the ϕ-emission probability associated with any $\Delta Y \neq 0$ weak reactions. From existing data, we know that

$$f^2 \ll 1. \tag{86}$$

Indeed, it seems that $(4\pi)^{-1}f^2$ can be as big as the fine structure constant without violating any known observations.

For ϕ quantum of sufficiently high energy, reactions such as $\phi + p \to \Lambda^0 + \pi^+$ can occur. The corresponding cross section is expected to be $\sim G^2 f^2$. If the energy of ϕ is $\lesssim 175$ MeV, then to the lowest order of G^2 (but to arbitrary orders in f^2 and e^2) the ϕ quantum cannot be absorbed by a nucleon at rest. Thus, the mean free path of ϕ in a medium is, in general, much longer than that of the neutrino.

So far, we consider the case that ϕ is related to the hypercharge gauge transformation (or strangeness-gauge transformation). Therefore, for weak decays which satisfy $\Delta Y = 0$, the emission probability for ϕ is zero. On the other hand, if ϕ is connected with the gauge transformation associated with the z component of isotopic spin, I_z, then reactions such as

$$n \to p + e^- + \bar{\nu}_e + \phi \tag{87}$$

or

$$\phi + p \to n + l^+ + \nu_l \tag{88}$$

become possible ($l = e$ or μ).

6. REMARKS

(i). The possibility that our system (or universe) is in a state with $\pi_0 \neq 0$ evokes many questions concerning its nature. From Eqs. (49), (66), and (74), we notice that the energy density due to π_0 is given by

$$\tfrac{1}{2}\Omega^{-1}\pi_0{}^2 = (8f^2)^{-1} V^2 m_N{}^2 > 10^7 \text{ BeV/cm}^3. \tag{89}$$

The total energy in the $\mathbf{k}=0$ mode is proportional to the entire volume Ω of the system (which is much greater than the rest-mass energy of the baryons in the universe). The corresponding energy contained in any other mode in the ground state of the system is the well-known zero-point energy $\frac{1}{2}|\mathbf{k}|$. The zero-point energy density contained in all the $\mathbf{k}\neq 0$ modes diverges in the ultraviolet region. If we introduce a momentum cutoff $k_{\max} \gg m_N$, the energy density in all the $\mathbf{k}\neq 0$ states is

$$(8\pi^2)^{-1} k_{\max}{}^4 \gg 10^{34} (\tfrac{1}{2}\Omega^{-1}\pi_0{}^2). \tag{90}$$

Although the fractional energy contained in the $\mathbf{k}=0$ mode is extremely small, the fact that its average value is of a macroscopic nature gives rise to the many striking physical effects discussed in this paper.

The absorption and scattering of the ϕ quantum in matter have some similarities with neutrinos. Thus, it is of some interest to compare the energy density contained in the $\mathbf{k}=0$ mode of the ϕ field with the degenerate neutrino energy density of the universe, which is, of course, not known. If we make the arbitrary assumption that these two energy densities should be comparable, then the Fermi momentum of the degenerate neutrinos would be ~ 10 eV. As has been discussed by Weinberg,[7] such a high value of Fermi momentum is, theoretically, not impossible. Similarly, an energy density of the magnitude given by Eq. (89) may also be possible for the ϕ field in some oscillatory cosmological models.

(ii). The present theory resembles the Bose-Einstein condensation of a system of Bose particles. In either case, the $\mathbf{k}=0$ mode acquires a macroscopic value for its occupation number. Such solutions are intrinsically different in both mathematical content and physical manifestation from the usual ones. It is possible that the existence of such eigensolutions is a general feature of any boson system. In this sense, the present theory may also serve as a simple model for a general class of field theories.

In the case of Bose gas, it is known that there are quasistationary states which correspond to the flow of superfluid. The simplest way is to divide the entire volume into many smaller but macroscopic boxes, and to allow variations between different boxes. The same can also be done in the present case by allowing the value for π_0 to change gradually from one box to the next one. In this way, we can construct a macroscopic field $\pi_0(x)$. The solution $\pi_0(x) \neq$ constant is not an eigenstate of the Hamiltonian of the entire system, but it might be regarded as a quasistationary solution. Such a macroscopic field, if it exists, should have some cosmological influences, and the present value of π_0 as determined by the observed $K_2{}^0 \to 2\pi$ rate may, then, fit into a general picture concerning the evolution of our universe.

(iii). An entirely different mechanism has been suggested[2,8] to account for the 2π decay mode of $K_2{}^0$ and, at the same time, to maintain CP invariance by introducing a long-range spin-1 field V_μ interacting with the hypercharge (or I_z) current density j_μ of the presently known particles.

Since j_μ is not conserved, V_μ must have a mass m.

[7] S. Weinberg, Phys. Rev. **128**, 1457 (1962).

[8] J. S. Bell and J. K. Perring, Phys. Rev. Letters **13**, 348 (1964). (There is an unpublished note by F. Gürsey and A. Pais in which they considered the possible existence of a pseudoscalar field in connection with the experiment by Christenson *et al.* The proposal by Gürsey and Pais seems to be totally different from the theory discussed in this paper and from the suggestion made by Bernstein, Cabbibo and Lee, and by Bell and Perring.)

It has been pointed out by Weinberg[9] that the mass, m cannot be too small. In order to account for the observed rate of $K_2^0 \to 2\pi$, the experimental absence of real emission of such V_μ quanta and the present experimental accuracy[10] of the equality between the observed gravitational mass and the inertial mass, the Compton wavelength m^{-1} of V_μ should be about (or less than) the radius of the earth. The corresponding (coupling constant)^2 is about (or larger than) 10^{-46}. Thus, such a field, if it exists, could also be detected by improving the present accuracy of the Eötvos-type experiment by another order of magnitude. In contrast, the theory discussed in this paper does not give any such observable effect in the Eötvos-type experiment.

If the vector field V_μ exists, its predictions on all apparent CP noninvariant phenomena are the same as that discussed in the present paper. These two different theories can be differentiated either by improving the present accuracy on the equality between the gravitational mass and inertial mass or by observing the real emission of either the V_μ or the ϕ quantum and determining its spin.

Another variation on the same theoretical idea is to couple V_μ with J_μ [Eq. (9)]. Since J_μ is conserved, V_μ could then also have zero mass. While such a model has some appeal because of its possible connection with the gauge invariance of the second kind, it does appear at the present time to be too speculative to warrant a full discussion.

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APPENDIX

In this Appendix we give a simple example of the interaction between an electron and the phonon field in a solid to illustrate the interplay between the gauge transformation, the canonical transformation, and the apparent asymmetry phenomenon.[4] Let the Hamiltonian be given by

$$H=\frac{p^2}{2m}+\sum \omega_{\mathbf{k}}(\alpha_{\mathbf{k}}^\dagger\alpha_{\mathbf{k}}+\tfrac{1}{2}) +\sum \left[V_{\mathbf{k}}\alpha_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}}+V_{\mathbf{k}}^*\alpha_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{r}}\right], \quad \text{(A1)}$$

where $\mathbf{r}$, $\mathbf{p}$ are the coordinate and momentum of the electron, $\alpha_{\mathbf{k}}$ and $\alpha_{\mathbf{k}}^\dagger$ are the annihilation and creation operators of the phonon, $\omega_{\mathbf{k}}$ is its frequency and $V_{\mathbf{k}}$ is the interaction form factor. The Hamiltonian (A1) is invariant under the transformation

$$\alpha_{\mathbf{k}} \to \alpha_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{d}}$$

and

$$\mathbf{r} \to \mathbf{r}-\mathbf{d}. \quad \text{(A2)}$$

[9] S. Weinberg, Phys. Rev. Letters **13**, 495 (1964).
[10] R. H. Dicke, Phys. Rev. **126**, 1580 (1962).

This invariance is connected with the conservation law that the total-momentum operator

$$\mathbf{p}+\textstyle\sum_{\mathbf{k}} \alpha_{\mathbf{k}}^\dagger\alpha_{\mathbf{k}}\mathbf{k} \quad \text{(A3)}$$

commutes with H.

We may introduce a canonical transformation generated by a unitary matrix

$$U\equiv\exp[i \textstyle\sum \alpha_{\mathbf{k}}^\dagger\alpha_{\mathbf{k}}\mathbf{k}\cdot\mathbf{r}] \quad \text{(A4)}$$

which transforms

$$U\alpha_{\mathbf{k}}U^\dagger=\alpha_{\mathbf{k}}e^{-i\mathbf{k}\cdot\mathbf{r}},$$

$$U\mathbf{r}U^\dagger=\mathbf{r},$$

and

$$U\mathbf{p}U^\dagger=\mathbf{p}-\textstyle\sum \alpha_{\mathbf{k}}^\dagger\alpha_{\mathbf{k}}\mathbf{k}. \quad \text{(A5)}$$

Thus, the transformed Hamiltonian becomes

$$UHU^\dagger=\frac{1}{2m}(\mathbf{p}-\textstyle\sum \alpha_{\mathbf{k}}^\dagger\alpha_{\mathbf{k}}\mathbf{k})^2+\sum \omega_{\mathbf{k}}(\alpha_{\mathbf{k}}^\dagger\alpha_{\mathbf{k}}+\tfrac{1}{2}) +\sum \left[V_{\mathbf{k}}\alpha_{\mathbf{k}}+V_{\mathbf{k}}^*\alpha_{\mathbf{k}}^\dagger\right]. \quad \text{(A6)}$$

In the transformed system, $UHU^\dagger$ is independent of $\mathbf{r}$; therefore $\dot{\mathbf{p}}=0$, and the conservation of momentum becomes simply

$$\mathbf{p}=\text{constant}. \quad \text{(A7)}$$

Furthermore, after the canonical transformation, the gauge transformation (A2) becomes an identity transformation for the transformed $\alpha_{\mathbf{k}}$.

This simple example shares all the features of the theory discussed in the paper. Indeed, if the system (after the canonical transformation) is in a state with

$$\mathbf{p}\neq 0,$$

the first term on the right hand side of (A6) contains a part which is of the form

$$-\frac{1}{m}\sum (\mathbf{p}\cdot\mathbf{k})\alpha_{\mathbf{k}}^\dagger\alpha_{\mathbf{k}}. \quad \text{(A8)}$$

This implies that the energy of a phonon with momentum $\mathbf{k}$ is different from that of $-\mathbf{k}$.

We may carry the analogy further and imagine that for some observational reasons it is easy to detect the phonons, but the existence of the extra electron is not known. Thus, it would appear that in this solid phonons are created and annihilated spontaneously and there is an apparent nonconservation of momentum. By postulating the existence of an additional electron it is possible to construct the Hamiltonian, Eq. (A1), which is invariant under the gauge transformation (A2), thus conserving momentum, and which is invariant under the discrete space inversion transformation $\mathbf{k}\to-\mathbf{k}$. However, if the system is in a state with a nonzero value of total momentum, there would exist an apparent noninvariance under space inversion as well, which is in complete analogy with the apparent CP noninvariance discussed in the paper.