upper bound α_2 which coincides exactly with the lower bound α_1 obtained earlier, both agreeing with the correct value of the exponent obtained by Nakanishi. For the general model, we observe that in the strong coupling limit[6]

$$\alpha_2 \underset{g\to\infty}{\longrightarrow} \frac{g}{4\pi m}+(n-\tfrac{5}{2})+\cdots. \tag{36}$$

The leading term in α_2 is identical to the leading term for α_1; i.e., the ratio of upper to lower bound on the exponent approaches unity in the strong coupling limit. So the correct exponent is fully determined as regards the leading behavior in the strong coupling limit. The result, in this limit, is common to *all* the models under discussion.

Finally, let us briefly consider including more than one irreducible graph in the kernel. For instance, we can include all such graphs of the same order $2n$. It is easily seen that there are at least $(n-1)!$ irreducible graphs of order $2n$ (within the class considered in this paper). We can, therefore, employ the procedure used in Sec. III for the $2n$th order kernel, the only difference being a factor of $(n-1)!$. The lower bound on the corresponding amplitude will then be of the form s^α with

$$f(\alpha)=(n-1)!(g/4\pi m)^{2n},$$

so that $\alpha\to n^{1/2}(g/4\pi m)$ in the strong coupling limit. This shows that for the amplitude $A_{\rm tot}$ generated by a kernel including all irreducible graphs of all orders the exponent α must grow faster than linearly in g. Since, clearly, a lower bound for this kernel is

$$K>\sum_{n=1}^{\infty}(n-1)!\times 32\pi^3\left(\frac{g}{4\pi}\right)^{2n}\frac{1}{(n-1)!}\frac{y_0^{\,n-1}}{D_{\max}^{\,2n-2}} =2\pi g^2\exp[(g/4\pi)^2y_0D_{\max}^{\,-2}],$$

a minorizing integral inequality for $A_{\rm tot}$ will then be

$$A_{\rm tot}(u,x)<\pi g^2\delta(s-\mu^2) +16\pi^3\left(\frac{g}{4\pi}\right)^4\frac{1}{(m^2+u)^2}\exp\left[\left(\frac{g}{4\pi}\right)^2\frac{(u-x)}{(m^2+u)^2}\right] +\left(\frac{g}{4\pi}\right)^2\int_{\mu^2}^{u}\frac{du'}{u}\int_{x(u'/u)}^{u'}dx'\frac{A_{\rm tot}(u',x')}{(m^2+x')^2} \times\exp\left[\left(\frac{g}{4\pi}\right)^2\frac{(u-u')(x'/u'-x/u)}{(m^2+x'u/u')^2}\right].$$

One can obtain a lower bound on $A_{\rm tot}$ of the form $u^\alpha(x+m^2)^{-\beta}$ in which α grows *quadratically* with g in the strong coupling limit. However, we have no reason to think that this bound cannot be considerably improved. The point is, of course, that the exponential kernel is too complicated to permit one to carry out the integrations even for the simplest kinds of trial functions.

[6] For $n>1$, we do not expect α_2 to be a good approximation to the true exponent for small values of g. In fact in the weak coupling limit we find $\alpha_2\to n-2$, whereas one expects that $\alpha\to -1$. More generally, it is plausible that $\alpha\leqslant\alpha_L$ where $\alpha_L=-\frac{3}{2}+[\frac{1}{4}+(g/4\pi m)^2]^{1/2}$ is the known exponent for the straight ladder model. At least for a subclass of our models, one can in fact show rigorously that the forward absorptive amplitude is everywhere bounded from above by that of the straight ladder.

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Corrections to the Octuplet Spurion in the Nonleptonic Decays of the Hyperons

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Corrections due to the 27-plet spurion and the second-order electromagnetic effects are calculated to the octuplet spurion in the nonleptonic hyperon decays. The 27-plet spurion predicts a relation among small deviations from the $\Delta I=\frac{1}{2}$ rule, namely $(\langle\Lambda|p\pi^-\rangle+\sqrt{2}\langle\Lambda|n\pi^0\rangle)=-(\langle\Xi^-|\Lambda\pi^-\rangle+\sqrt{2}\langle\Xi^0|\Lambda\pi^0\rangle)$ for the parity-violating amplitudes. This holds as it stands if the second-order electromagnetic effects are introduced on the assumption of the octuplet tadpole mechanism. A test of this relation, although still not possible with present experimental data, has a deep significance for the structure of the weak interactions.

1. INTRODUCTION

UNITARY symmetry[1–3] predicts a relation

$$2\langle\Xi^-|\Lambda\pi^-\rangle-\sqrt{3}\langle\Sigma^+|p\pi^0\rangle+\langle\Lambda|p\pi^-\rangle=0 \tag{1}$$

among the parity-violating amplitudes of the nonleptonic hyperon decays on the following assumptions[4]:

(a) The strong interactions are fully invariant under SU_3.

(b) The weak interactions are of the current × current type[5,6] and are CP-invariant.

(c) Among the spurions arising from the current

[1] Y. Yamaguchi, Progr. Theoret. Phys. Suppl. (Kyoto) **11**, 1, 37 (1960).
[2] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).
[3] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).
[4] M. Gell-Mann, Phys. Rev. Letters **12**, 155 (1964).
[5] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[6] For its unitary symmetric version, see, for example, S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962); N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

$\times$ current interactions, the octuplet spurion is dominant over the 27-plet one.

This relation seems to hold with reasonable accuracy if we neglect the 27-plet spurion and the electromagnetic corrections as well as the T_3^3 violation of the strong interactions. According to the present experimental data, however, the $\Delta I=\frac{1}{2}$ rule does not hold with satisfactory accuracy; correction terms certainly appear to exist.

In the present paper we shall investigate these corrections in detail from the group-theoretical viewpoint. We turn our attention to the deviations from the $\Delta I=\frac{1}{2}$ rule within the respective isomultiplets. The independent quantities to be compared are

$$\langle\Lambda|p\pi^-\rangle+\sqrt{2}\langle\Lambda|n\pi^0\rangle, \tag{2}$$

$$\sqrt{2}\langle\Sigma^+|p\pi^0\rangle-\langle\Sigma^+|n\pi^+\rangle+\langle\Sigma^-|n\pi^-\rangle, \tag{3}$$

$$\langle\Xi^-|\Lambda\pi^-\rangle+\sqrt{2}\langle\Xi^0|\Lambda\pi^0\rangle. \tag{4}$$

We intend to find a relation among them. It should be remarked here that the T_3^3-type violation of unitary symmetry in the strong interactions need not be considered in the first approximation. Indeed, the T_3^3 violation of the octuplet spurion respects the $\Delta I=\frac{1}{2}$ rule, since it maintains the charge independence of the strong interactions. Therefore, the effects cancel each other within each isomultiplet. The situation is just the same as in the electromagnetic mass differences. The T_3^3 violation contributes only as a correction to the 27-plet spurion.

2. THE 27-PLET SPURION

Let us consider the 27-plet spurion arising from the bilinear weak currents, neglecting the electromagnetic corrections for the time being. Our starting assumptions are therefore:

(a) The strong interactions are approximately invariant under SU_3, and the violation is characterized by a mixed tensor T_3^3 to lowest order.

(b) The weak interactions are of the current $\times$ current type and CP invariant.

The weak interactions are described by

$$H^{(w)}=(G/\sqrt{2})(J_2^1\cos\theta+J_3^1\sin\theta)\times(J_1^2\cos\theta+J_1^3\sin\theta) \tag{5}$$

in tensor notation, where the leptonic parts are omitted. Only the cross terms are responsible for the nonleptonic decays

$$(G/\sqrt{2})\cos\theta\sin\theta(J_2^1J_1^3+J_3^1J_1^2). \tag{6}$$

The second-rank mixed tensor within the bracket decomposes into two irreducible tensors, octuplet, and 27-plet. Decay amplitudes of the hyperons due to the 27-plet spurion are written as follows;

$$\begin{aligned}\mathfrak{M}=\;&a[(\bar{B}_2^3B_i^1M_1^i+\bar{B}_2^1B_i^3M_1^i+\bar{B}_1^3B_i^1M_2^i+\bar{B}_1^1B_i^3M_2^i)\pm(\bar{B}_1^iB_3^2M_i^1+\bar{B}_3^iB_1^2M_i^1+\bar{B}_1^iB_3^1M_i^2+\bar{B}_3^iB_1^1M_i^2)]\\&+b[(\bar{B}_2^3B_1^iM_i^1+\bar{B}_2^1B_1^iM_i^3+\bar{B}_1^3B_2^iM_i^1+\bar{B}_1^1B_2^iM_i^3)\pm(\bar{B}_i^1B_3^2M_1^i+\bar{B}_i^1B_1^2M_3^i+\bar{B}_i^2B_3^1M_1^i+\bar{B}_i^2B_1^1M_3^i)]\\&+c[(\bar{B}_i^3B_2^1M_1^i+\bar{B}_i^1B_2^3M_1^i+\bar{B}_i^3B_1^1M_2^i+\bar{B}_i^1B_1^3M_2^i)\pm(\bar{B}_1^2B_3^iM_i^1+\bar{B}_3^2B_1^iM_i^1+\bar{B}_1^1B_3^iM_i^2+\bar{B}_3^1B_1^iM_i^2)]\\&+d[(\bar{B}_2^iB_1^3M_i^1+\bar{B}_2^iB_1^1M_i^3+\bar{B}_1^iB_2^3M_i^1+\bar{B}_1^iB_2^1M_i^3)\pm(\bar{B}_3^1B_i^2M_1^i+\bar{B}_1^1B_i^2M_3^i+\bar{B}_3^2B_i^1M_1^i+\bar{B}_1^2B_i^1M_3^i)]\\&+e[(\bar{B}_i^3B_2^iM_1^1+\bar{B}_i^1B_2^iM_1^3+\bar{B}_i^3B_1^iM_2^1+\bar{B}_i^1B_1^iM_2^3)\pm(\bar{B}_i^2B_3^iM_1^1+\bar{B}_i^2B_1^iM_3^1+\bar{B}_i^1B_3^iM_1^2+\bar{B}_i^1B_1^iM_3^2)]\\&+f[(\bar{B}_2^iB_i^3M_1^1+\bar{B}_2^iB_i^1M_1^3+\bar{B}_1^iB_i^3M_2^1+\bar{B}_1^iB_i^1M_2^3)\pm(\bar{B}_3^iB_i^2M_1^1+\bar{B}_1^iB_i^2M_3^1+\bar{B}_3^iB_i^1M_1^2+\bar{B}_1^iB_i^1M_3^2)],\end{aligned} \tag{7}$$

where B, $\bar{B}$, and M represent the wave functions of the octuplet baryons, antibaryons, and pseudoscalar mesons. The double sign before the second parenthesis within each square bracket corresponds to the parity-conserving $(+)$ and -violating $(-)$ amplitudes. To be strict, the above expression contains a part of the contributions from the octuplet spurion, but this is irrelevant to the following arguments. The T_3^3 correction to the 27-plet spurion has been omitted here as a higher order correction.

We should notice that the interaction given by Eq. (6) is invariant under the interchange of 2 and 3. Since we may regard the strong interactions as fully invariant as far as we are concerned with deviations from the $\Delta I=\frac{1}{2}$ rule, the same invariance under $2\leftrightarrows 3$ is maintained in the decay amplitudes given above. This invariance together with the CP invariance imposes the restrictions

$$a=d,\quad b=c \tag{8}$$

for the parity-conserving amplitudes, and

$$a=-d,\quad b=-c,\quad e=f=0 \tag{9}$$

for the parity-violating amplitudes. We have no relation for the parity-conserving amplitudes. In contrast, we find

$$\langle\Lambda|p\pi^-\rangle+\sqrt{2}\langle\Lambda|n\pi^0\rangle=(\sqrt{\tfrac{2}{3}})(a+b), \tag{10}$$

$$\sqrt{2}\langle\Sigma^+|p\pi^0\rangle-\langle\Sigma^+|n\pi^+\rangle+\langle\Sigma^-|n\pi^-\rangle=2(-a+b), \tag{11}$$

$$\langle\Xi^-|\Lambda\pi^-\rangle+\sqrt{2}\langle\Xi^0|\Lambda\pi^0\rangle=-(\sqrt{\tfrac{2}{3}})(a+b), \tag{12}$$

for the parity-violating amplitudes. The relation

$$\langle\Lambda|p\pi^-\rangle+\sqrt{2}\langle\Lambda|n\pi^0\rangle=-(\langle\Xi^-|\Lambda\pi^-\rangle+\sqrt{2}\langle\Xi^0|\Lambda\pi^0\rangle) \tag{13}$$

cannot be tested with the available experimental data. We have no knowledge of the asymmetry parameter of the process $\Xi^0\to\Lambda+\pi^0$, though the other asymmetry parameters and all the lifetimes have been measured. However, experimental capability will surely suffice to provide us with data accurate enough for the test in the near future.

3. THE ELECTROMAGNETIC CORRECTIONS

Another correction to the octuplet spurion is due to the second-order electromagnetic effects. The T_3^3 correction to the 27-plet spurion may compete with it in magnitude. We have no idea which is dominant over the other. If we make the $T_3{}^3$ correction to the 27-plet spurion, the relation (13) does not hold any longer. In contrast we can develop some arguments on the electromagnetic corrections using a few dynamical assumptions which are quite plausible.

Since most general arguments from the group-theoretical viewpoint do not give any relation among the decay amplitudes, we supplement the foregoing assumptions (a) and (b) with the following:

(c) Octuplet dominance works for the second-order electromagnetic processes. That is, the tensor $T_1{}^1$ describes the second-order electromagnetic processes as well as the first-order ones.[7,8]

(d) The strong-interaction spurion ($T_3{}^3$), the second-order electromagnetic spurion ($T_1{}^1$), and the nonleptonic-decay spurion ($T_2{}^3$) can be described universally by the tadpole mechanism due to the octuplet matter.[7–10]

The assumption (c) describes accurately the electromagnetic mass differences of the baryons.[7,8] The assumption (d) can adjust simultaneously the mass splitting of the four isomultiplets and the electromagnetic mass differences of the baryons.[7,8]

The electromagnetic corrections arise from the interaction

$$(G/\sqrt{2})e^2 \cos\theta \sin\theta \, (J_2{}^1 J_1{}^3 + J_3{}^1 J_1{}^2)(j_1{}^1 j_1{}^1)\,, \qquad (14)$$

where $j_1{}^1$ is the electromagnetic current. We may pick up the octuplet part of the tensor within the first parenthesis; the electromagnetic corrections to the 27-plet spurion are of higher order. The assumption (c) requires us to pick up the octuplet part of the tensor within the second parenthesis also. Thus, the above interaction is approximated as

$$\approx (G/\sqrt{2})e^2 \cos\theta \sin\theta (J_2{}^i J_i{}^3 + J_3{}^i J_i{}^2)(j_1{}^k j_k{}^1)\,. \qquad (15)$$

If we invoke assumption (d), this is further rewritten as

$$\approx (G/\sqrt{2})e^2 \cos\theta \sin\theta (S_2{}^3 + S_3{}^2) S_1{}^1\,, \qquad (16)$$

where $S_j{}^i$ is the octuplet scalar matter. Since the spurions have no angular momentum, the system composed of two spurions must be symmetric under their exchange. The tensor $(S_2{}^3+S_3{}^2)S_1{}^1$, therefore, acts as the 27-plet spurion. Then we have the same decay amplitudes as given in Eq. (7). The resulting relation is just the same as Eq. (13). In this way we have shown that the electromagnetic corrections give the same relation as Eq. (13) on the dynamical assumptions given above.

[7] S. Coleman and S. L. Glashow, Phys. Rev. **133**, B671 (1964).
[8] M. Suzuki, Progr. Theoret. Phys. (Kyoto) **32**, 166 (1964).
[9] M. Suzuki, Progr. Theoret. Phys. (Kyoto) **31**, 1073 (1964).
[10] M. Suzuki, Progr. Theoret. Phys. (Kyoto) **31**, 1090 (1964).

4. PHYSICAL SIGNIFICANCE OF THE RELATION

We need to explain the reason why we should stick so closely to the relation derived here [Eq. (13)] in spite of the experimental difficulties in measuring the decay asymmetry parameter of the Ξ^0 particle and improving the accuracy of the data. First of all, this provides us with a test for the current × current picture of the weak interactions. Next, it will reveal whether the octuplet spurion of the nonleptonic decays may be due to dynamical enhancement[7–12] or have some more profound reasons such as triplet W mesons.[13,14] Let us make a few comments on all the possible cases.

A. The Relation Holds

(*i*) *The magnitude of both sides,* $|\langle\Lambda|p\pi^-\rangle+\sqrt{2}\langle\Lambda|n\pi^0\rangle|$, *is an appreciable fraction of* $|\langle\Lambda|p\pi^-\rangle|$. The contributions from the 27-plet spurion certainly exist. The octuplet spurion is due to dynamical enhancement. It can be shown in a certain approximation that the dynamical enhancement necessarily follows the octuplet scalar mesons.[9,10]

(*ii*) $|\langle\Lambda|p\pi^-\rangle+\sqrt{2}\langle\Lambda|n\pi^0\rangle|$ *is sufficiently small, probably of the order of 1/137, in comparison with the octuplet spurion terms.* The deviations from the octuplet spurion are due to the electromagnetic effects. The 27-plet spurion must be ruled out as a fundamental structure of the weak interactions. The most natural way to realize this in the SU_3 scheme is to assume the existence of triplet W mesons.

B. The Relation is Wrong

(*i*) *The deviations from the* $\Delta I=\frac{1}{2}$ *rule are appreciable.* Unless there is some reason to expect a large $T_3{}^3$ violation of the 27-plet spurion, the current × current picture of the weak interactions is wrong, at least in the form of Cabibbo theory.[6]

(*ii*) *The deviations from the* $\Delta I=\frac{1}{2}$ *rule are small.* The current × current picture is correct. The dynamical assumptions (c) and (d) in the preceding section are wrong.

We know that the well-known triangular relation for the Σ decays is appreciably violated. Insofar as we adopt the central values of the experimental data, the deviation from the $\Delta I=\frac{1}{2}$ rule seems to be more than the electromagnetic violation; cases A(*ii*) and B(*ii*) may be excluded. However, it is too soon for us to pass judgment, partly because the experimental errors are not small and partly because the order of magnitude of the electromagnetic violations has not yet been estimated in a sufficiently reliable manner.

[11] A. Salam and J. C. Ward, Phys. Rev. Letters **5**, 390 (1960).
[12] R. F. Dashen, S. C. Frautschi, M. Gell-Mann, and Y. Hara, in *Proceedings of the 1964 International Conference on High-Energy Physics* (Joint Institute for Nuclear Research, Dubna, U.S.S.R., to be published).
[13] B. d'Espagnat, Phys. Letters **7**, 209 (1963).
[14] S. Okubo, Phys. Letters **8**, 362 (1964).