Dynamical Model for Nonleptonic Decays of Hyperons*

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A dynamical model based on broken unitary symmetry is developed for nonleptonic decays of hyperons. The contributions to these decays are taken from the primary current-current interaction and from poles due to the baryons, the K meson, Y_0^* (1405), and the decuplet of baryon resonances (B_{10}^*). The contributions from the baryon and K-meson poles are calculated on the assumption that weak baryon-baryon and K-π transitions transform like divergences of the relevant weak currents, which are members of octets. The contributions from poles due to Y_0^* and the B_{10}^* resonances must be included to explain the p-wave amplitudes. Both s- and p-wave amplitudes then satisfy the triangular relations and fit the experimental values for the signs and magnitudes. The model predicts the correct sign of the K_2^0-K_1^0 mass difference.

INTRODUCTION

In a previous paper,[1] we have discussed the nonleptonic decays on the basis of the current-current picture. We have shown that the s-wave amplitudes for the nonleptonic decays of hyperons can be rather well understood on this picture. In particular, we have shown that the triangular relation

$$2A_{\Xi_-^-}+A_{\Lambda_-^0}=\sqrt{3}A_{\Sigma_0^+} \qquad (1)$$

is satisfied by the s-wave amplitudes for the nonleptonic decays of hyperons. Also, in this picture, it is natural to take the decay $\Sigma^-\to n+\pi^-$ as pure s wave, and we find the effective coupling constant for the decay rate of $\Sigma^-\to n+\pi^-$ to be $2.25\times10^{-15}m_\pi^{-2}$, which is to be compared with the experimental number $4.5\times10^{-15}m_\pi^{-2}$. Finally, in this picture, we get quantitative agreement with experiment for the total decay rate of $K_1^0\to\pi^+\pi^-$. In this analysis, we have followed the method discussed by one of us (R) in an earlier paper[2] in connection with the strangeness-changing leptonic processes.

This current-current picture for hyperon nonleptonic decays is incomplete for the following reasons: (1) The decay rate for $\Sigma^-\to n+\pi^-$ is less than the experimental value. (2) We cannot obtain $\Sigma^+\to n+\pi^+$ (which we take as pure p wave) in a simple way. Therefore the current-current picture must be supplemented by additional mechanisms. Now the current-current picture is equivalent to boson-pole approximation. Lee and Swift[3] and Schwinger[4] have explicitly introduced the K^* pole for this purpose and have obtained the relation (1). Other contributions may come from baryon poles. It is the purpose of this paper to show that the contributions from the baryon poles and the K-meson pole to hyperon nonleptonic decays do round out the current-current picture, provided that we assume that the weak baryon-baryon and K-π transitions transform like the divergences of the $\Delta S=1$ vector and axial-vector currents for p-wave and s-wave amplitudes, respectively. In particular, if we take the F/D ratio for the strong vertices to be the same as for the weak axial-vector vertices (an assumption justified by the consistency[5,6] of Goldberger-Treiman-type relations for baryons) we show that (1) s-wave amplitudes continue to satisfy the relation (1); (2) p-wave amplitudes also satisfy the triangular relation

$$2B_{\Xi_-^-}+B_{\Lambda_-^0}=\sqrt{3}B_{\Sigma_0^+}. \qquad (2)$$

Further, if we insist that the decay $\Sigma^-\to n+\pi^-$ is pure s wave, we get uniquely the F/D ratio as $\pm1/\sqrt{3}$; the value $1/\sqrt{3}$ is consistent with other determinations of this ratio.[6-8] It is, in fact, in remarkable agreement with the recent determination[8] of this ratio from the experimental analysis of Σ leptonic decays.

Up to this point, the p-wave amplitude $B_{\Sigma_+^+}$ for

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[1] Riazuddin, A. H. Zimerman, and Fayyazuddin, Nuovo Cimento 32, 1819 (1964). In this reference and in Ref. 2, K' was denoted by κ, $\Delta S=I$ vector current by V_α, and $\Delta S=0$ axial vector current by P_α.

[2] Riazuddin, Nuovo Cimento 32, 1122 (1964).

[3] B. W. Lee and A. R. Swift, Phys. Rev. 136, B228 (1964).

[4] J. Schwinger, Phys. Rev. Letters 13, 348 (1964); 13, 500 (1964).

[5] J. J. Sakurai, Phys. Rev. Letters 12, 79 (1964).

[6] Riazuddin, Phys. Rev. 136, B268 (1964).

[7] C. Ryan, reported by R. E. Marshak in Proceedings of the Conference on symmetry Principles at High Energy at Coral Gables, Florida, (1964), W. H. Freeman and Company, London.

[8] W. Willis, H. Courant, and H. Filthuth *et al.*, Phys. Rev. Letters 13, 291 (1964); N. Brene, B. Hellesen, and M. Ross, Phys. Letters 11, 344 (1964).

TABLE I. The contributions from the primary current-current interaction to the s-wave amplitudes. The notation is as follows: $f_\pi = -(\sqrt{2}m_N/g)(-G_A/G)$, $G_A \approx -1.2G$, $G = 10^{-5}/m_N^2$, g is the pion-nucleon coupling constant ($g^2/4\pi \approx 14$). G' is the constant associated with the divergence of $g_\mu{}^V$ (defined in Refs. 1 and 2) and is taken to be $-1.25G$. $A_{\Sigma_0{}^+} = (A_{\Sigma_+{}^+} - A_{\Sigma_-{}^-})/\sqrt{2}$.

Amplitude	Current-current contribution	Relative strengths
$A_{\Lambda_-{}^0}$	$-\frac{1}{2}\sqrt{3} f_\pi G'(m_\Lambda - m_N) = A_{\Lambda_-{}^0}{}^{(1)}$	$A_{\Sigma_0{}^+}{}^{(1)}/A_{\Lambda_-{}^0}{}^{(1)} = -0.84$
$A_{\Sigma_+{}^+}$	0	$A_{\Xi_-{}^-}{}^{(1)}/A_{\Lambda_-{}^0}{}^{(1)} = -1.23$
$A_{\Sigma_-{}^-}$	$-(1/\sqrt{2}) f_\pi G'(m_\Sigma - m_N) = A_{\Sigma_-{}^-}{}^{(1)}$	$A_{\Sigma_-{}^-}{}^{(1)} = h_2{}^{(1)}(m_\Sigma - m_N)$,
$A_{\Xi_-{}^-}$	$\frac{1}{2}\sqrt{3} f_\pi G'(m_\Xi - m_\Lambda) = A_{\Xi_-{}^-}{}^{(1)}$	with $h_2{}^{(1)2}/4\pi = 2.25 \times 10^{-15} m_\pi{}^{-2}$

$\Sigma^+ \to n + \pi^+$ is still zero. Since we are taking this decay to be pure p wave, we must look for another mechanism. Such a mechanism, as suggested by Schwinger,[4] can be provided by the pole due to the $Y_0{}^*$ (1405) resonance which contributes to the $\Sigma^+ \to n + \pi^+$ and $\Sigma^- \to n + \pi^-$ decays only, and in equal amount. However, with this contribution alone and with our value of the F/D ratio ($1/\sqrt{3}$), neither the relative signs of $B_{\Lambda_-{}^0}$, $B_{\Xi_-{}^-}$, $B_{\Sigma_0{}^+}$ nor their relative strengths come out to be right, although the triangular relations are still satisfied. We, therefore, also have to include contributions from poles due to the decuplet of baryon resonances ($B_{10}{}^*$). These contributions themselves satisfy the triangular relations for both s- and p-wave amplitudes, and therefore the relations (1) and (2) are not affected, while it is possible now to fit all the experimental numbers pertinent to the hyperon nonleptonic decays.

In our model, the $K_2{}^0$ meson comes out to be heavier than the $K_1{}^0$ meson, which is in agreement with recent experimental indications.[9] Below we give the details of our calculations.

CURRENT-CURRENT CONTRIBUTION TO S-WAVE AMPLITUDES

We write the matrix element for nonleptonic decay of a hyperon Y into a pion and a baryon as follows:

$$S = (2\pi)^4 \delta(p' + k - p)(m_Y m_B / 2k_0 p_0 p_0')^{1/2} \bar{U}_B(p') \times [A_Y + B_Y \gamma_5] U_Y(p). \quad (3)$$

The contribution from a primary current-current-type interaction to the s-wave amplitude A_Y can be written as

$$A_Y{}^{(1)} = (G/\sqrt{2}) \langle 0 | j_\mu{}^A | \pi^- \rangle \langle B | g_\mu{}^V | Y \rangle, \quad (4)$$

where $j_\mu{}^A$ and $g_\mu{}^V$ are, respectively, the $\Delta S = 0$ axial-vector and $\Delta S = 1$ vector currents and G is the universal Fermi constant. The currents are assumed to be members of octets. The vector current is assumed to be of F type, while the axial-vector current is a mixture of F and D types. The matrix elements appearing in (4) have been explicitly evaluated in Ref. 1. For future reference, we summarize them in Table I.

[9] For sign of the ($K_2{}^0$-$K_1{}^0$) mass difference, see G. W. Meisner, R. L. Golden, B. B. Crawford, and F. S. Crawford, Jr., Proc. Intern. Conf. Fundamental Aspects Weak Interactions, BNL 837, 1963 (unpublished); and Bull. Am. Phys. Soc. 9, 443 (1964). For magnitude of the mass difference, see T. Fujii, J. V. Jouanovich, and F. Turkot, Phys. Rev. Letters 13, 253 (1964); 13, 324 (E) (1964), where other references are also given.

CONTRIBUTIONS FROM POLES DUE TO BARYONS AND K MESON

In order to calculate the contributions to various amplitudes from poles due to the baryons and the K meson, we need the weak baryon-baryon and K-π transitions. We define these transitions as

$$\langle B | H_W(0) | Y \rangle = \bar{u}_B (a_{YB} + \gamma_5 b_{YB}) u_Y, \qquad \langle \pi | H_W(0) | K \rangle = a_{K\pi}. \quad (5)$$

We now assume that a_{YB}, $a_{K\pi}$, and b_{YB} are given by

$$\alpha_V G \langle B | \partial_\mu g_{\mu_0}{}^V | Y \rangle = a_{YB}/m_{K'}{}^2, \qquad \alpha_V G \langle \pi | \partial_\mu g_{\mu_0}{}^V | K \rangle = a_{K\pi}/m_{K'}{}^2, \quad (6)$$

$$\beta_A G \langle B | \partial_\mu g_{\mu_0}{}^A | Y \rangle = b_{YB}/m_K{}^2, \quad (7)$$

where α_V and β_A are dimensionless constants of proportionality, while $g_{\mu_0}{}^V$ and $g_{\mu_0}{}^A$ are the neutral counterparts of the $\Delta S = 1$ charged currents $g_\mu{}^V$ and $g_\mu{}^A$, respectively. m_K is the mass of the pseudoscalar meson K associated with the divergence of the $\Delta S = 1$ axial-vector current $g_\mu{}^A$, while $m_{K'}$ is that of the corresponding scalar meson K' associated with the divergence of the $\Delta S = 1$ vector current $g_\mu{}^V$. As discussed in Ref. 2, we divide the $\Delta S = 1$ vector current into two parts:

$$g_\mu{}^V = g_\mu{}^{V(1)} + g_\mu{}^{V(2)}.$$

The divergence of $g_\mu{}^{V(1)}$ is always taken to be zero; however, in doing so, the mass differences $(m_K{}^2 - m_\pi{}^2)$, $(m_\Lambda - m_N)$, etc., are to be taken equal to zero. The effect of mass differences is supposed to be taken care of by $g_\mu{}^{V(2)}$, which acts only when we do not neglect the mass differences. We take the divergence of $g_{\mu_0}{}^{V(2)}$ as nonzero and, in fact, proportional to the $I = \frac{1}{2}$, $S = 1$ scalar meson K_0'. Similarly, we associate $\partial_\mu g_{\mu_0}{}^A$ with the pseudoscalar meson K_0. Then from Eqs. (5) and (6), we obtain

$$\begin{aligned} a_{\Lambda^0 n} &= (\sqrt{\tfrac{3}{2}}) C_V (m_\Lambda - m_N), \\ a_{\Sigma^0 n} &= -(\sqrt{\tfrac{1}{2}}) C_V (m_\Sigma - m_N), \\ a_{\Xi^0 \Lambda^0} &= -(\sqrt{\tfrac{3}{2}}) C_V (m_\Xi - m_\Lambda), \\ a_{\Xi^0 \Sigma^0} &= (\sqrt{\tfrac{1}{2}}) C_V (m_\Xi - m_\Sigma), \end{aligned} \quad (8)$$

$$\begin{aligned} a_{K^-\pi^-} &= C_V (m_K{}^2 - m_\pi{}^2) = -\sqrt{2} a_{\bar{K}^0 \pi^0}, \\ |a_{K_2{}^0 \pi^0}| &= |a_{K^-\pi^-}|, \\ |a_{K_2{}^0 \eta^0} / a_{K_2{}^0 \pi^0}| &= \sqrt{3}(m_\eta{}^2 - m_K{}^2)/(m_K{}^2 - m_\pi{}^2), \end{aligned} \quad (9)$$

where $C_V = \alpha_V G' m_{K'}{}^2$ and we have assumed the universality of the coupling constants for the baryon and

TABLE II. The contributions from the baryon poles and the K-meson pole. $C_A{}^D=\beta_A G_A{}^{D\prime} m_K{}^2$, $C_V=\alpha_V G' m_{K'}{}^2$, $X=G_A{}^{F\prime}/G_A{}^{D\prime}$. $x=g^F/g^D$, gF and gD are strong F- and D-type pseudoscalar meson-baryon coupling constants. $g^F+g^D=g$, $g^D=[1/(1+x)]g$, $g^2/4\pi\approx 14$, At strong pseudoscalar meson-baryon vertices, the coupling constants given by the octet version of unitary symmetry are used. With these coupling constants, the contributions to the p-wave amplitudes from the K-meson pole in our model are exactly the same as those from the baryon poles, and it is the combined contribution of these poles to each amplitude which is listed in the table.

Amplitude	Contribution	Contribution with the assumption $x=X$
$A_{\Lambda_-{}^0}$	$(1/\sqrt{3})C_A{}^D g^D[-(1+x)(1+3X)+2(1-X)]$	$(1/\sqrt{3})C_A{}^D g^D(1-6x-3x^2)$
$A_{\Sigma_+{}^+}$	$\sqrt{2}C_A{}^D g^D[(1+2x)(1-X)-\frac{1}{3}(1+3X)]$	$2\sqrt{2}C_A{}^D g^D(\frac{1}{3}-x^2)$
$A_{\Sigma_-{}^-}$	$\sqrt{2}C_A{}^D g^D[-\frac{1}{3}(1+3X)-x(1-X)]$	$\sqrt{2}C_A{}^D g^D(-\frac{1}{3}-2x+x^2)$
$A_{\Xi_-{}^-}$	$(1/\sqrt{3})C_A{}^D g^D[2(1+X)-(1-x)(1-3X)]$	$(1/\sqrt{3})C_A{}^D g^D(1+6x-3x^2)$
$B_{\Lambda_-{}^0}$	$(2/\sqrt{3})C_V g^D(1+3x)$	$(2/\sqrt{3})C_V g^D(1+3x)$
$B_{\Sigma_+{}^+}$	0	0
$B_{\Sigma_-{}^-}$	$-2\sqrt{2}C_V g^D(1-x)$	$-2\sqrt{2}C_V g^D(1-x)$
$B_{\Xi_-{}^-}$	$(2/\sqrt{3})C_V g^D(1-3x)$	$(2/\sqrt{3})C_V g^D(1-3x)$

meson currents [that is why the same C_V appears in (8) and (9)]. G' is the strength associated with $g_\mu{}^{V(2)}$ and is defined in Ref. 2. From the analysis of K_{e3} and $K_{\mu 3}$ discussed[10] in Ref. 2, $G'=-1.25G$. It is this value of G' which gives the quantitative agreement for the decay rate of $K_1{}^0\to\pi^+\pi^-$ discussed in Ref. 1 on the basis of the current-current picture. Similarly, from (5) and (7), we have

$$\begin{aligned} b_{\Lambda^0 n}&=(\sqrt{\tfrac{1}{6}})C_A{}^D(1+3X)(m_\Lambda+m_N)\,,\\ b_{\Sigma^0 n}&=(\sqrt{\tfrac{1}{2}})C_A{}^D(1-X)(m_\Sigma+m_N)\,,\\ b_{\Xi^0\Lambda^0}&=(\sqrt{\tfrac{1}{6}})C_A{}^D(1-3X)(m_\Xi+m_\Lambda)\,,\\ b_{\Xi^0\Sigma^0}&=(\sqrt{\tfrac{1}{2}})C_A{}^D(1+X)(m_\Xi+m_\Sigma)\,, \end{aligned} \tag{10}$$

where $C_A{}^D=\beta_A G_A{}^{D\prime} m_K{}^2$, $X=G_A{}^{F\prime}/G_A{}^{D\prime}$. $G_A{}^{F\prime}$, and $G_A{}^{D\prime}$ are the weak coupling constants associated, respectively, with the F and D types of the $\Delta S=1$ axial vector current. From the leptonic decays of the hyperons, we know that[6]

$$|G_A{}^{D\prime}/G_A{}^D|=|G_A{}^{F\prime}/G_A{}^F|=|f_K/f_\pi|\approx\tfrac{1}{4}, \tag{11}$$

where $G_A{}^D+G_A{}^F=G_A\approx-1.2G$; f_π and f_K are decay constants for $\pi_{\mu 2}$ and $K_{\mu 2}$ decays.

Using Eqs. (8), (9), and (10), the contributions from various baryon poles and the K-meson pole are summarized in Table II. In writing down these contributions, we have used the octet version of unitary symmetry for the pseudoscalar meson-baryon strong coupling constants. In Column 2 of this table, we write these contributions under the assumption that x, the F/D ratio for the strong pseudoscalar meson-baryon coupling constants, is equal to X, the F/D ratio for the weak axial-vector constants. This assumption is justified by the consistency of the generalized Goldberger-Treiman relations for baryons.[5,6]

We observe from Column 2 of Table II that both the s- and p-wave amplitudes satisfy the triangular relations (1) and (2). As remarked in the introduction, we take the decay $\Sigma^+\to n+\pi^+$ as pure p wave, so that $A_{\Sigma_+{}^+}=0$. Hence, from Column 2 and Row 2 of Table II, we have

$$x^2=\tfrac{1}{3},\quad\text{or}\quad x=\pm 1/\sqrt{3}.$$

We take the positive sign. The value $+1/\sqrt{3}$ for the F/D ratio is in excellent agreement with the recent determination of this ratio from the analysis of Σ leptonic decays.

CONTRIBUTIONS FROM POLES DUE TO Y_0^* AND B_{10}^* RESONANCES

From Table II, we see that $B_{\Sigma_+{}^+}$ is zero. Therefore, we must look for other mechanisms to generate the p-wave amplitude for the decay $\Sigma^+\to n+\pi^+$. One such mechanism, as suggested by Schwinger,[4] may be provided by the Y_0^* pole which contributes only to $\Sigma^+\to n+\pi^+$ and $\Sigma^-\to n+\pi^-$ in equal magnitude. Let its s-wave and p-wave contributions to the above decays be, respectively, denoted by $\sqrt{2}a_1$ and $\sqrt{2}b_1$. It can be easily seen that inclusion of the contribution b_1 from the Y_0^* pole alone to the p-wave amplitudes (with b_1 determined from the condition $B_{\Sigma_-{}^-}=0$) cannot provide the right relative signs for $B_{\Lambda_-{}^0}$, $B_{\Xi_-{}^-}$, $B_{\Sigma_0{}^+}$ when we use our value of the F/D ratio, namely $1/\sqrt{3}$. We, therefore, include the contributions from the poles due to the B_{10}^* resonances. Using Table I of Hara,[11] we summarize the sum of the contributions from the poles due to Y_0^* and B_{10}^* resonances in Table III. These contributions themselves satisfy the triangular relations. Now with the F/D ratio $1/\sqrt{3}$, the baryon pole contributions yield $A_{\Sigma_+{}^+}=0$. We, therefore, want the sum of the contributions from Y_0^* and the B_{10}^* poles to this amplitude also to be zero, since this decay is taken to be pure p wave. Therefore, from Table III, we obtain $a_1=\frac{1}{4}a_2$. Also from Table II (with $x=X=1/\sqrt{3}$) and from Table III, we get

$$B_{\Sigma_-{}^-}=\sqrt{2}\{-2C_V g^D[1-(1/\sqrt{3})]+b_1+(7/12)b_2\}.$$

Since $\Sigma^-\to n+\pi^-$ is taken to be pure s wave, therefore $B_{\Sigma_-{}^-}=0$. This gives

$$b_1=2C_V g^D[1-(1/\sqrt{3})]-(7/12)b_2.$$

[10] The effective coupling constant which appears in the total decay rates for $\Delta S=1$ leptonic processes is $G_{\rm eff}=G+G'=-0.25G$.

[11] Y. Hara, Phys. Rev. Letters 12, 378 (1964). The C and D in Hara's Table I are denoted by $\sqrt{2}a_2$ and $\sqrt{2}b_2$ in our Table III, and we have multiplied his amplitudes for Λ and Ξ decays by (-1) in order to conform to our convention.

Using these values of a_1 and b_1 in terms of a_2 and b_2, we list in Table IV the combined contributions from poles due to the baryons, K meson, Y_0^* and B_{10}^* with the F/D ratio $1/\sqrt{3}$.

TABLE IV. Sum of contributions from poles due to baryons, K meson, Y_0^*, and B_{10}^*.

Amplitude	Sum of contributions
$A_{\Lambda_-^0}$	$-\frac{\sqrt{3}}{2}\left[\frac{4}{\sqrt{3}}C_A{}^D g^D-\frac{1}{3}a_2\right]=A_{\Lambda_-^0}{}^{(2)}$
$A_{\Sigma_+^+}$	0
$A_{\Sigma_-^-}$	$-\frac{1}{\sqrt{2}}\left[\frac{4}{\sqrt{3}}C_A{}^D g^D+\frac{1}{3}a_2\right]=A_{\Sigma_-^-}{}^{(2)}$
$A_{\Xi_-^-}$	$\frac{\sqrt{3}}{2}\left[\frac{4}{\sqrt{3}}C_A{}^D g^D\right]=A_{\Xi_-^-}{}^{(2)}$
$B_{\Lambda_-^0}$	$\frac{1}{\sqrt{3}}[2g^D C_V(1+\sqrt{3})+\frac{1}{2}b_2]=B_{\Lambda_-^0}{}^{(2)}$
$B_{\Sigma_+^+}$	$\sqrt{2}\left[2g^D C_V\left(1-\frac{1}{\sqrt{3}}\right)-\frac{1}{2}b_2\right]=B_{\Sigma_+^+}{}^{(2)}$
$B_{\Sigma_-^-}$	0
$B_{\Xi_-^-}$	$\frac{1}{\sqrt{3}}[2g^D C_V(1-\sqrt{3})-b_2]=B_{\Xi_-^-}{}^{(2)}$

COMPARISON WITH EXPERIMENT

We first consider the s-wave amplitudes listed in Tables I and IV. Experimentally,

$$\frac{A_{\Sigma_0^+}}{A_{\Lambda_-^0}}=-\frac{1}{\sqrt{3}}\frac{m_\Sigma-m_N}{m_\Lambda-m_N}. \tag{12}$$

We see from Table I that our current-current contributions $A_{\Lambda_-^0}{}^{(1)}$, $A_{\Sigma_0^+}{}^{(1)}$ already satisfy this relation. Therefore, we want the contributions $A_{\Lambda_-^0}{}^{(2)}$ and $A_{\Sigma_0^+}{}^{(2)}$ listed in Table IV also to satisfy the relation (12). This condition determines

$$a_2=4\sqrt{3}g^D C_A[(m_\Sigma-m_\Lambda)/(m_\Sigma+m_\Lambda-2m_N)]. \tag{13}$$

Now if we write $A_{\Sigma_-^-}=h_2(m_\Sigma-m_N)$, we see from Table I that the current-current contribution alone gives $h_2{}^2/4\pi$ as $2.25\times10^{-15}m_\pi{}^{-2}$ to be compared with the experimental number $4.5\times10^{-15}m_\pi{}^{-2}$. Therefore, we must have

$$A_{\Sigma_-^-}{}^{(2)}=(\sqrt{2}-1)A_{\Sigma_-^-}{}^{(1)}$$

in order to secure agreement with experiment for the decay rate of $\Sigma^-\to n+\pi^-$. This condition, on using Tables I and IV and Eq. (13), leads to

$$2C_A g^D=(\sqrt{3}/2)f_\pi G'\{[(m_\Lambda+m_\Sigma)/2]-m_N\}(\sqrt{2}-1). \tag{14}$$

TABLE III. The contributions from poles due to Y_0^* and the B_{10}^* baryon resonances.

Amplitude	From Y_0^*	From B_{10}^*	Sum
$A_{\Lambda_-^0}$	0	$\frac{1}{2\sqrt{3}}a_2$	$\frac{1}{2\sqrt{3}}a_2$
$A_{\Sigma_+^+}$	$\sqrt{2}a_1$	$-\frac{\sqrt{2}}{4}a_2$	$\sqrt{2}(a_1-\frac{1}{4}a_2)$
$A_{\Sigma_-^-}$	$\sqrt{2}a_1$	$-\frac{5\sqrt{2}}{12}a_2$	$\sqrt{2}\left(a_1-\frac{5}{12}a_2\right)$
$A_{\Xi_-^-}$	0	0	0
$B_{\Lambda_-^0}$	0	$\frac{1}{2\sqrt{3}}b_2$	$\frac{1}{2\sqrt{3}}b_2$
$B_{\Sigma_+^+}$	$\sqrt{2}b_1$	$\frac{\sqrt{2}}{12}b_2$	$\sqrt{2}\left(b_1+\frac{1}{12}b_2\right)$
$B_{\Sigma_-^-}$	$\sqrt{2}b_1$	$\frac{7}{12}\sqrt{2}b_2$	$\sqrt{2}\left(b_1+\frac{7}{12}b_2\right)$
$B_{\Xi_-^-}$	0	$-\frac{1}{\sqrt{3}}b_2$	$-\frac{1}{\sqrt{3}}b_2$

Using Eqs. (13) and (14), we get from Tables I and IV

$$A_{\Lambda_-^0}{}^{(2)}=(\sqrt{2}-1)A_{\Lambda_-^0}{}^{(1)},\quad A_{\Sigma_-^-}{}^{(2)}=(\sqrt{2}-1)A_{\Sigma_-^-}{}^{(1)},$$

$$A_{\Xi_-^-}{}^{(2)}=(\sqrt{2}-1)A_{\Xi_-^-}{}^{(1)}[(m_\Lambda+m_\Sigma-2m_N)/2(m_\Xi-m_\Lambda)],$$

the last equation being the prediction. Because of the Gell-Mann–Okubo mass formula $(m_\Lambda+m_\Sigma-2m_N)/2(m_\Xi-m_\Lambda)=1$, so that

$$A_{\Xi_-^-}{}^{(2)}=(\sqrt{2}-1)A_{\Xi_-^-}{}^{(1)}.$$

Hence, finally,

$$A_{\Lambda_-^0}=\sqrt{2}A_{\Lambda_-^0}{}^{(1)},\quad A_{\Sigma_-^-}=\sqrt{2}A_{\Sigma_-^-}{}^{(1)},$$

$$A_{\Sigma_+^+}=0,\quad A_{\Xi_-^-}=\sqrt{2}A_{\Xi_-^-}{}^{(1)},$$

which, on using Table I, give

$$A_{\Sigma_0^+}/A_{\Lambda_-^0}=-0.84,\quad A_{\Xi_-^-}/A_{\Lambda_-^0}=-1.23,$$

$$h_2{}^2/4\pi=4.5\times10^{-15}m_\pi{}^{-2},$$

in agreement with experiment. Last, since $C_A{}^D=\beta_A G_A{}'^D m_K{}^2$, we get from (14) [using $g^D=\sqrt{3}g/(1+\sqrt{3})$, $g^2/4\pi\approx14$, $G_A{}^D=\sqrt{3}G_A/(1+\sqrt{3})$, Eq. (11) and values of f_π and G' listed in Table I]

$$\beta_A\approx1/70. \tag{15}$$

We now consider the p-wave amplitudes. Experimentally, $B_{\Lambda_-^0}/B_{\Xi_-^-}\approx\frac{3}{2}$. Therefore, using Table IV, this condition yields

$$b_2=-[(5\sqrt{3}-1)/4]2g^D C_V. \tag{16}$$

With this value of b_2, we predict, on using Table IV,

$$B_{\Sigma_0^+}/B_{\Lambda_-^0}\approx 1.34\,,$$

in agreement with experiment. Using (16) and Table IV, we get

$$B_{\Sigma_+^+}[\equiv g_2(m_\Sigma+m_N)]$$
$$=2\sqrt{2}g^D C_V\frac{\sqrt{3}-1}{\sqrt{3}}\left(1+\frac{5\sqrt{3}-1}{8}\frac{\sqrt{3}}{\sqrt{3}-1}\right). \quad (17)$$

Experimentally, $|g_2|\approx 2.8\times 10^{-7}m_\pi^{-1}$. Using this value of $|g_2|$, and noting that $C_V=\alpha_V G' m_{K'}^2$, we obtain from (17) (using $m_{K'}^2\approx 28m_\pi^2$, $G'=-1.25G$),

$$|\alpha_V|\approx 1/63\,. \quad (18)$$

Note that the dimensionless constants of proportionality β_A and α_V appearing in Eqs. (6) and (7) are nearly the same.

$K_2^0-K_1^0$ MASS DIFFERENCE

We calculate the contributions of the pion and the η poles to the mass difference in our model. These contributions are given by

$$\delta m=\delta m_{K_2^0}-\delta m_{K_1^0}$$
$$=\delta m_{K_2^0}=\frac{1}{2m_K}\left[\frac{a_{K_2^0\pi^0{}^2}}{m_K^2-m_\pi^2}+\frac{a_{K_2^0\eta^0{}^2}}{m_K^2-m_\eta^2}\right].$$

Using Eqs. (9), we get

$$\delta m=\alpha_V^2 G'^2 m_{K'}^4\left[\frac{4m_K^2-3m_\eta^2-m_\pi^2}{2m_K}\right]$$
$$=\alpha_V^2 G'^2 m_{K'}^4(73\text{ MeV})\,. \quad (19)$$

Note that the mass difference is positive, i.e., K_2^0 is heavier than K_1^0 for which there is some experimental evidence.[9] In order to calculate the magnitude, we use $m_{K'}^2\approx 28m_\pi^2$, $G'=-1.25G$, $|\alpha_V|\approx 1/63$ [Eq. (18)]. Then

$$\delta m\approx 0.11\times 10^{-5}\text{ eV}\,. \quad (20)$$

DISCUSSION AND SUMMARY OF CONCLUSIONS

To summarize, we have considered a dynamical model for nonleptonic decays based on broken unitary symmetry. The contributions considered to these decays are from the primary current-current interaction and the poles due to baryons, K meson, Y_0^* and decuplet of baryon resonances B_{10}^*. Our basic assumption for calculating the contributions from the baryons and the K-meson poles are Eqs. (6) and (7). Equation (6) is, in fact, equivalent to introducing a scalar tadpole provided that the scalar meson (in our case K') is coupled to other particles through the vector K^* meson, i.e., the scalar particle, whose $CP=1$ component gives rise to the scalar tadpole, is abnormal under charge conjugation. It is this fact which distinguishes our model from that of Schwinger[4] for the p-wave amplitudes. In Schwinger's model the scalar particle has a direct nonderivative coupling with hadrons and his scalar particle giving rise to the scalar tadpole is normal under charge conjugation. It is owing to this difference that in our model the contributions to the p-wave amplitudes from the baryon poles and the K-meson pole are equal and of the same sign and are therefore added, whereas in Schwinger's model they tend to cancel each other. In fact, they cancel exactly in Schwinger's model, if pseudoscalar meson-baryon coupling constants as given by unitary symmetry are used. In order to avoid this exact cancellation, Schwinger must use different pion-baryon and kaon-baryon coupling constants. Another consequence of this difference is that the contributions of the pion and the η poles to the K_2^0-K_1^0 mass difference is positive in our model, whereas this will be negative if one follows Schwinger's model as discussed by Okubo and two of us (R and Z) in a previous paper.[12]

Equation (7) is also equivalent to introducing a pseudoscalar tadpole generated by the K_1^0 component of the pseudoscalar K meson, provided that the coupling of K_1^0 to the baryons is induced through an axial-vector meson analogous to K^*. We have evaluated Eqs. (6) and (7) by following a method discussed in Ref. 2 in connection with $\Delta S=1$ leptonic processes. The advantage of this method is that we have a natural explanation of why the pseudoscalar tadpole should be suppressed compared to the scalar tadpole as indicated by experimental data on hyperon nonleptonic decays. The reason for this suppression is that in our way of evaluating Eqs. (6) and (7), the constant $G'(=-1.25G)$ appearing in Eqs. (8) and (9) is larger than the corresponding constants $G_A'^D$ and $G_A'^F$ [given in Eq. (11)] which appear in Eq. (10). [Note that the constants of proportionality α_V and β_A in Eqs. (6) and (7) are nearly the same (cf. Eqs. (15) and (18)]. For the same reason, we have not included in the p-wave amplitudes the contributions from the primary current-current interaction similar to the ones calculated from Eq. (4) and listed in Table I for the s-wave amplitudes. These contributions to the p-wave amplitudes involve $G_A'^D$ and $G_A'^F$, which are smaller than the corresponding constant G' appearing in the s-wave amplitudes of Table I, and as such are small compared to the other contributions considered for the p-wave amplitudes.

In our evaluation of the baryon poles and the K-meson pole contributions, we have made the assumption that the F/D ratio is the same for the weak and strong vertices. Then these contributions satisfy the triangular relations (1) and (2) and our insistance that the decay $\Sigma^+\to n+\pi^+$ should be pure p-wave fixes the F/D ratio as $1/\sqrt{3}$ in agreement with that determined recently in Ref. 8. Inclusion of the Y_0^* and B_{10}^* poles is necessitated by the p-wave considerations, since the baryon and

[12] A. H. Zimerman, Riazuddin, and S. Okubo, Nuovo Cimento (to be published).

K-meson poles alone, although satisfying the triangular relations, cannot explain the experimental data for the p-wave amplitudes. With these contributions included, it has been shown that all experimental numbers regarding the hyperon nonleptonic decays are fitted. In addition, the contribution of the pion and the η poles to the K_2^0-K_1^0 mass difference is predicted to be positive and about 0.11×10^{-5} eV. If we add this contribution to other contributions to the mass difference calculated by two of us (R and Z) in a previous paper,[13] we get the total magnitude to be about 0.4×10^{-5} eV, in fair agreement with experiment.[9]

[13] Riazuddin and A. H. Zimerman, Phys. Rev. 135, B1211 (1964).

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Forbidden Transitions in Pole Models with Unitary Symmetry*

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Two selection rules are derived which explain vanishing transition matrix elements found for many processes. A decay is forbidden in a pole model having a momentum-independent symmetry-breaking vertex and a symmetry-conserving vertex with arbitrary form factors if either (1) all propagators are equal in magnitude and the matrix elements of the symmetry-breaking vertex are proportional to those of a generator of the symmetry group, or (2) the propagators involve only known mass differences described by the Gell-Mann–Okubo mass formula and the matrix elements of the symmetry-breaking vertex are described by the D coupling of three unitary octets. Applications to K decays and nonleptonic Σ decays are discussed.

I. A SIMPLE SELECTION RULE

VANISHING–TRANSITION probabilities have been found in a number of calculations of nonleptonic K-meson decays using a boson-pole model with unitary symmetry.[1] A simple example is the $K^+\to\pi^+\pi^+\pi^-$ decay which is described in the boson-pole model by two diagrams:

$$K^+\to\pi^+\to\pi^++\pi^++\pi^-, \quad (1a)$$

$$K^+\to K^++\pi^++\pi^-\to\pi^++\pi^++\pi^-. \quad (1b)$$

The calculations of this decay by use of unitary symmetry show that the contributions of the two modes (1a) and (1b) are equal and opposite and just cancel. Similar cancellations have been found for all other $K\to3\pi$ decays and for combined weak and electromagnetic decays of kaons into combinations of pions and photons. The purpose of this paper is to show how all these cancellations can be described by a single selection rule based on SU_3 algebra, and to establish very general sufficient conditions for the vanishing of the transition amplitude in these and related processes.

When the decay (1) is treated by a boson-pole model with unitary symmetry, the following assumptions are usually made:

(1) The process goes via an intermediate state and has two vertices—a symmetry-breaking vertex which violates unitary symmetry and a symmetry-conserving vertex.

(2) The symmetry-breaking vertex is a two-point vertex, a single transition ($K^+\to\pi^+$) between two one-particle states. No form factor is assumed for this vertex; i.e., the matrix element is taken to be momentum-independent.

A characteristic feature of pole diagrams of the type (1) is that each contains only one particle in the intermediate state that is off the mass shell, namely the one that is connected either to the initial or to the final state by the weak vertex. For the case (1), the propagator for the intermediate state has the same absolute magnitude for all diagrams that contribute to the process. The sign of the propagator depends upon whether the symmetry-breaking vertex comes before or after the symmetry-

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[1] S. Hori, S. Oneda, S. Chiba, and A. Wasaka, Phys. Letters 5, 339 (1963); S. Oneda, Y. S. Kim, and D. Korff, University of Maryland Technical Report No. 385, 1964 (unpublished); K. Tanaka, Phys. Rev. 136, B1813 (1964). References to earlier works are given in these papers. Oneda *et al.* use a specific strong-interaction vertex that satisfies unitary symmetry. Tanaka uses the most general vertex consistent with unitary symmetry but considers only those cases in which all propagators have the same magnitude. These treatments find vanishing transition matrix elements for each individual decay by explicit calculations of contributions from all diagrams.