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Single-Particle Exchange Models for the Reactions $\pi p \to \rho p$, $\bar{p}p \to \bar{Y}Y$, and $np \to pn$*

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The predictions of single-particle exchange models for particle reactions at high energies have been shown by several authors to be altered significantly by the effects of the strong absorption from the entrance and exit channels into the generally numerous competing channels. The necessary modifications of the usual single-particle exchange models have been discussed by Sopkovich, Durand and Chiu, and by Gottfried and Jackson. In the present paper, the predictions of the modified models for the reactions $\pi^\pm p(\pi^0, \omega)\,\rho^\pm p$, $\bar{p}p(K, K^*)\bar{Y}Y$, and $np(\pi, \rho)pn$ are compared with the results of recent experiments. [We denote by $ab(e,f, \cdots)cd$ the reaction $a+b \to c+d$, assumed to proceed through the exchange of particles $e,f, \cdots$.] The effects of the particle spins, including the decay angular distribution of the $\rho^\pm$ in the first reaction and the two-particle spin correlations in the $\bar{Y}Y$ system in the second, are discussed in detail. The rather striking success of the modified theories for the foregoing reactions suggests strongly that ideas underlying the modifications are correct in principle, if not in detail.

THE modifications of single-particle exchange models for high-energy interactions which result from the presence of strong absorption in the initial and final states have been considered recently by a number of authors.[1–9] However, the calculations reported thus far have, for the most part, treated the participating particles as spinless,[10] and have dealt only with absorptive corrections to the propagator of the exchanged particle. Although this procedure is sufficient to demonstrate qualitatively the remarkable sharpening of production angular distributions which can result from the absorption, and may cause little difficulty for reactions such as $\bar{p}p(K^*)\bar{\Lambda}\Lambda$ for which the main part of the assumed interaction is spin independent, the neglect of spin can lead to serious difficulties in other cases, for example, in the reactions $\pi p(\pi,\omega)\rho p$, $np(\pi,\rho)pn$, and $\bar{p}p(\pi)\bar{N}^*N^*$.[11] In general, the transition amplitudes for different spin states are affected differently by the competition from other inelastic processes, even if the absorption is assumed to be spin independent. As a consequence, the predicted reaction cross sections may differ in magnitude and show considerably less diffraction structure than would be suggested by some simple models.[3,6,9] Furthermore, the predictions for such specifically spin-dependent features of the reactions as the decay angular distributions of unstable reaction products, distributions in the Treiman-Yang angle, and two-particle spin correlation parameters, may differ significantly from the predictions of the unmodified single particle exchange models,[8,12] and thus provide a sensitive test of the theory. In the present note, we wish to present some results on the reactions $\pi^\pm p \to \rho^\pm p$, $\bar{p}p \to \bar{\Lambda}\Lambda$, $\bar{\Lambda}\Sigma^0$, and $np \to pn$, which illustrate the foregoing points in a rather striking manner, and suggest strongly that the ideas involved in the modified single particle exchange models are correct in principle, if not in detail.

The calculations followed the procedure discussed elsewhere, and we shall only note the main points. The transition matrix M for the reaction $a+b \to c+d$ can be written in the helicity representation in the form[13]

$$M_{\lambda_c\lambda_d;\lambda_a\lambda_b}(s,t)=\sum_j(2j+1)M_{\lambda_c\lambda_d;\lambda_a\lambda_b}{}^j(s)d_{\lambda\mu}{}^j(x), \quad (1)$$

where the λ's label the helicities of the particles, $\lambda=\lambda_a-\lambda_b$, $\mu=\lambda_c-\lambda_d$, and $d_{\lambda\mu}{}^j(x)$ is the usual rotation coefficient with argument $x=\cos\theta$. Direct calculation of M in the single-particle exchange approximation yields matrix elements of the form

$$M_{\lambda_c\lambda_d;\lambda_a\lambda_b}{}^B(s,t)=(\tfrac{1}{2}(1-x))^{|\lambda-\mu|/2}(\tfrac{1}{2}(1+x))^{|\lambda+\mu|/2}\,, \times\{(B/2pp')(z-x)^{-1}+\text{polynomial in } x\}\,, \quad (2)$$

where B depends on the coupling constants and the particle helicities and momenta. The variable z is given by

$$z=1+(\mu^2-t)/2pp'\,|_{\theta=0}\,, \quad (3)$$

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[1] N. J. Sopkovich, dissertation, Carnegie Institute of Technology, 1962, and Nuovo Cimento **26**, 186 (1962).

[2] M. Baker and R. Blankenbecler, Phys. Rev. **128**, 415 (1962).

[3] A. Dar, M. Kugler, Y. Dothan, and S. Nussinov, Phys. Rev. Letters **12**, 82 (1964).

[4] L. Durand, III, and Y. T. Chiu, Phys. Rev. Letters **12**, 399 (1964).

[5] L. Durand, III, and Y. T. Chiu, Proceedings of the 1964 Boulder Conference on Particles and High Energy Physics (to be published). A more comprehensive account of this work is in preparation for the Physical Review.

[6] A. Dar and W. Tobocman, Phys. Rev. Letters **12**, 511 (1964).

[7] M. H. Ross and G. L. Shaw, Phys. Rev. Letters **12**, 627 (1964).

[8] K. Gottfried and J. D. Jackson (to be published).

[9] A. Dar, Phys. Rev. Letters **13**, 91 (1964).

[10] Exceptions are the papers of Sopkovich, Ref. 1, on the reaction $\bar{p}p(K^*,K)\bar{\Lambda}\Lambda$ (these results are incorrect in detail because of an error in the treatment of the spins of the antiparticles), the paper of Gottfried and Jackson, Ref. 8, on the reaction $\pi^-p(\pi)\rho^-p$, and the paper of Ref. 5.

[11] We denote by $ab(e,f,\cdots)cd$ the reaction $a+b \to c+d$, assumed to proceed through the exchange of particles e, f, $\cdots$.

[12] It should be emphasized in this connection that the introduction of form factors at the vertices or on the propagator of the exchange particle in the usual unmodified single-particle exchange models will not change the predictions of the models for the foregoing spin dependent quantities. Thus, the experimental examination of these quantities provides a much more sensitive test of the models than is provided by the differential reaction cross section alone: the latter can often be reproduced by an appropriate choice of form factors.

[13] M. Jacob and G. C. Wick, Ann. Phys. (N. Y.) **7**, 404 (1959).

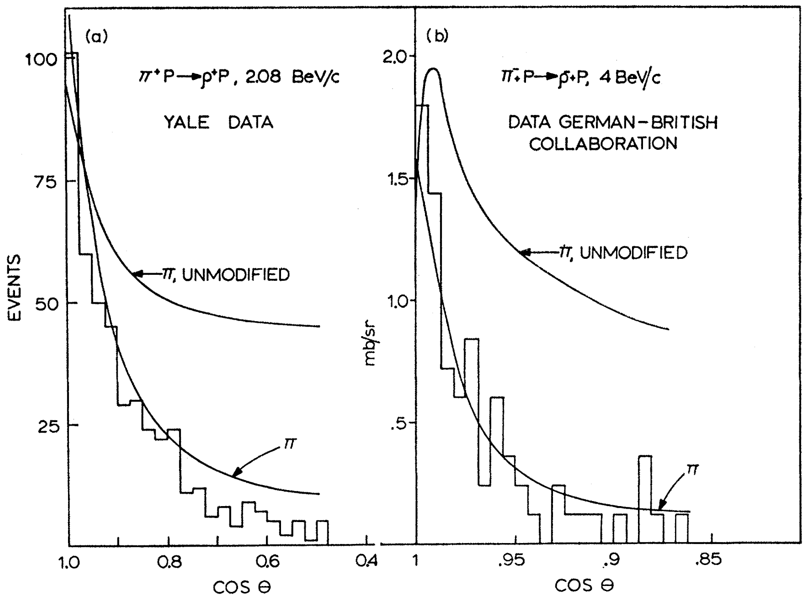

FIG. 1. Comparison of the production angular distributions in the reactions $\pi^\pm p \to \rho^\pm p$ with the predictions of the modified single-pion-exchange model: (a) 2.08 BeV/c π^+p data from Ref. 16, for $M_{\pi\pi}=$ (760±100) MeV; (b) 4 BeV/c π^-p data from Ref. 17, $M_{\pi\pi}=$ (760±60) MeV. The theoretical curves have been adjusted to the indicated slices of the ρ mass spectrum. The theory contains no adjustable parameters.

where μ is the mass of the particle exchanged, and p and p' are the three momenta of the initial and final particles in the overall center of mass system. The partial wave transition amplitudes $M^{j,B}$ are readily calculated from Eq. (2) using the orthogonality properties of the rotation coefficients; the results are expressible in terms of the rotation coefficients of the second kind,[14] $e_{\lambda\mu}{}^j(z)$,

$$M_{\lambda_c\lambda_d;\lambda_a\lambda_b}{}^{j,B}(s)=\epsilon(\lambda,\mu)\frac{B}{2pp'}\times\left(\tfrac{1}{2}(z-1)\right)^{|\lambda-\mu|/2}\left(\tfrac{1}{2}(z+1)\right)^{|\lambda+\mu|/2}e_{\lambda\mu}{}^j(z)\,,\qquad(4)$$

where $\epsilon(\lambda,\mu)$ is a signature factor determined by the relative signs and magnitudes of λ and μ. The polynomial in Eq. (2) leads to extra terms which affect only the low partial-wave amplitudes. Since these exceptional terms generally exceed the limits imposed by unitarity even when corrected for absorption in the initial and final states, and are in any case not significant within the confines of the model because of our neglect of the shorter range parts of the interactions, they have been omitted in the following calculations. However, we have retained the "normal" contributions to the low partial wave amplitudes, Eq. (4), so as to obtain some reasonable estimate of their effect on the reaction.

The modification of the partial-wave transition amplitudes $M^{j,B}$ for the effects of strong absorption in the initial and final states has been discussed elsewhere. Under the assumptions (i) that the particle wave lengths in the entrance and exit channels are small compared to the characteristic ranges of interaction in those channels and the range of the single particle exchange interaction, (ii) that the entrance and exit channels are strongly coupled to a number of inelastic channels, and (iii), that the range of the exchange interaction is small compared to the ranges of the other interactions, it may be shown that the distorted wave Born approximation result for M^j is related to $M^{j,B}$ as[5]

$$M^j=[S_{cd}{}^j]^{1/2}M^{j,B}[S_{ab}{}^j]^{1/2}.\qquad(5)$$

Here the S's are the S matrices for elastic scattering in the initial and final states, expressible as $S=ODO^{-1}$, D diagonal and O a (complex) orthogonal matrix, and $S^{1/2}=OD^{1/2}O^{-1}$. If instead of (iii), it is assumed that the range of the exchange interaction is long compared to the other ranges, Eq. (5) is replaced by[5]

$$M^j\approx\tfrac{1}{2}[S_{cd}{}^jM^{j,B}+M^{j,B}S_{ab}{}^j].\qquad(6)$$

In the present, largely exploratory, calculations, we have assumed for lack of better information that $S_{ab}{}^j$ and $S_{cd}{}^j$ are diagonal matrices with elements which depend only on j; Eqs. (5) and (6) are then equivalent. It was assumed furthermore that the elastic scattering in the initial and final state was exponential diffraction scattering, completely absorptive for the lowest particle waves, corresponding to the scattering amplitudes[15]

$$S_{ab}\approx S_{cd}\approx[1-e^{-(\nu^2/pp')j(j+1)}].\qquad(7)$$

The partial wave expansion for M, Eq. (1), is then readily reconstructed using the modified amplitudes.

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[14] L. Durand, III and Y. T. Chiu, Ref. 5. Also M. Andrews and J. Gunson, J. Math. Phys. 5, 1391 (1964). The phase convention for the $e_{\lambda\mu}{}^j$ used by Andrews and Gunson differs from that of Ref. 5 by the factor $(-1)^{|\lambda-\mu|/2}$.

[15] To take some account of the difference between the initial and final momenta p and p' without overly complicating the calculation, we have used the product pp' rather than p^2 or p'^2 in Eq. (7).

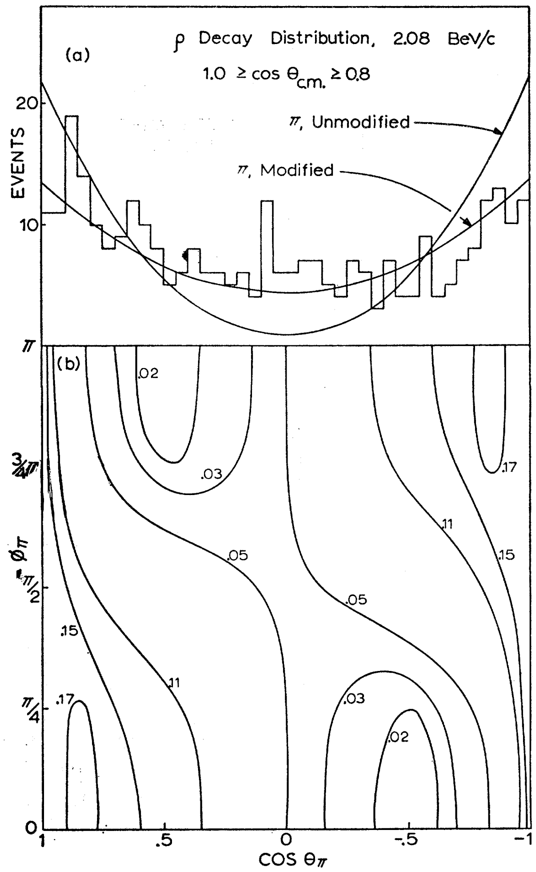

FIG. 2. (a) Comparison of the predicted ρ^+-decay angular distribution in the reaction $\pi^+p \to \rho^+p$ at 2.08 BeV/c with the data of Ref. 16 for $M_{\pi\pi}=(760\pm100)$ MeV, $1.0\geq\cos\theta_{\text{c.m.}}\geq0.8$. The prediction of the unmodified single-pion exchange theory, superposed on the estimated background of 1 event per bin, is shown for comparison. Both theoretical curves are normalized to the total number of events. (b) Contour plot of the normalized ρ^+-decay distribution $I(\theta_\pi,\phi_\pi)$ predicted by the modified single pion exchange theory at 2.08 BeV/c.

However, it is convenient to convert the sum over j into an integral using the Bessel function approximations for the rotation coefficients,[5] valid for z near unity,

$$d_{m'm}{}^j(z)\to(-1)^{m'-m}J_{m'-m}([2j(j+1)(1-z)]^{1/2}),\quad(8)$$

$$e_{m'm}{}^j(z)\to(-1)^{m'-m}K_{m'-m}([2j(j+1)(z-1)]^{1/2}).\quad(9)$$

After some manipulation, the modified matrix elements can be reduced to the form

$$M_{\lambda_c\lambda_d;\lambda_a\lambda_b}(s,t)=(\tfrac{1}{2}(1-x))^{|\lambda-\mu|/2}(\tfrac{1}{2}(1+x))^{|\lambda+\mu|/2}\times B(\mu^2-t)^{-1}[1-L_{|\lambda-\mu|}],\quad(10)$$

where the functions L_α, readily evaluated numerically, are defined by the integral

$$L_\alpha=\int_0^\infty e^{-y+g(y)}\left[1+\frac{4\nu^2}{\mu^2-t}y\right]^{-\alpha-1}dy,\quad(11)$$

$$g(y)=\frac{2pp'(1-x)}{\mu^2-t}\,\frac{4\nu^2y^2}{\mu^2-t+4\nu^2y}.\quad(12)$$

This result for M involves just the normal single-particle exchange amplitude multiplied by a correction factor, the general form of which is clear from Eq. (11). (We could also add the corrected polynomial terms from Eq. (2); however, as noted, we do not feel that these are to be taken seriously.)

The foregoing model has been applied to the reactions $\pi^\pm p(\pi,\omega)\rho^+p$, $\bar{p}p(K^*,K)\bar{Y}Y$, and $np(\pi)\,pn$. The results for $\rho^\pm$ production are shown in Figs. 1 and 2. In the analysis of the 2.08-BeV/c data on the reaction $\pi^+p\to\rho^+p$,[16] the value of the parameter ν, Eq. (7), was taken as $\nu=0.29$ BeV in accord with an exponential fit to the π^+p elastic scattering cross section; the value $\nu=0.24$ BeV derived from elastic π^-p scattering at 4 BeV/c was used in the analysis of the ρ^- production data at that momentum.[17] In each case, it was assumed that the same value of ν could be used to describe the $\rho^\pm p$ scattering in the final state. It may be shown that the use of a somewhat different value of ν for the final state interactions affects mainly the over-all magnitude of M. The possibilities of π and ω exchange were both considered; as would be expected for geometrical reasons, the latter was found to contribute negligibly to $\rho^\pm$ production for reasonable $\pi\rho\omega$ and $pp\omega$ coupling constants. The coupling constants in the case of single π exchange are known: $g_{\pi NN}{}^2/4\pi=14$, $g_{\rho\pi\pi}{}^2/4\pi\approx2.3$ $[\Gamma_\rho\approx120\text{ MeV}]$.

The absolute predictions for the production angular distributions for ρ^+ and ρ^- in the π^+p interactions at 2.08 and the π^-p interaction at 4 BeV/c are compared to the corresponding experimental results in Figs. 1(a) and 1(b). The fit in each case is reasonably good. The predicted cross section is somewhat too large at 2.08 BeV/c, especially for production angles with $\cos\theta_{\text{c.m.}}<0.8$. However, the discrepancy at the larger angles may not be significant, since it can be traced to the transition amplitudes having the largest values of λ and μ. The Fourier-Bessel integral approximation for the partial-wave expansion is least accurate for these amplitudes, and generally overestimates the correct result. Nevertheless, the apparent discrepancy suggests that the lower partial waves should be more strongly suppressed. This would be in accord with the observation of some apparent diffraction structure in elastic $\pi-p$ scattering in this energy range, and in addition, with the expectation that the absorption should properly depend on l rather than on j, as we have assumed. The background at 2.08 BeV/c is small, $\approx$3–4 events per bin, and should cause no difficulty.[16] The predicted cross section at 4 BeV/c is somewhat too small. However, the background problem is much more severe at this momentum, and we have not attempted to

[16] F. E. James and H. Kraybill, Proceedings of the 1964 International Conference on High Energy Physics at Dubna (to be published). F. E. James, dissertation, Yale University, 1964 (unpublished). The authors would like to thank Dr. James and Prof. Kraybill for supplying the data in Figs. 1 and 2.

[17] Aachen-Berlin-Birmingham-Bonn-Hamburg-London(I.C.)-Munich Collaboration, Nuovo Cimento 31, 729 (1964).

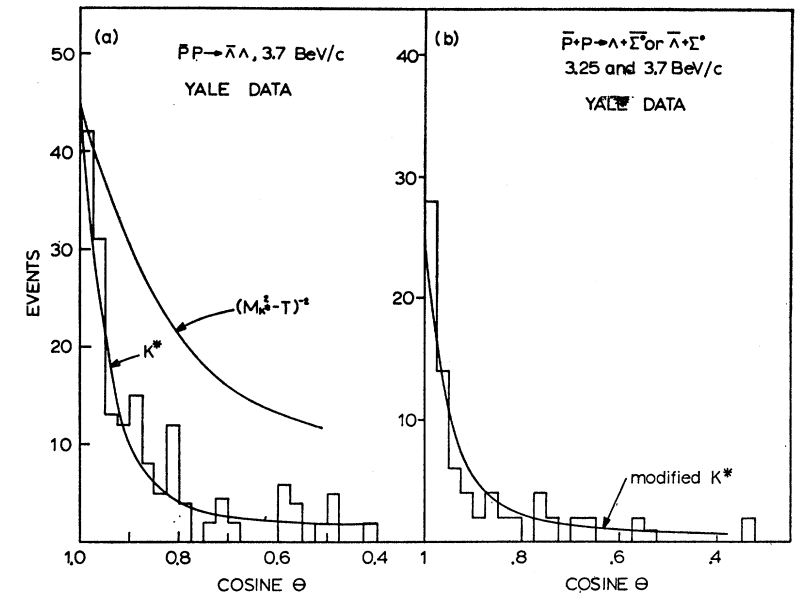

FIG. 3. Comparison of the differential reaction cross section predicted by the modified K^* exchange theory with the data of Ref. 21 for the reactions (a) $\bar{p}p \rightarrow \bar{\Lambda}\Lambda$ at 3.7 BeV/c, $g_{N\Lambda K^*}{}^2/4\pi \approx 2.2$, and (b) $\bar{p}p \rightarrow \bar{\Lambda}\Sigma^0$, $\bar{\Sigma}^0\Lambda$ at 3.25 and 3.7 BeV/c, $g_{N\Sigma K^*}{}^2/4\pi \approx 1.6$.

make a background subtraction. The predicted decay angular distribution of the ρ^+ is compared with the 2.08 BeV/c data and the predictions of the unmodified single particle exchange model in Fig. 2; similar studies of the decay angular distributions at 4 BeV/c have been made by Derado *et al.*,[18] for the reaction $\pi^- p \rightarrow \rho^- p$, and by the British-German group for the reaction $\pi^+ p \rightarrow \rho^+ p$.[19] The decay angular distribution of the ρ in its rest system is of the form[20]

$$I(\theta_\pi,\phi_\pi)=[\rho_{00}\cos^2\theta_\pi+\rho_{11}\sin^2\theta_\pi -\sqrt{2}\mathrm{Re}\rho_{10}\sin 2\theta_\pi\cos\phi_\pi-\rho_{1,-1}\sin^2\theta_\pi\cos 2\phi_\pi], \quad (13)$$

where the $\rho_{mm'}$ are the elements of the ρ spin density matrix, and θ_π and ϕ_π are the polar and azimuthal angles of either decay pion measured relative to the direction of the incoming beam and production plane, respectively. In the unmodified single-pion exchange theory, $\rho_{00}=1$, while all other elements of the density matrix vanish.[12] The decay distribution predicted by that model therefore behaves as $\cos^2\theta_\pi$ as a function of θ_π, and is isotropic in ϕ_π. At 2.08 BeV/c, the modified single pion exchange theory also predicts an isotropic decay distribution in the Treiman-Yang angle ϕ_π because of an accidental near vanishing of $\rho_{1,-1}$ at this energy. The experimental distribution is isotropic within the statistics,[16] and provides no test of the models. On the other hand, the decay distribution in θ_π predicted by the modified theory differs significantly from $\cos^2\theta_\pi$. This prediction is compared with the experimental results in Fig. 2(a) for ρ^+ production angles in the range $1 \geq \cos\theta_\pi \geq 0.8$. The experimental background is estimated to be less than one event per bin. For comparison, we have included the decay distribution predicted by the unmodified single pion exchange theory superposed on this background. The marked improvement in the fit obtained with the present theory is evident. It should be emphasized in this connection that the results in Figs. 1 and 2 were obtained without the introduction of any adjustable parameters. A contour map of the complete (normalized) decay distribution $I(\theta_\pi,\phi_\pi)$ is shown in Fig. 2(b) for $|\cos\theta_\pi| \leq 1$, $\theta \leq \phi_\pi \leq \pi$; becacuse of parity conservation, the distribution for $\pi \leq \phi_\pi \leq 2\pi$ is a mirror image of that shown. The effect of the ρ_{10} term in Eq. (13) is quite striking: If the figure is divided into quarters, the integrated intensity in

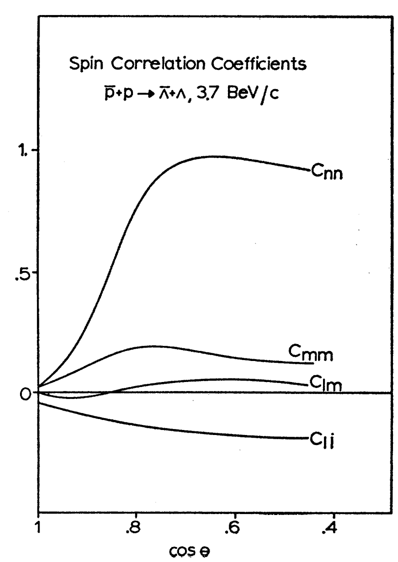

FIG. 4. Predictions of the two-particle spin-correlation coefficients in the reaction $\bar{p}p \rightarrow \bar{\Lambda}\Lambda$ obtained at 3.7 BeV/c using the modified K^*-exchange model.

[18] I. Derado, V. P. Kenney, and W. D. Shephard, Phys. Rev. Letters **13**, 505 (1964).
[19] Aachen-Berlin-Birmingham-Bonn-Hamburg-London(I.C.)-Munich Collaboration, Phys. Letters (to be published).
[20] K. Gottfried and J. D. Jackson, Nuovo Cimento **33**, 309 (1964).

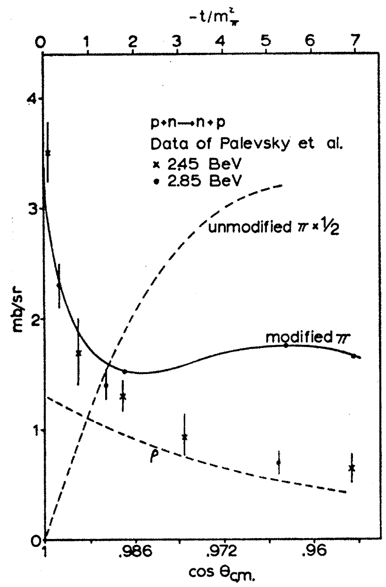

FIG. 5. Predictions of the modified single pion-exchange model for the differential $n-p$ charge exchange reaction at 2.85 BeV. The predictions of the unmodified theory, and of the modified single ρ exchange theory ($g_{\rho NN}^2/4\pi=1$) are shown for comparison. The data are from Ref. 23.

adjacent quarters is in the ratio 2.2:1; the resulting checkerboard pattern should be readily observable. No data have been published on the complete decay distribution; we would strongly urge that such data be included in future experimental papers.

Our results for the reactions $\bar{p}p \rightarrow \bar{\Lambda}\Lambda$, and $\bar{p}p \rightarrow \Lambda\bar{\Sigma}^0$ or $\bar{\Sigma}^0\Lambda$ at 3.7 BeV/c are shown in Figs. 3 and 4. The parameter ν was determined from the Yale-BNL-NYU data[21] on elastic $\bar{p}p$ scattering, $\nu=0.20$ BeV. We have examined the predictions of both K and K^* exchange. For reasonable values of the coupling constants, the latter is more important, and K exchange has been neglected in our results. The predictions for the production angular distributions for the $\bar{\Lambda}\Lambda$ and $\bar{\Lambda}\Sigma^0$, $\bar{\Sigma}^0\Lambda$ reactions are compared to the data[21] in Figs. 3(a) and 3(b). We have included a slowly varying form factor on the K^* propagator, essentially a one pole form with an effective mass at the pole of 1.8 BeV. This affects only the large-angle cross section. Such a form factor would clearly be unnecessary if the absorption were slightly stronger than we have assumed (however, the problem may well arise from the inaccuracy of our approximations at the larger angles). The coupling constants which result from this analysis are $g_{N\Lambda K^*}^2/4\pi\approx 2.2$, $g_{N\Sigma K^*}^2/4\pi\approx 1.6$. These results would suggest a value for the vector F/D mixing parameter in the unitary symmetry model of $\alpha'\sim 0.8$, in reasonable agreement the value expected from other considerations. The predictions of the K^* exchange model for the two-particle spin correlation parameters are shown in Fig. 4. The conventions are those of Durand and Sandweiss.[22] We note in particular that the parameter C_{nn} attains measurably large values close to the forward direction. An experimental determination of this parameter would provide a sensitive test of the theory, hence, some detailed information on the reaction dynamics.

Finally, we show in Fig. 5 the results obtained for the differential np charge exchange reaction at 2.85 BeV.[23] The parameter ν in this case was derived from the pp elastic scattering data at 3 BeV, $\nu=0.23$ BeV. The theoretical curve for single π exchange shown in Fig. 5 is the absolute prediction of the modified theory,[24] using the usual pion-nucleon coupling constant. The prediction of the unmodified single π exchange theory is shown for comparison. The modified result fits the experimental data quite well at the smaller angles. The secondary maximum in the curve at larger angles arises from the contributions of the lowest few double-helicity flip amplitudes.[24] These are quite sensitive to the degree of absorption, and to the shorter range parts of the interaction, and are not given reliably by the present theory. The predicted contribution of ρ-meson exchange is also shown in Fig. 5 for a ρ-nucleon vector coupling constant $g_{\rho NN}^2/4\pi\approx 1$. Althought there is no evidence for ρ exchange in the present experiment, the existence of some contribution cannot be precluded because of the uncertainties in the low partial-wave amplitudes.

One of the authors (L.D.) would like to thank the Physics Division of the Aspen Institute for Humanistic Studies for the hospitality afforded him while part of this work was performed.

[21] C. Baltay, T. Ferbel, J. Sandweiss, H. Taft, B. Culwick *et al.*, in *Nucleon Structure, Proceedings of the International Conference at Stanford, 1963*. (Stanford University Press, Stanford, California, 1964), p. 267 ff.

[22] L. Durand, III and J. Sandweiss, Phys. Rev. **135**, B540 (1964).

[23] H. Palevsky, J. A. Moore, R. L. Stearns, H. R. Muether, R. J. Sutter *et al.*, Phys. Rev. Letters **9**, 509 (1962).

[24] This reaction has also been considered by G. A. Ringland and R. J. N. Phillips, Phys. Letters **12**, 62 (1964). The results of those authors for the π-exchange contribution are similar to those reported here; however, the ρ-exchange contribution appears to be incorrect.