Decay of Long-Lived Ho^{166}†

C. W. REICH* AND J. E. CLINE
National Reactor Testing Station, Phillips Petroleum Company, Idaho Falls, Idaho
(Received 9 November 1964)

The gamma radiation following the decay of the long-lived Ho^{166} activity has been studied using lithium-ion-drifted germanium counters and NaI(Tl) scintillation detectors. In addition to providing more precise energy measurements for many of the transitions than was possible in our earlier study, the higher resolution of the germanium detector revealed that two of the previously reported high-energy gamma rays are doublets. Thus, the level previously reported at 1680 keV has been revealed as two levels, separated by 26 keV. Gamma-gamma coincidence studies have revealed the existence of transitions having energies of 96, 122, 137, and 163 keV which excite these two levels. Gamma-gamma directional correlation measurements have been carried out on prominent cascades. The results of the correlations confirm the spins previously reported. The existence of levels in Er^{166} having the following energies in keV and spin and parity assignments is proposed: 0, 0+; 90, 2+; 265, 4+; 544, 6+; 787, 2+; 861, 3+; 910, 8+; 957, 4+; 1074, 5+; 1215, 6+; 1374, 7+; 1662, 5−; 1688, 5−; 1784, 6−; and 1825, 6−. The gamma-ray relative intensities indicate that there is no appreciable beta feeding of the two 5- states. The mixing-ratio parameters δ for the interband transitions of 529, 830, and 810 keV are all positive and large. For the first two of these, the M1 admixture is less than 0.1%, while for the last, it is about 0.1%. An interpretation of the properties of the positive-parity states is given in terms of the strong-coupling nuclear model with band mixing and in terms of a generalized asymmetric-rotator model. Possible interpretations of the nature of the negative-parity states are presented.

I. INTRODUCTION

THE results of an investigation at this laboratory of the decay of the long-lived Ho^{166} activity have recently been published.[1] From the analysis of the gamma-ray and conversion-electron spectra which were observed using scintillation spectrometers and silicon surface-barrier detectors, respectively, the existence of twenty-seven gamma-ray transitions in Er^{166} was definitely established. In the proposed decay scheme these transitions were placed between thirteen states in Er^{166}. These states may be conveniently arranged into three groups. The first of these is a $K=0$ band, in which all the states up through the 8+ member are observed. The second of these is a $K=2$ band, in which all states up through the 7+ member are observed. The third group consists of negative-parity states.

The spin and parity assignments for the Er^{166} states were previously made on the basis of relative gamma-ray intensities, collective-structure considerations and, in some instances, relative intensities of internal-conversion electrons. It was felt that directional-correlation measurements on the gamma-ray cascades in Er^{166} would be of considerable importance. Such measurements not only would provide independent determinations of the angular momenta but also would provide values for the mixing parameters δ for those transitions which consist of a multipole mixture. These parameters can be important in providing information concerning the purity of the collective wave functions which describe the low-lying positive-parity states and in making configuration assignments to the observed intrinsic states. Grace *et al.*[2] have studied the directional correlations of the 710–810-keV, the 710–184-keV, and the 810–184-keV cascades. They found that the first two of these were isotropic to within 1 and 5%, respectively. For the third cascade, they reported the following Legendre-polynomial expansion coefficients: $A_2/A_0 = -(0.19\pm0.06)$ and $A_4/A_0 = +(0.01\pm0.06)$, and concluded that unambiguous angular-momentum assignments were not permitted by these data. Quite recently, Gerdau *et al.*[3] have reported a series of directional-correlation measurements on some of the prominent gamma-ray cascades in Er^{166}.

Extensive gamma-gamma directional-correlation measurements have been made at this laboratory on many of the cascades in Er^{166}. These measurements provide unique angular-momentum assignments for most of the observed states. In all cases studied, these assignments agree with those proposed in Ref. 1. In addition, the gamma-ray spectrum from the long-lived Ho^{166} activity has been investigated using a lithium-drifted germanium detector. These data indicate that the previously reported level at 1680 keV is actually a doublet. Four low-energy transitions which populate the members of this doublet have been observed from coincidence measurements. The positive-parity states are interpreted in terms of the strong-coupling and asymmetric-rotator models of the nucleus. Various possible interpretations of the nature of the negative-parity states are also presented.

† Work performed under the auspices of the U. S. Atomic Energy Commission.
* Temporary address: Institute for Theoretical Physics, Copenhagen, Denmark.
[1] J. E. Cline and C. W. Reich, Phys. Rev. 129, 2152 (1963).
[2] M. A. Grace, R. T. Taylor, and P. B. Treacy, Phil. Mag. 3, 90 (1958).
[3] E. Gerdau, W. Krull, L. Mayer, J. Braunsfurth, J. Heisenberg, P. Steiner, and E. Bodenstedt, Z. Physik 174, 389 (1963). We are grateful to these authors for kindly sending us a copy of their manuscript prior to publication.

II. EXPERIMENTAL TECHNIQUES AND DATA

A. Source Preparation

The Ho^{166} activity was produced by a three-month irradiation of 10 mg of holmium metal powder in a high-flux facility of the Materials Testing Reactor ($\approx 6\times 10^{14} n/cm^2$-sec). After irradiation, the sample was removed and allowed to decay for about one month in order to reduce the 27-h Ho^{166} activity to a negligible level. At the end of this period, the sample was dissolved in HNO_3 and the activities not following rare-earth chemistry were removed. These activities consisted mainly of Ta^{182} and Sc^{46}. The purified sample was loaded on a Dowex 50-X4 column and the various rare earths were eluted, using α-hydroxyisobutyric acid as the eluting agent. During this elution, observable amounts of the rare-earth activities Tm^{170}, Er^{169}, and Tb^{160} were separated. In order to minimize the amount of possible contaminants, the Ho^{166} activity used in the experiment was selected from the center of the holmium elution curve. This center-cut solution was treated with fuming nitric acid and perchloric acid to destroy the organic matter. The activity was then taken up in concentrated HCl and passed through a Dowex 2-X10 anion column as an additional step to further insure that trace amounts of Ta^{182} would not be present in the Ho^{166} source. The resulting solution was then concentrated until a saturated solution of $HoCl_3$ was produced.

B. Gamma-Ray Studies

The spectrum of gamma rays emitted in the decay of the Ho^{166} activity was studied using a lithium-drifted germanium semiconductor detector.[4] This counter had a depletion depth of 1 mm and a surface area of 1 cm^2. The full width at half-maximum of the gamma-ray peaks was found to be ≈ 5 keV when the detector was operated at 80°K. A typical gamma-ray spectrum obtained is shown in Fig. 1. Of particular interest is the region above the Compton distributions from the strong 710- and 810-keV gamma rays. The good resolution afforded by this detector allows one to resolve clearly the photopeaks of the weaker gamma rays having energies above about 600 keV. The presence of these gamma rays has been established previously,[1] although precise energies were not obtained for them because they were not well resolved in that study. The energy scale for the spectrum shown in Fig. 1 was established using the energies reported by Geiger *et al.*[5] for the strong 184-, 280-, 710-, 810-, and 830-keV peaks. It was found that the observed pulse height was a linear function of gamma-ray energy in this region. To establish the energies of the gamma-ray transitions above ≈ 850 keV, sources of Bi^{207}, Co^{60}, and Y^{88} were used. The photopeak efficiency of this detector was measured at three energies using calibrated sources of Ce^{139} (166 keV) and Na^{22} (511 and 1280 keV). In order to obtain relative gamma-ray intensities, the data shown in Fig. 1 were divided into three energy regions: below 600 keV, between 600 and 850 keV, and greater than 850 keV. The relative intensities of the gamma rays within each energy region were obtained from the observed photopeak intensities using interpolated values for the photopeak efficiencies. These relative intensities and energies were used as an aid in the unfolding of a gamma-ray spectrum taken with a 3 in.$\times$3 in. NaI(Tl) scintillation detector. The spectrum which was unfolded is shown in Fig. 2. The energies and transition intensities obtained from this analysis are given in Table I. From

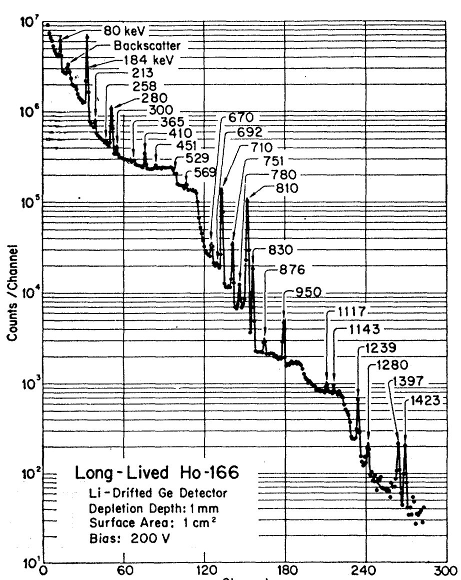

FIG. 1. Gamma-ray spectrum of the long-lived Ho^{166} activity taken using a lithium-ion-drifted germanium detector.

[4] This detector was obtained from Solid State Radiations, Inc. We wish to thank Dr. F. P. Ziemba and the staff at SSR for making this detector available to us.

TABLE I. Transition energies and intensities.

E_γ (keV)	I_t (%/β^- transition)	E_γ (keV)	I_t (%/β^- transition)
80± 2	100%	610±10	2.5±1.0
96[a]	0.49[b]	670± 2	5.7±1.0
122[a]	0.56[b]	692± 4	1.4±0.8
136[a]	0.14[b]	710± 1	57.8±6.0
163[a]	0.16[b]	751± 1	14.4±2.0
184± 1	97.8±6.0	780± 2	3.7±1.5
213± 3	4.5±2.0	810± 1	59.6±6.0
258± 5	$1.0_{-0.5}^{+2.0}$	830± 1	10.6±2.0
280± 1	31.6±3	876± 2	0.8±0.2
300± 2	3.7±1.0	950± 2	3.3±0.5
365± 3	1.7±1.0	1117± 3	0.3±0.1
410± 1	12.5±2.0	1143± 3	0.2±0.1
451± 3	2.7±1.0	1239± 3	1.0±0.3
465±10	1.7±1.0	1280± 3	0.3±0.1
529± 5	11.8±1.0	1397± 3	0.7±0.3
569± 5	7.1±2.0	1423± 3	0.6±0.3
596±10	0.8±0.4		

[a] The energies have been obtained from energy differences of previously established levels as explained in the text.
[b] These transition intensities represent lower limits as explained in the text. However, the uncertainties in the intensities of these transitions relative to one another are estimated to be about 10%.

[5] J. S. Geiger, R. L. Graham, and G. T. Ewan, Nucl. Phys. **30**, 409 (1962).

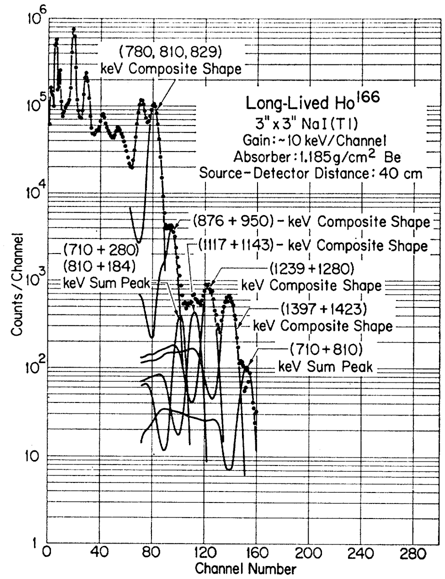

FIG. 2. Gamma-ray spectrum of the long-lived Ho^{166} activity taken using a 3-in.×3-in. NaI(Tl) detector. The composite shapes used in the analysis of the high-energy region are also indicated.

the spectrum shown in Fig. 1, one may observe that each of the previously reported peaks at 1415 and 1135 keV is composed of two gamma rays, separated by ≈25 keV. A reanalysis was made of some of the coincidence data previously obtained.[1] The coincidence relationships of each member of the two gamma-ray pairs was the same, indicating the existence of levels at 1662 and 1688 keV instead of a single level at 1680 keV as was previously reported.[1]

C. Coincidence Studies of Low-Energy Transitions

Gamma-gamma coincidence studies were made in an attempt to observe transitions to the levels at 1662 and 1688 keV from the upper two levels. These measurements used a 256×256 channel multiparameter analyzer. Signals from two 3 in.×3 in. NaI(Tl) detectors, each located 12 cm from the source, were amplified and routed into the two sides of the analyzer. The detectors were placed at 90° with respect to one another and a 20-mil thickness of Ta was placed between them to reduce the probability that low-energy Compton-scattered gamma rays created in one detector would be detected in the other. Spectra were taken coincident with 16 channels spanning the region from 1000 to 1600 keV. Figure 3 shows some of these coincidence spectra. The energy regions with respect to which these spectra are coincident are shown in Fig. 4. Spectrum D of Fig. 3 is coincident with the coincidence sum peak at 1520 keV. This peak is composed mainly of (810–710)-keV coincidence summing and is therefore coincident with 184- and 80-keV transitions. Spectra A, B, and C are coincident with the peak containing the unresolved gamma rays of 1397 and 1423 keV, which are transitions from the two levels at 1662 and 1688 keV to the 4+ level at 265 keV. Thus, these spectra also contain the 184- and 80-keV transitions along with their associated K x ray. In addition, these spectra show the presence of four gamma rays of 96, 122, 136, and 163 keV. That these peaks do not arise from some anomalous scattering effect is demonstrated by their absence in spectrum D as well as in spectra taken coincident with the 1240-keV peak. Furthermore, spectra taken coincident with the unresolved doublet at 1130 keV show these same four transitions. This doublet represents the transitions from the same two levels to the 6+ state at 545 keV. These four new transitions have been incorporated into the level scheme presented in Fig. 5. The energies reported for the transitions represent differences between known energy levels. The location of the peaks in the coincidence spectra, although somewhat uncertain, are consistent with these values.

These transitions appear in spectra A, B, and C with different relative intensities, since different relative amounts of the 1397- and 1423-keV component are included in the "single-channel" windows. After removal of the contribution from random coincidences,

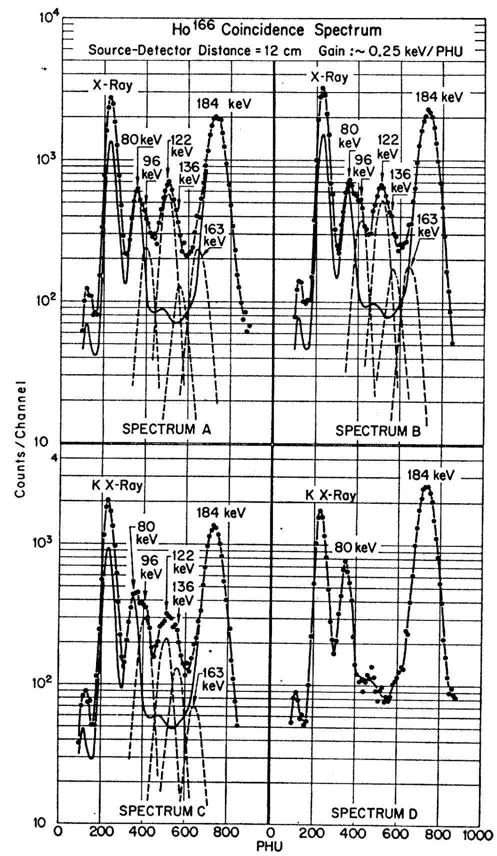

FIG. 3. Coincidence spectra showing the existence of four weak low-energy transitions which are believed to excite the states at 1662 and 1668 keV. The abscissa are in arbitrary pulse-height units.

TABLE II. K-conversion coefficient data for the weak low-energy transitions in Er^{166}.

E_γ (keV)	Measured[a] K-conversion coefficient	$E1$ α_K	$E1$ α_T	$M1$ α_K	$M1$ α_T	$E2$ α_K	$E2$ α_T	$M2$ α_K	$M2$ α_T
96		0.31	0.37	2.6	3.2	1.1	3.6	23	32
136		0.11	0.14	0.95	1.17	0.48	1.09	6.2	9.8
$[\alpha_K(96)+0.56\alpha_K(136)]$	2.8								
122		0.16	0.20	1.35	1.66	0.65	1.61	9.5	14.1
163		0.08	0.10	0.60	0.69	0.30	0.51	3.4	5.7
$[\alpha_K(122)+0.45\alpha_K(163)]$	1.0								

[a] As is mentioned in the text, the uncertainties on these values are difficult to obtain, but are estimated to be of the order of 20%.

each spectrum was analyzed by the method of successive subtractions. The 184- and 80-keV transitions and their associated K x ray were removed using the shape given in D. The resultant spectra contained only the four new transitions and their associated K x ray. From the analysis of these three spectra, values were obtained for the intensity of the 136-keV gamma ray relative to that of the 96-keV gamma ray and for the intensity of the 163-keV gamma ray relative to that of the 122-keV gamma ray. Using these ratios, effective K-conversion coefficients were determined for the 96–136-keV pair and for the 122–163-keV pair. These values are given in Table II along with the theoretical conversion coefficients assuming $E1$, $M1$, $E2$, and $M2$ multipolarity assignments for these transitions. The uncertainties associated with the experimental values are difficult to estimate, but it is felt that they are of the order of 20%.

From a comparison of the measured coefficients with the theoretical values, it is apparent that the 96- and 122-keV transitions are either $M1$ or $E2$. An $E1$ assignment to either of these transitions would require an

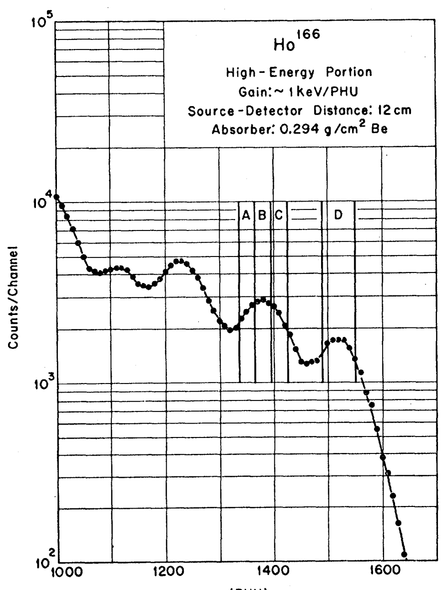

FIG. 4. A portion of the Ho^{166} gamma-ray spectrum showing the energy regions with respect to which the spectra of Fig. 3 are coincident.

$M2$ admixture of about 10%, an admixture considered quite unlikely. Furthermore, it appears that the 96-keV transition is predominantly $M1$, while the 122-keV transition is predominantly $E2$. The data do not rule out the possibility that either or both of them are pure multipoles. No multipolarity assignments were made for the 136- or 163-keV transitions although the data are more consistent with an $M1$ assignment for both if the 96- and 122-keV transitions are pure $M1$ and $E2$ transitions, respectively.

Relative transition intensities for the four transitions were obtained by adding the intensities from the analyses of spectra A, B, and C, and correcting for internal conversion. It is noted from Table II that the total conversion coefficients are nearly the same whether the transitions are $M1$ or $E2$. Absolute transition intensities, listed in Table I, presume that there is no β^- feeding of the levels at 1688 and 1662 keV and that the 1117-, 1143-, 1397-, and 1423-keV transitions represent the sole mechanism by which these levels are depopulated. Justification for the former assumption arises from the fact that the sum of the transition intensities for the 96-, 122-, 136-, and 163-keV transitions equals, to within 5%, that of the 184-keV transition in each of the first three spectra of Fig. 3. Correction was made for directional-correlation effects, internal conversion, and inclusion in the "single-channel" windows of a portion of the distribution from the coincidence sum peak at 1520 keV.

The second assumption made in obtaining the absolute intensities for these transitions, that the levels at 1688 and 1662 keV do not feed the spin 4, 5, and 6 members of the $K=2$ band, is probably not justified. If the two levels have $K>2$, it is probable that such transitions do exist and may, in fact, represent the dominant mode of de-excitation. Coincidence measurements were made in an attempt to observe such transitions. Spectra were taken coincident with 10-keV windows from 50 to 180 keV with two different experimental arrangements. One of these used a 3-in.×3-in. NaI(Tl) crystal as the "single-channel" detector while the other used a $\frac{1}{8}$-in.×2-in. NaI(Tl) detector. The latter detector was used in an attempt to reduce the magnitude of the Compton distribution included in the "single-channel" windows from higher energy

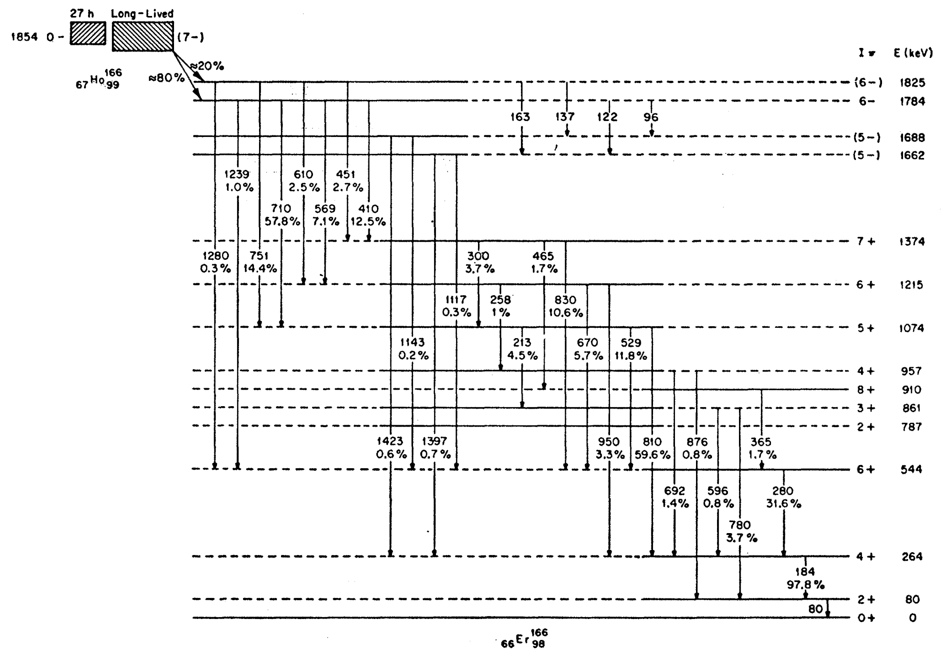

FIG. 5. Proposed decay scheme for the long lived Ho^{166} activity.

gamma rays. In both experiments the contribution to the window counting rates from the 184- and 280-keV transitions were such that the region of interest in the coincidence spectra represented coincidences with these two components. No peaks were observed which could not be attributed to this effect. An enhancement of the 1130- and 1410-keV peaks in those spectra coincident with the region from 90 to 170 keV indicated the presence of the 96-, 122-, 136-, and 163-keV transitions. Transitions to the $K=2$ band would have to have been $\sim$20 times more intense than the 1397- and 1423-keV transitions in order to have been observed in the spectra. Thus, if these transitions exist, their intensities must be less than this amount. Since little knowledge has been gained on possible transitions to the $K=2$ band from the states at 1688 and 1662 keV, the intensities listed for the transitions which populate these states represent only lower limits.

D. Directional-Correlation Measurements

The apparatus and general procedures employed at this laboratory for carrying out gamma-gamma directional-correlation measurements have been discussed previously.[6] Because of the relatively low specific activity of the source, the internal dimensions of the Lucite source holder were increased to a diameter of $\frac{1}{8}$ in. and a height of $\frac{3}{8}$ in. The gamma-ray detectors were 3-×3-in. NaI(Tl) crystals mounted on Dumont 6363 photomultiplier tubes. At a source-detector distance of 10 cm, the gamma rays from the Ho^{166} source produced a counting rate of $\approx 7.5\times 10^3$/sec in the detectors. The resolving time (2τ) of the coincidence circuit was 0.1 μsec. Random coincidence rates encountered in the measurements were typically of the order of a few percent of the total coincidence rate.

Coincidence data were taken in one quadrant at equally spaced angles from 90° to 180°. Generally, 6 angles were used, although, for the 280–184-keV cascade, data were taken at 11 angles. For directional-correlation measurements involving the weak, high-energy gamma rays, counting times of the order of 80 000 sec were necessary in order to accumulate sufficient counts in each coincidence spectrum to make a reasonable fit to the various peaks. For this reason only a relatively small number of measurements were carried out for each correlation. In order that these measurements yield the most precise estimates possible of the Legendre-polynomial expansion coefficients, the coincidence data were taken at three angles: 90°, 135°, and 180°. It has been shown[7] that this choice of angles, for properly chosen relative frequencies of measurement, provides more precise estimates of the coefficients than configurations consisting of a larger number of angles and a correspondingly smaller number of observations per angle.

Typical coincidence spectra obtained in these experiments are shown in Figs. 6(a) and 6(b). The intensities of the various prominent gamma rays in the coincidence spectra were obtained by the customary unfolding technique, using known detector response functions. The heights of the full-energy peaks of the gamma rays obtained were used as the coincidence intensities to be analyzed. These data were fitted by least-squares to a function of the form $W(\theta)=A_0+A_2P_2(\cos\theta)+A_4P_4(\cos\theta)$.

[6] C. W. Reich, R. P. Schuman, J. R. Berreth, M. K. Brice, and R. L. Heath, Phys. Rev. **127**, 192 (1962).

[7] C. W. Reich, J. A. Merrill, and E. D. Klema, Nucl. Instr. Methods **23**, 36 (1963).

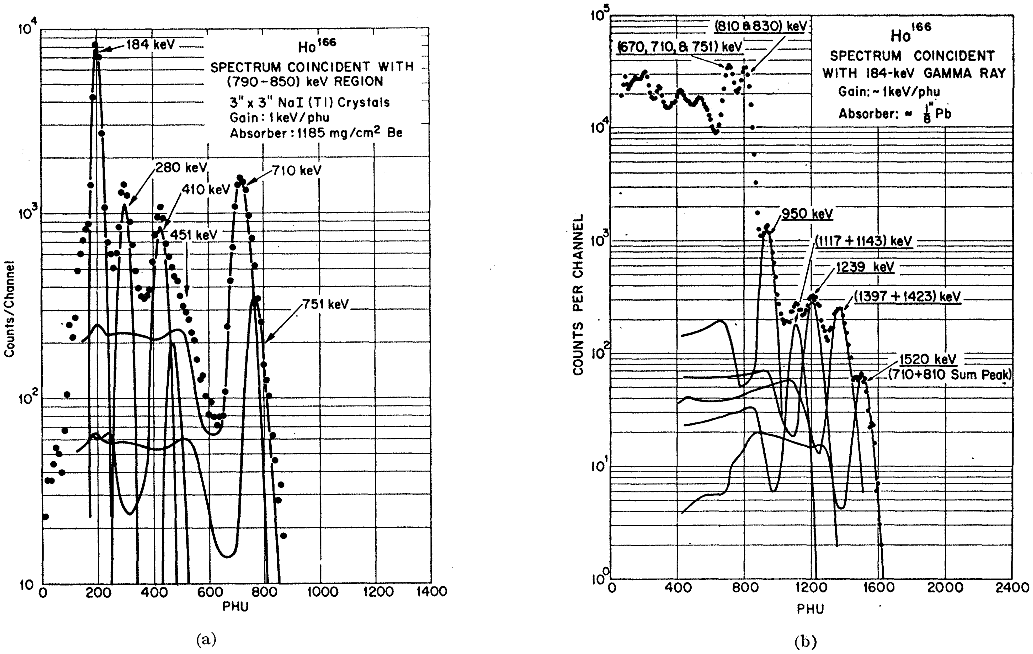

FIG. 6. (a) Spectrum of gamma radiation coincident with the composite peak at ≈820 keV. The distance from the source to each detector is 10 cm. Because of the uncertainties involved in the unfolding process, no attempt was made to obtain correlation data for the weak 451- and 751-keV transitions. (b) Spectrum of gamma radiation coincident with the 184-keV peak. In order to reduce the coincidence summing effects, the distance from the source to the detector whose spectrum is displayed above was increased to 17.5-cm and a $\frac{1}{8}$-in. thickness of Pb was placed over the detector.

From the resulting values of the coefficients and their variances, the ratios $r_\lambda(\equiv A_\lambda/A_0)$ and their standard deviations were obtained. No correction of the r_λ for the "bias" inherent in this manner of estimating them was made, since it has been shown[7] that for these experiments this effect is many times smaller than the statistical uncertainties involved. The ratios were corrected for the finite angular resolution of the detectors, using experimentally determined correction factors in which the variation of photopeak efficiency across the face of the crystal was included.

Since many of the cascades in Er^{166} proceed through the 184-keV transition, it was convenient in making the measurements to include a portion of the photopeak of this gamma ray in the single-channel window and to record the coincidence spectra in the multichannel analyzer. For these measurements it was necessary to correct the resultant data for effects arising from the inclusion, in the single-channel window, of Compton distributions from the higher energy gamma rays in the spectrum. Contributions from this effect were measured by placing the single-channel window below the 184-keV photopeak where the contribution from this gamma ray was negligible. Owing to the large range of energies and intensities of the transitions coincident with the 184-keV gamma ray, it was necessary to vary the experimental conditions in order to study the various correlations. To measure the directional correlation functions for the 80–184-, 280–184-, and 810–184-keV cascades, data were taken with 10-cm source-detector distances and with Pb shields placed between the detectors to reduce the effects of spurious coincidences arising from large-angle scattering of photons between the two crystals. For the 950–184-keV and (1397+1423)–184-keV cascades, the detector used to provide the coincidence spectrum was moved to a distance of 17.5 cm from the source and was covered with a $\frac{1}{4}$-in.-thick Pb shield. These procedures were necessary to reduce the coincidence summing effects sufficiently so that the correlation data could be meaningfully analyzed.

Similar measurements were made with the single-channel window including a portion of the 280-keV photopeak. Correlation functions were measured for the 529–280- and 1239–280-keV cascades, and for the composite (1117+1143)–280-keV cascade. As in the measurements described above, it was necessary to reduce the solid-angle subtended by one of the detectors and to shield it from the low-energy gamma rays for the last two measurements. Corrections to the data for the effects of Compton distributions in the single-channel window were made as described above.

Several measurements were made with the single-channel analyzer set on the 810–830-keV peak. From these data, directional correlations were measured for the 810–710-, 810–184-, 830–280-, and 830–410-keV cascades. The intensity of the 810-keV gamma ray is much larger than that of the 830-keV gamma ray. Consequently, no correction of the coincidence data for fluctuations in the single-channel counting rate was

TABLE III. Summary of directional-correlation data for Er^{166}.

Cascade (keV)	Single-channel position (keV)	Number of angles, number of runs/angle, and counting time/measurement (sec)	r_2[a]	r_4[a]
184–80	184	11, 7, 2000	$+0.065\pm0.006$	$+0.001\pm0.012$
280–184	184	11, 6, 800	$+0.116\pm0.010$	$+0.010\pm0.018$
280–184	184	11, 7, 2000	$+0.100\pm0.005$	-0.003 ± 0.009
410–830	(810,830)	6, 8, 4000	$+0.053\pm0.008$	$+0.030\pm0.012$
410–830	(810,830)	3, –, 4000[b]	$+0.056\pm0.039$	-0.025 ± 0.050
529–280	280	6, 6, 4000	-0.003 ± 0.009	-0.101 ± 0.016
569–950	950	3, 4, 40 000	$+0.163\pm0.023$	-0.004 ± 0.034
710–529	710	6, 5, 20 000	$+0.033\pm0.011$	-0.019 ± 0.019
710–810	(810,830)	6, 8, 4000	$+0.018\pm0.008$	-0.002 ± 0.011
710–810	710	6, 13, 4000	$+0.011\pm0.008$	$+0.006\pm0.012$
710–810	710	6, 5, 20 000	$+0.012\pm0.004$	$+0.009\pm0.006$
810–184	184	11, 6, 800	-0.138 ± 0.009	-0.052 ± 0.016
810–184	(810,830)	6, 8, 4000	-0.148 ± 0.006	-0.037 ± 0.010
810–184	(810,830)	6, 10, 4000	-0.151 ± 0.004	-0.042 ± 0.007
830–280	(810,830)	3, 10, 4000	-0.108 ± 0.013	-0.048 ± 0.017
830–280	(810,830)	6, 8, 4000	-0.118 ± 0.009	-0.047 ± 0.015
830–280	(810,830)	6, 10, 4000	-0.106 ± 0.013	-0.037 ± 0.022
950–184	184	3, 10, 80 000	$+0.102\pm0.008$	-0.009 ± 0.013
(1117+1143)–280	280	3, 5, 80 000	-0.122 ± 0.022	$+0.011\pm0.030$
1239–280	280	3, 5, 80 000	$+0.159\pm0.013$	$+0.015\pm0.019$
(1397+1423)–184	184	3, 10, 80 000	-0.053 ± 0.014	-0.014 ± 0.019

[a] After correction for the finite angular resolution of the detectors.
[b] Relative frequencies of 8, 14, 8 at 90°, 135°, and 180°, respectively.

made for the correlations involving the 830-keV gamma ray. These fluctuations were small and did not appear to exhibit any long-term systematic trend. Hence, it was assumed that no significant error in the average counting rate at a given angle would be incurred through this procedure. In order to obtain the correlation function of the 830–410-keV cascade, it was necessary to correct the observed intensity of the 410-keV peak for events arising from the 410–300–810-keV triple cascade. This contribution was calculated to be isotropic and amounted to about 20% of the average intensity of the 410-keV peak. In the analysis of the intensity of the 184-keV peak for the correlations of the 810–184-keV gamma-ray cascade, the contribution to this peak from those events which arose from the 830–280–184-keV cascade were removed. About 16% of the intensity of the 184-keV peak arose from this effect. The angular dependence of these events was inferred from a knowledge of the spin sequences involved (see below).

In order to measure the correlation function of the 569–950-keV cascade it was necessary to reduce the effects of coincidence summing in the region of 950 keV. Consequently, a $\frac{1}{4}$-in.-thick Pb shield was placed over one of the detectors and the 950-keV peak was scanned with that detector.

With the single-channel analyzer set to span the 710-keV peak, measurements were made of the correlation functions for the 710–810- and 710–529-keV cascades. The results of all of the directional correlation measurements carried out on the Er^{166} gamma-ray cascades are summarized in Table III. Two typical correlation functions are shown in Fig. 7.

III. DISCUSSION OF DIRECTIONAL CORRELATION RESULTS

In an interpretation of the results of directional correlation measurements involving transitions of mixed multipolarities, it is necessary to specify the definition of the mixing ratio, $\delta^2 \equiv I_{L+1}/I_L$. The convention used here is that of Biedenharn and Rose, in which the spin of the intermediate state always appears on the right in the reduced matrix elements regardless of whether this state is the initial or final one for the transition.[8,9] Since the multipole mixtures most commonly encountered are dipole quadrupole, the measured sign of δ for such a transition which is the first member of a cascade will differ from that in which the same transition is the second member of a cascade. Several experiments in which this sign change has been observed have been reported.[10] In the correlations measured in the present work, the 810-keV transition is encountered both as the first member of a cascade and as a second member of a cascade. The values of δ observed for this transition in these two instances do exhibit this change of sign.

The spin and parity assignments of the ground, first, and second excited states of Er^{166} are 0+, 2+, and 4+, respectively. The $E2$ characters of the 80- and 184-keV transitions have been established from the measured

[8] L. C. Biedenharn and M. E. Rose, Rev. Mod. Phys. **25**, 729 (1953).
[9] L. C. Biedenharn, in *Nuclear Spectroscopy Part B*, edited by F. Ajzenberg-Selove (Academic Press, Inc., New York, 1960), Chap. V.C.
[10] S. Ofer, Phys. Rev. **114**, 870 (1959); R. W. Grant and D. A. Shirley, Phys. Rev. **130**, 1100 (1963).

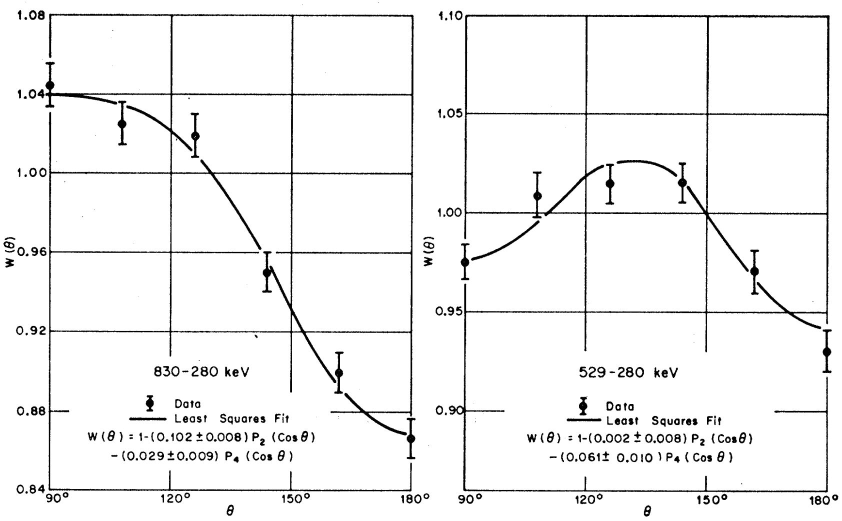

FIG. 7. Typical measurements of the directional correlation of two of the gamma-ray cascades in Er^{166}. The solid lines represent the least-squares fit using a function of the form $W(\theta)=A_0+A_2P_2(\cos\theta)+A_4P_4(\cos\theta)$.

internal-conversion coefficients.[11] Measurements[3,12] of the lifetimes of the 80- and 265-keV states have yielded $B(E2)$ values for the decay of these states[12] which indicate a 4+ assignment for the second excited state. The consistency of the experimentally determined g factors for the 80-keV state[13] and the 265-keV state[3] also support 2+ and 4+ assignments, respectively, for these states. These assignments are also to be expected on the basis of the collective nature of the low-lying states in deformed even-even nuclei.[14] The measured Legendre-polynomial coefficients r_2 and r_4 in the directional correlation of the 184–80-keV cascade are smaller than the calculated values for a 4(2)2(2)0 cascade. This is an expected result, since the directional correlations of other gamma-ray cascades[13,15] involving the 80-keV state as an intermediate state have shown marked attenuation arising apparently from the magnetic interaction between the $4f$ electron shell and the magnetic moment of the nucleus in this state.[13] For an assumed 4(2)2(2)0 cascade, the present data indicate an attenuation factor G_2 of 0.64 ± 0.06. (The uncertainties in the experimental value of r_4 for this cascade are such that no useful estimate of G_4 is afforded by these data.) This value is in agreement with the value of 0.63 ± 0.03 obtained by Marklund *et al.*[15] and with the value of 0.80 ± 0.08 of Bodenstedt *et al.*[13] derived from measurements on the 1380–80-keV, 0(2)2(2)0, cascade in the decay of the 27-h activity in Ho^{166}. The measured directional correlation of the 184–80-keV cascade hence is consistent with the assumption of a 4(2)2(2)0 cascade.

Several of the measured directional correlations involving the 184-keV, 4(2)2, transition as the second member of the cascade are shown in Fig. 8. Also shown are several theoretical parametric plots, calculated for various assumed values of the spin of the initial state. From an inspection of the figure it is apparent that the spin of the state at 545 keV is either 4, 5, or 6. The first two assignments (which require $\delta\approx-0.26$ and ≈-0.31, respectively, for the 280-keV transition) lead to difficulty in the interpretation of the correlation of the 830–280- and the 529–280-keV cascades. These relatively small values of δ would make it impossible to explain the large values observed for r_4, particularly that of the 529–280-keV correlation. No such difficulties, however, are encountered with a spin-6 assignment.

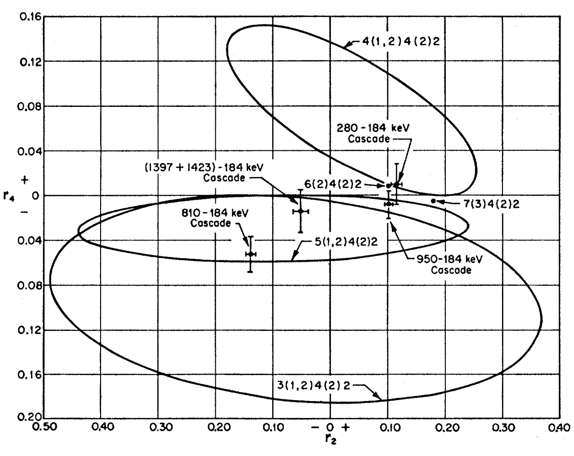

FIG. 8. A plot of the ratio r_4 versus r_2 for various correlation functions in which the final transition takes place from a state of spin 4 to a state of spin 2. A function of the form $W(\theta)=1+r_2P_2(\cos\theta)+r_4P_4(\cos\theta)$ is assumed. Also included are some of the experimentally determined correlation coefficients, after correction for the finite angular resolution of the detectors.

---

[11] K. P. Jacob, J. W. Mihelich, B. Harmatz, and T. H. Handley, Phys. Rev. **117**, 1102 (1960); B. Harmatz, T. H. Handley, and J. W. Mihelich, *ibid.* **123**, 1758 (1961).

[12] Angela C. Li and A. Schwarzschild, Phys. Rev. **129**, 2664 (1963).

[13] E. Bodenstedt, H. J. Korner, C. Gunther, and J. Radeloff, Nucl. Phys. **22**, 145 (1961).

[14] K. Alder, A. Bohr, T. Huus, B. R. Mottelson, and A. Winther, Rev. Mod. Phys. **28**, 432 (1956).

[15] I. Marklund, B. van Nooijen, and Z. Grabowski, Nucl. Phys. **15**, 533 (1960).

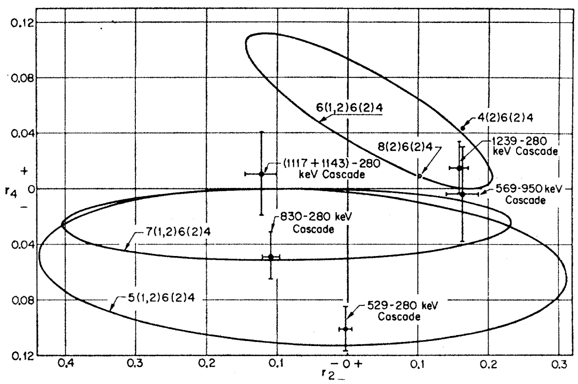

FIG. 9. An r_4-r_2 plot for spin sequences in which the second member of the cascade is a 6(2)4 transition. The experimental coefficients, after correction for the finite angular resolution of the detectors, are also shown.

Consequently, the 545-keV state is assumed to have a spin of 6. It is also apparent from Fig. 8 that the spin of the state at 1074 keV is 5. Using an average value of $-(0.146\pm0.004)$ for r_2, one finds a value of $+37_{-7}^{+10}$ for δ for the 810-keV transition. The composite (1397+1423)–184-keV correlation is consistent with a spin-3 or a spin-5 assignment to the states at 1662 and 1688 keV. Since these states both presumably de-excite to states of spin 4 and spin 6, the spin-5 assignment is preferred.

Several of the measured coefficients for those cascades for which the 280-keV, 6(2)4, transition is the second member are plotted in Fig. 9. Also shown are the theoretical parametric plots for several pertinent values of the spin of the initial state. From an inspection of Fig. 9, there appears to be little doubt that the spins of the states at 1074 and 1374 keV are 5 and 7, respectively. For the 529-keV transition, the value of δ consistent with the experimentally observed r_2 is $+85_{-45}^{+\infty}$. Using a value of $-(0.111\pm0.007)$ for the coefficient r_2, obtained by averaging the three measurements of the directional correlation of the 830–280-keV cascade, one finds a value of $\delta=+70_{-30}^{+260}$ for the 830-keV transition. The assignment of spin 5 to the states at 1662 and 1688 keV, discussed above, is consistent with the measured correlation of the composite (1117+1143)–280-keV cascade.

Information concerning the spin of the state at 1784 keV is provided by the correlations of the 710–810-, the 710–529-, the 1239–280-, and the 410–830-keV gamma-ray cascades. The results of the 710–810-keV correlation rule out the possibility of spin assignments of 3, 5, 7, and 8 for this state. Spin assignments of 4 and 6, however, are consistent with the results of this correlation as well as with those from measurements on the 710–529-keV cascade. The spin-4 assignment, however, is ruled out by the results of the correlation of the 410–830-keV cascade. Such an assignment would require that the multipole order of the 410-keV transition be at least 3, which is highly unlikely in view of the large intensity of this transition. If a value of $\delta=-70$ is used for the 830-keV transition and if the 410-keV transition is a pure multipole of any reasonable order (i.e., <7), then the spin-4 assignment requires a negative r_2 for the 410–830-keV correlation. Since r_2 for this correlation is positive, the spin of the 1784-keV state must be 6. The analysis of the correlation of the 710–529-keV cascade on this basis, including the uncertainty in the value of r_2 as well as in the value of δ for the 529-keV transition, yields $\delta=-(0.024\pm0.029)$ for the 710-keV transition. The directional correlation of the 1239–280-keV cascade provides $\delta=-(0.09\pm0.06)$ for the 1239-keV transition. If one includes both the uncertainty in the value of r_2 from the 410–830-keV correlation and the uncertainty in δ for the 830-keV transition as obtained from the 830–280-keV correlation, $\delta=-(0.27\pm0.18)$ is obtained for the 410-keV transition.

The analysis of the directional correlation of the 710–810-keV cascade is interesting for two reasons. Owing to the "accidental" vanishing of the Racah coefficient $W(5522;24)$, the directional correlation of any gamma-ray cascade in which the second gamma ray is a pure quadrupole transition from a state of spin 5 to a state of spin 4 must have a $P_2(\cos\theta)$ term which is identically zero.[16] The fact that the directional correlation of the 710–810-keV cascade possesses a finite $P_2(\cos\theta)$ term thus indicates that the 810-keV gamma ray must of necessity contain some dipole admixture. The second feature of the 710–810-keV correlation is the fact that it provides a check on the consistency of the other observations involving the 710- and 810-keV transitions. For various assumed values of δ_{710}, values of δ_{810} consistent with the experimental value of $r_2=+(0.013\pm0.004)$ for the 710–810-keV correlation were calculated. The results of this calculation are shown in Fig. 10. Also shown are the values of δ_{810} and δ_{710} obtained from the values of r_2 measured for the 810–184- and the 710–529-keV correlations, respectively. Some overlap exists in the magnitudes of the parameters δ as determined from the three experiments.

The directional correlations of the 950–184- and the 569–950-keV cascades provide information about the spin of the state at 1215 keV. The measured coefficients of the former cascade, shown in Fig. 8, are consistent with spins of 6, 5, or 4 for this state. The latter two assignments require $\delta_{950}=-0.28$ and 0.26, respectively. For an assumed value of -0.28 for δ_{950} in an analysis of the 569–950-keV correlation, the possibility of a spin-5 assignment appears to be eliminated, since this assignment cannot yield a positive value of r_2 that is sufficiently large to agree with the measured one. For a spin-4 assignment, with $\delta_{950}=+0.26$, a value of $+0.13$ is obtained for r_2. While this value is outside the quoted experimental uncertainty, it is not sufficiently far

[16] The coefficient $W(5522;44)$, however, does not vanish and, hence, such a cascade may in general have a nonvanishing $P_4(\cos\theta)$ term.

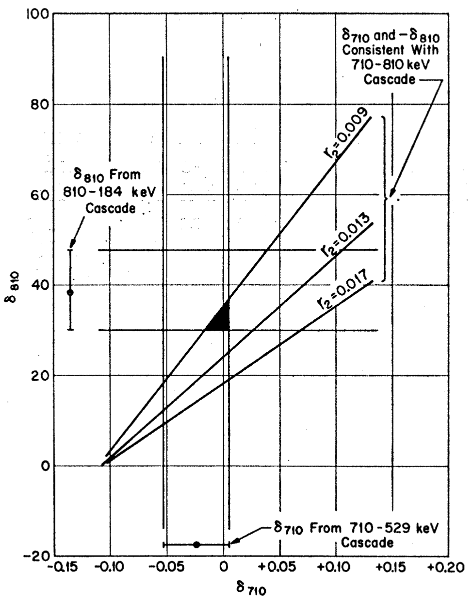

FIG. 10. A comparison of the mixing-ratio parameters δ for the 710- and 810-keV gamma rays as obtained from the various directional correlation experiments. Note that, while the magnitudes of δ_{810} from the two experiments are in fairly good agreement, the signs are different. This sign difference is a consequence of the definition of δ and arises in this instance since different intermediate states are involved in the two measurements which provide information on the 810-keV transition.

outside that one could conclude that the spin-4 assignment can be definitely excluded. The most reasonable spin assignment, however, is 6. In this case a value of $-0.08_{-0.08}^{+0.12}$ is obtained for δ^{569}.

For those cascades for which directional correlation measurements were carried out, the data confirm the spin assignments proposed previously.[1] Because of the complexity of the gamma-ray spectrum, the present measurements provide no information concerning the spins of the states at 787, 861, 910, and 957 keV. No attempt was made to obtain directional correlations

TABLE IV. Summary of mixing-ratio data from directional-correlation experiments.

Transition ($I\pi \to I'\pi'$)	Energy (keV)	Correlation from which δ was obtained (keV cascade)	δ[a]
$6^- \to 7^+$	410	410–830	$-(0.27 \pm 0.18)$
$6^- \to 6^+$	569	569–950	$-0.08_{-0.08}^{+0.12}$
$6^- \to 5^+$	710	710–529	$-(0.024 \pm 0.029)$
$6^- \to 6^+$	1239	1239–280	$-(0.09 \pm 0.06)$
$7^+ \to 6^+$	830	830–280	$+70_{-30}^{+260}$
$5^+ \to 4^+$	810	810–184	$+37_{-7}^{+10}$
$5^+ \to 6^+$	529	529–280	$+85_{-45}^{+\infty}$

[a] In the reduced matrix elements which occur here, the state from which the transition originates appears on the left and the state at which it terminates appears on the right.

for any of the gamma rays which de-excite the state at 1825 keV. It was felt that the uncertainties in such data would have been sufficiently great that no meaningful analysis could be carried out. For those transitions which de-excite the 1784-keV state, the data indicate negative values for δ, although the possibility that all the transitions are pure $E1$ cannot be eliminated. For the 529-, 810-, and 830-keV transitions, the values of δ are all positive and quite large. In the directional correlations involving the 529- and 830-keV gamma rays, the mixing ratios are such sensitive functions of r_2 that one should not rule out the possibility that both of these transitions are pure $E2$. For the 810-keV gamma ray, however, the apparent existence of a $P_2(\cos\theta)$ term in the 710–810-keV correlation implies that this gamma ray cannot be a pure $E2$ transition. The values of δ obtained for the various transitions are summarized in Table IV.

IV. DISCUSSION OF THE OBSERVED Er^{166} STATES

A. Positive-Parity States

Ratios of the $B(E2)$ values, calculated from the measured relative gamma-ray intensities, are summarized in Table V. Implicit in this calculation is the

TABLE V. Comparison of experimental and calculated $B(E2)$ ratios for some Er^{166} transitions.

Transition ($I'K' \to IK$)	Experimental $B(E2)$ ratio	Calculated $B(E2)$ Ratios: Band-mixing model ($z=0.047$)	Asymmetric-rotator model[a]
Interband transitions only			
$(72 \to 80)/(72 \to 60)$	2.9 ± 1.8	2.5	3.0
$(62 \to 60)/(62 \to 40)$	9.7 ± 2.2	13.4	16
$(52 \to 60)/(52 \to 40)$	1.7 ± 0.2	1.46	1.7[b]
$(42 \to 20)/(42 \to 40)$	0.17[c]	0.17	0.17
$(32 \to 20)/(32 \to 40)$	1.3[c]	1.28	1.26
$(22 \to 00)/(22 \to 20)$	0.56[c]	0.53	0.50
Intraband-interband transitions		calculated using $(Q_{22}/Q_{20})^2=40$	
$(72 \to 52)/(72 \to 60)$	55 ± 18	60	69
$(62 \to 42)/(62 \to 40)$	$(1.8_{-1.1}^{+3.4}) \times 10^2$	288	3.0×10^2
$(52 \to 32)/(52 \to 40)$	49 ± 22	33	38

[a] The parameters used here differ slightly from those of Ref. 1. The asymmetry parameter k has been changed from -0.96 to -0.961, the intrinsic quadrupole moment k_0 from $2.28e \times 10^{-24}$ cm^2 to $2.24e \times 10^{-24}$ cm^2, and the parameter r from 0.18 to 0.16.
[b] Ratio used to obtain the parameter r.
[c] From Harmatz *et al.*, Ref. 11.

TABLE VI. Comparison of observed energies for the positive-parity states in Er^{166} with the calculated energies.

		Calculated energies	
IK	Experimental energy (keV)	Asymmetric-rotator model[a]	Strong-coupling model with band mixing
20	80.3[b]	80.3	80.3
40	264.5[b]	264.3	264.4
60	544.5[b]	544.0	544.6
80	910	907.5	908.8
22	787.1[c]	786.9	787.1[d]
32	860.4[c]	859.9	860.4
42	957.3[c]	957.1	956.8
52	1074	1074.9	1076.4
62	1215	1218.0	1214.9
72	1374	1373.8	1373.5

[a] The model parameters are those reported in Table IV.
[b] Energies reported by Geiger *et al.* (Ref. 5).
[c] Energies reported by Harmatz *et al.* (Ref. 11).
[d] Used as reference point for the energies in the fit.

assumption that the $M1$ admixtures in these transitions are small. For the stronger transitions, the directional correlation experiments indicate that this assumption is justified. The measured energies of the Er^{166} positive-parity states are summarized in Table VI.

Interpretation in Terms of an Asymmetric-Rotator Model

An interpretation of the observed positive-parity states in Er^{166} in terms of a generalized asymmetric-rotator model of the nucleus has previously been presented.[1] In this model four independent parameters are needed to fit the rotational energy levels of the even-even nucleus. These parameters are the three principal moments of inertia (I_A, I_B, and I_C) and a rotation-vibration parameter (b). In the calculations it is convenient to introduce rotational constants A, B, and C, where $A=\hbar/4\pi I_A$, etc. The fit is then carried out using the parameters $k\equiv(2B-A-C)/(A-C)$, A/C, hC, and b. For Er^{166}, a satisfactory fit to the observed states was found for the following values of these parameters[1]: $k=0.96$, $A/C=17.14$, $hC=11.62$ keV, and $b=-6.3\times10^{-4}$ keV^{-1}. Two additional parameters, the intrinsic quadrupole moments k_0 and k_2, are required to calculate the reduced transition probabilities. In the previous work,[1] these $B(E2)$ values were calculated using $k_0=2.28\times10^{-24}$ cm^2 and $r=(k_2/k_0)\sqrt{2}=+0.18$; and fairly good agreement with experiment was obtained. Li and Schwarzschild[12] have recently remeasured the lifetime of the first 2+ state in Er^{166}. Since the lifetime of this state was used to derive a value for k_0 and since the present work has given rise to improved values for the level energies and relative gamma-ray intensities, it was necessary to make slight changes in the values of some of the model parameters. These changes are given in Table V. The $B(E2)$ ratios and level energies calculated using them are summarized in Tables V and VI, respectively. Although some discrepancy exists in the ratio of the $B(E2)$ values for the $(62\rightarrow60)$ and $(62\rightarrow40)$ transitions, the agreement between the calculated and the measured quantities is good.

Strong-Coupling Model with Band Mixing

It is of interest to discuss the properties of these states in terms of the strong-coupling model of Bohr and Mottelson.[14,17–21] The inclusion of the rotation-vibration interaction in the unperturbed strong coupling model leads to a mixing of the $K=0$ and $K=2$ bands and to a deviation of the energy levels from the $I(I+1)$ rule. The similarity of this approach and that of the asymmetric rotator in predicting the ratios of interband transition probabilities in strongly deformed nuclei has been demonstrated by Emery *et al.*[21] The general similarity of the predictions of the asymmetric-rotator model to those of the unified model has been discussed quite recently by Yamazaki,[22] who concludes that these predictions are equivalent so far as the $K=2$ band is concerned.

The mixing of the K values of the ground state and the gamma-vibrational bands is expressed in terms of a parameter, ϵ, which is the amplitude of $K=2$ admixture in the wave function of the 2+ state of the ground-state band.[21] The $K=2$ admixture in the wave function of the state of spin I in the ground-state band is then $[(I-1)I(I+1)(I+2)/24]^{1/2}\epsilon$. To first order, the intraband transition probabilities are unaffected by the mixing of the bands while the interband transition probabilities are multiplied by a factor $f_{I'I}(z)$, where $z=(Q_{22}/Q_{20})\epsilon$. Q_{22} and Q_{20} are the intrinsic $E2$ transition amplitudes for intraband and interband transitions, respectively. The functions $f_{I'I}(z)$ for $E2$ transitions from a state of spin I' in the $K=2$ band to a state of spin I in the $K=0$ band have been tabulated.[18,19] The experimentally measured ratios of reduced $E2$ transition probabilities for interband transitions may hence be used to determine the parameter z. A value of 0.047 for z appears to give the best agreement with all of these data. The results of the calculated $B(E2)$ ratios using this value of z are summarized in Table V. The $B(E2)$ ratios involving an interband and an intraband transition also depend on the quantity $(Q_{22}/Q_{20})^2$. The best agreement for these calculated ratios appears to be obtained for $(Q_{22}/Q_{20})^2\approx40$. The $B(E2)$ ratios calculated for this $(Q_{22}/Q_{20})^2$ and z are summarized in Table V. From these parameters, ϵ is calculated to be ≈0.0075, assuming a positive sign for Q_{22}/Q_{20}. Again, with the exception of the $B(E2)$ ratios for the $(62\rightarrow60)$ and $(62\rightarrow40)$ transitions, the

[17] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **26**, No. 14 (1952); A. Bohr and B. R. Mottelson, *ibid.* **27**, No. 16 (1953).
[18] R. K. Sheline, Rev. Mod. Phys. **32**, 1 (1960).
[19] P. Gregers Hansen, O. B. Nielsen, and R. K. Sheline, Nucl. Phys. **12**, 389 (1959).
[20] O. B. Nielsen, in *Proceedings of the Rutherford Jubilee Conference, Manchester, 1961* (Academic Press Inc., New York, 1961), p. 317.
[21] G. T. Emery, W. R. Kane, M. McKeown, M. L. Perlman, and G. Scharff-Goldhaber, Phys. Rev. **129**, 2597 (1963).
[22] T. Yamazaki, Nucl. Phys. **49**, 1 (1963).

agreement between the calculations and the measured values is good.

From the measured lifetime of the 80.3-keV state, Li and Schwarzschild[12] obtain a value of $1.19e^2\times10^{-48}$ cm^4 for $B(E2;20\to00)$. This gives a value of $2.44e\times10^{-24}$ cm^2 for Q_{00} and, hence, a value of 7.73×10^{-24} cm^2 for the intrinsic quadrupole moment of the ground-state configuration of Er^{166}. This corresponds to a deformation parameter β of 0.33.

The experimentally determined energies[1,5,11] for the positive-parity states in Er^{166} are given in Table VI. The energies within the ground-state band were fitted to a function of the form

$$E_I=AI(I+1)+BI^2(I+1)^2,$$

and the coefficients $A(=\hbar^2/2I)$ and B were determined from a least-squares fit. Values of (13.45 ± 0.01) keV and $-(11.5\pm0.4)$ eV were obtained for A and B, respectively. To within the precision of the energy measurements, the levels of the $K=2$ band of Er^{166} do not exhibit an "odd-even" shift such as has been reported[21] for Os^{186}. Accordingly, the energies of these levels were fitted to a function of the form

$$E_{I2}=E_{22}+A'[I(I+1)-6]+B'[I^2(I+1)^2-36],$$

where E_{22} is the energy of the $2+$, $K=2$, state. Values of (12.41 ± 0.05) keV and $-(11\pm1)$ eV were obtained for A' and B', respectively. It appears that the moment of inertia associated with the gamma-vibrational band is somewhat larger than that of the ground-state band. The parameters B and B' appear to be equal, although the uncertainties associated with them are somewhat large. Again, the agreement between the calculated and measured energies is good.

In addition to mixing the $K=0$ and $K=2$ bands, the rotation-vibration interaction gives rise to a perturbation in the energies of the states. For the ground-state band, this perturbation is of the form $-(\epsilon^2/24)I^2(I+1)^2\hbar\omega_\gamma$, where $\hbar\omega_\gamma$ is the energy of the $2+$ gamma-vibrational state. Using the value of ϵ determined above, one calculates a value of ~1.8 eV for the coefficient of $I^2(I+1)^2$. Hence, this interaction accounts for only $\approx16\%$ of the observed perturbation. This has previously been pointed out by Nielsen,[20] who, in a survey of the available data on deformed even-even nuclei, concludes that for all such nuclei studied these deviations are not primarily due to the coupling of the ground-state band to the $K=2$ band. Recently, Greenberg *et al.*[23] have shown that, in Sm^{152}, while the coupling of the gamma vibrational band to the ground-state band can account for at most $\sim10\%$ of the observed perturbation, the coupling of the beta-vibrational band can account for the remainder. While the necessary data on the beta-vibrational band in Er^{166} are at present lacking, it is possible that the observed energy displacements in the ground-state band may be similarly explained.

B. Negative-Parity States

Summary of Experimental Results

The results of our work indicate that the parities of the states shown in Fig. 5 at 1662, 1688, and 1784 keV are negative. For the 1784-keV state, this assignment is made on the basis of the predominantly $E1$ multipolarity which the directional-correlation and conversion-electron[1] studies require for the 710-keV transition. Since the multipolarities assigned to the 96- and 122-keV transitions indicate no parity change, the 1662- and 1688-keV states must have negative parity also. The multipolarities of the 137- and 163-keV transitions have not been established in this work and consequently they provide no information concerning the parity of the state at 1825 keV. For reasons previously discussed,[1] however, the most likely assignment for this state is $6-$.

Previous studies[1,5] of the long-lived Ho^{166} activity have failed to observe its associated beta radiation and have succeeded only in establishing an upper limit for the end-point energy of the beta transition to the 1784-keV state. From the work of Ref. 1, it was also possible to set a lower limit to this energy. As a result of more precise gamma-ray energy measurements (see II, B above), the energy separation of the two $6-$ states, and hence the lower limit for the energy of the beta transition to the 1784-keV level, has been reduced to ≈41 keV. The upper limit of the energy of the beta transition to the 1825-keV state is thus 25 keV. On the basis of a 20% beta branch to this state and the recently measured value[24] of 1.2×10^3 years for the half-life of long-lived Ho^{166}, the $\log ft$ calculated for this beta transition varies from ≈5.0 for an end-point energy of 5 keV to ≈7.9 for an end-point energy of 25 keV. The calculated $\log ft$ for the beta transition to the 1784-keV state (assuming an 80% beta branch) varies from ≈8.1 to 8.4 as the assumed end-point energy varies from 45 to 65 keV.

Experimental $B(E1)$ ratios for the transitions which de-excite the four negative-parity states are summarized in Table VII. Although the experimental uncertainties are large, the transitions from the $6-$ states to the $6+$ members of the $K=0$ band are hindered by factors of ≈75 relative to the transitions to the $6+$ number of the $K=2$ band. Included in this Table are ratios of $B(E1)$ values calculated assuming various K values for the initial state. For those transitions which are not K forbidden, the $B(E1)$ ratios were calculated using the intensity rules of Alaga *et al.*[25] For the K-forbidden transitions, the expression given by Bohr and

[23] J. S. Greenberg, G. G. Seaman, E. V. Bishop, and D. A. Bromley, Phys. Rev. Letters **11**, 211 (1963).

[24] K. T. Faler, J. Inorg. Nucl. Chem. (to be published). We are indebted to Dr. Faler for making his results available to us prior to their publication.

[25] G. Alaga, K. Alder, A. Bohr, and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 9 (1955).

TABLE VII. Reduced transition-probability ratios for certain $E1$ transitions in Er^{166}.

Transition	Experimental $B(E1)$ ratio		Theoretical ratios[a]				
$(IK \rightarrow I'K')$	Initial state						
	1784 keV	1825 keV	$K=1$	$K=2$	$K=3$	$K=4$	$K=6$
$(6K \rightarrow 72)/(6K \rightarrow 52)$	1.12 ± 0.21	0.87 ± 0.34	3.09	1.20	0.24	0.24	0.24
$(6K \rightarrow 62)/(6K \rightarrow 52)$	0.24 ± 0.07	0.33 ± 0.14	3.72	0.23	0.93	0.93	0.93
$(6K \rightarrow 62)/(6K \rightarrow 60)$	72 ± 30	77 ± 40	...	...	...	...	...
	Initial state		Theoretical ratios[a]				
	1662 keV	1688 keV	$K=0$	$K=1$	$K=2$	$K=3$	$K=4$
$(5K \rightarrow 60)/(5K \rightarrow 40)$	0.84 ± 0.57	0.63 ± 0.44	1.2	0.83	0.83	0.83	0.83

[a] The theoretical ratios for transitions that are not K forbidden are calculated using the expressions given by Alaga *et al.*, Ref. 25. For the K-forbidden transitions, the expression [Eq. (7)] given by Bohr and Mottelson, Ref. 26, was used.

Mottelson[26] was applied. Some comments concerning the validity of these calculations should be made. First, there is some question concerning the assumption of a single K value for these states. The fact that two states of the same spin and parity lie so close together implies that they do not interact strongly and hence, arise from different configurations. It is reasonable to expect, however, that their K values will be mixed to some extent, thus affecting the $B(E1)$ values of the transitions which de-excite them. Second, the transition rates, particularly for K-forbidden transitions, may depend sensitively on small admixtures of other states in the wave function. In light of this, a comparison of the calculated $B(E1)$ ratios with the experimental ones may not be meaningful. Nevertheless, as is apparent from Table VII, the $B(E1)$ ratios for the transitions from the two 6− states agree quite well (perhaps fortuitously) with those calculated assuming $K=2$. The experimental uncertainties in the transition data from the 5− levels are sufficiently large that perhaps one ought not to draw any conclusions concerning these levels.

Possible Configuration Assignments

Many of the data necessary for an understanding of the negative-parity states observed in Er^{166} are presently lacking. For this reason, their nature cannot be established with certainty at this time. However, it is of interest to mention several possible mechanisms which can give rise to these states. These include quasiparticle and g-vibrational excitations.

The effect of the pairing-correlation on certain of the properties of nuclides in the region of large deformation $(156 \leq A \leq 188)$ has been considered by Soloviev.[27] Gallagher and Soloviev[28] have used this "superfluid" nuclear model to discuss the experimental data on intrinsic levels in even-mass nuclei in this region. According to this model, the lowest lying excited intrinsic states of a deformed even-even nucleus are two-quasiparticle excitations. The lowest of these occurs when the two quasiparticles (either proton or neutron) are in the K and $K+1$[29] states of the average field, and may lie below the formal energy gap.[27] For Er^{166}, the K and $K+1$ proton states are the Nilsson orbitals $[523]\frac{7}{2}-$ and $[411]\frac{1}{2}+$, respectively.[27,28] The K and $K+1$ neutron states are the Nilsson orbitals $[523]\frac{5}{2}-$ and $[633]\frac{7}{2}+$, respectively. Lipas and Davidson[30] have considered the effect of introducing small amounts of the spherical harmonics $Y_3{}^0$ and $Y_3{}^{\pm 2}$ into the shape of the nuclear surface and have shown that odd-parity bands having $K=0$ and $K=2$ result. The latter band is termed a g-vibrational band in analogy with the gamma-vibrational band, which also has $K=2$. These various possibilities are shown schematically in Fig. 11. Also shown is a partial level scheme for Er^{166} in which the negative-parity states excited in the decay of both Ho^{166} isomers are included. The 1− level at 1660, observed in the decay of the 27-h isomer has been interpreted[31,32] as the $K\pi=0-$ octopole vibrational state. It has been included for the sake of completeness.

The 27-h (0−) and 1200-year (7−) states in Ho^{166} presumably arise from the two different couplings of the $[633]\frac{7}{2}+$ neutron and $[523]\frac{7}{2}-$ proton orbitals. The two states of the $(K,K+1)$ two-neutron quasiparticle doublet in Er^{166} have $K\pi=1-$ and $6-$. A 1− state at $\sim$1.83 MeV, presumably the $K=1$ member of this doublet, has been observed[31,32] in the decay of the 0− state of Ho^{166}. The beta transition to this state takes place from the $[523]\frac{5}{2}-$ neutron orbital to the $[523]\frac{7}{2}-$ proton orbital and has a measured log*ft* value of $5.2_{-0.4}{}^{+0.3}$. A beta transition from the 7− state of Ho^{166} to the 6− member of the two-neutron doublet would also take place between these same two orbitals. In fact, for these two beta transitions, the two initial states as well as the two final states would be identical except for the coupling of the intrinsic spins. Except

[26] A. Bohr and B. R. Mottelson, At. Energ. (USSR) **14**, 36 (1963).

[27] V. G. Soloviev, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter **1**, No. 11 (1961).

[28] C. J. Gallagher, Jr., and V. G. Soloviev, Kgl. Danske Videnskab. Selskab, Mat. Fys. Skrifter **2**, No. 2 (1962).

[29] The notation here is that of Refs. 27 and 28. The K level denotes the last filled Nilsson orbital (in the absence of pairing correlations) of the average field, $K+1$ denotes the first unfilled orbital, etc. This is not to be confused with K, the projection of the total angular momentum along the nuclear symmetry axis.

[30] P. O. Lipas and J. P. Davidson, Nucl. Phys. **26**, 80 (1961).

[31] P. G. Hansen, K. Wilsky, D. J. Horen, and Lung-Wen Chiao, Nucl. Phys. **24**, 519 (1961).

[32] J. E. Cline, E. C. Yates, and E. H. Turk, Nucl. Phys. **30**, 154 (1962).

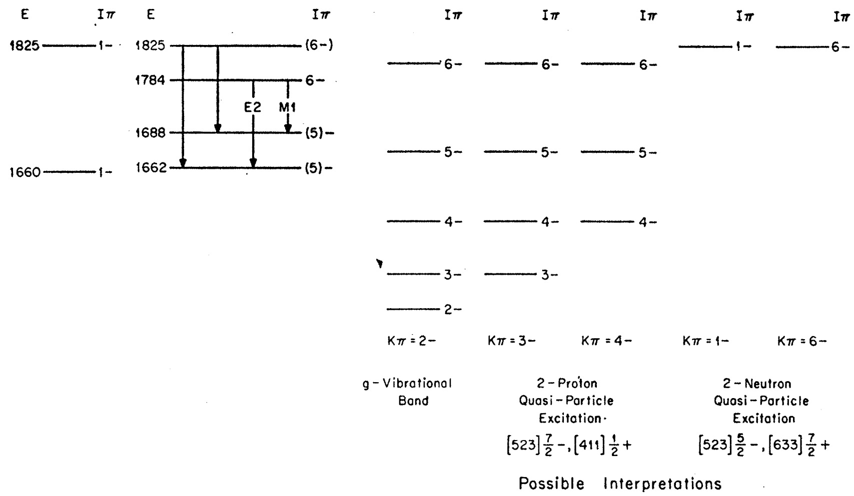

FIG. 11. Partial level scheme of Er^{166} showing negative-parity states excited in decay of Ho^{166} and Ho^{166m}. Shown schematically on the right are some of the possible interpretations of these states.

for the correction for the statistical factors, the ft values for the two transitions should be the same. If, therefore, it is energetically possible for the 6− member of the $(K,K+1)$ two-neutron quasiparticle excitation in Er^{166} to be excited in the beta decay of the long-lived Ho^{166} activity, the beta transition feeding it should have a $\log ft$ value of the order of 5.

It is probable that the 6− state at 1784 keV does not arise from this mechanism, since the beta transition feeding it has a $\log ft$ value somewhat larger than 8. Which, if any, of the other possible modes of excitation gives rise to this state is not clear at present. From the $B(E1)$ ratios given in Table VII, one might conclude that it was a member of the g vibrational band ($K=2$). The $\log ft$ for such a highly K-forbidden beta transition, however, would be expected to be larger than the observed value. The higher K values associated with the two-proton quasiparticles excitation would give a somewhat less K-forbidden beta transition with a $\log ft$ more nearly in agreement with the experimental range, but the $B(E1)$ ratios would than not agree with the experiment. However, the $\log ft$ does not provide a reliable method of choosing among the various possibilities, since it is quite sensitive to small admixtures of states having different K values (particularly $K=6$). In spite of the uncertainty concerning the nature of this state, it is most reasonable to suppose that one of the 5− states observed in the present work is a member of the same rotational band as the 1784-keV state. The 122-keV transition which connects the 1784- and 1662-keV states is predominantly $E2$ while the 96-keV transition connecting the 1784- and 1688-keV states is predominantly $M1$. On this basis it appears more likely that the 1662-keV state is the one associated with the 1784-keV state.

While the nature of the 1825-keV state is at present as uncertain as that of the other negative-parity states mentioned above, it may be possible to make a definite assignment once the $\log ft$ value of the beta transition exciting it is known. A value of ~ 5 is quite possible and if indeed, it is the correct value, then this state might quite reasonably be considered to be the $K\pi=6-$ two-neutron quasiparticle excitation. Furthermore, the splitting of the 1− and 6− members of the $(K,K+1)$ two-neutron quasiparticle excitation in Er^{166} would then be $\lesssim 5$ keV. The two states would be essentially degenerate. Logft values of ≈ 5 for the beta transitions which excite these two states would require that the 7− state in Ho^{166} lie below the 0− state. Struble *et al.*,[33] however, report that the 0− state is apparently the ground state, lying about 12 keV below the 7− state. If this be the case, then the above interpretation of the 1825-keV state as a two-neutron quasiparticle excitation is incorrect.

More experimental information on the level structure of Er^{166} is clearly necessary before the nature of the negative-parity states can be ascertained. In particular, a knowledge of the energies and decay properties of any additional negative-parity states with spins of 4 or less in the region below ~ 1.6 MeV would be most valuable.

[33] G. L. Strubel, N. Shelton, and R. K. Sheline, Phys. Rev. Letters **10**, 58 (1963).