Neutrino Theory of Photons*

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In this paper the neutrino is described by a particular four-component theory built from two two-component theories. This procedure, which eliminates the arbitrariness that exists in the usual four-component theory, is shown to be useful in formulating a neutrino theory of photons. A photon is then composed of a neutrino and antineutrino which have the appropriate helicities. The photon field is constructed by describing the annihilation and creation of photons in terms of neutrino and antineutrino processes. The electric and magnetic fields so formed are shown to satisfy Maxwell's equations. The operations of space inversion and charge conjugation are defined in terms of the neutrino operators in such a way that the electric and magnetic fields transform in the usual manner under these symmetry operations. The photon operators do not satisfy Bose commutation relations, since additional terms arise. Because of these additional terms, the electric and magnetic fields do not satisfy the usual commutation relations either. However, Planck's radiation law still follows. Although the consequences of the non-Bose commutation relations have not been explored, some experimental implications of the theory are discussed.

I. INTRODUCTION

IN 1928, the composite nature of the photon was inferred by Jordan[1] on the basis of a statistical argument. In 1932, de Broglie[2] put forth the idea that a photon is composed of a neutrino and an antineutrino. de Broglie's idea gives a good account of the creation and annihilation of photons and makes plausible the great difference in physical characteristics exhibited by a spin 1, mass 0 photon and a spin $\frac{1}{2}$, mass 0 neutrino. A neutrino-antineutrino pair is formed when a photon is emitted. The neutrino and antineutrino, being antiparticles, annihilate when a photon is absorbed. In terms of the hole theory, when a photon is emitted a neutrino makes a transition from the negative energy states. Then the neutrino and antineutrino (or hole) travel together until the photon is absorbed, at which time the neutrino makes a transition back into the hole.

According to de Broglie's original idea the photon is composed of a neutrino and antineutrino bound together in some way. This interaction as it was developed by de Broglie led to the result that the neutrinos had equal momentum so that if the state of the photon were known, the state of its components could be determined. This meant that it would be impossible for two photons to be in the same state, because of the underlying Fermi-Dirac statistics of the components. Therefore, the photons themselves would have to obey Fermi-Dirac statistics. This type of reasoning led Jordan to suggest[3] that it is not the interaction between the neutrinos and antineutrinos that binds them together into photons, but instead it is the manner in which they interact with other particles that leads to the simplified description of light in terms of photons. To account for the emission of a photon of momentum $\mathbf{p}$ he proposed, for instance, that an atom would simultaneously emit a neutrino with momentum $\mathbf{k}$ and an antineutrino of momentum $\mathbf{p}-\mathbf{k}$ in exactly the same direction. Actually, the neutrino and antineutrino are not emitted into a state in which the momentum of each is definite, but into a superposition of such states with definite weights and phases (see Secs. III and IV).

Jordan[3] also postulated that the absorption of a photon of momentum $\mathbf{p}$ could be simulated by a Raman effect of neutrinos or antineutrinos (i.e., one neutrino or antineutrino with momentum $\mathbf{p}+\mathbf{k}$ is absorbed while another of the same energy state, opposite spin, and momentum $\mathbf{k}$ is emitted) as well as the simultaneous absorption of neutrino-antineutrino pairs. Work on this theory by Jordan, Kronig, and others continued until 1938. It was brought to a halt when Pryce[4] discovered that these theories (which employ the old four-component neutrino theory) are not invariant under a spatial rotation of the coordinate system.

In describing the neutrino, we shall take both Weyl equations (each set of which describes a two-component neutrino) and assume that two neutrinos and two antineutrinos exist. The neutrino with spin *parallel* to its momentum will be designated ν_1, and the neutrino with spin *antiparallel*, ν_2. A photon would then be composed of the pair $(\nu_1\bar{\nu}_2)$ or $(\nu_2\bar{\nu}_1)$, resulting in a spin of $+1$ or -1 along the direction of propagation.

In β decay, experimental results have shown that only ν_2 and $\bar{\nu}_2$ appear. In the decays $\pi^{\pm}\rightarrow\mu^{\pm}+$(neutrino or antineutrino), experiment[5] indicates that a different type of neutrino occurs. This would presumably be ν_1 or $\bar{\nu}_1$. (This will be discussed further in Sec. VI.)

By using this particular four-component theory formed from two two-component neutrino theories and

* Work done under the auspices of the U. S. Atomic Energy Commission.
[1] P. Jordan, Ergeb. Exakt. Naturw. **7**, 158 (1928).
[2] L. de Broglie, Compt. Rend. **195**, 536, 862 (1932); *Une novelle conception de la lumiere* (Hermann et Cie., Paris, 1934).
[3] P. Jordan, Z. Physik **93**, 464 (1935).
[4] M. H. L. Pryce, Proc. Roy. Soc. (London) **A165**, 247 (1938). This paper contains a good list of references to prior work.
[5] G. Danby, J-M. Gaillard, K. Goulianos, L. M. Lederman, N. Mistry, M. Schwartz, and J. Steinberger, Phys. Rev. Letters **9**, 36 (1962).

omitting Jordan's Raman-effect hypothesis, one can form a theory which is invariant under a spatial rotation of the coordinate system. The construction of the photon field from the neutrino field is similar to the formulation used by Kronig[6] in 1936 with the two modifications noted above. Kronig's theory is not invariant under spatial rotations as is discussed by Pryce. Invariance of the present theory under a spatial rotation of the coordinate system is proven in Appendix B of this paper. However, the photon operators in the present theory do not obey Bose-Einstein commutation relations, as additional terms arise. The results of these new commutation relations are not investigated in this paper.

II. TWO-COMPONENT NEUTRINO PLANE-WAVE SOLUTIONS

Starting with the two-component theory of Lee and Yang,[7] we shall obtain the plane-wave solutions for both neutrinos. The Weyl equation can be written, with $\hbar=c=1$, as

$$H\phi_\nu=i\partial\phi_\nu/\partial t. \tag{1}$$

With $H=+(\boldsymbol{\sigma}\cdot\mathbf{p})$ (where σ_1, σ_2, and σ_3 are the usual 2×2 Pauli spin matrices), Eq. (1) gives rise to:

(a) ν_1, the particle (positive energy state) with spin *parallel* to its momentum.

(b) $\bar{\nu}_1$, the antiparticle (hole in the negative energy states) with spin *antiparallel* to its momentum.

With $H=-(\boldsymbol{\sigma}\cdot\mathbf{p})$, Eq. (1) gives rise to:

(a) ν_2, the particle (positive energy state) with spin *antiparallel* to its momentum.

(b) $\bar{\nu}_2$, the antiparticle (hole in the negative energy states) with spin *parallel* to its momentum.

We now obtain the plane-wave solution of Eq. (1), with $H=+\boldsymbol{\sigma}\cdot\mathbf{p}$.

$$\phi_\nu=\begin{pmatrix}u_1\\u_2\end{pmatrix}e^{i(\mathbf{p}\cdot\mathbf{x}-pt)}, \tag{2}$$

so with this substitution in (1) we have

$$(\boldsymbol{\sigma}\cdot\mathbf{p}+ip_4)\phi_\nu=0, \tag{3}$$

where

$$p_4=iW=i|\mathbf{p}|=ip=i(p_1^2+p_2^2+p_3^2)^{1/2},$$

so

$$(p_1-ip_2)u_2+(p_3+ip_4)u_1=0, \tag{4}$$

$$(p_1+ip_2)u_1+(-p_3+ip_4)u_2=0. \tag{5}$$

The condition for these equations to have a nontrivial solution is

$$p_1^2+p_2^2+p_3^2+p_4^2=0. \tag{6}$$

We obtain for the positive energy solution

$$u_1=1, \quad u_2=\frac{p_1+ip_2}{p_3-ip_4}, \tag{7}$$

$$\phi_\nu=\begin{bmatrix}1\\ \dfrac{p_1+ip_2}{p_3-ip_4}\end{bmatrix}e^{i(\mathbf{p}\cdot\mathbf{x}-pt)}. \tag{8}$$

Now we want to enlarge the matrices to a four-component formalism. We define

$$\boldsymbol{\alpha}\equiv\begin{pmatrix}\boldsymbol{\sigma}&0\\0&-\boldsymbol{\sigma}\end{pmatrix},\quad \gamma_4\equiv\beta\equiv\begin{pmatrix}0&1\\1&0\end{pmatrix},\quad \boldsymbol{\sigma}\equiv\begin{pmatrix}\boldsymbol{\sigma}&0\\0&\boldsymbol{\sigma}\end{pmatrix},$$
$$\boldsymbol{\gamma}\equiv-i\beta\boldsymbol{\alpha}\equiv i\begin{pmatrix}0&\boldsymbol{\sigma}\\-\boldsymbol{\sigma}&0\end{pmatrix},\quad \gamma_5\equiv\gamma_1\gamma_2\gamma_3\gamma_4\equiv\begin{pmatrix}-1&0\\0&1\end{pmatrix}, \tag{9}$$

where 1 represents a 2×2 unit matrix.

The γ's are Hermitian and obey the relation

$$\gamma_\mu\gamma_\lambda+\gamma_\lambda\gamma_\mu=2\delta_{\mu\lambda}. \tag{10}$$

The neutrino wave function becomes

$$\Phi_{\nu1}=\begin{pmatrix}\phi_\nu\\0\end{pmatrix}. \tag{11}$$

If we take $H=-\boldsymbol{\sigma}\cdot\mathbf{p}$, we get

$$\Phi_{\nu2}=\begin{pmatrix}0\\ \sigma_2\phi_\nu^\dagger\end{pmatrix}. \tag{12}$$

Here, we deviate from Lee and Yang[7] in interpretation. They said that the wave functions of both (11) and (12) are possible, and experiment should determine which exists. In order to make a composite photon with two-component neutrinos we must take both. Although we now have both neutrinos with spins parallel and antiparallel to their momentum vector, this result [Eqs. (11) and (12)] is *not* the same result that we would have obtained by starting with a four-component theory. The wave functions $\Phi_{\nu1}$ and $\Phi_{\nu2}$, of course, satisfy the four-component Dirac equation, but they are not determined uniquely by it. From Eq. (8) we obtain the plane-wave solutions

$$\Phi_{\nu1}=\begin{bmatrix}1\\ \dfrac{p_1+ip_2}{p_3-ip_4}\\0\\0\end{bmatrix}e^{i(\mathbf{p}\cdot\mathbf{x}-pt)},$$
$$\Phi_{2\nu}=i\begin{bmatrix}0\\0\\ -\dfrac{(p_1-ip_2)}{p_3-ip_4}\\1\end{bmatrix}e^{i(\mathbf{p}\cdot\mathbf{x}-pt)}. \tag{13}$$

[6] R. de L. Kronig, Physica 3, 1120 (1936).

[7] T. D. Lee and C. N. Yang, Phys. Rev. 105, 1671 (1957).

The spinors u and v are defined by

$$\Phi_{\nu 1}=ue^{i(\mathbf{p}\cdot\mathbf{x}-pt)},\quad \Phi_{\nu 2}=ve^{i(\mathbf{p}\cdot\mathbf{x}-pt)}. \tag{14}$$

If they are normalized in the sense that

$$\Phi_{\nu 1}^{\dagger}\Phi_{\nu 1}=\Phi_{\nu 2}^{\dagger}\Phi_{\nu 2}=1$$

(i.e., $u^{\dagger}u=v^{\dagger}v=1$), we get

$$u=\left[\frac{(p_4+ip_3)}{2p_4}\right]^{1/2}\begin{bmatrix}1\\ \dfrac{p_1+ip_2}{p_3-ip_4}\\ 0\\ 0\end{bmatrix},\qquad v=\left[\frac{(p_4+ip_3)}{2p_4}\right]^{1/2}\begin{bmatrix}0\\ 0\\ -\dfrac{(p_1-ip_2)}{p_3-ip_4}\\ 1\end{bmatrix}. \tag{15}$$

The spin operator is

$$S=\boldsymbol{\sigma}\cdot\mathbf{p}/p=(1/p_4)(\gamma_2\gamma_3p_1+\gamma_3\gamma_1p_2+\gamma_1\gamma_2p_3).$$

Operating on u and v, we obtain

$$Su=u,\quad Sv=-v. \tag{16}$$

Therefore, u is the solution with spin parallel to the direction of propagation, and v is the solution with spin antiparallel to the direction of propagation, as claimed above.

III. CONSTRUCTION OF PHOTON FIELD FROM NEUTRINO FIELD

The general neutrino field ψ is a superposition of the solutions in Eq. (14):

$$\psi=\int_0^{\infty}d^3k[a_1(\mathbf{k})ue^{i(\mathbf{k}\cdot\mathbf{x}-kt)}+c_1^{\dagger}(\mathbf{k})ue^{-i(\mathbf{k}\cdot\mathbf{x}-kt)}+a_2(\mathbf{k})ve^{i(\mathbf{k}\cdot\mathbf{x}-kt)}+c_2^{\dagger}(\mathbf{k})ve^{-i(\mathbf{k}\cdot\mathbf{x}-kt)}]. \tag{17}$$

The quantities $a_1(\mathbf{k})$, $c_1(\mathbf{k})$, $a_2(\mathbf{k})$, and $c_2(\mathbf{k})$ are the annihilation operators for ν_1, $\bar{\nu}_1$, ν_2, and $\bar{\nu}_2$, respectively, and do not depend upon $\mathbf{x}$ and t. Note that:

$$\psi_{\nu 1}=\int_0^{\infty}d^3ka_1(\mathbf{k})ue^{i(\mathbf{k}\cdot\mathbf{x}-kt)}\text{ annihilates }\nu_1, \tag{18}$$

$$\psi_{\bar{\nu}1}=\int_0^{\infty}d^3kc_1^{\dagger}(\mathbf{k})ue^{-i(\mathbf{k}\cdot\mathbf{x}-kt)}\text{ creates }\bar{\nu}_1, \tag{19}$$

$$\psi_{\nu 2}=\int_0^{\infty}d^3ka_2(\mathbf{k})ve^{i(\mathbf{k}\cdot\mathbf{x}-kt)}\text{ annihilates }\nu_2, \tag{20}$$

$$\psi_{\bar{\nu}2}=\int_0^{\infty}d^3kc_2^{\dagger}(\mathbf{k})ve^{-i(\mathbf{k}\cdot\mathbf{x}-kt)}\text{ creates }\bar{\nu}_2. \tag{21}$$

Following Jordan[3] and Kronig,[6] we shall assume that the neutrino and antineutrino are emitted in exactly the same direction. This hypothesis will be discussed further in Sec. VII. For simplicity, we shall try not to invent any new interaction between neutrino-antineutrino pairs and other particles, but instead, assume that once their direction is determined they are ejected by means of the same interaction mechanism as that of weak interaction processes. For example, the emission of a neutrino-antineutrino pair would be similar to a weak interaction decay. Processes such as the absorption and emission of photons by an electron would then be represented by the interaction Hamiltonian,

$$H_{\text{int}}=\text{const}\times[(\psi_e^{\dagger}O_{\text{int}}\psi_e)(\psi_{\bar{\nu}2}^{\dagger}O_{\text{int}}\psi_{\nu 1})+(\psi_e^{\dagger}O_{\text{int}}\psi_e)(\psi_{\bar{\nu}1}^{\dagger}O_{\text{int}}\psi_{\nu 2})]+\text{Hermitian conjugate}, \tag{22}$$

or we can introduce F_{int} by

$$H_{\text{int}}=\text{const}\times(\psi_e^{\dagger}O_{\text{int}}\psi_e)F_{\text{int}}. \tag{23}$$

Substituting Eqs. (18)–(21), we obtain

$$F_{\text{int}}=\text{const}\times\int_0^{\infty}d^3p\int_0^{\mathbf{p}}d^3k\delta(\mathbf{n}_p-\mathbf{n}_k)\{c_2(\mathbf{k})v^{\dagger}e^{i(\mathbf{k}\cdot\mathbf{x}-kt)}O_{\text{int}}a_1(\mathbf{p}-\mathbf{k})ue^{i[(\mathbf{p}-\mathbf{k})\cdot\mathbf{x}-(p-k)t]}+c_1(\mathbf{k})u^{\dagger}e^{i(\mathbf{k}\cdot\mathbf{x}-kt)}O_{\text{int}}a_2(\mathbf{p}-\mathbf{k})ve^{i[(\mathbf{p}-\mathbf{k})\cdot\mathbf{x}-(p-k)t]}\}+\text{H.c.}, \tag{24}$$

where $\mathbf{n}_p=\mathbf{p}/p$ and $\mathbf{n}_k=\mathbf{k}/k$.

Since the neutrino and antineutrino are assumed to be emitted in the same direction, $\mathbf{k}$ and $\mathbf{p}-\mathbf{k}$ differ from $\mathbf{p}$ by only a multiplicative constant. Therefore, there is no need to attach an index such as $\mathbf{k}$ to u and v, which are normalized [see Eq. (15)] so as to depend only upon direction. From Eq. (24) we see that F_{int} essentially describes the four-photon processes (i.e., absorption and emission of right- and left-handed photons).

The possible choices for O_{int} are

$$O_S=\gamma_4,\quad O_V=\gamma_4\gamma_\mu,\quad O_T=i\gamma_4(\gamma_\lambda\gamma_\mu-\gamma_\mu\gamma_\lambda),\quad O_A=i\gamma_4\gamma_\mu\gamma_5,\quad O_P=\gamma_4\gamma_5. \tag{25}$$

The only coupling for which F_{int} does not vanish identically is O_T. Substituting O_T of (25) in (24) results in

$$F_{\text{int}}=F_{\mu\lambda}(\mathbf{x},t)=\frac{1}{4\sqrt{2}\pi}\int_0^{\infty}d^3p\int_0^{\mathbf{p}}d^3k\delta(\mathbf{n}_p-\mathbf{n}_k)\{[ic_2(\mathbf{k})a_1(\mathbf{p}-\mathbf{k})v^{\dagger}\gamma_4(\gamma_\mu\gamma_\lambda-\gamma_\lambda\gamma_\mu)u+ic_1(\mathbf{k})a_2(\mathbf{p}-\mathbf{k})u^{\dagger}\gamma_4(\gamma_\mu\gamma_\lambda-\gamma_\lambda\gamma_\mu)v]e^{i(\mathbf{p}\cdot\mathbf{x}-pt)}\}+\text{H.c.} \tag{26}$$

It has been pointed out[2,6,8] that a six-vector results from any two solutions w and y of Dirac's equation by forming the products

$$w^\dagger\gamma_4(\gamma_\mu\gamma_\lambda-\gamma_\lambda\gamma_\mu)y.$$

Therefore, we see that $F_{\mu\lambda}$ as defined by Eq. (26) has the correct transformation properties. There are some similarities between Eq. (26) and Kronig's equations [Eqs. (36)–(38) in Ref. 6], with his A and C corresponding to u and v. The normalization factor $(4\sqrt{2}\pi)^{-1}$ is chosen so that the commutation relations, Eqs. (A18)–(A23) in Appendix A, will have the correct numerical factor. Equation (26) can be written in the form

$$F_{\mu\lambda}(\mathbf{x},t)=\frac{1}{4\sqrt{2}\pi}\int_0^\infty d^3p\, p^{1/2}\{[\xi(\mathbf{p})v^\dagger\gamma_4(\gamma_\mu\gamma_\lambda-\gamma_\lambda\gamma_\mu)u+\eta(\mathbf{p})u^\dagger\gamma_4(\gamma_\mu\gamma_\lambda-\gamma_\lambda\gamma_\mu)v]e^{i(\mathbf{p}\cdot\mathbf{x}-pt)}$$
$$-[\xi^\dagger(\mathbf{p})u^\dagger\gamma_4(\gamma_\mu\gamma_\lambda-\gamma_\lambda\gamma_\mu)v+\eta^\dagger(\mathbf{p})v^\dagger\gamma_4(\gamma_\mu\gamma_\lambda-\gamma_\lambda\gamma_\mu)u]e^{-i(\mathbf{p}\cdot\mathbf{x}-pt)}\}, \quad (27)$$

where $\xi(\mathbf{p})$ and $\eta(\mathbf{p})$ are

$$\xi(\mathbf{p})=\frac{i}{\sqrt{p}}\int_0^{\mathbf{p}} c_2(\mathbf{k})a_1(\mathbf{p}-\mathbf{k})\delta(\mathbf{n}_p-\mathbf{n}_k)d^3k, \quad (28)$$

$$\eta(\mathbf{p})=\frac{i}{\sqrt{p}}\int_0^{\mathbf{p}} c_1(\mathbf{k})a_2(\mathbf{p}-\mathbf{k})\delta(\mathbf{n}_p-\mathbf{n}_k)d^3k. \quad (29)$$

From Eqs. (27)–(29), it appears that $\xi(\mathbf{p})$ and $\eta(\mathbf{p})$ should be interpreted as absorption operators for right- and left-handed photons, respectively. In Sec. IV, $\xi(\mathbf{p})$ and $\eta(\mathbf{p})$ will be shown to obey commutation relations similar to Bose-Einstein commutation relations with some additional terms.

We define $\mathbf{E}$ and $\mathbf{H}$ by

$$F_{\mu\lambda}=\begin{pmatrix}0 & H_3 & -H_2 & -iE_1\\ -H_3 & 0 & H_1 & -iE_2\\ H_2 & -H_1 & 0 & -iE_3\\ iE_1 & iE_2 & iE_3 & 0\end{pmatrix}. \quad (30)$$

From (30),

$$E_s=-iF_{4s}. \quad (31)$$

Using Eqs. (27) and (10), one finds the components of $\mathbf{E}$ are

$$E_s(\mathbf{x},t)=\frac{-i}{2\sqrt{2}\pi}\int_0^\infty d^3p$$
$$\times p^{1/2}\{[\xi(\mathbf{p})v^\dagger\gamma_s u+\eta(\mathbf{p})u^\dagger\gamma_s v]e^{i(\mathbf{p}\cdot\mathbf{x}-pt)}$$
$$-[\xi^\dagger(\mathbf{p})u^\dagger\gamma_s v+\eta^\dagger(\mathbf{p})v^\dagger\gamma_s u]e^{-i(\mathbf{p}\cdot\mathbf{x}-pt)}\}. \quad (32)$$

From (30),

$$H_1=F_{23},\quad H_2=-F_{13},\quad \text{and}\quad H_3=F_{12}. \quad (33)$$

Using Eqs. (27) and (10), and the identities of Eq. (A2) of Appendix A,

$$H_s(\mathbf{x},t)=\frac{-1}{2\sqrt{2}\pi}\int_0^\infty d^3p$$
$$\times p^{1/2}\{[\xi(\mathbf{p})v^\dagger\gamma_s u-\eta(\mathbf{p})u^\dagger\gamma_s v]e^{i(\mathbf{p}\cdot\mathbf{x}-pt)}$$
$$+[\xi^\dagger(\mathbf{p})u^\dagger\gamma_s v-\eta^\dagger(\mathbf{p})v^\dagger\gamma_s u]e^{-i(\mathbf{p}\cdot\mathbf{x}-pt)}\}. \quad (34)$$

The vectors $\mathbf{E}$ and $\mathbf{H}$ are real, as $E_s^\dagger=E_s$ and $H_s^\dagger=H_s$, and they satisfy Maxwell's equations, but new commutation relations, as is shown in Appendix A. It is also shown in Sec. V that E_s and H_s transform under the parity and charge conjugation operations in the usual way.

IV. COMMUTATION RELATIONS FOR PHOTON OPERATORS

The neutrino operators obey the Fermi-Dirac commutation relations:

$$[a_1(k\mathbf{n}),a_1^\dagger(k'\mathbf{n}')]_+ =[a_2(k\mathbf{n}),a_2^\dagger(k'\mathbf{n}')]_+=[c_1(k\mathbf{n}),c_1^\dagger(k'\mathbf{n}')]_+ =[c_2(k\mathbf{n}),c_2^\dagger(k'\mathbf{n}')]_+=\delta(k-k')\delta(\mathbf{n}-\mathbf{n}') \quad (35)$$

while all other combinations anticommute.

The operators $\xi(\mathbf{p})$ and $\eta(\mathbf{p})$ of Eqs. (28) and (29) will be shown to obey the commutation relations:

$$[\xi(\mathbf{p}),\xi(\mathbf{q})]_-=0, \quad (36)$$

$$[\xi(\mathbf{p}),\xi^\dagger(\mathbf{q})]_-=\delta(\mathbf{p}-\mathbf{q})[1-\alpha_{12}(p)], \quad (37)$$

$$[\eta(\mathbf{p}),\eta(\mathbf{q})]_-=0, \quad (38)$$

$$[\eta(\mathbf{p}),\eta^\dagger(\mathbf{q})]_-=\delta(\mathbf{p}-\mathbf{q})[1-\alpha_{21}(p)], \quad (39)$$

$$[\xi(\mathbf{p}),\eta(\mathbf{q})]_-=0, \quad (40)$$

$$[\xi(\mathbf{p}),\eta^\dagger(\mathbf{q})]_-=0, \quad (41)$$

where

$$\alpha_{12}(p)=\frac{1}{p}\int_0^p [a_1^\dagger(k)a_1(k)+c_2^\dagger(k)c_2(k)]dk.$$

Since all neutrino operators anticommute for different $\mathbf{n}$, all photon operators will commute for different $\mathbf{n}$ as this just involves an even number of interchanges of the neutrino operators. Therefore we need consider only the absorption operators for the same $\mathbf{n}$, and Eqs. (28) and (29) reduce to

$$\xi(p)=\frac{i}{\sqrt{p}}\int_0^p c_2(k)a_1(p-k)dk, \quad (42)$$

$$\eta(p)=\frac{i}{\sqrt{p}}\int_0^p c_1(k)a_2(p-k)dk. \quad (43)$$

[8] S. S. Schweber, H. A. Bethe, and F. de Hoffmann, *Mesons and Fields* (Row, Peterson, and Company, Evanston, Illinois, 1955), Vol. 1, pp. 29–30.

These are similar to the one-dimensional operator defined by Jordan [see Eq. (19) in Ref. 3]. However, he let the limits of integration go from $-\infty$ to $+\infty$ and interpreted these extra terms as a Raman effect of neutrinos or antineutrinos (i.e., to simulate the absorption of a photon of energy p, one neutrino or antineutrino with energy $k+p$ is absorbed while another of the same energy state, opposite spin, and energy k is emitted). Nowadays, this Raman effect of neutrinos or antineutrinos is experimentally ruled out as it would easily have been observed in the inverse β-decay experiments.[5,9] For a one-dimensional model in which spin is neglected,[3,10,11] or for a spin-zero particle,[12] one can obtain the Bose-Einstein commutation relations with the Raman-effect terms providing a cancellation of some unwanted terms. However, if spin is included the cancellation does not occur for a spin-one particle.[4]

Proof of (36) for $\mathbf{n}_p=\mathbf{n}_q$ is as follows:

$$-(pq)^{1/2}[\xi(p),\xi(q)]_- =\int_0^p dk\int_0^q dk'[c_2(k)a_1(p-k),\, c_2(k')a_1(q-k')]_-. \quad (44)$$

However, using the fact that all four operators anticommute, it can be quickly shown that

$$[c_2(k)a_1(p-k),\, c_2(k')a_1(q-k')]_-=0 \quad (45)$$

and (36) is proven. Equations (38), (40), and (41) are proven in the same manner.

Proof of (37) for $\mathbf{n}_p=\mathbf{n}_q$ is as follows:

$$(pq)^{1/2}[\xi(p),\xi^\dagger(q)]_- =\int_0^p dk\int_0^q dk'[c_2(k)a_1(p-k),\, \times a_1^\dagger(q-k')c_2^\dagger(k')]_-. \quad (46)$$

Using (35) we obtain,

$$\begin{aligned}[c_2(k)a_1(p-k),\, a_1^\dagger(q-k')c_2^\dagger(k')]_- &=\delta(p-q)\delta(k-k')-a_1^\dagger(q-k')a_1(p-k)\delta(k-k') \\ &\quad -c_2^\dagger(k')c_2(k)\delta(q-p+k-k'). \quad (47)\end{aligned}$$

Substituting (47) in (46) results in

$$\begin{aligned}[\xi(p),\xi^\dagger(q)]_- &=\frac{1}{(pq)^{1/2}}\int_0^p \delta(p-q)dk \\ &\quad -\frac{1}{(pq)^{1/2}}\int_0^{\inf(p,q)} a_1^\dagger(q-k)a_1(p-k)dk \\ &\quad -\frac{1}{(pq)^{1/2}}\int_{\sup(0,p-q)}^p c_2^\dagger(q-p+k)c_2(k)dk, \quad (48)\end{aligned}$$

where $\inf(a,b)$ = smaller of a and b and $\sup(a,b)$ = larger of a and b. In previous calculations with the inclusion of the Raman effect, the last two integrals of (48) can be cancelled out[10] for a one-dimensional model in which spin is neglected.

The expectation value for the last two integrals of (48) is zero for $p\neq q$ since for any state

$$|\Phi\rangle=a_1^\dagger(m)a_1^\dagger(n)\cdots a_1^\dagger(z)|0\rangle,$$

$$\left\langle\Phi\left|-\frac{1}{(pq)^{1/2}}\int_0^{\inf(p,q)} a_1^\dagger(q-k)a_1(p-k)dk\right|\Phi\right\rangle=0 \quad (49)$$

if $p\neq q$. Therefore, (48) becomes

$$[\xi(p),\xi^\dagger(q)]_- =\delta(p-q)\left\{1-\frac{1}{p}\int_0^p [a_1^\dagger(k)a_1(k)+c_2^\dagger(k)c_2(k)]dk\right\}. \quad (50)$$

Equation (39) is proven in an identical manner.

If we assume that the neutrino momenta can only take on discrete values which are multiples of some fundamental momentum, i.e.,

$$k=\epsilon/2,\ 3\epsilon/2,\ 5\epsilon/2,\ \cdots$$

where, for convenience, we enumerate k with the help of half-integers and the photon momenta take on the values

$$p=\epsilon,\ 2\epsilon,\ 3\epsilon,\ \cdots,$$

we can replace the integral of Eq. (42) by a sum,

$$\xi(p)=\frac{i\epsilon}{\sqrt{p}}\sum_{\bar{k}=1/2}^{\bar{p}-1/2} c_2(k)a_1(p-k), \quad (51)$$

where $\bar{p}\epsilon=p$, $\bar{k}\epsilon=k$, and $\Delta k=\epsilon$.

Equation (50) now becomes

$$[\xi(p),\xi^\dagger(q)]_- =\delta(p-q)\left\{1-\frac{1}{\bar{p}}\sum_{\bar{k}=1/2}^{\bar{p}-1/2} [a_1^\dagger(k)a_1(k)+c_2^\dagger(k)c_2(k)]\right\}. \quad (52)$$

The sum in Eq. (52) is the number of ν_1 plus the number of $\bar{\nu}_2$, which equals twice the number of photons. For a state with a few photons and $p\gg\epsilon$, the last term of Eq. (52) can be neglected and Eq. (52) reduces to the Bose-Einstein commutation relation.

It can be seen from Eq. (51) that only one photon can be in the state with momentum $\bar{p}=1$, two photons in the state with $\bar{p}=2$, three photons in the state with $\bar{p}=3$, etc. Although it is usually stated that in order to obtain Planck's law the number of photons allowed in any state must be unlimited, the above conditions (the momentum of the photons in the cells which

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[9] C. L. Cowan, Jr., F. Reines, F. B. Harrison, H. W. Kruse, and A. D. McGuire, Science **124**, 103 (1956).

[10] P. Jordan, Z. Physik **99**, 109 (1936).

[11] M. Born and N. S. Nagendra Nath, Proc. Indian Acad. Sci. **A3**, 318 (1936).

[12] K. M. Case, Phys. Rev. **106**, 1316 (1957).

contain j photons is $jh\nu/c$) will lead to Planck's radiation law as was shown by Bose.[13]

V. PARITY AND CHARGE CONJUGATION OPERATORS

In this section we consider the transformation of the electric and magnetic fields under the parity P and charge conjugation C operations. It will be shown that E_s and H_s transform in the usual way. We define the parity operator such that

$$Pa_1^\dagger(\mathbf{k})P^{-1}=\epsilon_p{}^*a_2^\dagger(-\mathbf{k}), \tag{53}$$

$$Pa_2^\dagger(\mathbf{k})P^{-1}=\epsilon_p{}^*a_1^\dagger(-\mathbf{k}), \tag{54}$$

$$Pc_1^\dagger(\mathbf{k})P^{-1}=\epsilon_p c_2^\dagger(-\mathbf{k}), \tag{55}$$

$$Pc_2^\dagger(\mathbf{k})P^{-1}=\epsilon_p c_1^\dagger(-\mathbf{k}). \tag{56}$$

With these definitions, $\psi(\mathbf{x},t)$ of Eq. (17) transforms such that

$$P\psi(\mathbf{x},t)P^{-1}=\epsilon_p\gamma_4\psi(-\mathbf{x},t), \tag{57}$$

where we have used the fact that

$$\gamma_4u(-\mathbf{p})=v(\mathbf{p}) \quad\text{and}\quad \gamma_4v(-\mathbf{p})=u(\mathbf{p}) \tag{58}$$

which can be seen from (3), (11), and (12).

First, we consider the transformation of $\xi(\mathbf{p})$ and $\eta(\mathbf{p})$ of Eqs. (28) and (29).

$$P\xi^\dagger(\mathbf{p})P^{-1}=\frac{-i}{\sqrt{p}}\int_0^{\mathbf{p}}Pa_1^\dagger(\mathbf{p}-\mathbf{k})P^{-1}Pc_2^\dagger(\mathbf{k})P^{-1}\times\delta(\mathbf{n}_p-\mathbf{n}_k)d^3k$$
$$=\frac{-i\epsilon_p{}^*\epsilon_p}{\sqrt{p}}\int_0^{\mathbf{p}}a_2^\dagger(-\mathbf{p}+\mathbf{k})c_1^\dagger(-\mathbf{k})\delta(\mathbf{n}_p-\mathbf{n}_k)d^3k.$$

Let $\mathbf{k}\to-\mathbf{k}$:

$$P\xi^\dagger(\mathbf{p})P^{-1}=\frac{-i}{\sqrt{p}}\int_0^{-\mathbf{p}}a_2^\dagger(-\mathbf{p}-\mathbf{k})c_1^\dagger(\mathbf{k})\delta(\mathbf{n}_p+\mathbf{n}_k)d^3k,$$

thus

$$P\xi^\dagger(\mathbf{p})P^{-1}=\eta^\dagger(-\mathbf{p}). \tag{59}$$

Similarly,

$$P\eta^\dagger(\mathbf{p})P^{-1}=\xi^\dagger(-\mathbf{p}). \tag{60}$$

We now consider how E_s and H_s of Eqs. (32) and (34) transform.

$$PE_s(\mathbf{x},t)P^{-1}=\frac{-i}{2(2\pi)^{1/2}}\int d^3p\,p^{1/2}\{[P\xi(\mathbf{p})P^{-1}v^\dagger(\mathbf{p})\gamma_su(\mathbf{p})+P\eta(\mathbf{p})P^{-1}u^\dagger(\mathbf{p})\gamma_sv(\mathbf{p})]e^{i(\mathbf{p}\cdot\mathbf{x}-pt)}$$
$$-[P\xi^\dagger(\mathbf{p})P^{-1}u^\dagger(\mathbf{p})\gamma_sv(\mathbf{p})+P\eta^\dagger(\mathbf{p})P^{-1}v^\dagger(\mathbf{p})\gamma_su(\mathbf{p})]e^{-i(\mathbf{p}\cdot\mathbf{x}-pt)}\}$$
$$=\frac{-i}{2(2\pi)^{1/2}}\int d^3p\,p^{1/2}\{[\eta(-\mathbf{p})v^\dagger(\mathbf{p})\gamma_su(\mathbf{p})+\xi(-\mathbf{p})u^\dagger(\mathbf{p})\gamma_sv(\mathbf{p})]e^{i(\mathbf{p}\cdot\mathbf{x}-pt)}$$
$$-[\eta^\dagger(-\mathbf{p})u^\dagger(\mathbf{p})\gamma_sv(\mathbf{p})+\xi^\dagger(-\mathbf{p})v^\dagger(\mathbf{p})\gamma_su(\mathbf{p})]e^{-i(\mathbf{p}\cdot\mathbf{x}-pt)}\}.$$

Let $\mathbf{p}\to-\mathbf{p}$ and insert $\gamma_4{}^2=1$,

$$PE_s(\mathbf{x},t)P^{-1}=\frac{i}{2(2\pi)^{1/2}}\int d^3p\,p^{1/2}\{[\eta(\mathbf{p})v^\dagger(-\mathbf{p})\gamma_4\gamma_s\gamma_4u(-\mathbf{p})+\xi(\mathbf{p})u^\dagger(-\mathbf{p})\gamma_4\gamma_s\gamma_4v(-\mathbf{p})]e^{i(-\mathbf{p}\cdot\mathbf{x}-pt)}$$
$$-[\eta^\dagger(\mathbf{p})u^\dagger(-\mathbf{p})\gamma_4\gamma_s\gamma_4v(-\mathbf{p})+\xi^\dagger(\mathbf{p})v^\dagger(-\mathbf{p})\gamma_4\gamma_s\gamma_4u(-\mathbf{p})]e^{-i(-\mathbf{p}\cdot\mathbf{x}-pt)}\}.$$

Using Eq. (58),

$$PE_s(\mathbf{x},t)P^{-1}=\frac{i}{2(2\pi)^{1/2}}\int d^3p\,p^{1/2}\{[\eta(\mathbf{p})u^\dagger(\mathbf{p})\gamma_sv(\mathbf{p})+\xi(\mathbf{p})v^\dagger(\mathbf{p})\gamma_su(\mathbf{p})]e^{i(-\mathbf{p}\cdot\mathbf{x}-pt)}$$
$$-[\eta^\dagger(\mathbf{p})v^\dagger(\mathbf{p})\gamma_su(\mathbf{p})+\xi^\dagger(\mathbf{p})u^\dagger(\mathbf{p})\gamma_sv(\mathbf{p})]e^{-i(-\mathbf{p}\cdot\mathbf{x}-pt)}\}.$$

Thus,

$$PE_s(\mathbf{x},t)P^{-1}=-E_s(-\mathbf{x},t). \tag{61}$$

In a similar manner one obtains

$$PH_s(\mathbf{x},t)P^{-1}=H_s(-\mathbf{x},t). \tag{62}$$

We define the charge conjugation operator such that

$$Ca_1(\mathbf{k})C^{-1}=\epsilon_cc_2(\mathbf{k}), \tag{63}$$

$$Ca_2(\mathbf{k})C^{-1}=\epsilon_cc_1(\mathbf{k}), \tag{64}$$

$$Cc_1(\mathbf{k})C^{-1}=\epsilon_c{}^*a_2(\mathbf{k}), \tag{65}$$

$$Cc_2(\mathbf{k})C^{-1}=\epsilon_c{}^*a_1(\mathbf{k}). \tag{66}$$

With these definitions $\psi(\mathbf{x},t)$ of Eq. (17) transforms such that

$$C\psi(\mathbf{x},t)C^{-1}=\epsilon_c\gamma_2\psi^\dagger(\mathbf{x},t) \tag{67}$$

where we have used the fact that

$$\gamma_2u(\mathbf{p})=v^*(\mathbf{p});\quad \gamma_2v(\mathbf{p})=u^*(\mathbf{p}). \tag{68}$$

Now, we consider the transformation of $\xi(\mathbf{p})$ and $\eta(\mathbf{p})$ of Eqs. (28) and (29).

$$C\xi(\mathbf{p})C^{-1}=\frac{i}{\sqrt{p}}\int_0^{\mathbf{p}}Cc_2(\mathbf{k})C^{-1}Ca_1(\mathbf{p}-\mathbf{k})C^{-1}\delta(\mathbf{n}_p-\mathbf{n}_k)d^3k$$
$$=\frac{i\epsilon_c{}^*\epsilon_c}{\sqrt{p}}\int_0^{\mathbf{p}}a_1(\mathbf{k})c_2(\mathbf{p}-\mathbf{k})\delta(\mathbf{n}_p-\mathbf{n}_k)d^3k$$

[13] S. N. Bose, Z. Physik **26**, 178 (1924); **27**, 384 (1924); or see R. B. Lindsay, *Introduction to Physical Statistics* (John Wiley & Sons, Inc., New York, 1941), pp. 224–226.

$$=\frac{-i}{\sqrt{p}}\int_0^{\mathbf{p}} c_2(\mathbf{p}-\mathbf{k})a_1(\mathbf{k})\delta(\mathbf{n}_p-\mathbf{n}_k)d^3k$$

$$=\frac{-i}{\sqrt{p}}\int_0^{\mathbf{p}} c_2(\mathbf{k})a_1(\mathbf{p}-\mathbf{k})\delta(\mathbf{n}_p-\mathbf{n}_k)d^3k.$$

Thus,

$$C\xi(\mathbf{p})C^{-1}=-\xi(\mathbf{p}). \tag{69}$$

Similarly,

$$C\eta(\mathbf{p})C^{-1}=-\eta(\mathbf{p}), \tag{70}$$

$$C\xi^\dagger(\mathbf{p})C^{-1}=-\xi^\dagger(\mathbf{p}), \tag{71}$$

$$C\eta^\dagger(\mathbf{p})C^{-1}=-\eta^\dagger(\mathbf{p}). \tag{72}$$

With the use of Eqs. (69)–(72) and (32) and (34), one quickly obtains:

$$CE_s(\mathbf{x},t)C^{-1}=-E_s(\mathbf{x},t), \tag{73}$$

$$CH_s(\mathbf{x},t)C^{-1}=-H_s(\mathbf{x},t). \tag{74}$$

VI. EXPERIMENTAL IMPLICATIONS

It was necessary to use neutrinos with both right-handed and left-handed helicities to form the composite photon from neutrinos. This means that if the theory is correct, the four entities ν_1, ν_2, $\bar{\nu}_1$, and $\bar{\nu}_2$ must exist in nature. It has been determined experimentally that the neutrinos connected with β decay are ν_2 and $\bar{\nu}_2$. If the two neutrinos ν_1 and $\bar{\nu}_1$ exist, they may appear in some meson and hyperon decay modes. The experiment of Danby *et al.*[5] indicates that the neutrinos occurring in $\pi-\mu$ decay are *not* ν_2 or $\bar{\nu}_2$. Coupling that result[5] with the results[14] of polarization experiments, we obtain the pion decay modes

$$\begin{aligned}\pi^+&\rightarrow\mu^++\bar{\nu}_1,\\ \pi^-&\rightarrow\mu^-+\nu_1.\end{aligned} \tag{75}$$

This means that the μ^+ is the particle and μ^- the antiparticle. It then follows that in muon decay

$$\begin{aligned}\mu^+&\rightarrow e^++\nu_1+\nu_2,\\ \mu^-&\rightarrow e^-+\bar{\nu}_1+\bar{\nu}_2.\end{aligned} \tag{76}$$

These decay modes are consistent with polarization experiments and with a Michel parameter $\rho=\frac{3}{4}$, as the neutrinos have different spin orientation. Similar decay schemes with μ^+ as the particle have been suggested by Schwinger[15] to remove the $\mu-e$ quantum number degeneracy and by others[16] to give selection rules which forbid several unobserved transitions. Indeed, now with the results of Danby *et al.*, this appears[17] to be the most logical decay sequence. It would be unsatisfactory in terms of a neutrino theory of photons if

$$\mu^\pm\rightarrow e^\pm+\nu+\bar{\nu} \tag{77}$$

occurred, for then

$$\mu^\pm\rightarrow e^\pm+\gamma \tag{78}$$

should occur readily. This decay mode (78) has not been observed[18] experimentally. Experiments with high-energy neutrinos could shed some light on the neutrinos involved in μ decay. According to (76) the neutrinos from μ^+-decay should participate in the reactions

$$\begin{aligned}\nu_1+p&\rightarrow n+\mu^+,\\ \nu_2+n&\rightarrow p+e^-.\end{aligned} \tag{79}$$

Therefore, only positive muons and negative electrons should be observed as the final products from the decay of positive muons.

Similarly,

$$\begin{aligned}\bar{\nu}_2+p&\rightarrow n+e^+,\\ \bar{\nu}_1+n&\rightarrow p+\mu^-.\end{aligned} \tag{80}$$

So the neutrinos from μ^- decay produce only negative muons and positrons.

VII. DISCUSSIONS

The general effort on the neutrino theory of photons (or light, as it was earlier called) stopped in 1938 when Pryce showed[4] that those theories, based on Jordan's hypothesis[3] and using the old four-component neutrino theory, were not invariant under a spatial rotation of the coordinate system, and there have been only a few papers[12,19–21] since then. In this paper we have made two modifications to the Jordan-Kronig mathematical formalism: (1) The neutrino is described by a particular four-component theory. (2) The neutrino Raman-effect terms are omitted.

Concerning modification (1), as Pryce noted,[4] the theories using the old four-component neutrino were too arbitrary. Kronig,[6] by his Eq. (17) (which is not invariant under spatial rotations), attempted to elimi-

[14] A. I. Alikhanov, Yu. V. Galaktionov, Yu. V. Gorodkov, G. P. Eliseev, and V. A. Lyubimov, Zh. Eksperim. i Teor. Fiz. **38**, 1918 (1960) [English transl.: Soviet Phys.—JETP **11**, 1380 (1960)]; G. Backenstoss, B. D. Hyams, G. Knop, P. C. Marin, and U. Stierlin, Phys. Rev. Letters **6**, 415 (1961); M. Bardon, P. Franzini, and J. Lee, *ibid.* **7**, 23 (1961).

[15] J. Schwinger, Ann. Phys. **2**, 407 (1957). ν_1 and ν_2 are reversed, as ν_1 was thought to be the neutrino connected with electrons at that time.

[16] E. J. Konopinski and H. M. Mahmoud, Phys. Rev. **92**, 1045 (1953); K. Nishijima, *ibid.* **108**, 907 (1957); B. F. Touschek, Nuovo Cimento **5**, 754, 1281 (1957); Y. Katayama, Progr. Theoret. Phys. (Kyoto) **17**, 510 (1957); I. Kawakami, *ibid.* **19**, 459 (1958); H. Umezawa and A. Visconti, Nucl. Phys. **4**, 224 (1957); M. Konuma, *ibid.* **5**, 504 (1958).

[17] S. A. Bludman, Nuovo Cimento **27**, 751 (1963); A. A. Sokolov, Phys. Letters **3**, 211 (1963).

[18] D. Bartlett, S. Devons, and A. M. Sachs, Phys. Rev. Letters **8**, 120 (1962); S. Frankel, J. Halpern, L. Holloway, W. Wales, M. Yearian, O. Chamberlain, A. Lemonick, and F. M. Pipkin, *ibid.* **8**, 123 (1962).

[19] See L. de Broglie, Phys. Rev. **76**, 862 (1949); J. Phys. Radium **12**, 509 (1951).

[20] N. Rosen and P. Singer, Bull. Res. Counc. Israel **8F**, 51 (1959).

[21] I. M. Barbour, A. Bietti, and B. F. Touschek, Nuovo Cimento **28**, 452 (1963).

nate this arbitrariness in such a manner that the usual commutation relations would follow. He was unable to do so because $u^\dagger\gamma v$ and $v^\dagger\gamma u$ are not completely determined by the four-component Dirac equation. With this particular four-component theory for the neutrino, one is uniquely led to Eq. (A6) of Appendix A, and thereby commutation relations which differ from the usual ones only in terms involving $\alpha_{12}(p)$ and $\alpha_{21}(p)$. The fact that the usual commutation relations are not obtained is not due to $u^\dagger\gamma v$ and $v^\dagger\gamma u$.

Modification (2) can be argued as necessary on experimental grounds, as has been done earlier (see Sec. IV). Indeed, the Raman-effect terms are only useful in forming a spin-zero particle. Kronig's combination of neutrino creation and destruction operations [Eqs. (37) and (38) of Ref. 6] would result in no spin change upon emission and absorption of a photon [Kronig's Eqs. (37) and (38) as well as his (17) are not invariant under spatial rotations, but for different reasons]. In Sec. IV, the commutation relations for the photon operators were derived and additional terms appeared in the Bose-Einstein commutation relations. The consequences of these new commutation relations have not been explored other than to note that Planck's radiation law still follows. One of the important problems of the future is to find an experimental test to differentiate between the conventional and composite particle pictures of the photon and these commutation relations may lead to such a check.

Another problem still confronting this theory is the neutrino-antineutrino interaction question. de Broglie originally envisioned some type of interaction binding the neutrino and antineutrino into the photon. Jordan with his inclusion of a neutrino Raman effect had to give up all ideas of such an interaction. The present theory comes closer to de Broglie's original idea and such an interaction seems desirable because: (1) If photons, the "quanta" of the electromagnetic field, are composite particles, then the "quanta" of the nuclear force field and gravitation force field should also be composite particles. An interaction seems essential for the formation of a neutrino theory of pions in an analogous manner to the photon theory. (2) A new ejection mechanism (different from that in weak interactions) would have to be postulated for the neutrino and antineutrino to be emitted in exactly the same direction without an interaction, Whereas a neutrino-antineutrino interaction could force these particles to be emitted in exactly the same direction.

We postulate an interaction model that is intuitively described as follows: When a nucleus tries to emit the neutrino and antineutrino in different directions, the neutrino and antineutrino annihilate. The pair would be continually recreating and annihilating until they are created with parallel momentum and then they would escape together. In terms of the hole theory, neutrinos from the negative energy states are continually making transitions to positive energy states and then back again until one of them can make that transition back again to the negative energy state and still leave the atom. The interaction is essentially an energy interaction. Experiment would not exclude a rare escape of neutrinos in different directions, and in this model these cases would be the weak interaction decays. It appears, therefore, that a solution of the interaction problem should result in a relationship between the coupling constant for weak interactions and the coupling constant for electromagnetic interactions.

It is generally suspected that not all the "elementary particles" are really elementary, but that some are composite particles. The photon seems simple in comparison with the other "elementary particles." However, by learning how to form the photon (perhaps the simplest composite particle), we can develop a method which by modification can be extended to other composite particles. If the photon is a composite particle, gravitons and pions must also be composite particles and they should also be described by similar mathematical formulations.

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APPENDIX A: MAXWELL'S EQUATIONS AND THE COMMUTATION RELATIONS FOR THE PHOTON FIELD

In this Appendix we shall show that the electric and magnetic fields of Eqs. (32) and (34) satisfy Maxwell's equations for a charge-free, current-free region and commutation relations which differ from the usual commutation relations first derived by Jordan and Pauli.[22]

The spinors u and v satisfy the following identities (A2)–(A8) and those obtained by taking the Hermitian conjugate of both sides. These identities follow directly from (9) and (15). Equations (A2)–(A5) can alternatively be obtained[6] from (10), (16), and the four-component Dirac equation:

$$\begin{aligned}(\gamma_1p_1+\gamma_2p_2+\gamma_3p_3+\gamma_4p_4)u=0,\\(\gamma_1p_1+\gamma_2p_2+\gamma_3p_3+\gamma_4p_4)v=0;\end{aligned}\qquad\text{(A1)}$$

$$\begin{aligned}u^\dagger\gamma_4\gamma_2\gamma_3v=u^\dagger\gamma_1v,\\u^\dagger\gamma_4\gamma_3\gamma_1v=u^\dagger\gamma_2v,\\u^\dagger\gamma_4\gamma_1\gamma_2v=u^\dagger\gamma_3v;\end{aligned}\qquad\text{(A2)}$$

$$\begin{aligned}u^\dagger(\gamma_1p_1+\gamma_2p_2+\gamma_3p_3)v=0,\\u^\dagger\gamma_4v=0;\end{aligned}\qquad\text{(A3)}$$

$$\begin{aligned}u^\dagger(\gamma_3p_2-\gamma_2p_3)v=u^\dagger\gamma_1p_4v,\\u^\dagger(\gamma_1p_3-\gamma_3p_1)v=u^\dagger\gamma_2p_4v,\\u^\dagger(\gamma_2p_1-\gamma_1p_2)v=u^\dagger\gamma_3p_4v.\end{aligned}\qquad\text{(A4)}$$

[22] P. Jordan and W. Pauli, Z. Physik 47, 151 (1928).

Or (A4) can be combined and written as

$$u^\dagger(\gamma_\lambda p_\mu-\gamma_\mu p_\lambda)v=\tfrac{1}{2}p_4 u^\dagger\gamma_4(\gamma_\mu\gamma_\lambda-\gamma_\lambda\gamma_\mu)v; \tag{A5}$$

$$u^\dagger\gamma_1 v=\frac{1}{p_4(p_4+ip_3)}\times[i(p_4^2+p_1^2)+p_1p_2-p_3p_4],$$

$$u^\dagger\gamma_2 v=\frac{1}{p_4(p_4+ip_3)}\times[p_4^2+p_2^2+i(p_1p_2+p_3p_4)],$$

$$u^\dagger\gamma_3 v=\frac{1}{p_4(p_4+ip_3)}\times[p_1p_4+p_2p_3+i(p_1p_3-p_2p_4)]; \tag{A6}$$

$$u^\dagger\gamma_s v\cdot v^\dagger\gamma_s u=\frac{1}{p_4^2}(p_4^2+p_s^2),$$

$$u^\dagger\gamma_3 v\cdot v^\dagger\gamma_2 u=\frac{1}{p_4^2}(p_1p_4+p_2p_3),$$

$$u^\dagger\gamma_1 v\cdot v^\dagger\gamma_3 u=\frac{1}{p_4^2}(p_2p_4+p_3p_1),$$

$$u^\dagger\gamma_2 v\cdot v^\dagger\gamma_1 u=\frac{1}{p_4^2}(p_3p_4+p_1p_2). \tag{A7}$$

From (A7) and its Hermitian conjugate,

$$u^\dagger\gamma_r v\cdot v^\dagger\gamma_s u+u^\dagger\gamma_s v\cdot v^\dagger\gamma_r u=\frac{2}{p_4^2}(\delta_{rs}p_4^2+p_rp_s),$$

$$u^\dagger\gamma_3 v\cdot v^\dagger\gamma_2 u-u^\dagger\gamma_2 v\cdot v^\dagger\gamma_3 u=\frac{2}{p_4^2}p_1p_4,$$

$$u^\dagger\gamma_1 v\cdot v^\dagger\gamma_3 u-u^\dagger\gamma_3 v\cdot v^\dagger\gamma_1 u=\frac{2}{p_4^2}p_2p_4,$$

$$u^\dagger\gamma_2 v\cdot v^\dagger\gamma_1 u-u^\dagger\gamma_1 v\cdot v^\dagger\gamma_2 u=\frac{2}{p_4^2}p_3p_4, \tag{A8}$$

with r, s=1, 2, 3.

Direct differentiation of Eq. (32) yields

$$\frac{\partial E_1}{\partial x_1}=\frac{1}{2\sqrt{2}\pi}\int_0^\infty d^3p \times p^{1/2}\{[\xi(\mathbf{p})v^\dagger\gamma_1 u+\eta(\mathbf{p})u^\dagger\gamma_1 v]p_1e^{i(\mathbf{p}\cdot\mathbf{x}-pt)} +[\xi^\dagger(\mathbf{p})u^\dagger\gamma_1 v+\eta^\dagger(\mathbf{p})v^\dagger\gamma_1 u]p_1e^{-i(\mathbf{p}\cdot\mathbf{x}-pt)}\}, \tag{A9}$$

with similar expressions for $\partial E_2/\partial x_2$ and $\partial E_3/\partial x_3$. With the use of (A3), we immediately obtain

$$\nabla\cdot\mathbf{E}=\frac{\partial E_1}{\partial x_1}+\frac{\partial E_2}{\partial x_2}+\frac{\partial E_3}{\partial x_3}=0. \tag{A10}$$

Similarly, one can show from (34) and (A3) that

$$\nabla\cdot\mathbf{H}=0. \tag{A11}$$

The x_1 component of

$$\nabla\times\mathbf{E}=-\partial\mathbf{H}/\partial t \tag{A12}$$

is

$$\frac{\partial E_3}{\partial x_2}-\frac{\partial E_2}{\partial x_3}=-\frac{\partial H_1}{\partial t}. \tag{A13}$$

Equation (A13) is obtained directly by differentiating (32) and substituting the first of (A4). The other two components followed by use of the second and third of (A4).

In a similar manner, using (A4) again, one obtains

$$\nabla\times\mathbf{H}=\partial\mathbf{E}/\partial t. \tag{A14}$$

We next obtain the commutation relations for **E** and **H**. Using (32) and the commutation relations for the photon operators, (36)–(41), one easily obtains

$$\begin{aligned}&[E_s(\mathbf{x},t),E_{s'}(\mathbf{x}',t')]_-\\&=\frac{1}{8\pi^2}\int_0^\infty d^3p\, p(u^\dagger\gamma_{s'}v\cdot v^\dagger\gamma_s u+u^\dagger\gamma_s v\cdot v^\dagger\gamma_{s'}u)(\exp\{i[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]\}-\exp\{-i[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]\})\\&\quad-\frac{1}{8\pi^2}\int_0^\infty d^3p\, p([\alpha_{12}(p)v^\dagger\gamma_s u\cdot u^\dagger\gamma_{s'}v+\alpha_{21}(p)u^\dagger\gamma_s v\cdot v^\dagger\gamma_{s'}u]\exp\{i[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]\}\\&\qquad-[\alpha_{12}(p)u^\dagger\gamma_s v\cdot v^\dagger\gamma_{s'}u+\alpha_{21}(p)v^\dagger\gamma_s u\cdot u^\dagger\gamma_{s'}v]\exp\{-i[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]\}).\end{aligned} \tag{A15}$$

Using the first of (A7) and first of (A8) for $s=s'$ results in

$$[E_s(\mathbf{x},t),E_s(\mathbf{x}',t')]_-=\frac{-1}{4\pi^2}\int_0^\infty d^3p\, p^{-1}(p_4^2+p_s^2)(\exp\{i[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]\}-\exp\{-i[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]\})$$
$$+\frac{1}{8\pi^2}\int_0^\infty d^3p\, p^{-1}[\alpha_{12}(p)+\alpha_{21}(p)](p_4^2+p_s^2)(\exp\{i[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]\}-\exp\{-i[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]\})$$
$$=\frac{i}{2\pi^2}\int_0^\infty d^3p\, p^{-1}\left(\frac{\partial}{\partial t}\frac{\partial}{\partial t'}-\frac{\partial}{\partial x_s}\frac{\partial}{\partial x_s'}\right)\sin[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]$$
$$-\frac{i}{4\pi^2}\int_0^\infty d^3p\, p^{-1}[\alpha_{12}(p)+\alpha_{21}(p)]\left(\frac{\partial}{\partial t}\frac{\partial}{\partial t'}-\frac{\partial}{\partial x_s}\frac{\partial}{\partial x_s'}\right)\sin[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]. \quad \text{(A16)}$$

For $s=1$, $s'=2$ and by the use of the fourth of (A7) and the first of (A8), Eq. (A15) becomes

$$[E_1(\mathbf{x},t),E_2(\mathbf{x}',t')]_-=\frac{-i}{2\pi^2}\int_0^\infty d^3p\, p^{-1}\left(\frac{\partial}{\partial x_1}\frac{\partial}{\partial x_2'}\right)\sin[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]$$
$$+\frac{i}{4\pi^2}\int_0^\infty d^3p\, p^{-1}\left\{[\alpha_{12}(p)+\alpha_{21}(p)]\left(\frac{\partial}{\partial x_1}\frac{\partial}{\partial x_2'}\right)\sin[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]-[\alpha_{12}(p)-\alpha_{21}(p)]\left(\frac{\partial}{\partial x_3}\frac{\partial}{\partial t'}\right)\cos[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]\right\}. \quad \text{(A17)}$$

Following Schiff,[23] we can reduce further the first term of (A16) and the first term of (A17),

$$[E_s(\mathbf{x},t),E_s(\mathbf{x}',t')]_-=-4\pi i\left(\frac{\partial}{\partial t}\frac{\partial}{\partial t'}-\frac{\partial}{\partial x_s}\frac{\partial}{\partial x_s'}\right)D_0(\mathbf{x}-\mathbf{x}',\,t-t')-\frac{i}{4\pi^2}\left(\frac{\partial}{\partial t}\frac{\partial}{\partial t'}-\frac{\partial}{\partial x_s}\frac{\partial}{\partial x_s'}\right)\int_0^\infty d^3p\, p^{-1}[\alpha_{12}(p)+\alpha_{21}(p)]\times\sin[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')], \quad \text{(A18)}$$

$$[E_1(\mathbf{x},t),E_2(\mathbf{x}',t')]_-=4\pi i\frac{\partial}{\partial x_1}\frac{\partial}{\partial x_2'}D_0(\mathbf{x}-\mathbf{x}',\,t-t')+\frac{i}{4\pi^2}\int_0^\infty d^3p\, p^{-1}\left\{[\alpha_{12}(p)+\alpha_{21}(p)]\left(\frac{\partial}{\partial x_1}\frac{\partial}{\partial x_2'}\right)\sin[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]-[\alpha_{12}(p)-\alpha_{21}(p)]\left(\frac{\partial}{\partial x_3}\frac{\partial}{\partial t'}\right)\cos[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]\right\}, \quad \text{(A19)}$$

where

$$D_0(\mathbf{x},t)=(4\pi|\mathbf{x}|)^{-1}[\delta(|\mathbf{x}|-t)-\delta(|\mathbf{x}|+t)]. \quad \text{(A20)}$$

In a similar manner one can obtain

$$[H_s(\mathbf{x},t),H_{s'}(\mathbf{x}',t')]_-=[E_s(\mathbf{x},t),E_{s'}(\mathbf{x}',t')]_- \quad \text{(A21)}$$

$$[E_s(\mathbf{x},t),H_s(\mathbf{x}',t')]_-=\frac{-i}{4\pi^2}\left(\frac{\partial}{\partial t}\frac{\partial}{\partial t'}-\frac{\partial}{\partial x_s}\frac{\partial}{\partial x_s'}\right)\int_0^\infty d^3p\, p^{-1}[\alpha_{12}(p)-\alpha_{21}(p)]\cos[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')], \quad \text{(A22)}$$

[23] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1955), 2nd ed., pp. 384–385.

$$[E_1(\mathbf{x},t),H_2(\mathbf{x}',t')]_-=4\pi i\frac{\partial}{\partial x_3}\frac{\partial}{\partial t'}D_0(\mathbf{x}-\mathbf{x}',t-t')$$
$$+\frac{i}{4\pi^2}\int_0^\infty d^3p\,p^{-1}\left\{[\alpha_{12}(p)+\alpha_{21}(p)]\left(\frac{\partial}{\partial x_3}\frac{\partial}{\partial t'}\right)\sin[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]\right.$$
$$\left.+[\alpha_{12}(p)-\alpha_{21}(p)]\left(\frac{\partial}{\partial x_1}\frac{\partial}{\partial x_2'}\right)\cos[\mathbf{p}\cdot(\mathbf{x}-\mathbf{x}')-p(t-t')]\right\}. \quad \text{(A23)}$$

The cyclic permutation of the indices 1, 2, 3 gives the other parts of Eqs. (A19) and (A23).

These commutation relations for the electromagnetic field (A18)–(A23) differ from the usual commutation relations in the terms involving $\alpha_{12}(p)$ and $\alpha_{21}(p)$. It should be noted that these commutation relations do not satisfy space-like commutativity.

APPENDIX B: INVARIANCE OF E AND H UNDER A ROTATION OF THE COORDINATE SYSTEM

Pryce[4] in 1938 showed that with the Jordan-Kronig theory (using the old four-component neutrino theory) one could not construct an electromagnetic field which is invariant under a rotation of the coordinate system. For that reason it is important to prove that this formulation with a particular four-component neutrino theory is invariant under a rotation of the coordinate system.

What we must prove is that **E** and **H** of Eqs. (32) and (34) are invariant. **E** and **H** will be invariant under the rotation if the four vectors $\xi(\mathbf{p})v^\dagger\gamma u$, $\eta(\mathbf{p})u^\dagger\gamma v$, $\xi^\dagger(\mathbf{p})u^\dagger\gamma v$, and $\eta^\dagger(\mathbf{p})v^\dagger\gamma v$ separately remain invariant.

First we shall examine how the neutrino operators transform under a rotation of the coordinate system. $a_1(\mathbf{k})$ and $a_2(\mathbf{k})$ transform as[4,12]

$$a_1(\mathbf{k})\to a_1'(\mathbf{k})=e^{-is\theta}a_1(\mathbf{k}),$$
$$a_2(\mathbf{k})\to a_2'(\mathbf{k})=e^{is\theta}a_2(\mathbf{k}), \quad \text{(B1)}$$

where $s=\frac{1}{2}$ is the spin of the neutrino.

The other operators then transform so that

$$a_1'^\dagger(\mathbf{k})=e^{is\theta}a_1^\dagger(\mathbf{k}),$$
$$a_2'^\dagger(\mathbf{k})=e^{-is\theta}a_2^\dagger(\mathbf{k}),$$
$$c_1'(\mathbf{k})=e^{is\theta}c_1(\mathbf{k}),$$
$$c_2'(\mathbf{k})=e^{-is\theta}c_2(\mathbf{k}),$$
$$c_1'^\dagger(\mathbf{k})=e^{-is\theta}c_1^\dagger(\mathbf{k}),$$
$$c_2'^\dagger(\mathbf{k})=e^{is\theta}c_2^\dagger(\mathbf{k}). \quad \text{(B2)}$$

We thus see from Eqs. (28), (29), (B1), and (B2) that the photon operators transform so that

$$\xi'(\mathbf{p})=e^{-i2s\theta}\xi(\mathbf{p}),$$
$$\xi'^\dagger(\mathbf{p})=e^{i2s\theta}\xi^\dagger(\mathbf{p}),$$
$$\eta'(\mathbf{p})=e^{i2s\theta}\eta(\mathbf{p}),$$
$$\eta'^\dagger(\mathbf{p})=e^{-i2s\theta}\eta^\dagger(\mathbf{p}), \quad \text{(B3)}$$

as they must be for a particle in the spin states $m_s=\pm1$.

Next we shall determine the transformation properties of $u^\dagger\gamma v$ and $v^\dagger\gamma v$. Unlike the old four-component neutrino theory, here $u^\dagger\gamma v$ and $v^\dagger\gamma u$ are given uniquely by (A6) and its Hermitian conjugate. One failure of the old theory can be traced to the arbitrariness of these quantities.

For convenience, let $\mathbf{w}=u^\dagger\gamma v$. If we rotate about the x_3 axis by an angle θ, the components of $\mathbf{w}$ transform so that

$$w_1\to w_1'=w_1\cos\theta+w_2\sin\theta,$$
$$w_2\to w_2'=-w_1\sin\theta+w_2\cos\theta,$$
$$w_3\to w_3'=w_3. \quad \text{(B4)}$$

In terms of the new (prime) coordinates,

$$p_1=p_1'\cos\theta-p_2'\sin\theta,$$
$$p_2=p_1'\sin\theta+p_2'\cos\theta,$$
$$p_3=p_3'. \quad \text{(B5)}$$

Substituting (B5) in (A6) and then (A6) in (B4) yields

$$\mathbf{w}\to\mathbf{w}'=e^{-i\theta}\mathbf{w}. \quad \text{(B6)}$$

By taking the Hermitian conjugate of $\mathbf{w}$, one also obtains the transformation properties of $v^\dagger\gamma u$,

$$u^\dagger\gamma v\to(u^\dagger\gamma v)'=e^{-i\theta}u^\dagger\gamma v,$$
$$v^\dagger\gamma u\to(v^\dagger\gamma v)'=e^{i\theta}v^\dagger\gamma u. \quad \text{(B7)}$$

Combining (B3) and (B7) results in

$$\xi(\mathbf{p})v^\dagger\gamma u\to[\xi(\mathbf{p})v^\dagger\gamma u]'=\xi(\mathbf{p})v^\dagger\gamma u,$$
$$\eta(\mathbf{p})u^\dagger\gamma v\to[\eta(\mathbf{p})u^\dagger\gamma v]'=\eta(\mathbf{p})u^\dagger\gamma v,$$
$$\xi^\dagger(\mathbf{p})u^\dagger\gamma v\to[\xi^\dagger(\mathbf{p})u^\dagger\gamma v]'=\xi^\dagger(\mathbf{p})u^\dagger\gamma v,$$
$$\eta^\dagger(\mathbf{p})v^\dagger\gamma u\to[\eta^\dagger(\mathbf{p})v^\dagger\gamma u]'=\eta^\dagger(\mathbf{p})v^\dagger\gamma u. \quad \text{(B8)}$$

It then follows directly from Eq. (B8) that **E** and **H** will remain invariant under a rotation of the coordinate system.