Some Consequences of Possible Quadratic Intermediate-Vector-Boson Interactions with Hadrons*

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Some consequences of a hypothesis that the intermediate-vector boson supposed to be responsible for the weak interactions may have quadratic strong interactions with the nucleon are investigated. No contradiction is found with the existing experimental data if the mass of the vector boson is greater than 2.5 BeV. In this scheme, it is possible to explain the small difference between renormalized vector coupling constants of β and μ decays. Some experiments are suggested to test the validity of the postulated strong quadratic interaction.

1. INTRODUCTION

A RECENT paper[1] on the role of intermediate bosons in the unitary symmetry scheme suggested that, in a theory where the intermediate bosons constitute a unitary triplet,[2] it is possible for these particles to interact strongly among themselves or with hadrons (strongly interacting particles). This suggestion was first motivated by the consideration of the concept of the triality quantum number[3] and, secondly, by the need to find some mechanism to explain[4,4a] the large mass (>1.5 BeV) of the conjectured intermediate boson. The essential feature of the theory was the need to introduce a new additive quantum number[5–7] (which we call triality and some call "charm") in connection with the $SU(3)$ group, in order to ensure integral eigenvalues of the charges and hypercharges of a unitary triplet of particles. It was emphasized[3] that this device is equivalent to the introduction of the $U(3)$ group in place of its $SU(3)$ subgroup.

The point now is that a $U(3)$ triplet possesses unit triality, while all the conventional hadrons have zero triality. Since the strong interaction Hamiltonian is assumed to consist of a unitary singlet plus a unitary octet (the symmetry-breaking term), it follows that any strong interaction must conserve triality. One may conclude that any strong interaction between an intermediate boson triplet and hadrons must involve the bosons in pairs and that even in the presence of such a strong interaction, the intermediate boson would still be stable against strong decays. It is the purpose of this paper to spell out some of the consequences of this proposal. In particular, we examine whether the postulated strong "pair interactions" of intermediate bosons actually conflict with existing experimental data. For a reasonable value of the strong coupling constant [see Eq. (2.10)], we find no contradictions with the present data if the mass of the intermediate boson is larger than ≈ 2.5 BeV. For example, our interaction is compatible with the results of the CERN neutrino experiment.

An interesting sidelight of this theory bears on the problem of the renormalization of the vector coupling constant in β decay. There is now strong evidence[8] that there is an unexplained difference of about 2% between the vector coupling constants G_β and G_μ characterizing β decay and μ decay, respectively. We ordinarily expect no renormalization because of the conserved vector current hypothesis. However, in the presence of a strong pair interaction between the intermediate bosons and the hadrons, this argument no longer holds, and indeed we find a renormalization effect for the β decay constant G_β. This mechanism could then explain the discrepancy between G_β and G_μ, although, as we show below, we can also construct an alternative theory of strong quadratic interactions of the intermediate boson which is compatible with a Cabibbo-type theory[9] of weak interactions, namely, one in which there is no renormalization of G_β and the difference between G_β and G_μ is considered to be an inherent one.

Since strong pair production of the intermediate bosons is now possible in our theory, we have estimated the cross sections for strong reactions like $\bar{N}N \to \bar{W}W$, $\pi N \to \bar{W}WN$, and $NN \to \bar{W}WNN$. Experimental tests of these predictions would be of great interest.

* Supported by the U. S. Atomic Energy Commission.

[1] C. Ryan, S. Okubo, and R. E. Marshak, Nuovo Cimento 34, 753 (1964).

[2] For example, B. D'Espagnat, Phys. Letters 7, 204 (1963).

[3] S. Okubo, C. Ryan, and R. E. Marshak, Nuovo Cimento (to be published).

[4] G. Feinberg, Phys. Rev. 134, B1295 (1964), first proposed the idea of a 4-boson interaction for the intermediate vector boson; he did not consider a possible quadratic interaction between such bosons and hadrons based on the triality concept in the unitary symmetry scheme.

[4a] *Note added in proof.* T. Ericson and S. L. Glashow, Phys. Rev. 133, B130 (1964) have briefly considered the possibility of a strong quadratic interaction between W and hadrons.

[5] S. Okubo *et al.*, Ref. 3, where the $U(3)$ group has been employed instead of the conventional $SU(3)$ group.

[6] Cf. M. Gell-Mann, 1964 Dubna Conference on High Energy Physics and L. B. Okun, *ibid.* These authors have considered the $SU(3)\otimes U_1$ group. However, as has been emphasized in Ref. 3, we are dealing, strictly speaking, with the $U(3)$ group rather than $SU(3)\otimes U_1$, since the latter group still allows nonintegral eigenvalues for the charge and hypercharge.

[7] C. R. Hagen and A. J. Macfarlane, Phys. Rev. 135, B432 (1964). D. Amati, H. Bacry, J. Nuyts, and J. Prentki, CERN (unpublished). B. J. Bjorken and S. L. Glashow, Phys. Letters 11, 255 (1964). All these and other authors use the $SU(4)$ group instead of $SU(3)$. The terminology "charm" (essentially the same as our triality) has been used by the last-mentioned authors.

[8] L. Durand, L. Landovitz, and R. Marr, Phys. Rev. 130, 1188 (1963); C. S. Wu, Rev. Mod. Phys. 36, 618 (1964).

[9] N. Cabibbo, Phys. Rev. Letters 10, 531 (1963). For the extension of his ideas to an intermediate vector-meson theory see Refs. 1 and 3 and also S. Okubo, Phys. Letters 8, 362 (1964).

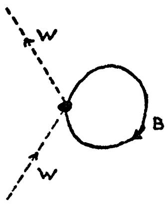


FIG. 1. Self energy of the intermediate boson in lowest order of strong interactions. The black circle denotes the strong quadratic interaction hereafter.

2. STRONG QUADRATIC INTERACTION OF VECTOR BOSONS

In this section, we discuss possible forms of a strong quadratic interaction of the intermediate vector boson. We designate by the symbol $W_{\lambda,a}$ the fields describing a unitary triplet of massive vector mesons, where the Greek subscript $\lambda=(1,2,3,4)$ refers to Lorentz space and the Latin subscript $a=(1,2,3)$ to unitary space. If one wishes to construct a unitary singlet interaction between a pair of baryons and a pair of intermediate bosons, there are, of course, many ways to achieve this. However, since we are only interested in the gross features of such a theory, we concentrate on some rather simple forms of interaction.

Let us first consider the following nonderivative interactions:

$$L_S=(f_S/m_W)\bar{N}_b{}^aN_a{}^b\bar{W}_{\lambda,c}W_{\lambda,c}\,, \tag{2.1}$$

$$L_T=(f_T/m_W)\bar{N}_b{}^a\sigma_{\mu\nu}N_a{}^b\bar{W}_{\mu,c}W_{\nu,c}\,, \tag{2.2}$$

where $N_a{}^b$ and $\bar{N}_b{}^a$ denote the baryon and antibaryon octets, respectively, m_W is the mass of the intermediate boson, and f_S and f_T are dimensionless coupling constants of the scalar and tensor interactions, respectively. The interactions (2.1) and (2.2) contain the following terms involving the nucleon:

$$L_S=(f_S/m_W)(\bar{p}p+\bar{n}n)\bar{W}_\lambda W_\lambda\,, \tag{2.3}$$

$$L_T=(f_T/m_W)(\bar{p}\sigma_{\mu\nu}p+\bar{n}\sigma_{\mu\nu}n)\bar{W}_\mu W_\nu\,, \tag{2.4}$$

where by W_λ we mean $W_{\lambda,1}$, which describes the first (charged) member of the intermediate boson triplet. In this paper, we mainly concentrate on the consequences of these particular terms. Actually, in order to maintain charge independence, we have to add a term involving $\bar{W}_{\lambda,2}W_{\lambda,2}$ or $\bar{W}_{\mu,2}W_{\nu,2}$ to the right-hand side of Eqs. (2.3) and (2.4), respectively, and such a modification is understood to be implied automatically if necessary. We remark that our predictions are based upon Eqs. (2.3) and (2.4) rather than Eqs. (2.1) and (2.2), and hence they are essentially independent of the validity of the $U(3)$ scheme.

For comparison, let us consider the case of derivative coupling of the following forms:

$$L_{S'}=(f_S'/m_W{}^3)(\bar{p}p+\bar{n}n)\bar{U}_{\mu\nu}U_{\mu\nu}\,, \tag{2.5}$$

$$L_{T'}=(f_T'/m_W{}^3)(\bar{p}\sigma_{\mu\nu}p+\bar{n}\sigma_{\mu\nu}n)\bar{U}_{\lambda\mu}U_{\lambda\nu}\,, \tag{2.6}$$

where $U_{\mu\nu}$ is defined as

$$U_{\mu\nu}=\partial_\mu W_\nu-\partial_\nu W_\mu\,. \tag{2.7}$$

Note that Eqs. (2.5) and (2.6) are invariant under the gauge transformation $W_\lambda\to W_\lambda-\partial\Lambda/\partial x_\lambda$ for an arbitrary Λ. Hence, we may call Eqs. (2.5) and (2.6) the gauge-independent interactions in contrast to the gauge-dependent ones of Eqs. (2.3) and (2.4). As we see below, there are some substantial differences in our predictions, depending upon whether we assume gauge-dependent interactions.

Of course, in addition to the above strong interactions, we have the customary weak interactions linear in the intermediate boson field:

$$L_{\text{weak}}=g\bar{W}_\mu l_\mu+g\bar{W}_\mu J_\mu\,, \tag{2.8}$$

where now W_μ is the *charged* member of our $U(3)$ triplet, l_μ is the lepton current and J_μ is the hadron current.

As stated in the beginning, one of the reasons which led us to postulate a strong interaction for the intermediate boson was the desire to provide a mechanism which could generate its large mass as a self-energy effect. Equation (2.1) can give rise to such a self-energy to first order in f_S through the Feynman diagram of Fig. 1, and one finds the following:

$$\delta(m_W{}^2)=(8f_S/m_W)\,\mathrm{tr}S_F(0)$$
$$=\frac{2f_S}{\pi^2}\frac{m_N}{m_W}\left[\Lambda^2-m_N{}^2\ln\left(\frac{\Lambda^2}{m_N{}^2}+1\right)\right], \tag{2.9}$$

where

$$S_F(x)=\frac{i}{(2\pi)^4}\int\frac{k+m}{k^2-m^2}e^{-ikx}d^4k\,,$$

Λ is the cutoff, and the factor 8 comes from the eight baryons in the $\frac{1}{2}^+$ baryon octet, whose mass differences we have neglected. We assume that this contribution to the mass of the intermediate boson is positive and, in fact, accounts for the bulk of its mass. If we choose $m_W\approx 2.5m_N$ and $f_S{}^2=4\pi$, we obtain $\Lambda\approx 5m_N$. Thus, it is not difficult to generate an intermediate boson mass of the order of 2–3 BeV, if one uses "reasonable" strong interaction parameters. Due to the quadratic cutoff, nothing more quantitative may be said about the magnitude of f_S. For the purpose of making estimates, we adopt hereafter the values

$$\frac{f_S{}^2}{4\pi}=\frac{f_T{}^2}{4\pi}=\frac{(f_S')^2}{4\pi}=\frac{(f_T')^2}{4\pi}=1\,. \tag{2.10}$$

An important observation with respect to Eq. (2.9) is that the sign of the coupling constant f_S is fixed by the requirement that $\delta(m_W{}^2)>0$. This rather unique possibility of being able to determine the sign of the coupling is due to the first-order nature of the perturbation that gave rise to the Feynman diagram of Fig. 1 and is very useful (cf. Sec. 4).

As for the tensor interaction (2.2), we note that the first-order contribution vanishes, but that higher order

TABLE I. W-meson production cross sections (in units of 10^{-36} cm^2) in the reaction $\nu+N\rightarrow\mu+W+N$.

$(E_\nu/m_N)_{\text{lab}}$ \ m_W/m_N	1.4	1.7	2.0	2.2	2.6	1.4	1.7	2.0	2.2	2.6
			Scalar					Tensor		
3.0	0.031					0.076				
3.5	0.21					0.61				
4.0	0.55	0.024				1.8	0.053			
4.5	1.1	0.11				3.7	0.35			
5.0	1.9	0.27	0.015			6.5	0.91	0.044		
5.5	2.9	0.52	0.054	0.0044		10.1	1.8	0.18	0.012	
6.0	4.1	0.86	0.13	0.022		15	3.0	0.43	0.072	
6.5	5.6	1.3	0.25	0.061		20	4.7	0.85	0.20	
7.0	7.2	1.8	0.41	0.12	0.0023	27	6.8	1.5	0.43	0.0074
7.5	9.1	2.5	0.61	0.21	0.010	34	9.2	2.3	0.74	0.032
8.0	11	3.2	0.86	0.33	0.026	42	12	3.2	1.2	0.090
8.5	14	4.0	1.1	0.47	0.052	51	15	4.4	1.8	0.18
9.0	16	5.0	1.5	0.65	0.089	61	19	5.8	2.5	0.33
9.5	19	6.0	1.9	0.85	0.14	72	23	7.4	3.3	0.51
10.0	22	7.1	2.3	1.1	0.20	84	28	9.3	4.3	0.76
10.5	25	8.2	2.8	1.4	0.28	97	33	11	5.4	1.1
			Gauge-Invariant Scalar					Gauge-Invariant Tensor		
3.0	0.34					0.035				
3.5	2.8					0.31				
4.0	9.3	0.29				1.1	0.031			
4.5	22	1.5				2.6	0.17			
5.0	41	4.1	0.15			5.1	0.41	0.018		
5.5	68	8.6	0.65	0.044		8.9	1.8	0.081	0.0054	
6.0	105	15	1.7	0.24		14	2.0	0.22	0.030	
6.5	150	25	3.5	0.71		21	3.3	0.46	0.091	
7.0	210	38	6.3	1.6	0.022	30	5.3	0.82	0.20	0.0027
7.5	280	55	10	2.8	0.10	42	7.7	1.4	0.38	0.013
8.0	370	76	15	4.8	0.28	54	11	2.1	0.65	0.036
8.5	490	100	22	7.3	0.59	70	15	3.0	1.0	0.071
9.0	580	130	30	11	1.1	88	19	4.5	1.5	0.14
9.5	710	170	40	15	1.7	110	25	5.8	2.1	0.25
10.0	860	200	51	20	2.7	130	31	7.6	2.9	0.38
10.5	1020	250	65	26	3.8	160	39	10	3.9	0.56

contributions will be nonzero. For the gauge-invariant interactions, perturbation theory does not generate a self-mass of the intermediate boson, although nonperturbative calculations might yield a nonvanishing self-mass, as has been conjectured by some authors.[10] One may also consider Feinberg's postulate[4] that the mass of the intermediate boson is due to a four-boson self-coupling. At any rate, one may explain an intermediate boson mass of the order of 2–3 BeV by the hypothesis of a strong quadratic interaction, and we choose the coupling constants f_T, f_S', and f_T' to be of the same order as f_S.

3. PRODUCTION OF THE INTERMEDIATE BOSON IN NEUTRINO-NUCLEON COLLISIONS

Interactions such as (2.3)–(2.6) will give rise to rather copious production of the intermediate boson in neutrino-nucleon collisions. Such reactions may be represented graphically in Fig. 2, where the lepton vertex is semiweak, while the hadron vertex is the quadratic strong interaction. We have computed $\sigma(E_\nu,m_W)$, the incoherent cross section for this process, for various values of the incident energy and the intermediate boson mass. The results of this calculation are presented in Table I.

[10] For example, J. Schwinger, Phys. Rev. **125**, 397 (1962).

The energy dependence of the cross sections is very similar, and the major differences between them are scale changes due to the different factors introduced by the spin summations. For a given mass m_W of the intermediate boson, the cross sections are between two and three orders of magnitude larger than those calculated by Lee *et al.*[11] on the basis of the incoherent electromagnetic mechanism.

We now wish to compare our theory with the results of the CERN experiment. We denote by $\sigma_{\text{EM}}(Z,A,E_\nu,m_W)$ the total cross section for W production via the electromagnetic mechanism for neutrinos of laboratory energy E_ν incident upon nuclei of total charge Z and mass number A. Since the strong quadratic boson interaction treats neutrons and protons on the same footing, the corresponding cross section will be $A\sigma(E_\nu,m_W)$. We

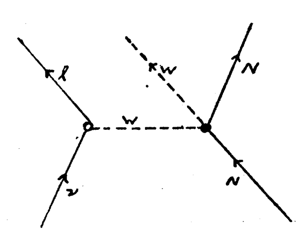


FIG. 2. Production of the intermediate boson by neutrino-nucleon collisions. The open circle denotes the semiweak interaction hereafter.

[11] T. D. Lee, P. Markstein, and C. N. Yang, Phys. Rev. Letters **7**, 429 (1961).

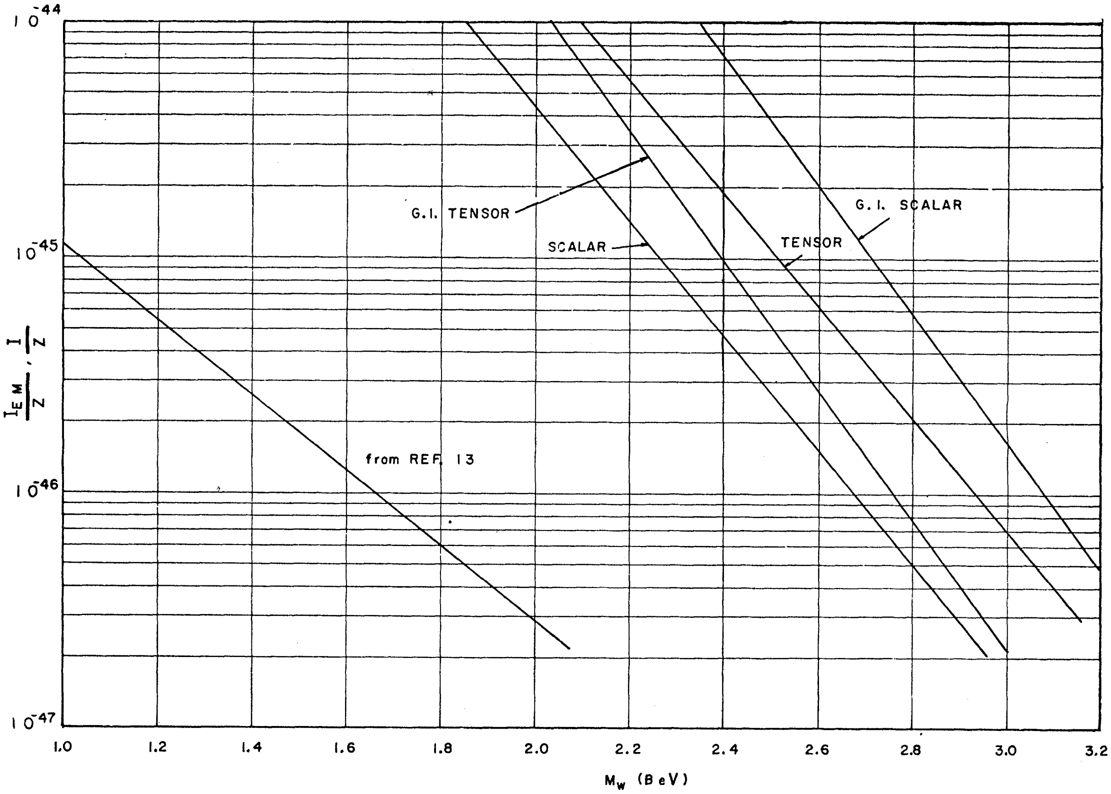


FIG. 3. The quantities I/Z and $I_{\rm EM}/Z$.

denote the CERN neutrino energy spectrum[12] by $\partial^2N/\partial E_\nu\partial S$ and define the following quantities that are relevant to the rate of production of the intermediate boson:

$$I(m_W)\equiv\int A\sigma(E_\nu,m_W)\frac{\partial^2N}{\partial E_\nu\partial S}dE_\nu\,, \tag{3.1}$$

$$I_{\rm EM}(Z,A,m_W)\equiv\int\sigma_{\rm EM}(Z,A,E_\nu,m_W)\frac{\partial^2N}{\partial E_\nu\partial S}dE_\nu\,. \tag{3.2}$$

Now the calculations of Wu *et al.*[13] imply that $(1/Z)\sigma_{\rm EM}(Z,A,E_\nu,m_W)$ is nearly independent of Z and A for materials in question: copper, aluminum, and freon. We therefore compare I/Z (with I given by (3.1) and $A\approx 2Z$) with $(1/Z)I_{\rm EM}$ [with $I_{\rm EM}$ given by (3.2)]. Figure 3 displays these quantities as a function of m_W using the CERN neutrino spectrum.[14]

The results of the CERN neutrino experiment have been recently reported by Bernardini.[15] Assuming the standard production mechanism and the preponderance of the pion decay modes, the bubble chamber part of the experiment[16] placed a lower limit of 1.5 BeV on the mass of the intermediate boson. If one now assumes that our mechanism dominates the boson production process, it is seen from Fig. 3 that the scalar interaction places a lower limit ≈ 2.5 BeV on m_W. The assumption that the decay of W is totally leptonic yielded a lower limit of 1.8 BeV and implies a correspondingly higher mass in our theory. The theoretically predicted[17] branching ratio lies between the two extremes and so does the corresponding lower limit on m_W.

The spark-chamber experiment[18] also yielded a lower limit of 1.8 BeV for m_W, under the assumption that the ratio of leptonic to pionic decay of W was unity. However, any comparison of our theory and the spark-chamber results by means of Fig. 3 implicitly assumes that the space and energy distributions for the muon pairs in our theory are the same as those in the standard theory.[15] This is not necessarily true and might make an important difference due to the geometric and kinematic limitations associated with the spark-chamber setup. We therefore rely on the bubble-chamber experiment

[12] We use the 1964 energy spectrum. See Ref. 15.

[13] A. C. T. Wu, C. Yang, K. Fuchel, and S. Heller, Phys. Rev. Letters **12**, 57 (1964). These calculations agree closely with the earlier work of J. S. Bell and M. Veltman, Phys. Letters **5**, 94 (1963).

[14] This plot has an additional feature: It turns out that if one uses the 1963 neutrino energy spectrum (cf. Ref. 15), the lines shift vertically by an amount that leaves the mass correlations unchanged.

[15] G. Bernardini, Proceedings of 1964 International Conference on High Energy Physics at Dubna (to be published).

[16] M. M. Block, H. Burmeister, D. C. Cundy, B. Eiben, C. Franzinetti *et al.*, Phys. Letters **12**, 281 (1964).

[17] Cf. H. S. Mani and J. C. Nearing, Phys. Rev. **135**, B1009 (1964).

[18] G. Bernardini, J. K. Bienlein, G. Von Dardel, H. Faissner, F. Ferrero *et al.*, Phys. Letters **13**, 86 (1964).

for the lower limit on m_W required by the mechanism which we propose.

In connection with the neutrino experiment, we must also inquire whether our theory predicts too large a cross section for elastic scattering of neutrinos by nucleons. From the postulate of strong interactions for the intermediate boson (cf. Fig. 4), this process can occur in the same order of the weak interaction as the ordinary process $\nu_\mu+p \rightarrow \mu^+ + n$. For the elastic process $\nu+p \rightarrow \nu+p$, we can easily find that the scalar interactions, Eqs. (2.3) and (2.5), give identically zero contributions, while the tensor interactions (2.4) and (2.6) give cross sections which are of the order of 1% of the cross sections for the normal reactions $\bar{\nu}_\mu+p \rightarrow \mu^+ + n$ for the incident neutrino energy spectrum of CERN. In making this estimate, we have taken the cutoff energy $\Lambda \approx 5$ BeV in the spirit of the preceding section. Hence, the elastic scattering process $\nu+p \rightarrow \nu+p$ induced by our theory does not contradict the present experimental data.[15] Although the predicted cross section for this process does not exceed 1% of that for the ordinary process, it would be of great interest to have more accurate experimental data on this point because the usual theory of weak interactions only allows the reaction $\nu+p \rightarrow \nu+p$ to take place in higher order (with respect to weak interactions) and hence predicts a cross section that is much smaller than 1% of the ordinary process.

Thus, a strong quadratic interaction of intermediate bosons with hadrons does not contradict the findings of the high-energy neutrino experiments performed until now. It does not predict too copious a W production rate nor does it induce a neutral current interaction of the type $(\bar{p}p)(\bar{\nu}\nu)$ with a strength inconsistent with the CERN data.

4. RENORMALIZATION EFFECT OF THE VECTOR COUPLING CONSTANT IN β DECAY

Perhaps the most intriguing feature of the present theory is that it may provide an explanation of the small discrepancy between the muon decay constant, G_μ, and the vector coupling constant in β decay, G_β, in terms of a renormalization effect. The fact that the intermediate boson, which is the carrier of the weak interaction, may have strong quadratic interactions implies that G_β, but not G_μ, is in general altered by strong interactions, even within the framework of the conserved vector current hypothesis.[19]

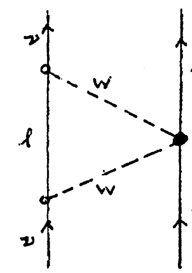


FIG. 4. Elastic neutrino-nucleon scattering.

One may see this most easily in the following manner: The matrix element for β decay contains.

$$M_\mu \equiv \langle p | W_\mu(0) | n \rangle . \tag{4.1}$$

The equation of motion of the W field may be written as

$$(\Box + m_W^2) W_\mu - \partial_\mu \partial_\nu W_\nu = g J_\nu , \tag{4.2}$$

where m_W is the physical mass of the intermediate boson and the current may contain renormalization counter-terms. Since for zero-momentum transfer we have

$$\langle p | \partial_\nu W_\nu | n \rangle = 0$$

and

$$\begin{aligned} \langle p | \Box W_\mu | n \rangle &= 0 , \\ M_\mu &= (g/m_W^2) \langle p | J_\nu(0) | n \rangle . \end{aligned} \tag{4.3}$$

Within the context of the ordinary conserved vector current hypothesis, we have $J_\nu{}^V = j_\nu{}^{(+)}$, where $J_\nu{}^V$ is the vector part of the current and $j_\nu{}^{(+)}$ is the conserved isotopic spin current. One may then show[19,20] that G_β suffers no renormalization. However, in our theory, $J_\nu{}^V$ contains an effective contribution from the strong interaction of the intermediate boson and the proof of nonrenormalization of G_β does not apply. We obtain $(f_S/m_W)\bar{N}NW_\mu$ as the contribution of the scalar strong interaction Eq. (2.1) to the current. Thus, in addition to the ordinary diagrams of β decay which are obtained from $\langle p | j_\nu{}^{(+)}(0) | n \rangle$ (cf. Fig. 5), we have diagrams like Fig. 6 which rise to a renormalization of G_β.

We have computed the renormalization effect due to the diagrams of Fig. 6 for the scalar interaction (the contribution from the tensor interaction Eq. (2.4)

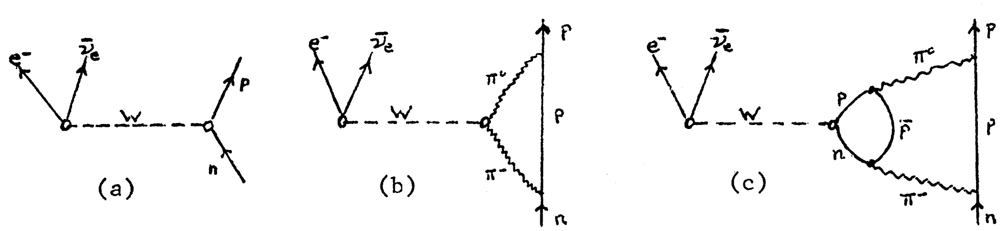


FIG. 5. Some standard diagrams contributing to the decay of the neutron.

[19] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).
[20] For example, S. Okubo, Nuovo Cimento **13**, 293 (1959).

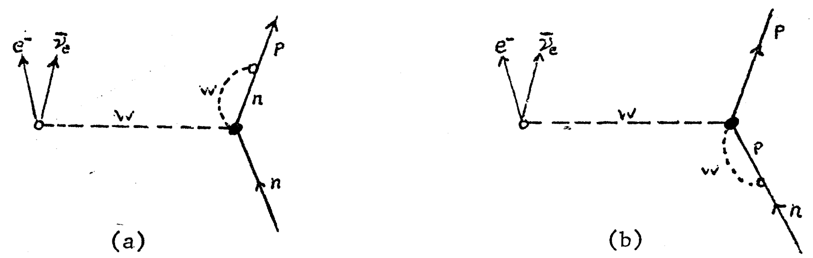


FIG. 6. Diagrams contributing to the decay of the neutron that arise from the postulated strong interaction of the intermediate boson.

turned out to be zero). Some of the details are given in Appendix A. The result for the renormalization of the vector coupling constant in beta decay may be expressed as

$$G_\beta \to G_\beta(1+x)\,,$$

where

$$x=\frac{f_S}{16\pi^2}\frac{m_N}{m_W}\left[-6\ln\frac{\Lambda}{m_N}+\frac{6\ln(m_W/m_N)}{1-(m_N/m_W)^2}-3.5\right]. \quad (4.4)$$

In the above equation, m_N is the nucleon mass and Λ is the cutoff. Note that the divergence is now only logarithmic in contrast to the quadratic divergence of the self-mass calculated in Sec. 2.

A number of authors have calculated the effects of radiative corrections on the determination of G_β and G_μ in a theory of weak interactions in which there is an intermediate boson.[21] They conclude that an intermediate boson of mass >1.5 BeV leads to a difference between the coupling constants that appear in the "bare" weak Lagrangian of about 2–3%, where $G_\beta < G_\mu$. In these theories, due to the conserved vector current (CVC) hypothesis, G_β suffers no strong interaction renormalization so that the vector part of the effective weak Lagrangian is the same as the vector part of the "bare" weak Lagrangian. We, on the other hand, find such a renormalization effect. An evaluation of Eq. (4.4) with $f_S=+(4\pi)^{1/2}$, $m_W=2.5m_N$ and $\ln(\Lambda/m_W)=1$ leads to $x\approx-0.027$, which is about the magnitude of the discrepancy. The important point is that the sign of x is fixed by the sign of f_S, which is determined unambiguously by the first-order self-mass of the intermediate boson (see Sec. 2). Thus we obtain the result that if universality holds, i.e., if $G_\beta{}^0=G_\mu{}^0$ in the "bare" weak Lagrangian, then the inclusion of a strong pair interaction of the intermediate boson predicts $(G_\mu-G_\beta)/G_\mu \approx +0.03$. The fact that we obtain the correct sign and magnitude of the renormalization correction in a manner that is consistent with the hypothesis of the generation of the boson mass is perhaps encouraging, even though the calculations are troubled by the customary ambiguities associated with the use of a cutoff.

The experiments on weak magnetism are usually taken as strong confirmation of the CVC hypothesis. The violation of the conservation of the vector current which is implied by the graphs of Fig. 6 gives rise to a correction to the magnetic form factor in beta decay. It is easily checked from an evaluation of the last equation of Appendix A that this amounts to a correction of $<1\%$ to $(\mu_p-\mu_n)$, the difference between the anomalous magnetic moments of the proton and neutron, and is consistent with the experimental evidence.[22]

So far, in this section, we have performed our computations assuming the gauge-dependent interactions (2.3) and (2.4). However, there is an important difference between the gauge-dependent interaction (2.3) or (2.4) and the gauge-independent interaction (2.5) or (2.6). For a gauge-independent theory based on (2.5) or (2.6), we find that the renormalization effect is zero not only in the lowest order diagrams of Fig. 6(a) and 6(b), but also to all orders of the perturbation with respect to f_S' or f_T'. The reason is the following. The interactions (2.5) and (2.6) are derivative interactions with respect to the intermediate vector meson. Because of this, matrix elements for the diagram Figs. 6(a) and 6(b) must be proportional to the momentum transfer between the proton and neutron. Therefore, for zero-momentum transfer, these matrix elements vanish identically. This reasoning applies equally to all other similar diagrams of higher order in f_S' or f_T'. Hence, for the gauge-independent theory, the only nonzero contributions come from the ordinary diagrams corresponding to the evaluation of the matrix element $\langle p|j_\mu{}^{(+)}(0)|n\rangle$. Therefore, we have no renormalization effect for G_β, even though we may have strong boson interactions.

Thus, in a gauge-independent theory of strong pair interactions of the intermediate boson, the experimental discrepancy between G_β and G_μ must be regarded as inherent from the beginning (if radiative effects are found to be too small). Actually, Cabibbo's[9] theory is based upon this point of view. If we wish to preserve the idea of Cabibbo, as well as the presence of a quadratic strong boson interaction, then we must use the gauge-independent interaction Eqs. (2.5) or (2.6) instead of (2.3) or (2.4). If a strong interaction of intermediate

[21] G. Dorman, Nuovo Cimento **32**, 1226 (1964); R. A. Schaffer, Phys. Rev. **128**, 1452 (1962); and D. Bailin, *ibid.* **135**, B166 (1964). These authors have calculated the radiative effects in β decay under the assumption of a structureless pnW vertex. We also assume that the radiative effects are essentially unchanged by the structure introduced by the strong interactions including Eq. (2.1). Riazuddin, Phys. Rev. **134**, B235 (1964), however, has assumed that the discrepancy between G_β and G_μ may be a consequence of just such a combined effect of electromagnetic and strong interactions.

[22] C. S. Wu, see Ref. 8.

bosons with hadrons is established, the gauge-dependent theory could be more appealing. We note that for the gauge-dependent theory, we would also expect a small difference between the β-decay coupling constants of the nucleon and the pion, since the renormalization effects would now be different for both cases. Preliminary experimental indications[23] can be reconciled with such a small difference, although the experimental error is still too large to allow for any definite conclusion.

Finally, it should be pointed out that the formulation of the conserved vector current hypothesis must be modified in our theory. For the gauge-independent theory, the equation expressing conservation of the vector current may be replaced by the conservation law

$$\partial_\mu W_\mu^{(V)}=0\,, \tag{4.5}$$

where $W_\mu^{(V)}$ is the part of W_μ which includes all contributions of strong interactions plus the part coming from the vector current of the weak interaction in the sense of the Yang-Feldman formalism.[24] Essentially, Eq. (4.5) is the consequence of the gauge invariance of the theory when we take account of all strong interactions plus the vector part of the weak interaction.

5. *W*-PAIR PRODUCTION IN STRONG REACTIONS

As we mention in Sec. 1, in the present theory, the intermediate boson can be produced in pairs by such strong reactions as

$$\bar{p}p\rightarrow \bar{W}W\,, \tag{5.1}$$

$$\pi N\rightarrow \bar{W}WN\,, \tag{5.2}$$

$$NN\rightarrow \bar{W}WNN\,. \tag{5.3}$$

Let us first estimate the cross section for reaction (5.1). For this purpose, we calculate the lowest order diagram for this reaction, as shown in Fig. 7. Corresponding to the interactions in Eqs. (2.3)–(2.6), we have, respectively:

$$\sigma_S(s)=\frac{f_S{}^2}{4\pi}\left(\frac{s-4m_W{}^2}{s-4m_N{}^2}\right)^{1/2}\frac{1}{32sm_W{}^6}[s^3-4(m_W{}^2+m_N{}^2)s^2$$
$$+4m_W{}^2(3m_W{}^2+4m^2)s-48m_N{}^2m_W{}^4]\,, \tag{5.4a}$$

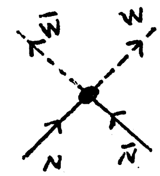
$$\sigma_T(s)=\frac{f_T{}^2}{4\pi}\left(\frac{s-4m_W{}^2}{s-4m_N{}^2}\right)^{1/2}\frac{1}{96sm_W{}^6}[s^3+4(2m_N{}^2+m_W{}^2)s^2$$
$$-8m_W{}^2(2m_N{}^2+m_W{}^2)s-160m_N{}^2m_W{}^4]\,, \tag{5.4b}$$

$$\sigma_{S'}(s)=\frac{(f_S{}')^2}{4\pi}\left(\frac{s-4m_W{}^2}{s-4m_N{}^2}\right)^{1/2}\frac{1}{4sm_W{}^6}[s^3-4(m_W{}^2+m_N{}^2)s^2$$
$$+2m_W{}^2(3m_W{}^2+8m_N{}^2)s-24m_N{}^2m_W{}^4]\,, \tag{5.4c}$$

[23] I. V. Chuvilo, 1964 Dubna Conference on High Energy Physics (to be published).

[24] C. N. Yang and D. Feldman, Phys. Rev. 79, 972 (1950).

FIG. 7. Nucleon-antinucleon annihilation into an intermediate boson pair.



$$\sigma_{T'}(s)=\frac{(f_T{}')^2}{4\pi}\left(\frac{s-4m_W{}^2}{s-4m_N{}^2}\right)^{1/2}\frac{1}{24sm_W{}^6}[s^3+2(m_N{}^2-m_W{}^2)s^2$$
$$-2m_W{}^2(2m_N{}^2+m_W{}^2)s-4m_N{}^2m_W{}^4]\,. \tag{5.4d}$$

In the above, s is the square of the total energy in the c.m. system. To get a rough idea for the cross section, we remark that for $m_W\approx 2.5m_N$ and $s=30m_N{}^2$, we obtain

$$\sigma_S=1.3\times10^{-29}\text{ cm}^2\,,$$
$$\sigma_T=1.1\times10^{-29}\text{ cm}^2\,,$$
$$\sigma_{S'}=6.6\times10^{-29}\text{ cm}^2\,,$$
$$\sigma_{T'}=1.6\times10^{-29}\text{ cm}^2\,.$$

For the reactions (5.2) and (5.3), even the lowest order Feynman diagrams are difficult to estimate and probably unreliable. For this reason, we have estimated the cross sections for these reactions by means of a relativistic statistical model, where apart from the relativistic invariant phase volume and statistical factors, we set the matrix elements of the reactions equal to unity in the natural units of $c=\hbar=m_\pi=1$. The results of these computations are tabulated in Appendix B for various energies and values of m_W. Here we simply remark that these cross sections range between 10^{-30} and 10^{-33} cm^2, depending upon m_W and the incident energy.

6. CONCLUSIONS

This completes our examination of the consequences of assuming that the hypothetical intermediate boson of weak interactions possesses strong quadratic interactions with nucleons (and other strongly interacting particles). Our analysis shows that this hypothesis is consistent with what is presently known about the production of the intermediate boson and its virtual effects (e.g., the enhancement of the cross section for the process $\nu+p\rightarrow\nu+p$) from experimental studies. In addition, the theory has the virtue of providing a mechanism for generating the large mass of the intermediate boson and suggesting a possible explanation of the $G_\beta-G_\mu$ discrepancy within the framework of the conserved vector current hypothesis.

From an experimental point of view, the theory raises the possibility that the intermediate boson may be strongly produced in pairs, and it also predicts that the elastic scattering of neutrinos by nucleons should occur with a measurable cross section. It would be of great interest to test these predictions.

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APPENDIX A

In this Appendix, we give some details of the calculation leading to Eq. (4.4). We start from the Lagrangian

$$\mathcal{L}=(f_S/m_W)(\bar{p}p+\bar{n}n)\bar{W}_\lambda W_\lambda+g\bar{W}_\mu l_\mu+\text{H.c.}+g\bar{p}\gamma_\mu nW_\mu+\text{H.c.},$$

where l_μ is the lepton current and g is the semiweak coupling constant. We denote the amplitudes obtained from Figs. 5(a), 6(a), and 6(b) as $T^{(1)}$, $T^{(2)}$, and $T^{(3)}$, respectively, and obtain for the sum of the three amplitudes

$$T=-\sum_{i=1}^{3}g^2\bar{l}_\mu\bar{u}(p)\Gamma_\alpha{}^{(i)}u(n)P_{\mu\alpha}(p-n)$$

where

$$P_{\mu\alpha}(q)=\frac{-g_{\mu\alpha}+q_\mu q_\alpha/m_W{}^2}{q^2-m_W{}^2},$$

$$\Gamma_\alpha{}^{(1)}=\gamma_\alpha,$$

$$\Gamma_\alpha{}^{(2)}=\frac{-if_S}{(2\pi)^4m_W}\int\gamma_\beta\frac{p-k+m_N}{(p-k)^2-m_N{}^2}P_{\beta\alpha}(k)d^4k,$$

$$\Gamma_\alpha{}^{(3)}=\frac{-if_S}{(2\pi)^4m_W}\int\frac{n+k+m_N}{(n+k)^2-m_N{}^2}\gamma_\beta P_{\beta\alpha}(k)d^4k.$$

and γ is the Dirac matrix, m_N the nucleon mass. Using standard methods,[25] we obtain

$$\Gamma_\alpha{}^{(2)}+\Gamma_\alpha{}^{(3)}=\gamma_\alpha\frac{if_Sm_N}{(2\pi)^4m_W}\left\{3\int\frac{d^4k}{[k^2-m_N{}^2]^2}+2i\pi^2\int_0^1(x-2)\ln\left[x^2+\frac{m_W{}^2}{m_N{}^2}(1-x)\right]dx+i\frac{\pi^2}{2}\right\}$$
$$+\sigma_{\alpha\beta}q_\beta\frac{if_S}{(2\pi)^4m_W}\left\{\int\frac{d^4k}{[k^2-m_N{}^2]^2}+2i\pi^2\int_0^1(x-1)\ln\left[x^2+\frac{m_W{}^2}{m_N{}^2}(1-x)\right]dx+i\frac{\pi^2}{2}\right\}.$$

The approximate expressions

$$\int_0^1(x-2)\ln\left[x^2+\frac{m_W{}^2}{m_N{}^2}(1-x)\right]dx\approx-\frac{3}{2}-\frac{\frac{3}{2}\ln(m_W{}^2/m_N{}^2)}{1-(m_N{}^2/m_W{}^2)},$$

$$\int_0^1(x-1)\ln\left[x^2+\frac{m_W{}^2}{m_N{}^2}(1-x)\right]dx\approx-\frac{1}{2}-\frac{\frac{1}{2}\ln(m_W{}^2/m_N{}^2)}{1-(m_N{}^2/m_W{}^2)},$$

$$\int\frac{d^4k}{[k^2-m_N{}^2]^2}\approx2\pi^2i\ln\frac{\Lambda}{m_N}$$

lead to the result

$$\Gamma_\alpha{}^{(2)}+\Gamma_\alpha{}^{(3)}=\gamma_\alpha\frac{f_S}{16\pi^2}\frac{m_N}{m_W}\left\{-6\ln\frac{\Lambda}{m_N}+\frac{3\ln(m_W{}^2/m_N{}^2)}{1-(m_N{}^2/m_W{}^2)}-3.5\right\}+\sigma_{\alpha\beta}\frac{q_\beta}{m_N}\frac{f_S}{16\pi^2}\frac{m_N}{m_W}\left\{-2\ln\frac{\Lambda}{m_N}+\frac{\ln(m_W{}^2/m_N{}^2)}{1-(m_N{}^2/m_W{}^2)}-1.5\right\}.$$

The corrections to the electric and magnetic form factors in beta decay now follow from the standard expressions.

APPENDIX B

We list in Tables II–III the results for the statistical model calculation of the reactions $\pi N\to\bar{W}WN$ and $NN\to\bar{W}WNN$.

TABLE II. Statistical model cross sections for $\pi N\to\bar{W}WN$ [in units of cm²].

$(E_\pi/m_N)_{\text{lab}}$ \ m_W/m_N	2.0	2.3	2.6
13	5.4×10^{-32}		
14	2.7×10^{-31}		
15	5.3×10^{-31}		
16	8.0×10^{-31}	5.0×10^{-32}	
17	1.1×10^{-30}	1.6×10^{-31}	
18	1.6×10^{-30}	3.3×10^{-31}	
19	2.0×10^{-30}	5.1×10^{-31}	1.9×10^{-34}
20	2.2×10^{-30}	7.9×10^{-31}	4.7×10^{-33}

TABLE III. Statistical model cross sections for $NN\to\bar{W}WNN$ [in units of cm²].

$(E_p/m_N)_{\text{lab}}$ \ m_W/m_N	2.0	2.3	2.6
20	3.9×10^{-32}		
25	7.3×10^{-31}	6.2×10^{-32}	
30	2.9×10^{-30}	7.7×10^{-31}	8.6×10^{-32}

[25] See, e.g., the Appendix of J. M. Jauch and F. Rohrlich, *Theory of Photons and Electrons* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1955).