

gave a value for $e_2 = P_2^2 = 0.081 \pm 0.127$, consistent with zero polarization. There is some depolarization of the antiprotons between the two elastic scatters due to the fact that the polarization vector precesses in the magnetic field of the bubble chamber. This was investigated and the effect of the precession was found to be on the average less than 2% of $\cos\phi$ and so was ignored. The distribution in the angle ϕ was consistent with isotropy, and the absence of any $\cos\phi$ dependence is consistent with a zero value for e_2 . From our values of $e_1 = P_1 P_2$ and $e_2 = P_2^2$, we obtain for the average $P_1^2 = 0.010 \pm 0.029$, consistent with zero beam polarization.

Our result of $e_2 = P_2^2 = 0.081 \pm 0.127$ is in agreement with the results of Czyzewski *et al.*¹⁴ at 3–4 BeV/c of $P^2 = 0.050 \pm 0.080$. These results should be compared

with the data of Button *et al.*,¹⁵ who obtained a value of $P^2 = 0.26 \pm 0.10$ at 1.6 BeV/c. As pointed out by Czyzewski *et al.*,¹⁴ there is a decrease in the polarization with increasing bombarding energy, when the polarization is averaged over a wide range of scattering angles.

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¹⁵ J. Button and B. Maglic, Phys. Rev. **127**, 1297 (1962).

Unitary or Octal Symmetry?

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Octal symmetry (R_8 =group of rotations in eight dimensions) is applied to the strong interactions. If the existence of an octet m and a singlet λ of spinless mesons is assumed, a bootstrap mechanism allows the existence of 36 vector mesons with the coupling constants given by the R_8 group. Under $R_8 \subset R_9$ the 36-plet of vector mesons splits in an octet v' [with η (780), K (730), π (560) as a possible identification] and a 28-plet v . From octal symmetry a multiplicative quantum number N (block parity) is obtained which forbids the decay $v' \rightarrow m+m$ and perhaps explains the small width of this octet. A reasonable mass formula allows us to predict the masses of the v, v' vector mesons and suitable candidates with the predicted masses are found. The partial widths for the $v \rightarrow m+m$ decays are consistent with the experimental data and in particular with $\Gamma_{K(1175) \rightarrow K\pi}$, $\Gamma_{\pi(1220) \rightarrow \pi\pi}$ being small.

I. INTRODUCTION

THE unitary symmetry octet model¹ has been applied recently with fair success in order to explain different features of the elementary particles.² The purpose of this work is to show that the experimental data are compatible (and in some cases show a better agreement) with the predictions of octal symmetry (R_8 =group of rotations in eight dimensions).

Gürsey,³ Ne'eman,⁴ and Gourdin⁵ have applied the R_8 group to the physics of the weak and strong interactions. In this paper, we will extend the physical applications, in order to include some of the newly found resonances

and their properties—quantum numbers, decay widths, etc.

II. OCTAL SYMMETRY AND BLOCK PARITY

The dimensions of the representations of the R_8 algebra are: 1, 8^i , 28, 35^i , 56^i , etc., where the index $i=0, 1, 2$, labels three “analogous nonequivalent representations.”⁶ In particular, 8^0 is the vector representation and 8^1 [8^2] is the first [second] kind of semispinor representation.

For the decomposition of some of the direct products we have^{6,7}:

$$8^i \otimes 8^i = 1 \oplus 28 \oplus 35^i, \quad (1)$$

$$8^{(1,2)} \otimes 8^0 = 8^{(2,1)} \oplus 56^{(2,1)}, \quad (2)$$

$$8^1 \otimes 8^2 = 8^0 \oplus 56^0, \quad (3)$$

$$8^i \otimes 28 = 8^i \oplus 56^i \oplus 160^i. \quad (4)$$

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¹ M. Gell-Mann, Report CTSL-20, 1961 (unpublished); Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

² Y. Ne'eman, Conference on Symmetry Principles at High Energies, Coral Gables, Florida, 1964 (unpublished).

³ F. Gürsey, Ann. Phys. (N.Y.) **12**, 91 (1961).

⁴ Y. Ne'eman, Phys. Letters **4**, 81 (1963).

⁵ M. Gourdin, Nuovo Cimento **30**, 587 (1963). This author discusses in an explicit way the algebra of $R_8 \supset SU_3/Z_3$.

⁶ E. Cartan, *Leçons sur la Théorie des Spineurs* (Herman et Cie., Paris, 1938).

⁷ R. Brauer and H. Weyl, Am. J. Math. **57**, 425 (1935).

Under SU_3

$$28 \rightarrow 8^- + 10 + \bar{10}, \quad (5)$$

$$56 \rightarrow 1 + 8 + 10 + \bar{10} + 27, \quad (6)$$

$$35 \rightarrow 8 + 27. \quad (7)$$

If $n_1 \geq n_2 \geq n_3 \geq n_4$ are four numbers which characterize a certain block diagram (Young diagram) we have

$$\{n_1 n_2 n_3 n_4\} \otimes \{n'_1 n'_2 n'_3 n'_4\} = \sum_k \{n_1^{(k)} n_2^{(k)} n_3^{(k)} n_4^{(k)}\}, \quad (8)$$

$$N^{(k)} = (-)^{\alpha(k)} = (-)^{\alpha} (-)^{\alpha'} = NN', \quad \alpha = \sum_{i=1}^4 n_i. \quad (9)$$

As an obvious way of checking (9) is by inspection of the block diagrams. We name this multiplicative quantum number N the block parity. In particular, the representations 1 and 28 have positive block parity and the representations 8 negative block parity.⁸

III. BOOTSTRAP MECHANISM AND VECTONS

In the Gell-Mann-Ne'eman octet model the primary objects are three-component spinors.¹ Although it is questionable if these objects actually exist,⁹ the spinless mesons and baryons are contained in the direct product $3 \otimes \bar{3} = 1 \oplus 8$ which belongs to the scalar and regular representations of SU_3 .

In addition, if we consider n spinless mesons it is possible to bootstrap $n(n-1)/2$ vectons (vector mesons) and we find the coupling constants predicted by the R_n group. Thus Chang-Hong-Mo *et al.*¹⁰ have shown that in order to bootstrap M vectons when we begin with n mesons, the coupling constants g^{rab} (antisymmetric in the meson indices a, b ; r is the vecton index) must obey the restrictions

$$\sum_{ab} g^{rab} g^{sab} = \delta^{rs}, \quad (a = 1 \cdots n, r = 1 \cdots M) \quad (10)$$

$$\sum_{rdc} g^{rac} g^{rbg} g^{sca} = \lambda g^{sab}, \quad \lambda \text{ positive.} \quad (11)$$

If we consider the case of maximal degeneracy [$M = n(n-1)/2$], then (10) implies that the coupling constants form a $n(n-1)/2 \times n(n-1)/2$ orthogonal matrix and the restriction (11) is obeyed identically with $\lambda = 1$. If we diagonalize this matrix, we obtain

$$\{g_{ab}^{(a'b')}\}^2 = (\delta_{aa'} \delta_{bb'} - \delta_{ab'} \delta_{ba'})^2 \quad (12)$$

which is the result we obtain from the R_n group.

If we consider the set of nine pseudoscalar mesons

⁸ Vectors, and semispinors of the first and second kind, exist in three "analogous" spaces and therefore we may define a block parity in each one of these spaces (see E. Cartan, Ref. 6).

⁹ D. R. O. Morrison, Phys. Letters 9, 199 (1964); H. H. Bingham, M. Dickinson, R. Diebold, W. Koch, D. W. G. Leith *et al.*, Phys. Letters 9, 201 (1964); L. P. Leipuner, N. T. Chu, R. C. Larsen, and R. K. Adair, Phys. Rev. Letters 12, 423 (1964).

¹⁰ Chan-Hong-Mo, P. C. DeCelles, and J. E. Paton, Phys. Rev. Letters 11, 521 (1963).

$\{\pi^+ \pi^0 \pi^-, K^+ K^0, \bar{K}^0 K^-, \eta; \lambda\}$, where λ is the singlet in the product $3 \otimes \bar{3} = 1 \oplus 8$, we expect the bootstrap mechanism to produce 36 vectons with the coupling constants given by the R_9 group.

Under $R_8 \subset R_9$, we have

$$9 \rightarrow 1 + 8 = \lambda + \{m\} \text{ mesons,} \quad (13)$$

$$36 \rightarrow 8 + 28 = \{v'\} + \{v\} \text{ vectons.} \quad (14)$$

From (9) and (13), (14), we observe that λ and v have positive block parity and m, v' have a negative one.

Therefore

$$v \rightarrow m + m, v \rightarrow m + \lambda, \text{ odd number of mesons,} \quad (15)$$

$$v' \rightarrow m + m, \text{ even number of mesons, } v' \rightarrow m + \lambda. \quad (16)$$

The $\{v \text{ or } v'\} \otimes \{v \text{ or } v'\} \cdot \{m \text{ or } \lambda\}$ vertex is not allowed by R_9 (nor by R_8); but the $\{m\} \otimes \{v\} \cdot \{v'\}$ or $\{v\} \otimes \{v'\} \cdot \{\lambda\}$ vertices are allowed by R_8 (although forbidden by R_9). If the available phase space in the reaction $v' \rightarrow m + \lambda$ is small or nonexistent, we expect to see the vecton v' as resonances with a small width, since they may decay into two mesons m only if they violate R_8 symmetry. This octet might be composed by η (790 MeV, 1^-), K (730 MeV, 1^-), and π'' ($?, 1^-$).

As possible candidates for the π'' position, we have the ζ particle¹¹ which is a π (560 MeV) and the $\pi^+ \pi^-$ resonance at 520 MeV found by Samios *et al.*,^{12,13} π'' might also be found around 300 to 370 MeV (see below).

As the η (790 MeV, 1^-) vecton has negative G , the meson λ must have negative G (and C).¹⁴

The 28-plet v would be composed by a singlet η (1^{--}), four doublets K, \bar{K} (1^-), two triplets π (1^{+-}), one triplet X (1^{--}), two quadruplets $\delta, \bar{\delta}$ (1^-), and two singlets $\sigma, \bar{\sigma}$ (1^-) [$Y = \pm 2$].

The physical particles π (1^{+-}), [K, \bar{K} (1^-)] might be a mixture of the elementary vectons ($\pi\pi'$), ($K\bar{K}$) [$(K\eta), (K\pi)^{1/2}$]. We name the mixing angle $\theta[\Phi]$.

In Table I we show the set of vectons v, v' , their quantum numbers and possible identifications. We also show the predictions of the mass formula (see below) and the possible decay modes.

IV. MASS FORMULA

If the elementary vecton v is composed of mesons with masses m, m' , we will assume the mass of the vecton $v_{\{mm'\}}$ is given by the formula

$$m_{v_{\{mm'\}}} = (m + m')^2 + f[I(I+1), Y^2], \quad (17)$$

¹¹ R. Barloutaud, J. Heughebaert, A. Leveque, J. Meyer, and R. Omnes, Phys. Rev. Letters 8, 32 (1962); T. Ferbel, J. Sandweiss, H. Taft, M. Gailloud, T. Morris *et al.*, Bull. Am. Phys. Soc. 9, 22 (1964).

¹² N. P. Samios, A. Bachman, R. Lea, T. Kalogeropoulos, and W. Shephard, Phys. Rev. Letters 9, 139 (1962).

¹³ Although only the $\pi^+ \pi^-$ mode has been observed, when R_8 is violated, isospin might be violated and the $\pi^+ \pi^-, \pi^+ \pi^0$ modes might have different partial widths. The total width ($\sim 70 \pm 30$ MeV) would come from the decay $\pi'' \rightarrow \pi + \lambda$.

¹⁴ If $m_\lambda < 3m_\pi$, λ will not decay through strong interactions and perhaps the dominant mode will be $\lambda \rightarrow \pi^0 + \gamma$.

TABLE I. Vecton quantum numbers and mass formula predictions.

Vecton	I	Y	G	N	Possible candidate	Mass (m_{π^2})	Linear combination of pure vectons	$m_{v^2} = (m+m')^2 + f[I(I+1)_1 Y^2]^a$	$v \rightarrow m+m^b$	Observed modes
π	1	0	+	+	$B(1220)$ (12)	76	$\cos\theta(K\bar{K})' + \sin\theta(\pi\pi)$	76	$\rightarrow K+\bar{K}$ $\rightarrow \pi+\pi$	$\pi\omega$ (Ref. 12)
π'	1	0	+	+	$\rho(750)$	29	$-\sin\theta(K\bar{K})' + \cos\theta(\pi\pi)$	29	$\rightarrow \pi+\pi$ $\rightarrow K+\bar{K}$	$\pi\pi$
X	1	0	-	+	(1000) (17)	50	$(\eta\pi)$	50	$\rightarrow \eta+\pi$	3π (Ref. 17)
η	0	0	-	+	(1040)	53	$(K\bar{K})^0$	53	$\rightarrow K+\bar{K}$	$K\bar{K}$
K	$\frac{1}{2}$	1	.	+	$K(890)$	40	$\cos\phi(K\pi)^{1/2} + \sin\phi(\phi K)$	40	$\rightarrow K+\pi$ $\rightarrow K+\eta$	$K\pi$
K'	$\frac{1}{2}$	1	.	+	$K(1175)$ (17)	72	$-\sin\phi(K\pi)^{1/2} + \cos\phi(\eta K)$	74	$\rightarrow K+\eta$ $\rightarrow K+\pi$	$\rightarrow K\pi\pi$ (Ref. 18) $\rightarrow K\pi$
δ	$\frac{3}{2}$	-1	.	+			$(K\pi)^{3/2}$	68 ^b	$\rightarrow K+\pi$	
σ	0	2	.	+			$(KK)^0$	85 ^b	$\rightarrow K+K$	
π''	1	0	+	-			$(\pi\lambda)$	7? ^c	$\rightarrow \pi+\pi$ $\rightarrow \pi+\lambda$	
K''	$\frac{1}{2}$	1	.	-	$K(730)$	27	$(K\lambda)$	27	$\rightarrow K+\lambda$ $\rightarrow K+\pi$	$K\pi$ small width
η'	0	0	-	-	$\eta(790)$	31	$(\eta\lambda)$	31	$\rightarrow \eta+\lambda$	3π

^a Assuming the physical particles are "pure" vectons: $\cos^2\theta = \cos^2\phi = 1$. If we want $SU_3/C_3 \subset R_8$, then $\cos^2\theta = \frac{3}{4}$ and $\cos^2\phi = \frac{1}{4}$.

^b If $f[I(I+1)_1 Y^2] = 3 + 11I(I+1) + 8Y^2$.

^c If $f[I(I+1)_1 Y^2] = -1.2$.

where I, Y are the isospin and the hypercharge. In this formula $(m+m')^2$ breaks R_8 symmetry (as the basic mesons have different masses) while $f[I(I+1), Y^2]$ breaks SU_3 symmetry. (If we consider a Gell-Mann-Okubo type formula,¹⁵ we expect

$$f[I(I+1), Y^2] \simeq a + bI(I+1) + cY^2.$$

In order to fit $\pi(750, 1^-)$, $\eta(1040, 1^-)$, and $K(890, 1^-)$, we need $f(2,0) \simeq 25$, $f(0,0) \simeq 3$, and $f(3/4) \simeq 19$; this allows us to predict a $\pi(1220, 1^{++})$, a $\pi(1000, 1^{--})$, and a $K(1200, 1^-)$. As possible candidates we have the $T=1$, $G=+B$ particle ($\pi\omega$ resonance),¹⁶ the $T \geq 1$, $G=-3$ π resonance discovered by Trebukhovsky *et al.*,¹⁷ and the $T=\frac{1}{2}$, $\frac{3}{2}$ $K\pi\pi$ resonance discovered by Wangler *et al.*¹⁸ In order to predict the mass of the δ, σ particles we need to know $f(15/4, 1)$, $f(0, 4)$. If we assume that the Okubo-type linear approximation is valid, we expect $m_\delta \simeq 1140$ MeV, $m_\sigma \simeq 1300$ MeV. (Although this linear approximation is questionable and one does not expect the δ, σ particles to have exactly these mass values, it would not be reasonable to look for them far away.)

For the v' octet we have

$$m_{v'}^2 = (m+\lambda)^2 + g[I(I+1), Y^2]. \quad (18)$$

The values of the masses of $\eta(780)$, $K(730)$ are consistent with $g[I(I+1), Y^2] \simeq -1.2m_{\pi^2}$, $m_\lambda \simeq 250$ MeV.

¹⁵ S. Okubo, Progr. Theoret. Phys. (Kyoto) **27**, 949 (1962).

¹⁶ M. Abolins, R. Lander, W. Mehlop, N. Xuong, and Ph. Yager, Phys. Rev. Letters **11**, 381 (1963); J. Kirz, in *Proceedings of the 1963 Sienna Conference on Elementary Particles* (Società Italiana di Fisica, Bologna, 1963).

¹⁷ Yu. Trebukhovsky, L. Yerofev, and G. Tikhomirov, Phys. Letters **6**, 190 (1963).

¹⁸ T. P. Wangler, A. R. Erwin, and W. D. Walker, Phys. Letters **9**, 71 (1964).

These values give us $m_{\pi'} \simeq 330$ MeV. (Of course we should obtain other values if we chose a different form for g , although the preceding simple choice is suggestive.)

V. COUPLING CONSTANTS

For the $v_{\{m_1 m_2\}} m m'$ vertex we have

$$\gamma_{\{m_1 m_2\} m m'}^2 = a(\delta_{m_1 m} \delta_{m_2 m'} - \delta_{m_1 m'} \delta_{m_2 m}). \quad (19)$$

In particular

$$(\gamma_{K^+ K^-})^2 = \frac{1}{2} a, \quad (20)$$

$$(\gamma_{\pi^+ \pi^-})^2 = a \cos^2\theta, \quad (21)$$

$$(\gamma_{\pi^+ \pi^- B})^2 = a \sin^2\theta, \quad (22)$$

$$(\gamma_{K^+ K^- \rho})^2 = \frac{1}{2} a \sin^2\theta, \quad (23)$$

$$(\gamma_{K^+ K^- B})^2 = \frac{1}{2} a \cos^2\theta, \quad (24)$$

$$(\gamma_{K\eta}^{K*})^2 = a \sin^2\phi, \quad (25)$$

$$(\gamma_{K\pi}^{K*})^2 = \frac{2}{3} a \cos^2\phi, \quad (26)$$

$$(\gamma_{K\eta}^{K**})^2 = a \cos^2\phi, \quad (27)$$

$$(\gamma_{K\pi}^{K**})^2 = \frac{2}{3} a \sin^2\phi. \quad (28)$$

In Table II we compute the partial widths for some decays, both for physical vectons taken as pure vectons and when they are linear combinations which transform as the 8, 10, $\bar{10}$ SU_3 representations. In the first case (pure vectons) we get a better value for the $K(890, 1^-)$ width, and a better agreement with $K(1175) \rightarrow K\pi$, $\Gamma_{\pi(1220) \rightarrow \pi\pi}$ small.

As we said before, the decays $v \rightarrow v+m$ (or λ), $v' \rightarrow v'+m$ (or λ) are forbidden by R_9, R_8 symmetries. However, the processes $v \rightarrow v'+m$, $v' \rightarrow v'+\lambda$, although

TABLE II. Two-meson decays of vectons.

Decay	Q^3/m^2	$\Gamma(SU_3/C_3 \subset R_8)^a$ (MeV)	$\Gamma(\text{pure } R_8)^b$ (MeV)	Γ_{exp} (MeV)	Observed modes
$\rho \rightarrow \pi + \pi$	0.54	120	120	120 ± 10	$\pi\pi$
$B \rightarrow K^+ + K^-$	0.218	24	24		$\pi\omega(16)$
$B \rightarrow \pi^+ + \pi^-$	1	110	0	small	$\Gamma_T \sim 100 \text{ MeV}$
$\phi \rightarrow K^+ + K^-$	0.014	2.3	1.55	3.1 ± 0.6	$K\bar{K}$
$\phi \rightarrow K^0 + \bar{K}^0$	0.0073	1.2	0.8		
$K^* \rightarrow K^+ + \pi^-$	0.192	$\begin{cases} 21.5 \\ \Gamma_T = 32 \end{cases}$	$\begin{cases} 28.7 \\ \Gamma_T = 43 \end{cases}$	50	$K\pi$
$X \rightarrow \pi + \eta$	0.236	82	55	120 ± 30	$3\pi(17)$
$K(1175) \rightarrow K + \eta$	0.111	19	24	40 ± 15	$K\pi\pi(18)$
$K(1175) \rightarrow K + \eta$	0.57	48	0	0	$\rightarrow K\pi$

^a $\cos^2\theta = \frac{2}{3}$; $\cos^2\phi = \frac{1}{2}$.
^b $\cos^2\theta = \cos^2\phi = 1$.

forbidden by R_9 , are allowed by R_8 . Therefore we expect $\gamma_{vv'm} \simeq 0$ and $\gamma_{v'v'm}$ to be small.

Experimentally,

$$\begin{aligned} \gamma_{K^*K} \phi \simeq \gamma_{\rho\pi} \phi \simeq 0 \\ \gamma_{B\pi^0}, \gamma_{\rho\pi^0}, \gamma_{K^*K^0} \neq 0 \end{aligned}$$

which seems consistent with the preceding consideration.¹⁹

VI. BARYONS

Experimentally, we observe an octet of baryons; therefore, it seems reasonable to try to introduce them into the R_8 scheme. We will assume that the mesons m belong to the vector representation 8^0 and the baryons (spin up \uparrow , spin down \downarrow)²⁰ to the semispinor representations $8^1, 8^2$. Now the Yukawa vertex $B\bar{B}m$ is given by $8^1 \otimes 8^2 \cdot 8^0$. This implies that the baryon-antibaryon is in a 1S_0 state ($\uparrow\downarrow$) and therefore *the mesons m must be pseudoscalar*.

The Yukawa interaction $B\bar{B}m$ may be obtained from the invariant form²¹

$$\mathcal{F} = \psi^{(1)} C X \psi^{(2)},$$

where $\psi^{(1)}$ and $\psi^{(2)}$ are semispinors of the first and second kind, X is the 8^0 vector matrix, and C is the R_8 metric matrix.²² When we substitute the physical baryons and

¹⁹ P. Schein, W. E. Slater, L. T. Smith, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters **10**, 368 (1963); H. K. Ticho, Proceedings of the Athens Conference of Resonant Particles, Ohio University, 1963 (unpublished); M. Abolins, *ibid.*; Y. Y. Lee, W. D. Moebs, Jr., B. P. Roe, D. Sinclair, and J. C. Vander Velde, Phys. Rev. Letters **11**, 508 (1963).

²⁰ F. Gürsey (Ref. 3) associated the two baryon helicity amplitudes with the $8^1, 8^2$ semispinor representations.

²¹ E. Cartan, Ref. 6.

²² The "improper" group for R_8 is the semidirect product $G = R_8 \otimes \zeta_3$ (E. Cartan, Ref. 6), where ζ_3 is a permutation group of order 3 which interchanges the $8^0, 8^1, 8^2$ representations. If we remember that the SU_3 group is essentially the product of the isospin by a ζ_3 group (Weyl group), this leads to think that perhaps

mesons, we obtain the expected old global symmetry:

$$\begin{aligned} \mathcal{F} = g [\bar{N} \gamma_s \tau \cdot \pi N + \bar{\Lambda} \gamma_s \pi \cdot \Sigma + \text{H.a.} + i \bar{\Sigma} \gamma_s \times \Sigma \cdot \pi \\ + \bar{\Xi} \gamma_s \tau \cdot \pi \Xi + \bar{N} \gamma_s K \Lambda + \text{H.a.} + \bar{N} \gamma_s \tau \cdot \Sigma K \\ + \bar{\Xi} \gamma_s \hat{K} \Lambda + \text{H.a.} + \bar{\Xi} \gamma_s \tau \cdot \Sigma \hat{K} + \text{H.a.} + \bar{\Lambda} \gamma_s \Lambda \eta \\ + \bar{N} \gamma_s N \eta + \bar{\Xi} \gamma_s \Xi \eta + \bar{\Sigma} \gamma_s \cdot \Sigma \eta], \quad (29) \end{aligned}$$

where

$$\hat{K} = -i\tau_2 K^*. \quad (30)$$

Perhaps the value of the coupling constants are affected by the presence of the SU_3 symmetry and the final answer may be a compromise between both symmetries.

VII. CONCLUSIONS

In this work we have shown that the octal (R_8) symmetry allows us:

(I) to explain the small width of the $\eta(780)$, $K(730)$, $\pi(560)$ octet;

(II) to obtain a reasonable mass formula which allows us to predict the mass of new particles (and to find suitable candidates with the predicted masses);

(III) to find a better value for the $K(890)$ width and to explain why $\Gamma_{K(1175) \rightarrow K\pi}$ and $\Gamma_{B \rightarrow \pi\pi}$ are small.

It should be worth while to look for the new δ, σ particles, to measure the quantum numbers of the X, K', B particles, and to check some of the decisive modes ($K' \rightarrow K + \eta, X \rightarrow \pi + \eta, B \rightarrow K + \bar{K}$). It could be also interesting to search for candidates for the hypothetical particles π'' and λ .

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there exists a parallelism between the isospin and the hypercharge on the one hand and the ordinary spin and the baryonic number of the other.