Resonance Model for Σ^+-K^+ Production*

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(Received 1 October 1964)

A model for the process $\pi^+ + p \rightarrow \Sigma^+ + K^+$ is proposed, consisting of hyperon- and K^*-exchange diagrams as well as the $p_{3/2}$ resonance, and a resonance of 1940 MeV, assumed to be $f_{7/2}$. The resulting four-parameter fit is compared with the cross-section, angular distribution, and polarization data at beam momenta from 1.04 to 1.76 BeV/c.

I. INTRODUCTION

ASSOCIATED production of hyperons in hydrogen by beams of positive pions has been the subject of various experiments over the past seven years.[1–8] In recent years there have been a few experiments with beam energies considerably above the threshold for $\Sigma^+ - K^+$ production; earlier experiments nearer threshold were concerned principally with determination of the hyperon decay parameters. Angular distributions are available at beam momenta up to 1.76 BeV/c, well above the $T=3/2$ $\pi - N$ resonance (usually assigned the quantum numbers $J^P = (7/2)^+$) at approximately 1920 MeV total energy in the center-of-mass system.

Since most of these experiments involve relatively small numbers of events, the probable errors in the angular distributions are large, but several striking changes in the shapes of these distributions can be seen as the energy increases. At the lowest energies there is strong backward peaking of the K's, with very few events in the extreme forward direction. At beam momenta from 1.2–1.4 BeV/c the extreme backward angles are suppressed and a forward peak appears. This forward peak persists up to the highest momentum we shall consider (1.76 BeV/c), while the backward peaking disappears almost entirely.

Polarization data are even more uncertain, and at only one momentum (1.17 BeV/c) is polarization given as a function of angle of production. The main feature in the polarization is the large positive average polarization up to 1.17 BeV/c with a sharp decline above that energy.

In this paper we present a model to account at least qualitatively for all these features.[9] We assume that the K^*-exchange and hyperon-exchange terms dominate the other "left-hand singularity" contributions, and that the amplitude can be represented by these plus terms representing the $T=3/2$, $J^P=3/2^+$ $\pi - N$ resonance at 1238 MeV and the 1920-MeV resonance mentioned above. Although in principle there are terms from both Σ and Λ exchange, we represent them by one term, using the mass of the Λ. We neglect the width of the K^*, take the position and width of the 3-3 resonance from the experimental values, and fix the upper resonance to be $f_{7/2}$ at 1920 MeV. We then have five parameters: a coupling constant product for each exchange term, the partial width of each resonance into the $\Sigma - K$ channel, and the width of the upper resonance (this latter is only partly adjustable since the width is known to be greater than 100 MeV).

In Sec. II we outline the kinematics and give the cross-section and polarization formulas. In Sec. III we present the best fits given by our model in comparison with the data. In Sec. IV we discuss the successes and failures of the model.

II. DESCRIPTION OF THE MODEL

A pion of momentum $k=(\mathbf{k},k_0)$ is incident on a proton of momentum $p=(\mathbf{p},p_0)$, producing a K meson of momentum $k'=(\mathbf{k}',k_0')$ and a Σ hyperon of momentum $p'=(\mathbf{p}',p_0')$. If χ_p and χ_Σ denote the spin states of the proton and hyperon, the amplitude for the process may be written

$$A=\chi_\Sigma^+ M \chi_p, \tag{1}$$

where

$$M=f+i\boldsymbol{\sigma}\cdot\mathbf{n}g\sin\theta, \tag{2}$$

$$\mathbf{n}=\mathbf{k}'\times\mathbf{k}/|\mathbf{k}'\times\mathbf{k}|, \tag{3}$$

$$\cos\theta=\mathbf{k}'\cdot\mathbf{k}/(|\mathbf{k}||\mathbf{k}'|). \tag{4}$$

θ is the angle in the c.m. system between the initial pion and the final K meson. In our model, the amplitudes f and g each consist of two parts, an exchange part, and a

* Supported in part by the U. S. Army Research Office (Durham), by the National Science Foundation, and the U. S. Office of Naval Research.

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[2] Bruce Cork, Leroy Kerth, W. A. Wenzel, James W. Cronin, and R. L. Cool, Phys. Rev. **120**, 1000 (1960).

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[7] Horst W. J. Foelsche, A. Lopez-Cepero, C.-Y. Chien, and H. L. Kraybill, copy of paper submitted to the 1964 International Conference on High Energy Physics at Dubna.

[8] E. C. Fowler, L. R. Fortney, J. W. Chapman, P. L. Connolly, E. L. Hart, P. V. C. Hough, and R. C. Strand, Bull. Am. Phys. Soc. **9**, 420 (1963).

[9] Similar resonances plus exchange models have been worked out for Λ production processes. See, for example, Akira Kanazawa, Phys. Rev. **123**, 997 (1961); G. T. Hoff, Phys. Rev. **131**, 1302 (1963); T. K. Kuo, *ibid.* **129**, 2264 (1963); and N. A. Beauchamp and W. G. Holladay, *ibid.* **131**, 2719 (1963).

resonant part. The exchange parts arise from the diagrams of Fig. 1. The contribution of these diagrams to f and g are obtained from the following formulas[10]:

$$A=-2\frac{G_{K^*}}{q_t^2+M^2}\left(\frac{\Sigma-N}{M^2}\right)(K^2-\pi^2)+\frac{G_\Lambda}{q_u^2+\Lambda^2}\left(\Lambda-\frac{N+\Sigma}{2}\right), \quad (5)$$

$$B=-4G_{K^*}/(q_t^2+M^2)+G_\Lambda/(q_u^2+\Lambda^2), \quad (6)$$

$$f-g\cos\theta=-\frac{[(p_0+N)(p_0'+\Sigma)]^{1/2}}{8\pi W}\times\left[A+B\left(W-\frac{N+\Sigma}{2}\right)\right], \quad (7)$$

$$g=\frac{[(p_0-N)(p_0'-\Sigma)]^{1/2}}{8\pi W}\left[A-B\left(W+\frac{N+\Sigma}{2}\right)\right]. \quad (8)$$

In these formulas, W is the total energy in the center-of-mass system; π, N, K, and Σ are the masses of the initial and final particles; and M and Λ are the masses of the K^* and Λ particles, q_t and q_u are the momentum transfer four-vectors for K^* and Λ exchange, respectively. G_{K^*} and G_Λ are coupling parameters which are products of two coupling constants, one for each vertex of the corresponding exchange diagram.

The resonant contribution to f and g is taken to be of the Breit-Wigner form:

$$\frac{1}{2(kk')^{1/2}}\frac{C_i(\Gamma_1\Gamma_2)^{1/2}}{W_r-W-i\Gamma/2},$$

where C_i is an adjustable strength factor, and i is 3 (7) for the $p_{3/2}$ ($f_{7/2}$) resonance. The partial widths for the two-particle channels are assumed to have a momentum dependence of the form[11]

$$\Gamma=\text{const}\times\left|\frac{|\mathbf{k}|^2}{|\mathbf{k}|^2+X^2}\right|^l\frac{|\mathbf{k}|}{M}, \quad (9)$$

where l and M are the "orbital angular momentum" and mass of the resonance, $\mathbf{k}$ the relative momentum of the two particles in the c.m. system, and X is a parameter characteristic of the radius of the interaction. The value of X we have used corresponds to a radius of 0.4-pion Compton wave length. The total width of the resonance is chosen to agree with the experimentally observed width, and the partial width for the $\Sigma-K$ channel enters the calculation as an adjustable parameter.

[10] The mathematical formulation of the ΣK production problem was discussed by M. Gourdin and M. Rimpault, Nuovo Cimento **20**, 1166 (1961).

[11] This form of the momentum dependence was used in comparing the predictions of unitary symmetry with experimental resonance widths by Sheldon L. Glashow and Arthur H. Rosenfeld, Phys. Rev. Letters **10**, 192 (1963).

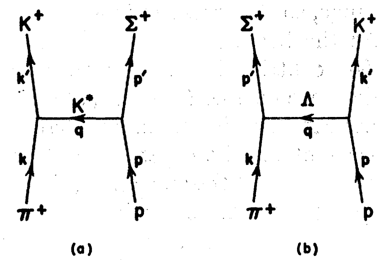

FIG. 1. Diagrams contributing to the process $\pi^++p\to\Sigma+K^+$. (a) shows the exchange of a K^*-resonance of mass 888 MeV, and (b) shows the exchange of a Λ hyperon of mass 1115 MeV.

Once f and g have been evaluated, the differential cross section and polarization are given by the formulas

$$d\sigma/d\Omega=(k'/k)(|f|^2+|g|^2\sin^2\theta), \quad (10)$$

$$P(\theta)=\frac{2\text{Im}fg^*\sin\theta}{|f|^2+|g|^2\sin^2\theta}. \quad (11)$$

III. NUMERICAL RESULTS

We have attempted to fit all the available data for this process up to 1.76 BeV/c by adjusting the four coupling constants and the position and width of the resonances. The last two parameters were adjusted only within the narrow range consistent with the experimentally observed peak in the cross section around 1.5 BeV/c. The theoretical curves were then normalized to the experimental value of the total production cross section at 1.39 BeV/c. Thus, there were effectively three adjustable coupling constants in the calculation.

In order to show qualitatively how our model accounts for the data, we will describe the contribution to the amplitude of each of the separate terms. The K^*-exchange and Λ-exchange terms are peaked in the forward and backward directions, respectively. The magnitude and sharpness of these peaks increase with energy. The contribution of the 3-3 resonance is an odd function of $\cos\theta$, and decreases slowly with increasing energy over our range. The $f_{7/2}$ resonance contributes an odd function of $\cos\theta$ whose magnitude varies fairly rapidly as the energy passes through resonance. The nature of our fit may be understood from Table I, which gives the signs of these contributions to the amplitude for forward and backward angles, and for energies below the $f_{7/2}$ resonance. For small energies, the large cross sections at backward angles are seen to arise from a constructive interference of the Λ and 3-3 contributions. The K^* exchange term is small in this energy

TABLE I. Signs of the different contributions to the Σ^+K^+ production amplitude for beam momenta between threshold and 1.49 BeV/c.

	$\cos\theta\approx-1$	$\cos\theta\approx+1$
K^*	−	−
Λ	+	+
Re3-3	+	−
Re$f_{7/2}$	−	+
Im3-3	−	+
Im$f_{7/2}$	−	+

range, so the cross sections are small for forward angles. As the beam momentum increases to 1.1 BeV/c, the $f_{7/2}$ contribution begins to cause the amplitude to decrease near $\cos\theta=-1.0$; at $\cos\theta=-0.5$, however, this contribution is small, so that the Λ and 3-3 terms account for the observed peak in the cross section at this angle. At the same time, the increasing contribution of the K^* term is partically cancelled by the $f_{7/2}$ term at forward angles. As the energy passes through resonance, the real part of the $f_{7/2}$ contribution changes sign, producing constructive interference with both the Λ, for backward angles, and the K^*, for forward angles. This accounts for the peaks shown by the theoretical curves for the highest energies.

The measured average polarizations are large for momenta below 1.17 BeV/c, and drop to a small value at 1.22 BeV/c. Our model shows this general trend, though not as markedly as the data. The decrease of polarization around 1.2 BeV/c is obtained in our model from a cancellation between the 3-3 and $f_{7/2}$ contributions to the amplitude. The energy at which the average polarization begins to decrease is a sensitive function of the position and width of the $f_{7/2}$ resonance, and further restricts the range over which these may be allowed to vary.

In Figs. 2 through 10 we show the best fit to the angular distribution, total cross section, and polarization data we were able to obtain. The $f_{7/2}$ resonance was placed at 1935 MeV, with a width of 125 MeV. The values of the coupling parameters are

$$G_{K^*}=-0.722\,,$$
$$G_\Lambda=-10.8\,,$$
$$C_3=\ \ 0.195\,,$$
$$C_7=\ \ 0.253\,.$$

The first of these implies $g_{\Sigma NK^*}{}^2/4\pi=0.00348$, using[12] $g_{K^*K\pi}{}^2/4\pi=0.95$. No simple inferences can be drawn from the value of G_Λ given, since it represents the combined contributions of the Λ and Σ^0 exchange terms. We expect that these values are somewhat high because we have neglected the effects of absorption, which would tend to decrease the cross section.

In addition to the model described up to now, we tried other parameters for the resonances. In particular, we attempted to fit the data using a $g_{7/2}$ resonance at 1.5 BeV/c. Because of the different parity of this state, we were unable to find any set of parameters which reproduced the experimental angular distributions. We also tried including, instead of the 3-3 resonance, a resonance

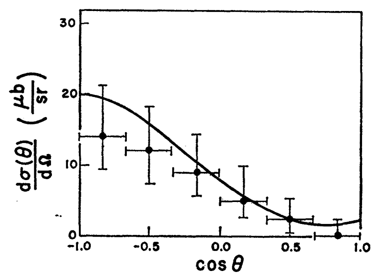

FIG. 2. Differential cross section versus $\cos\theta$ at beam momentum 1.11 BeV/c. The experimental points are from Baltay *et al.*, Ref. 3.

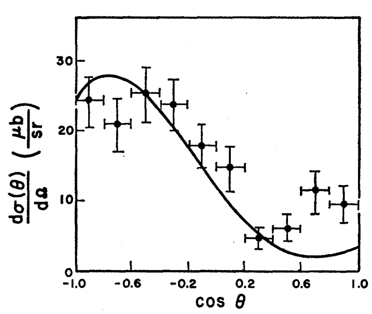

FIG. 3. Differential cross section versus $\cos\theta$ at 1.17 BeV/c. Data from Crawford, Grard, and Smith, Ref. 6.

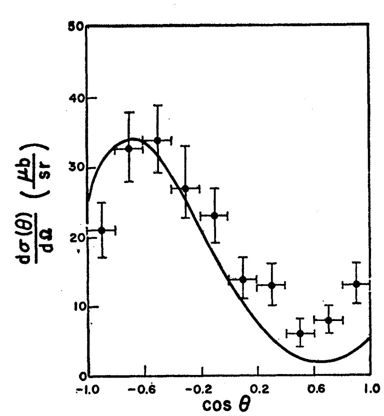

FIG. 4. Differential cross section versus $\cos\theta$ at 1.22 BeV/c. Data are from Baltay *et al.*, Ref. 3.

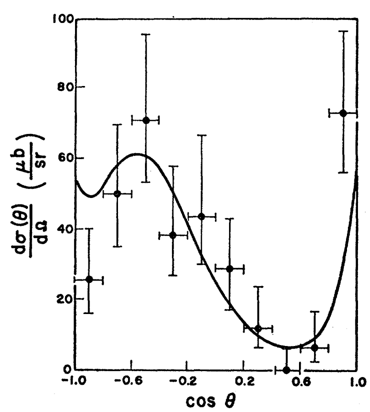

FIG. 5. Differential cross section versus $\cos\theta$ at 1.39 BeV/c beam momentum. Data from Foelsche, Lopez-Cepero, Chien and Kraybill, Ref. 7. Angular distributions at the same beam momentum from Ref. 3 are similar to those presented here.

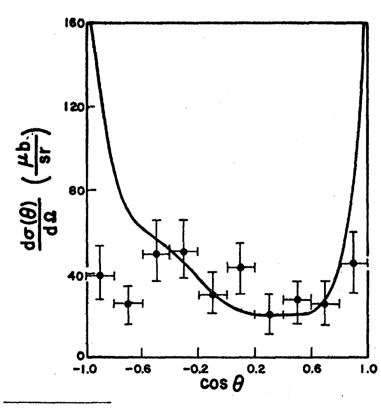

FIG. 6. Differential cross section versus $\cos\theta$ at 1.49 BeV/c. Experimental points are from Fowler, Fortney, Chapman, Connolly, Hart *et al.*, Ref. 8.

[12] R. H. Dalitz, Ann. Rev. Nucl. Sci. **13**, 339 (1963).

at about 0.93 BeV/c. This is the momentum at which there occurs "shoulder" in the π^+p total cross section, which could be due to a resonance at that energy. We were, however, unable to get any agreement with the data, although we tried various values of the spin and parity. An odd l is necessary to produce the characteristic tilted shape of the lowest energy angular distributions. A $p_{3/2}$ state gives the correct sign, but if its strength is increased sufficiently to give a significant magnitude of the polarization, the total cross section rises much too rapidly just above threshold. A resonance at this position also causes the angular distribution to vary too rapidly with energy. Therefore, none of these attempts led to agreement with the data.

IV. DISCUSSION

From the figures it is apparent that our model gives correctly the qualitative trends of all the data available. In the cases at the lower energies, the fits to the angular distributions are reasonably good, and the fact that so many pieces of data can be understood with so relatively simple a model is encouraging.

There are, of course, some discrepancies. The data in Fig. 3 at 1.17 BeV/c show a small peak in the forward direction, while our model gives no such behavior. If further experiment shows this peak, we would have to consider this case as a true discrepancy. The chief discrepancies in the angular distributions occur at 1.49 and 1.76 BeV/c where our predicted backward peaking is not seen. At these relatively high energies, the absorptive effects which we neglect could make this peaking much sharper, as has been suggested by several authors. If the peaks were quite sharp, they could be difficult to find experimentally. Only more data at these energies will resolve this ambiguity.

Probably the most serious failure of the model is that the average polarizations at the lower energies are considerably smaller than indicated by the data. We have been unable to find any choice of parameters for which $\bar{P}$ is greater than 0.5, while the data indicate $\bar{P}\approx 1.0$. We should emphasize, however, that we have reproduced the trend of $\bar{P}$ as energy increases, and that the experimental errors are quite large.

We acknowledge with gratitude helpful advice of Professor Earle Fowler on the experimental aspects of this work. We are grateful to Professor Wendell Holladay for providing us with the results of a calculation based on a simpler version of this model, and for pointing out an error in a preliminary version of this paper.

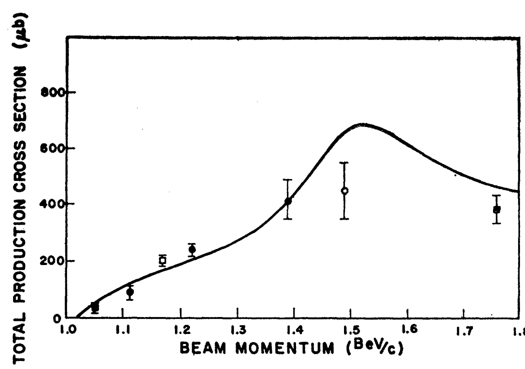

FIG. 9. Total Σ^+K^+ production cross section as a function of beam momentum. References to the data: (i) Solid circles: Baltay *et al.*, Ref. 3, (ii) Open circles: Fowler *et al.*, Ref. 8, (iii) Solid square: Foelsche *et al.*, Ref. 7, and (iv) Open square: Crawford *et al.*, Ref. 6.

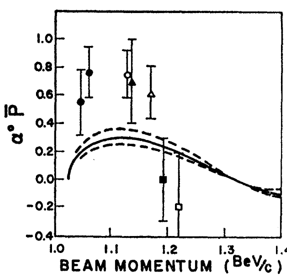

FIG. 10. $\alpha^0\bar{P}$ as a function of beam momentum. $\bar{P}$ is the average of the polarization over all angles, and α^0 is the asymmetry parameter for the decay $\Sigma^+ \rightarrow p+\pi^0$. The value of α^0, taken from Ref. 4, is $\alpha^0=0.73_{-0.11}^{+0.16}$. The solid curve is our calculated curve, and the dashed curves represent a standard deviation from the calculated value due to the uncertainty in α^0. References to the data: (i) solid circles: Grard and Smith, Ref. 5, (ii) open circle: Cork *et al.*, Ref. 2, (iii) solid triangle: Cool *et al.*, Ref. 1, (iv) open triangle: Crawford *et al.*, Ref. 6, (v) solid square: Beall *et al.*, Ref. 4, and (vi) open square: Baltay *et al.*, Ref. 3.

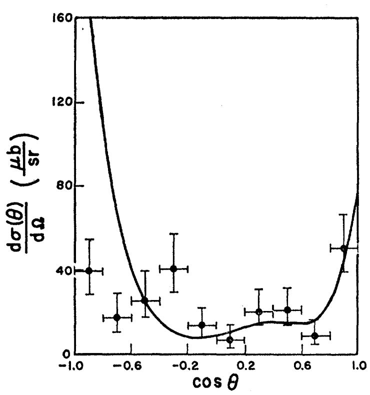

FIG. 7. Differential cross section versus cosθ at 1.76 BeV/c. Data from Foelsche, Lopez-Cepero, Chien, and Kraybill, Ref. 7.

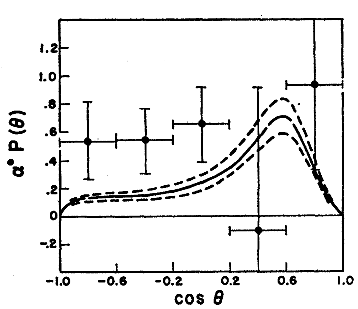

FIG. 8. $\alpha^0P(\theta)$ versus cosθ at 1.17 BeV/c. α^0 is the asymmetry parameters for the decay $\Sigma^+ \rightarrow p+\pi^0$. $\alpha^0P(\theta)$ is measured directly in the experiment of Crawford, Grard, and Smith, Ref. 6. The dashed curves show the standard deviation from our theoretical curve arising from the experimental uncertainty in α^0. The value $\alpha^0=0.73_{-0.11}^{+0.16}$ is taken from Beall, Cork, Keefe, Murphy, and Wenzel, Ref. 4.