Departures from the Eightfold Way. III. Pseudoscalar-Meson Electromagnetic Masses*†

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The leading contributions to the π^+-π^0 and K^+-K^0 mass differences are calculated. The contributions of pseudoscalar-meson and vector-meson intermediate states are considered, and the Feynman integrations are performed assuming very general momentum dependence at the vertices. Using form factors having poles at the vector-meson masses, and the unitary-symmetric vector mixing model of Coleman and Schnitzer, we find $m_{\pi^+}-m_{\pi^0}=4.9$ MeV and $m_{K^+}-m_{K^0}=2.9$ MeV. It is difficult to give a reliable estimate of the errors in these calculations; we believe they are correct to within 1 MeV. The uncertainty lies partly in the determination of the γ-ρ-π coupling constant and partly in the dynamical assumptions. When the scalar-meson contribution suggested by Coleman and Glashow is included, the π-meson mass difference is unchanged and the K-meson mass difference becomes $m_{K^+}-m_{K^0}=-1.4$ MeV. Our numerical values have been tabulated without discussion in two earlier papers of this series.

I. INTRODUCTION

ONE of the most tantalizing problems in particle physics is the problem of calculating the mass differences between particles belonging to the same isotopic multiplet. One usually assumes that these mass differences are entirely electromagnetic,[1] for otherwise, one would have to abandon one of the most cherished symmetries of the strongly interacting particles. In the example that concerns us here, the theorist has been confronted with the experimental fact that the π^+-π^0 and K^+-K^0 mass differences have opposite signs. To explain the observed mass differences, Bose and Marshak[2] postulated electromagnetic properties of the K mesons totally different from those of the π meson. In recent years, such efforts have been rendered unacceptable by the success of the $SU(3)$ supermultiplet theory of Gell-Mann[3] and Ne'eman.[4] The accuracy of the Coleman-Glashow formula[5] relating the electromagnetic masses of the spin-$\frac{1}{2}$ baryons leads one to expect the higher symmetry to dominate the electromagnetic properties of the particles in a supermultiplet, even in the presence of symmetry breakdown, and leads one to demand that all of the electromagnetic mass differences within a supermultiplet be explained at once, using common assumptions.

The earlier papers in this series[6,7] have addressed themselves to this challenge. The calculational scheme proposed in Ref. 6 has been successfully applied to the calculation of the electromagnetic mass differences within the spin-$\frac{1}{2}$ baryon octet[7]; it has also been extended to estimate the $Y^{*-}-Y^{*+}$ mass difference[8] and the decay rate for $\omega\rightarrow 2\pi$,[9] obtaining satisfactory agreement with somewhat unreliable experiments in both cases. The results of comparable calculations for the pseudoscalar mesons have been reported in Table II of Ref. 6 and in Table II of Ref. 7; the details of these calculations are published here.

The method of calculation builds on the work of Riazuddin,[10] who showed that the electromagnetic self-energy of a pseudoscalar meson can be related by dispersion theory to the amplitude for Compton scattering of unphysical photons. In diagrammatic language, the lowest order electromagnetic self-energy is expressed as a sum over Figs. 1(a)–1(c). Figures 1(a) and 1(b) together represent the contribution of the lowest mass intermediate state, which contains a single pseudoscalar meson. Figure 1(c) stands for an infinite number of diagrams in which the intermediate state is heavier than the single pseudoscalar meson. The theory allows a contribution from Fig. 1(d), which does not contribute to the absorptive part but adds a constant to the scattering amplitude. Historically, Riazuddin and later Bose and Marshak[2] only considered Figs. 1(a) and 1(b), neglecting all higher mass intermediate states. Their calculation is presented in a somewhat more general form in Sec. II.

In Sec. III, an estimate is made of the contribution of vector-meson intermediate states to the pseudoscalar meson mass differences. This contribution is a part of the contribution represented by Fig. 1(c). The author is aware of only one other attempt to calculate the contribution of Fig. 1(c).[11]

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‡ National Science Foundation Postdoctoral Fellow.

[1] The classic paper which first exploited this assumption was by R. P. Feynman and G. Speisman, Phys. Rev. **94**, 500 (1954).

[2] S. K. Bose and R. E. Marshak, Nuovo Cimento **25**, 529 (1962).

[3] M. Gell-Mann, California Institute of Technology, Report CTSL-20, March 1961 (unpublished); Phys. Rev. **125**, 1067 (1962).

[4] Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

[5] S. Coleman and S. L. Glashow, Phys. Rev. Letters **6**, 423 (1961).

[6] S. Coleman and S. L. Glashow, Phys. Rev. **134**, B671 (1964).

[7] S. Coleman and H. J. Schnitzer, Phys. Rev. **136**, B223 (1964).

[8] R. Socolow and S. Coleman, Phys. Rev. **135**, B1451 (1964).

[9] S. Coleman, S. L. Glashow, H. J. Schnitzer, and R. Socolow, Proceedings of the International Conference on High-Energy Physics, Dubna, USSR, 1964 (to be published).

[10] Riazuddin, Phys. Rev. **114**, 1184 (1959).

[11] J. K. Perring, (unpublished). The recent work by J. H. Wojtaszek, R. E. Marshak, and Riazuddin, Phys. Rev. **136**, B1053 (1964) divides the problem quite differently, and its connection with Riazuddin's original formulation is not clear.

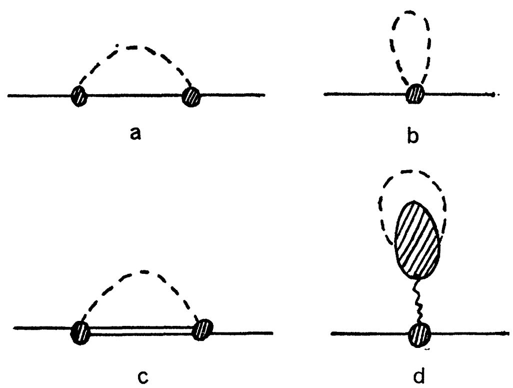

FIG. 1. Contributions to the electromagnetic mass differences of the pseudoscalar meson. The solid line represents a pseudoscalar meson, the dashed line represents a photon, the double line represents any intermediate state heavier than the pseudoscalar meson, and the wiggly line represents the $I=1$ scalar meson $\pi^{0\prime}$.

In Secs. II and III, the breaking of exact $SU(3)$ symmetry by the strong interactions is introduced by using the physical particle masses and allowing for ω-ϕ mixing.

A limited number of results may be obtained from the group transformation properties of a set of intermediate states, without knowing the details of their interactions. These results are summarized in Sec. IV.

The suggestion that Fig. 1(d) should contribute to the electromagnetic mass splittings was made by Coleman and Glashow, in the first paper of this series.[6] Coleman and Glashow call Fig. 1(d) the "tadpole" contribution to the mass splitting, and as a result Figs. 1(a)–1(c) become "nontadpole" contributions. The aspects of the tadpole theory relevant to our calculation are reviewed in Sec. V.

II. THE CONTRIBUTION OF THE PSEUDOSCALAR MESON INTERMEDIATE STATE

The contribution of the lowest mass intermediate state to the electromagnetic self-mass (dm) of a pseudoscalar meson has been written down by Riazuddin:

$$dm=\frac{i\alpha}{8\pi^3 m}\int d^4k\frac{3k^2-4k\cdot p-4m^2}{k^2(k^2-2k\cdot p)}[F(k^2)]^2, \qquad (1)$$

where p, m, and $F(k^2)$ are the momentum, mass, and electromagnetic form factor of the pseudoscalar meson; $\alpha=e^2/4\pi=1/137$ is the fine structure constant; and we use the metric $p_\mu p_\mu=p^2=m^2$. Figures 1(a) and 1(b) are not independently gauge-invariant. The two contributions have been combined in (1) in the only possible gauge-invariant manner. The integral in (1) is identical to the Born approximation obtained earlier by Feynman and Speisman from field theory,[1] except for the appearance of the form factors in (1). This method of combining Figs. 1(a) and 1(b) is equivalent to the assumption that the form factor in Fig. 1(b) is the square of the form factor in Fig. 1(a).

We will evaluate (1), making the assumption that the form factor $F(k^2)$ possesses the spectral representation

$$F(k^2)=\int_0^\infty dc\,\rho(c)\frac{cm^2}{cm^2-k^2}, \qquad (2a)$$

where ρ is a weight function restricted only to satisfy

$$\int_0^\infty \rho(c)dc=1. \qquad (2b)$$

A straightforward Feynman calculation gives the result[12]:

$$dm=\int_0^\infty dc_1\int_0^\infty dc_2\rho(c_1)\rho(c_2)\frac{m\alpha c_1c_2}{16\pi(c_2-c_1)}\times[f(c_2)-f(c_1)], \qquad (3a)$$

where

$$f(c)=c\ln c-(c-4)^2w(c), \qquad (3b)$$

and

$$w(c)=\int_0^1 dx(x^2-cx+c)^{-1}. \qquad (3c)$$

To evaluate (3), we need an explicit choice for the weight function appearing in (2). We make the approximation that the form factor may be written as a sum of poles at the masses of the ρ, ω, and ϕ mesons. The π^0 form factor vanishes, because the π^0 meson is its own charge conjugate. Because of the odd G parity of the π^+, the photon coupling is purely isovector and only the ρ pole can contribute to its form factor. Hence, we choose

$$\rho(c)=0 \quad (\pi^0 \text{ meson}), \qquad (4a)$$

$$\rho(c)=\delta(c-C), \quad \text{where} \quad C=m_\rho^2/m_\pi^2 \ (\pi^+ \text{ meson}). \qquad (4b)$$

This leads to the result

$$m_{\pi^+}-m_{\pi^0}=(m_\pi\alpha C/16\pi)[4+C\ln C-(C-4)(C+2)w(C)]=4.37 \text{ MeV}. \qquad (5)$$

This result is within 0.2 MeV of the experimental value. This felicitous feature of the ρ meson mass was discovered by Bose and Marshak.[2]

In calculating the electromagnetic masses of the K^+ and K^0 mesons, we have two alternatives. If we assume that the K-meson form factors are given in terms of the π-meson form factors by the predictions of the octet model of unitary symmetry, neglecting the medium-strong interactions entirely, then the K^0 form factor must vanish, and the K^+ form factor must be identical to the π^+ form factor. We prefer instead to

[12] Equations (3) and (21) are the analogs of the expressions written down by M. Cini, E. Ferrari, and R. Gatto, Phys. Rev. Letters 2, 7 (1959), who considered the lowest intermediate state contribution to the electromagnetic self-energy of the nucleon.

allow for the medium-strong interactions at this point. We assume that the isoscalar part of the K-meson form factor has poles at the ω and ϕ masses, and that the ω and the ϕ are eigenstates of an interaction which has mixed two isotopic singlet particles, one belonging to a unitary singlet (ω_1) and one belonging to a unitary octet (ω_8). To handle the ω-ϕ mixing, we turn to the vector mixing scheme of Coleman and Schnitzer.[13]

According to Coleman and Schnitzer,[13] the matrix propagator for vector mesons should be written in the form

$$\mathbf{D}^{-1}(k^2)=\mathbf{M}_0{}^2-k^2\mathbf{1}-k^2\mathbf{\Delta}\,, \tag{6}$$

where $\mathbf{\Delta}$ carries the coefficients which express the symmetry breaking. $\mathbf{M}_0{}^2$ is a diagonal mass matrix whose elements are the squares of the particle masses in the absence of symmetry breaking, and $\mathbf{1}$ is the unit matrix. The characteristic feature of (6) is that $\mathbf{\Delta}$ multiplies k^2; it is therefore suitable for electromagnetic calculations in which a photon couples to other particles by means of this propagator, since a real photon automatically couples to the static charge.

We arbitrarily restrict our consideration to the nine vector mesons, ρ, K^*, ω, ϕ, and assume that in the absence of symmetry breaking they form an octet and a singlet. If the usual octet-type symmetry breaking is assumed, then the matrix $\mathbf{\Delta}$ has the form

$$\mathbf{\Delta}=\begin{bmatrix}0 & \beta & 0 & 0\\ \beta & -2\epsilon & 0 & 0\\ 0 & 0 & 2\epsilon & 0\\ 0 & 0 & 0 & -\epsilon\end{bmatrix}, \tag{7}$$

where ϵ characterizes the splitting within the octet and β characterizes the octet-singlet mixing. The rows and columns label ω_1, ω_8, ρ, K^*, respectively. The propagator is required to have poles at the physical masses of the particles. The form of (6) is such that it is the *inverse* masses squared which are related by the Gell-Mann–Okubo formula; that is,

$$1/\rho+3/\omega_8=4/K^*\,, \tag{8}$$

where each symbol refers to the square of the corresponding particle mass. We may also obtain

$$1/3\rho+2/3K^*=1/M_0{}^2;\quad 1/3\rho-1/3K^*=\epsilon/M_0{}^2; \tag{9}$$

where M_0 is the mass of the vector-meson octet in the absence of symmetry breaking. (9) yields the value $M_0=837$ MeV.

Suppose that the octet and singlet masses are degenerate in the absence of symmetry breakdown.[14] Then

$$\mathbf{M}_0{}^2=M_0{}^2\cdot\mathbf{1}. \tag{10}$$

In the absence of octet-singlet mixing but in the presence of octet splitting this assumption leads to

$$1/K^*=\tfrac{1}{2}(1/\omega_8+1/\omega_1)\,, \tag{11}$$

since $\omega_1=M_0$. Thus, in the presence of octet-singlet mixing as well, we obtain the prediction

$$1/K^*=\tfrac{1}{2}(1/\omega+1/\phi)\,. \tag{12}$$

This relation is satisfied to 1.5%. [The right-hand side of (12) predicts a K^* mass of 878 MeV, compared with the observed value of 891 ± 1 MeV.] We therefore assume this degeneracy in what follows, as it enables us to write the propagator (6) in a simplified form.

From the structure of (6), (7), and (10), it is clear that we can write the propagator for the ω and ϕ mesons in the form:

$$\mathbf{D}(k^2)=\frac{1}{M_0{}^2}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\omega/(\omega-k^2) & 0\\ 0 & \phi/(\phi-k^2)\end{pmatrix}\times\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}, \tag{13}$$

where θ is called the ω-ϕ mixing angle. Taking the inverse of (13) and comparing the lower right matrix element with the corresponding element in (6), we obtain

$$\sin^2\theta/\omega+\cos^2\theta/\phi=(1-2\epsilon)/M_0{}^2. \tag{14}$$

With the help of (9), this yields $\theta=27°$.[15]

We use (13) to obtain the K-meson form factor. Since the ω_1-K-K and $\omega_1-\gamma$ couplings vanish in the limit of unitary symmetry, we obtain

$$\rho_K(c)=\tfrac{1}{2}[\delta(c-c_\rho)\pm(\sin^2\theta\,\delta(c-c_\omega)+\cos^2\theta\,\delta(c-c_\phi))], \tag{15}$$

where $c_i=m_i{}^2/m_K{}^2$, and the upper (lower) sign refers to the K^+ (K^0) form factor. In obtaining (15), we have assumed exact unitary symmetry at the vertices. When we insert (15) in (3), we obtain expressions for the K^+ and K^0 electromagnetic masses. Only the terms in which one electromagnetic vertex is isoscalar and one is isovector will contribute to the mass difference, which is found to be

$$m_{K^+}-m_{K^0}=\frac{m_K\alpha c_\rho}{16\pi}\left\{\frac{c_\omega\sin^2\theta}{c_\omega-c_\rho}[f(c_\omega)-f(c_\rho)]+\frac{c_\phi\cos^2\theta}{c_\phi-c_\rho}[f(c_\phi)-f(c_\rho)]\right\}=+2.17\text{ MeV}\,, \tag{16}$$

[13] S. Coleman and H. J. Schnitzer, Phys. Rev. **134**, B863 (1964).

[14] This octet-singlet degeneracy has often been assumed in investigations of larger symmetries than $SU(3)$. When traditional mixing procedures are used, one obtains $K^*=\frac{1}{2}(\omega+\phi)$, which predicts a K^* mass of 909 MeV.

[15] A matrix propagator having the same form as (13) was used by R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964). They obtain the value $\theta=39°$ from entirely different considerations. Our value, $\theta=27°$, can be obtained more properly as a best fit to the exact propagator of Coleman and Schnitzer (see Ref. 13), where our Eq. (10) is not assumed.

when $\theta=27°$.[16] The experimental K-meson mass difference has the opposite sign. In order to obtain agreement with the experimental value, one is forced to one of two alternatives. One may postulate exotic form factors for the K mesons, as was done by Bose and Marshak[2]; in the process one gives up the assumption that remnants of a higher symmetry should still be evident in the relation of the π-meson form factors to the K-meson form factors. Alternatively, one may look for other contributions to the K^+-K^0 mass difference. The latter course is followed below.

III. THE CONTRIBUTION OF THE VECTOR-MESON INTERMEDIATE STATES

To estimate the contribution to the pseudoscalar meson mass differences which comes from higher mass intermediate states, one requires some kind of approximation scheme. In the spirit of much of particle physics in recent years, we will exalt the importance of the resonant channels and will assume that the major part of this contribution comes from the most tractable of the intermediate states, those containing a vector meson. The phenomenological gauge-invariant electromagnetic couplings are

$$\mathcal{L}_1=(3ef_{\gamma\rho\pi}/m_\pi)\epsilon_{\mu\nu\lambda\tau}(\partial_\nu A_\lambda)\,\mathrm{Tr}[(\mathbf{V}_\tau\partial_\mu\mathbf{P}+(\partial_\mu\mathbf{P})\mathbf{V}_\tau)\mathbf{Q}], \quad (17)$$

and

$$\mathcal{L}_2=(\sqrt{2}ef_{\gamma\omega_1\pi}/m_\pi)\epsilon_{\mu\nu\lambda\tau}(\partial_\nu A_\lambda)S_\tau\,\mathrm{Tr}[\mathbf{Q}\partial_\mu\mathbf{P}], \quad (18)$$

where $\mathbf{V}_\tau$, $\mathbf{P}$, and $A_\lambda\mathbf{Q}$, the fields of the eight vector mesons, the eight pseudoscalar mesons, and the photon, respectively, are 3×3 matrices in the unitary spin space over which the traces (Tr) are taken. The explicit representation for $\mathbf{V}$, $\mathbf{P}$, and $\mathbf{Q}$ written down by Coleman and Glashow[5] is assumed in normalizing the two independent dimensionless coupling constants $f_{\gamma\rho\pi}$ and $f_{\gamma\omega_1\pi}$.[17] S_τ is the field of ω_1, the vector meson which is a unitary singlet. The relative plus sign in (17) is required to make the coupling invariant under charge conjugation.

The phenomenological Lagrangians (17) and (18) contribute in a formally similar way to the meson self-energy (dm). We obtain, upon evaluating the graph represented by Fig. 1(c):

$$dm=\left(\frac{f_i s}{m_\pi}\right)^2\frac{ie^2m}{(2\pi)^4}\int d^4k\frac{[(k\cdot p)^2-k^2m^2][F_{VP}(k^2)]^2}{k^2(k^2-2k\cdot p+m^2-M^2)}, \quad (19)$$

where $f_i=f_{\gamma\rho\pi}$ or $f_{\gamma\omega_1\pi}$, depending on whether (17) or (18) is used. s is a parameter, supplied by the 3×3 $SU(3)$ matrices, which expresses the relations among the coupling constants; it is unity in all cases occurring here except two: $s=\sqrt{3}$ for the γ-ω_8-π^0 coupling, and $s=2$ for the γ-$\bar{K}^{*0}$-K^0 coupling. $F_{VP}(k^2)$ is a form factor, normalized to unity at $k^2=0$, expressing the momentum dependence of the γ-V-P vertex when the photons are unphysical but the mesons are on the mass shell. M and m are the masses of the vector and pseudoscalar meson, respectively.

It is possible to show on very general grounds that the interactions (17) and (18) lower the mass of the pseudoscalar meson, i.e., that dm is negative, by making use of a theorem due to Lehmann.[18] Given Lehmann's spectral representation for the vacuum expectation value of the commutator of two spinless Heisenberg fields $\phi(x)$, and the field equation $(\Box^2+m_0^2)\phi(x)=J(x)$, it is straightforward to show that

$$\langle 0|[\phi(\mathbf{x},0),J(\mathbf{y},0)]|0\rangle=-i\delta^3(\mathbf{x}-\mathbf{y})\int_0^\infty da^2\rho(a^2)(m_0^2-a^2), \quad (20)$$

where m_0 is the mass of the meson in the absence of the interaction and $\rho(a^2)$ is a positive semidefinite function which is nonzero only for physically realizable states. In particular, $\rho(a^2)$ is zero below the mass m of the physical pseudoscalar meson and is greater than zero for at least some $a^2>m^2$. Hence $m_0^2>m^2$, whenever the left-hand side of (20) vanishes. Any interaction Lagrangian which is linear in the pseudoscalar field will have an associated current $J(x)$ which is independent of $\phi(x)$, and the commutator on the left-hand side of (20) will vanish. Hence, in particular, the interactions (17) and (18) lead to negative electromagnetic self-energies.

Knowing the sign of the vector meson contribution, we may immediately draw some qualitative conclusions, using only the information from unitary symmetry about the γ-V-P coupling constants. Because $f_{\gamma\text{-}\bar{K}^{*0}\text{-}K^0}=2f_{\gamma\text{-}K^{*-}\text{-}K^+}$ in the unitary symmetry theory, the mass of the K^0 is depressed four times as much as the mass of the K^+ by the contribution of the K^* intermediate state, so that the effect enlarges the mass excess of the K^+ over the K^0. Thus, one cannot possibly produce the experimental K^+-K^0 mass difference by combining this contribution with the one considered in the previous section; *both* have the wrong sign.

In the case of the π^+-π^0 mass difference, we will consider ρ, ω, and ϕ intermediate states. The ρ-meson intermediate state couples only to the isoscalar part of the electromagnetic field, and does not contribute to the π^+-π^0 mass difference. The ω and ϕ contributions lower the π^0 mass, and hence enlarge the π^+-π^0 mass difference. If the effect is large, the agreement between the experimental value and the theoretical calculation of Bose and Marshak will be disrupted.

[16] One gets $m_{K^+}-m_{K^0}=+2.10$ MeV if $\theta=39°$ is used.

[17] The normalizations in (17) and (18), coupled with the notation of Ref. 5, are consistent with the normalizations implied by the phenomenological Lagrangians:

$$\mathcal{L}_{\gamma\rho\pi}=em_\pi^{-1}f_{\gamma\rho\pi}\epsilon_{\mu\nu\lambda\tau}\boldsymbol{\rho}_\tau\partial_\mu\boldsymbol{\pi}\partial_\nu A_\lambda$$

and

$$\mathcal{L}_{\gamma\omega_1\pi^0}=em_\pi^{-1}f_{\gamma\omega_1\pi}\epsilon_{\mu\nu\lambda\tau}\omega_1\partial_\mu\pi^0\partial_\nu A_\lambda.$$

[18] H. Lehmann, Nuovo Cimento 11, 342 (1954).

In order to make quantitative estimates of the vector meson contribution, (19) is evaluated explicitly, using Feynman's techniques and assuming that the form factor possesses the spectral representation (2); this yields[12]

$$dm=\frac{-m\alpha}{16\pi}\left(\frac{f_i sm}{m_\pi}\right)^2\int_0^\infty dc_1\int_0^\infty dc_2\rho(c_1)\rho(c_2)$$
$$\times\frac{c_1c_2}{c_2-c_1}[U(b,c_2)-U(b,c_1)], \quad (21a)$$

where

$$2cU(b,c)=2c^2+(b-c)[6c-(b-c)^2]\ln[(1+b)/c]$$
$$+2b^3\ln(1+b^{-1})+[4c-(b-c)^2]^2w(b,c), \quad (21b)$$

$$w(b,c)=\int_0^1 dx(x^2+bx-cx+c)^{-1} \quad (21c)$$

and

$$b=-1+M^2/m^2. \quad (21d)$$

The calculation may be completed once the form factors are chosen. If the form factor is restricted to have the form of a sum of pole terms,

$$\rho(c)=\sum_i t_i\delta(c-c_i), \quad \sum_i t_i=1,$$

then

$$dm=(-m\alpha/16\pi)(f_i sm/m_\pi)^2\{\sum_j t_j^2V(b,c_j)$$
$$+\sum_{j<k}2c_jc_kt_jt_k(c_k-c_j)^{-1}[U(b,c_k)-U(b,c_j)]\}, \quad (22)$$

where

$$V(b,c)=c^2(dU(b,c)/dc)$$
$$=[-3c^2+\tfrac{1}{2}(c-b)^2(2c+b)]\ln[(1+b)/c]$$
$$+c(2c-b)-b^3\ln(1+b^{-1})-\tfrac{1}{2}[(c-b)^2-4c]$$
$$\times[(c-b)^2-3c(c-b)+2c]w(b,c). \quad (23)$$

To calculate the vector meson contribution to the π^+-π^0 mass difference, we use the vector-mixing propagator (13) for the ω and ϕ intermediate states and a form factor having a single pole at the ρ-meson mass. This yields

$$m_{\pi^+}-m_{\pi^0}$$
$$=3\alpha m_\pi f^2(16\pi)^{-1}[(a\cos\theta+\sin\theta)^2(m_\omega^2/M_0^2)V(b_\omega,c_\rho)$$
$$+(a\sin\theta-\cos\theta)^2(m_\phi^2/M_0^2)V(b_\phi,c_\rho)], \quad (24)$$

where f henceforth stands for $f_{\gamma\rho\pi}$, and where

$$b_\omega=m_\omega^2/m_\pi^2-1, \quad b_\phi=m_\phi^2/m_\pi^2-1, \quad c_\rho=m_\rho^2/m_\pi^2,$$

and

$$a=f_{\gamma\omega_1\pi}/\sqrt{3}f.$$

Similarly, the contribution of the K^* intermediate state to the K^+-K^0 mass difference may be found, using the mixed propagator for the isoscalar form factor and a ρ-pole approximation to the isovector form factor. Only parts of Fig. 1(c) which contain one isovector and one isoscalar vertex contribute to the mass difference, which is found to be

$$m_{K^+}-m_{K^0}=3\alpha m_K m_\rho^2 f^2 y(16\pi m_\pi^2)^{-1}, \quad (25a)$$

where

$$y=c_\omega(c_\omega-c_\rho)^{-1}(\sin^2\theta$$
$$-a\sin2\theta)[U(b',c_\omega)-U(b',c_\rho)]$$
$$+c_\phi(c_\phi-c_\rho)^{-1}(\cos^2\theta+a\sin2\theta)$$
$$\times[U(b',c_\phi)-U(b',c_\rho)], \quad (25b)$$

$$c_i=m_i^2/m_K^2, \quad\text{and}\quad b'=-1+m_{K^*}^2/m_K^2. \quad (25c)$$

If we continue to assume that the photon always couples to the strongly interacting particles by means of the neutral vector mesons, then we may rewrite the parameter a as a ratio of strong coupling constants, $a=G_{\omega_1\rho\pi}/G_{\omega_8\rho\pi}$. This ratio may be estimated by making use of the experimental result that the decay $\phi\to\rho+\pi$ is almost completely suppressed.[19] The amplitude for this decay vanishes if

$$a=\operatorname{ctn}\theta. \quad (26)$$

One consequence of (26) is that only the ω intermediate state contributes to the π^+-π^0 mass difference. Inserting (26) and $\theta=27°$ into (24) and (25) yields

$$m_{K^+}-m_{K^0}=+18.5f^2\text{ MeV}, \quad (27a)$$

and

$$m_{\pi^+}-m_{\pi^0}=+11.8f^2\text{ MeV}. \quad (27b)$$

An alternative path to (26) is found by imposing conservation of the A quantum number of Bronzan and Low.[20] If A is exactly conserved, however, the only allowed coupling is γ-ω-π^0, and there is no K^* contribution to the K^+-K^0 mass difference. In the case at hand, we have a consistent picture of a nearly conserved A quantum number if we consider (26) to be approximate. The ω-ϕ mixing measures the violation of A conservation and is small (ω is a nearly pure singlet). The presence of substantial A conservation in turn accounts for a qualitative feature of (27): The vector contributions to the K and π mass splittings are comparable in spite of the fact that the π has a relatively much heavier intermediate state ($b_\omega\gg b'$).

The estimate of the coupling constant f has been attempted several times; great disparities between the results of different methods still remain. Gell-Mann, Sharp, and Wagner[21] tried to estimate these coupling constants using models for the decays $\omega\to3\pi$ and $\pi^0\to2\gamma$. There was a distressing lack of agreement

[19] P. L. Connolly, E. L. Hart, K. W. Lai, G. London, G. C. Moneti *et al.*, Phys. Rev. Letters **10**, 371 (1963).
[20] J. B. Bronzan and F. E. Low, Phys. Rev. Letters **12**, 522 (1964).
[21] M. Gell-Mann, D. Sharp, and W. Wagner, Phys. Rev. Letters **8**, 261 (1962).

TABLE I. Contributions to the π^+-π^0 and K^+-K^0 mass differences (in MeV).

	Fig. 1(a) +Fig. 1(b)	Fig. 1(c)	Fig. 1(d)	Total	Experimental value
$\pi^+-\pi^0$	4.37	0.49±0.37	0.0	4.9±0.4	4.590±0.004
K^+-K^0	2.17	0.77±0.58	−4.2	−1.3±0.6	−4.2 ±0.5

between the two models; stated one way, the calculated π^0 decay rate was much too large. But when $\omega-\phi$ mixing was taken into account by Dashen and Sharp,[22] the agreement was improved. The reason is that the π^0-decay model involves the $\omega_8-\rho-\pi$ coupling, which is a factor of $\sin\theta$ smaller than the $\omega-\rho-\pi$ coupling of the ω-decay model. The vector mixing model predicts a smaller mixing angle than Dashen and Sharp have used, and consequently it achieves still better harmony between the two models. If we insert $\Gamma(\rho\rightarrow 2\pi)=110\pm10$ MeV, $\Gamma(\omega\rightarrow 3\pi)=8.5\pm1.9$ MeV, $\Gamma(\pi^0\rightarrow 2\gamma)=6.3\pm1.0$ MeV, and $\theta=27°$ into Dashen's and Sharp's expressions, we find

$$f=0.11\pm0.02 \quad (\omega\text{-decay model}), \tag{28a}$$

$$f=0.09\pm0.02 \quad (\pi^0\text{-decay model}). \tag{28b}$$

The two results are in agreement.[23] This may be considered a point in favor of the vector mixing approximation of Coleman and Schnitzer.

An entirely independent estimate of f has been made by Berman and Drell,[24] on the basis of an analysis of a "ρ^0 bump" in the photoproduction of pion pairs. In our notation, they find

$$f=0.27. \tag{29}$$

(28) and (29) may be considered representative estimates of f.[25] We helplessly repeat a sentiment recently expressed by Adler and Drell,[26] that a better knowledge of this coupling constant is essential. In Table I, we quote results in such a way that the upper limit agrees with (29) and the lower limit agrees with the average of (28a) and (28b).[27] Our most important result is that the vector intermediate state contribution is not large enough to spoil the agreement previously obtained by Bose and Marshak for the π^+-π^0 mass difference on the basis of Figs. 1(a) and 1(b) only.

The uncertainty in f is not the only possible source of error in this calculation; thus the numbers following the ± signs in Table I are not to be taken as the error bounds of our calculation. It is difficult to estimate realistic error bounds; we would guess (perhaps with excessive optimism) that the error is of the order of one MeV.

IV. GROUP THEORETIC RESULTS FOR OTHER INTERMEDIATE STATES

In the limit of exact $SU(3)$, all intermediate states which contribute to the π^+-π^0 and K^+-K^0 mass differences must belong to a restricted set of supermultiplets, 1, 8_D, 8_F, $10\oplus\overline{10}$, and 27. The subscripts D and F distinguish between the two types of 8-8-γ couplings; the properties of the states under charge conjugation generally force a given supermultiplet of intermediate states to couple either one way or the other, and forbid a linear combination of the two couplings. In Table II, we tabulate the signs of the contributions due to intermediate states in each channel, assuming only that the basic diagram, Fig. 1(c), contributes a *negative* electromagnetic self-energy. This is the sign of Fig. 1(c) whenever the interaction is such that the left-hand side of (20) vanishes; if the sign is opposite, as in the calculation of Sec. II, the signs in the first two rows of Table II must be reversed. On the bottom row of Table II, the ratio $(K^+-K^0)/(\pi^+-\pi^0)$ is given for each channel, assuming exact $SU(3)$. This ratio will be modified considerably, of course, if physical masses are used in a calculation.

Table II shows that the K^0 is made heavier than the K^+ by intermediate states transforming like a 27-plet or like an antisymmetrically coupled octet if the interaction contributes a negative self-energy, and by states transforming like $10\oplus\overline{10}$ or like a symmetrically coupled octet if the interaction contributes a positive self-energy. Even if contributions large enough to account for the physical K^+-K^0 mass difference were discovered, they would simultaneously have to conspire to cancel one another's effect on the π^+-π^0 mass difference. It appears unlikely, although not impossible, that the meson mass problem can be resolved by higher mass intermediate states alone.

TABLE II. Dependence of signs of $(\pi^+-\pi^0)$ and (K^+-K^0) and value of $(\pi^+-\pi^0)/(K^+-K^0)$ on the $SU(3)$ transformation property of the intermediate states, in the limit of exact symmetry. The signs are calculated assuming that the separate contributions of Fig. 1(c) *lower* the meson mass.

	Transformation property of intermediate states				
	1	8_D	8_F	$10\oplus\overline{10}$	27
Sign of $(\pi^+-\pi^0)$	+	−	+	−	+
Sign of (K^+-K^0)	None	−	+	+	−
$(K^+-K^0)/(\pi^+-\pi^0)$	0	+1	+1	−2	−8/7

[22] R. F. Dashen and D. H. Sharp, Phys. Rev. **133**, B1585 (1964). See also Ref. 15.

[23] The ω and π^0 decay rates are the same as those used by Dashen and Sharp, and references are given there. The ρ^0 decay rate is taken from M. Roos, Rev. Mod. Phys. **35**, 314 (1963), footnote 40, who compiled a weighted average of a number of experiments. (The masses of the particles used in our work have also been taken from Roos.) A ρ width of 110 MeV rather than 100 MeV, which was used in Ref. 22, improves the agreement found in (28); the ratio of (28b) to (28a) varies with the cube of this width.

[24] S. M. Berman and S. D. Drell, Phys. Rev. **133**, B791 (1964).

[25] For further estimates see W. Alles and D. Boccaletti, Nuovo Cimento **27**, 306 (1963), and S. Hatsukade and H. J. Schnitzer, Phys. Rev. **128**, 468 (1962).

[26] R. J. Adler and S. D. Drell, Phys. Rev. Letters **13**, 349 (1964).

[27] The results quoted in earlier papers of this series assumed that f was given by the average of (28a) and (28b).

V. THE CONTRIBUTION OF SCALAR MESON TRANSITIONS

Coleman and Glashow[6] have suggested that another contribution to the electromagnetic masses should be taken into account, in which the $\pi^{0\prime}$ of a scalar meson octet makes a virtual transition to the vacuum, thereby violating isotopic spin symmetry in a well-defined way. The magnitude of this symmetry breaking is fixed by their second assumption, to the effect that the transition to the vacuum of the η' member of the same scalar meson octet is responsible for *all* of the observed mass splitting between isotopic multiplets. The theory then contains one free parameter, the ratio of the expectation values of the $\pi^{0\prime}$ and η' transitions. The magnitude of the scalar meson contribution to the electromagnetic mass splittings of the baryons and mesons is tabulated in Table III. The normalization is arbitrarily chosen to be unity for the neutron-proton mass difference.[28] The lowest order contribution to the π^+-π^0 mass difference is zero, because this mass difference transforms like $I=2$, and there are no $I=2$ particles in an octet. The remaining coefficients of X are ratios of intermultiplet mass splittings, with numerical factors supplied by $SU(3)$. For example, the coefficient of X for the K^+-K^0 mass difference is

$$2(\pi-K)m_K^{-1}[2(\Xi-N)-3(\Sigma-\Lambda)]^{-1}\approx-1.75,$$

where the unsubscripted symbols stand for the corresponding baryon masses and squares of meson masses. There is an arbitrariness in this coefficient (and the others) to the extent that the Gell-Mann–Okubo formula is not satisfied exactly: one can add arbitrary multiples of $(4K-\pi-3\eta)$ to the numerator and of $(2\Xi+2N-\Sigma-3\Lambda)$ to the denominator. The coefficients in Table III are calculated with the guidance of a simplicity assumption.

Coleman and Schnitzer have attempted to fix the parameter X by calculating the nontadpole contributions to the baryon electromagnetic mass differences and then choosing X so that, when all contributions are combined, a best fit to the experimental values is obtained. Since their work, new experimental values for the Σ mass differences have been reported,[29] and these have an interesting feature. If we define $\delta=(\Sigma^--\Sigma^0)-(\Sigma^0-\Sigma^+)$ to express the deviation from equal mass splitting in the Σ triplet, then the new result is $\delta=1.9\pm0.3$ MeV. The tadpole contribution gives $\delta=0$. The nontadpole contribution was calculated by Coleman and Schnitzer in two ways, and they obtained $\delta=2.1$ MeV and $\delta=2.0$ MeV in the two cases.[30] As a result, it now turns out to be possible to obtain a quite consistent value for X on the basis of the well-determined Σ and nucleon mass differences. Coleman and Schnitzer, on the basis of earlier data, did not do this, but chose to accept somewhat larger discrepancies in the baryon mass differences in order to account for a larger part of the K-meson mass difference. The two alternatives are contrasted in Table IV. The nontadpole contributions to the baryon mass differences are taken from the second column of Table III of Ref. 7.[31] The choice of $X=2.4$ MeV achieves excellent agreement with the observed baryon splittings; the choice $X=3.0$ MeV achieves less good agreement for the baryons, but better agreement for the K mesons. We do not know whether to regard the accuracy in the Coleman-Schnitzer calculation of δ as an accident; on the chance that it is not, we have inserted into Table I a scalar

TABLE III. Scalar-meson contributions to the electromagnetic mass differences.

Mass difference	Scalar-meson contribution
$\pi^+-\pi^0$	None
K^+-K^0	$-1.75X$
$n-p$	$1.00X$
$\Sigma^0-\Sigma^+$	$1.45X$
$\Sigma^--\Sigma^0$	$1.45X$
$\Xi^--\Xi^0$	$1.90X$

TABLE IV. Total contribution to baryon and meson electromagnetic mass differences for two values of the parameter X. All values are in MeV.

		Total contribution (tadpole plus nontadpole)		
	Nontadpole contribution	$X=2.4$ MeV	$X=3.0$ MeV	Experimental value
$\pi^+-\pi^0$	4.9	4.9	4.9	4.6
K^+-K^0	2.8	-1.4	-2.4	-4.2 ± 0.5
$n-p$	-1.1	1.3	1.9	1.3
$\Sigma^0-\Sigma^+$	-0.7	2.9	3.8	2.8 ± 0.3
$\Sigma^--\Sigma^0$	1.4	5.0	5.9	4.8 ± 0.1
$\Xi^--\Xi^0$	1.2	6.0	7.2	6.1 ± 1.6

[28] The relation of X to the value of $\langle\pi^{0\prime}\rangle/\langle\eta'\rangle$ discussed in Ref. 6 is $\langle\pi^{0\prime}\rangle/\langle\eta'\rangle=X/(15\text{ MeV})$.

[29] R. A. Burnstein, T. B. Day, B. Kehoe, B. Sechi-Zorn, and G. A. Snow, Phys. Rev. Letters **13**, 66 (1964).

[30] An inspection of the results reported by Coleman and Schnitzer reveals that their two calculations of δ agree with one another more nearly than do their two calculations of $(\Sigma^--\Sigma^0)$ or $(\Sigma^0-\Sigma^+)$ separately. This is not surprising. The calculation of δ involves only the isovector form factors, since δ transforms like $I=2$; the calculations of the separate Σ mass differences, on the other hand, involve the isoscalar form factors as well. The two Coleman-Schnitzer calculations correspond to different approximations for the electromagnetic form factors; among other features which distinguish the two approximation schemes, in one case but not the other, ω-Φ mixing is taken into account. Since ω-Φ mixing is only relevant for the isoscalar form factors, an important distinction between the two calculations of the separate Σ mass differences is absent from the two calculations of δ.

[31] These are the contributions which result when the strange-baryon form factors are obtained from the nucleon form factors by the use of the exact relations provided by unitary symmetry (Ref. 5). Coleman's and Schnitzer's second estimate of the nontadpole contribution to the baryon mass differences associates with the poles of the form-factors vector mesons possessing definite $SU(3)$ transformation properties, and does not lead to quite as good agreement with experiment when combined with the scalar meson contribution. Since there is no apparent reason to prefer one of their estimates to the other, it seems advisable to regard as fortuitous any agreement to better than $\frac{1}{2}$ MeV in our Table IV.

meson contribution to the K^+-K^0 mass difference of -4.2 MeV, corresponding to $X=2.4$ MeV.

In conclusion, the scalar meson contribution to the K^+-K^0 mass difference has the right sign to agree with experiment, and, curiously, taken alone it also has the right magnitude. However, when the scalar meson contribution is combined with the other contributions, the sign, but somewhat less than half the magnitude, of the K^+-K^0 mass difference is accounted for. The predicted π^+-π^0 mass difference remains unchanged when the scalar meson contribution is included, and hence the agreement between theory and experiment obtained by previous authors persists.

We started out to explain the π- and K-meson mass differences simultaneously, by exploiting the supermultiplet properties of all particles participating in the interactions, and it is clear that we have been less than perfectly successful: The π-meson mass difference remains much better understood than the K-meson mass difference. If the scalar meson contribution were larger, we would indeed have a satisfactory explanation of both mass differences. One way that this could arise would be if the *nontadpole* contribution to the *intermultiplet* mass differences, neglected in finding the coefficients in Table III, were in fact substantial. The successes of the tadpole theory persist if these nontadpole contributions have octet transformation properties and if, in addition, they preserve the ratio $(\Xi-N)/(\Sigma-\Lambda)$ found in nature. Subject to these constraints, such nontadpole contributions improve our theory either if they split the mesons in the direction opposite to that observed in nature, or if they split the baryons in the same direction as that observed in nature, or if they do both. If nontadpole contributions with these signs exist, that would mean that we had underestimated the ratio of the scalar-meson–pseudoscalar-meson coupling to the scalar-meson–baryon couplings, and therefore had underestimated the tadpole contribution to the K-meson mass difference.

As was shown in Sec. IV, it is possible but very difficult for higher mass intermediate states to conspire to yield the experimental K-meson mass difference without upsetting the π-meson agreement. The tadpole theory, refined in the manner just described, presents an alternative mechanism for explaining the K^+-K^0 mass difference to the same accuracy as the π^+-π^0 mass difference.

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Deuteron Stripping at 3.54 GeV/c†

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This paper presents the angle and momentum distributions for protons stripped from deuterons at 3.54 GeV/c by aluminum, copper, and lead. The parameters of interest are summarized in the table. Roughly, the results are consistent with a cross section about $\frac{1}{2}$ geometric (where $r=1.22A^{1/3}\times10^{-13}$ cm) and a momentum distribution obtained by transforming the deuteron internal-momentum distribution to the laboratory frame. The results are: d^+ momentum, 3.54 ± 0.100 GeV/c; p^+ momentum, 1.77 ± 0.100 GeV/c; angle spread (full width at half-maximum) 3°; σ_s(Al), 290 mb$\pm$25%; σ_s(Cu), 550 mb$\pm$25%; σ_s(Pb), 950 mb$\pm$25%.

THIS paper presents the angle and momentum distributions for protons stripped from deuterons at 3.54 GeV/c by aluminum, copper, and lead. The parameters of interest are summarized in Table I. Roughly, the results are consistent with a cross section about $\frac{1}{2}$ geometric (where $r=1.22A^{1/3}\times10^{-13}$ cm) and momentum and angle distributions obtained by transforming the deuteron internal-momentum distribution to the laboratory frame (as though the deuteron were a decaying particle).

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