Mean Lives of Excited Rotational States of Heavy Even-Even Nuclei*

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The lifetimes of the 2+ and 4+ excited states of the ground-state rotational band of three even-even nuclides in the rotational region beyond Pb, and the 2+ first excited state of a nearby vibrational nuclide, were measured by means of delayed-coincidence techniques. The alphas to the excited state were detected with a semiconductor charged-particle detector, and the conversion electrons from the de-exciting state were detected with a plastic scintillator. The full width at half maximum of the resulting prompt spectrum was 0.5 nsec, with a limiting equivalent half-life of 60 psec. A system of multiple Peierls analyses was carried out on the MIT 7094 computer to obtain the best lifetime values, which are presented below, along with the ratio of the reduced transition probabilities in the case of rotational nuclides. The uncertainties quoted for the $B(E2)$ ratios do not include estimates of absolute uncertainties in the theoretical internal-conversion coefficients employed.

Nuclide	$T_{1/2}^{(2+)}$ (psec)	$T_{1/2}^{(4+)}$ (psec)	$\frac{B(E2;4+\rightarrow 2+)}{B(E2;2+\rightarrow 0+)}$
Rn^{220}	145±5	———	———
Ra^{224}	744±19	181±9	1.37±0.07
Th^{228}	402±10	161±5	1.46±0.05
Th^{230}	354±9	166±5	1.40±0.05

The ratios of the reduced transition probabilities are seen to be quite close to the value of 1.43 predicted by the collective model, with Ra^{224} being 4% low, Th^{228} 2% high, and Th^{230} 2% low.

INTRODUCTION AND THEORY

NUCLIDES in the rare-earth region and in the region beyond the doubly magic Pb^{208}, starting at about Ra, have long been known to possess large equilibrium deformations with resultant large quadrupole moments and $E2$ transition rates.[1] Even-even nuclei in these regions behave in many aspects like an axially-symmetric quantum-mechanical rotor, exhibiting the low-lying-level spin sequences of 0+, 2+, 4+, etc.,[2,3] with energies given (often to within 1 or 2%) by

$$E(I)=\frac{\hbar^2}{2\mathcal{I}_0}I(I+1),$$

which defines the experimental moment of inertia $\mathcal{I}_0$. To deal with such nuclei Bohr and Mottelson[4,5] developed the theory of the strong coupling model, and some pertinent results of it are given below.

For pure $E2$ radiation from the 2+ first excited level to the 0+ ground level it can be shown that the reduced transition probability[6] is given by

$$B(E2;2+\rightarrow 0+)=\frac{\ln 2}{(1+\alpha_T)T_{1/2}}\frac{75\hbar}{4\pi}\left(\frac{hc}{E}\right)^5, \quad (1)$$

where α_T is the total internal conversion coefficient, $T_{1/2}$ is the experimentally measured half-life of the level, and E is the transition energy. The collective model gives the reduced transition probability of a deformed nucleus of intrinsic quadrupole moment Q_0 as

$$B(E2;I_i,K\rightarrow I_f,K)=\frac{5}{16\pi}Q_0^2(K)(2I_f+1)\begin{pmatrix} I_f & 2 & I_i \\ -K & 0 & K\end{pmatrix}^2,$$

whereby the $K=0$ band intrinsic quadrupole moment Q_0 may be found from experiment

$$Q_0^2=(16\pi/5)B(E2;2+\rightarrow 0+).$$

In terms of the deformation parameter β, the intrinsic quadrupole moment Q_0 is given by

$$Q_0=\frac{3ZeR_0^2}{(5\pi)^{1/2}}\beta(1+0.36\beta+\cdots),$$

from which the deformation β may be calculated. Finally, with the parameter β one can calculate the ratio of the experimental moment of inertia $\mathcal{I}_0$ to the rigid-rotation moment of inertia (for a nucleus of noninteracting nucleons) given by

$$\mathcal{I}_{\text{rig}}=\tfrac{2}{5}\text{AMR}_0^2(1+0.31\beta+\cdots),$$

and the ratio of $\mathcal{I}_0$ to the irrotational-flow moment of inertia (for a nucleus of strongly interacting nucleons) given by

$$\mathcal{I}_{\text{irrot}}=0.894(\tfrac{2}{5}\text{AMR}_0^2)\beta^2[1+O(\beta^2)].$$

Besides enabling the evaluation of some of its parameters, the theory of Bohr and Mottelson predicts definite relationships between measurable quantities. One such prediction is the ratio of the reduced transition proba-

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† This work was done in partial fulfillment of the requirements for the Ph.D. degree at M.I.T.
[1] G. M. Temmer and N. P. Heydenburg, Phys. Rev. **99**, 1609 (1955).
[2] G. Scharff-Goldhaber, Phys. Rev. **90**, 587 (1953).
[3] G. Scharff-Goldhaber and J. Weneser, Phys. Rev. **98**, 212 (1955).
[4] A. Bohr, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **26**, No. 14 (1952).
[5] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **27**, No. 16 (1953).
[6] A. de-Shalit and I. Talmi, *Nuclear Shell Theory* (Academic Press Inc., New York, 1963).

bilities of successive $E2$ transitions within the ground state rotational band ($K=0$):

$$\frac{B(E2;I+4\rightarrow I+2)}{B(E2;I+2\rightarrow I)}=\frac{(I+3)(I+4)(2I+3)(2I+5)}{(I+1)(I+2)(2I+7)(2I+9)},$$

which is thus independent of intrinsic nuclear properties. For the $4+\rightarrow 2+\rightarrow 0+$ cascade, the theoretical $B(E2)$ ratio reduces to 10/7, whence

$$\frac{B(E2;4+\rightarrow 2+)}{B(E2;2+\rightarrow 0+)}=\frac{(1+\alpha_{2\rightarrow 0})E_{2\rightarrow 0}{}^5(T_{1/2})_{2\rightarrow 0}}{(1+\alpha_{4\rightarrow 2})E_{4\rightarrow 2}{}^5(T_{1/2})_{4\rightarrow 2}}=\frac{10}{7}.$$

Many nuclei do not possess an equilibrium deformation but can still perform a collective motion of shape vibration. In the theory of Bohr and Mottelson quadrupole vibrations may be described by a simple collective Hamiltonian containing the two parameters[7] B_2 and C_2 in terms of which the energy of the 2+ first excited level and the reduced transition probability $B(E_2;2+\rightarrow 0+)$ may be expressed. The experimentally obtained values of B_2 and C_2 may then be compared with the Bohr and Mottelson hydrodynamic-model values.[7]

There have been relatively few experimental tests of the predicted ratio of the reduced transition probabilities because, although many 2+ level lifetimes have been measured, very few 4+ level lifetimes have been done due to the shortness of the lifetimes, and the frequently very weak feeding of these levels by parent sources. The measurements of both 2+ and 4+ level lifetimes, previous to the present ones, are given in Table I. They are all in the rare-earth region.

The present measurements were intended to make a similar test of the predicted ratio in the rotational region beyond Pb. Those nuclides just away from the boundary separating rotational and vibrational nuclei are of special interest both because the similar measurements in the rare-earth region seem to differ from the predicted values, and because these nuclides have the larger 4+ level feeding fractions as well as anticipated half-lives. The rotational nuclides Ra^{224}, Th^{228}, and Th^{230}, together with the vibrational nuclide Rn^{220}, were selected for measurement.

TABLE I. Some previous measurements of 2+ and 4+ level halflives and the resulting $B(E2)$ ratios.*

Nuclide	$\frac{E_{4+}}{E_{2+}}$	$T_{1/2}{}^{2+}$ (nsec)	$T_{1/2}{}^{4+}$ (psec)	$\frac{B(E2;4+\rightarrow 2+)}{B(E2;2+\rightarrow 0+)}$
Gd^{154}	3.02	1.16±0.05	39± 5	1.77±0.25[a]
Gd^{156}	3.24	1.9 ±0.1	105±21	1.4 ±0.4[b]
Dy^{160}	3.26	1.99±0.05	107±15	1.4 ±0.23[c]
Dy^{162}	3.28	2.25±0.07	132± 8	1.42±0.10[c]
Er^{166}	3.27	1.83±0.06	120± 8	1.42±0.10[c]
Er^{168}	3.30	1.90±0.06	121± 8	1.46±0.10[c]
Hf^{180}	3.32	1.53±0.05	75±10	1.45±0.19[c]
Os^{190}	2.93	0.37±0.03	19±10	1.01±0.56[d]

* This is not intended to be a thorough survey. It comprises only 2+ and 4+ level measurements which were both done by the same worker.
[a] J. Burde, M. Rakavy, and G. Rakavy, Phys. Rev. 129, 2147 (1963).
[b] S. Ofer, Phys. Rev. 115, 412 (1959).
[c] A. C. Li and A. Schwarzschild, Phys. Rev. 129, 2664 (1963).
[d] A. W. Sunyar (private communication).

[7] M. A. Preston, *Physics of the Nucleus* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1962), p. 228.

EXPERIMENTAL DETAILS

Detectors Employed

Since the excited levels of the nuclides of interest are all populated by alpha decay, typically 30% to the 2+ level, and 0.2% to the 4+ level[8], the semiconductor charged-particle detector was chosen as the alpha detector because of its good energy resolution together with its capability for nsec timing. The detectors employed were made in this laboratory and were of the surface-barrier type. The base material used was 300 ohm-cm Si with a minority carrier lifetime of 2000 μsec or better, and was cut into 5 mm×5 mm square wafers. The finished diode was mounted in a BNC type connector, and after application of an edge protection coating, the active area of the diode was about 4 mm×4 mm.

It was found that from 50 to 200 V of bias was required for good rise time (2 or 3 nsec, or less), and only diodes with about 25 keV full-width-at-half-maximum (FWHM) energy resolution or less at these biases were used.

Since the levels of the nuclides used are strongly internally converted, $\frac{1}{16}$-in. thick, $\frac{1}{4}$-in. diam discs of Pilot B and a 56AVP photomultiplier (PM) tube were used to detect the conversion electrons. The 56AVP was magnetically shielded and run at about 2000 V using a linear divider string which put some 500 V across the photocathode and first dynode for good low-energy time resolution.

Electronics

A detailed block diagram of the electronics used is given in Fig. 1 which is basically a fast-slow delayed-coincidence system. The output current pulse from the 56AVP PM tube drives a GaAs tunnel diode discriminator with delay line reset. Use of such a discriminator at this point enables one to trigger on the fraction of the current pulse giving the best time resolution as discussed by Gatti and Svelto[9,10] and investigated by Bartl and Weinzierl[11] with 0.2 to 0.3 being the fraction used here.

A Nuvistor cathode follower was used to sample the diode pulse and to drive the low input impedance of the fast transistor amplifier. An Ortec 103 integrating preamp was connected through a 1 K buffering resistor to the diode so that the fast alpha signal would be larger during the first 10 or 20 nsec of the pulse. The present method of deriving the fast-timing signal slightly degrades the alpha-channel energy resolution; use of the

[8] I. Perlman and J. O. Rasmussen, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 42, p. 109.
[9] E. Gatti and V. Svelto, Nucl. Instr. Methods 4, 189 (1958).
[10] E. Gatti and V. Svelto, in *Symposium on Nuclear Instruments*, edited by J. B. Birks (Academic Press, Inc., New York, 1962), p. 35.
[11] W. Bartl and P. Weinzierl, Rev. Sci. Instr. 34, 252 (1963).

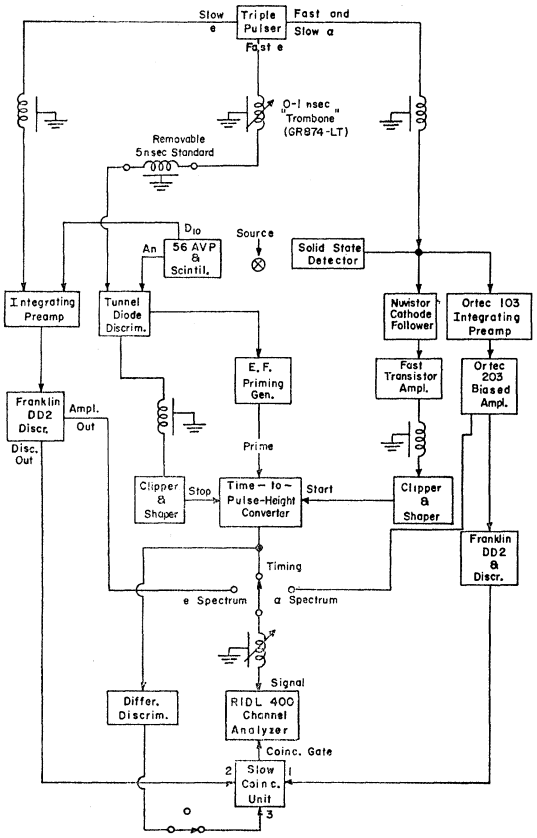

FIG. 1. Detailed block diagram of the electronics.

recently reported inductive pick-off technique[12] should considerably reduce this effect. The integrating preamp was followed by an Ortec 203 biased amplifier to exploit the high energy resolution of the alpha detector. The fast amplifier was an 8-stage transistor amplifier with a gain of 250 and rise time of 5 nsec.

The time-to-pulse-height converter used was the inductive fly-back design of Culligen and Lipman[13] requiring priming so that there will not be a large overflow-pulse count rate at the output due to "unstopped" "start" pulses. Each channel contained limiting and clipping amplifiers.

For time calibration purposes a mercury-wetted relay pulser delivering three shaped and relatively delayed synchronous pulses was permanently connected to the system with its own injection points so that calibration checks could be made with a timing run in progress without shutting anything down. When the pulser was on, the two fast signals entered the converter, and their fixed relative delay resulted in a fixed amplitude converter output pulse. However, although one slow pulse is set to enter the slow electron window, the other pulse to the alpha channel is made to fall outside the alpha window. Displacing the alpha window from the nuclear alpha peak and placing it on the pulser peak allowed only the fixed amplitude pulse from the converter to be accumulated. Delaying one pulser pulse with a calibrated delay results in another different, fixed amplitude pulse from the converter. The two 1- or 2-channel wide timing peaks provide the time calibration factor. Since it is easy to have both peaks maximize on two decade channels initially, a fast, easy (and thus frequent, if necessary) visual check can be made for tenths of a channel drift. Returning the alpha window to the nuclear alpha peak, accumulation of the coincidence spectrum resumes, with no essential changes having been made in the nuclear signal lines.

System Calibration

Time calibration was performed as just described using a calibrated 5 nsec length of cable (with GR type 874 connectors) made by Nanosecond Systems, Inc., who specify the delay to be accurate to 2%. Linearity tests were made by using equal lengths of cable with the triple output pulser and obtaining plots of inserted delay versus analyzer channel. The result was a straight line fitting the points to within one channel for over 200 channels (20 nsec). With a time calibration factor of 0.1 nsec/channel, drift was observed to be less than $\frac{1}{2}$ channel in a day and the time calibration factor changed less than $\frac{1}{2}$% in a day.

Data Reduction

The exponential slopes of the delayed coincidence data curves were analyzed using Peierls' method[14,15] which is equivalent to the use of the method of maximum likelihood[16,17] when dealing with exponential distributions. A program was written to solve the resulting transcendental equation, and the computations were performed on the MIT Computation Center 7094 computer. The program computes the lifetime from the data in a fixed number of consecutive channels but with different starting points on the decay spectrum. As this "analysis interval" moves over the peak of the decay spectrum, down its exponential decay, and then into the background, the plot of successive lifetimes should become rather constant while the interval is on the exponential portion of the curve, should increase when the interval includes some of the "nonexponential" counts near the maximum of the curve, and should either increase or decrease when the interval is near the bottom of the decay curve, depending upon whether too little or too much background has been removed.

The plot of successive lifetimes then indicates the largest range of data that should be used in a final calculation of the lifetime where the maximum permissible number of counts should be used. Also, when a

[12] C. W. Williams and J. A. Biggerstaff, Nucl. Instr. Methods, **25**, 370 (1964).
[13] G. Culligen and N. H. Lipman, Rev. Sci. Instr. **31**, 1209 (1960).
[14] R. Peierls, Proc. Roy. Soc. (London) **A149**, 467 (1935).
[15] R. D. Evans, *The Atomic Nucleus* (McGraw-Hill Book Company, Inc., New York, 1955).
[16] E. Segre, in *Experimental Nuclear Physics*, edited by E. Segre (John Wiley & Sons, Inc., New York, 1959), Vol. 3, p. 40.
[17] M. S. Bartlett, Proc. Roy. Soc. (London) **A154**, 124 (1936).

nonconstant background is to be stripped out, the flattening of the plot of successive lifetimes is a fair criterion for estimating the amount of the background spectrum to be removed in the absence of any other such knowledge. An estimate of the error introduced by the uncertainty in the exact amount to be stripped out may be obtained from noting the change in the calculated lifetime value in the high-count region of the decay curve for what is deemed to be "obviously" too much or too little background subtraction as judged from the downward or upward trend of the low-count region of the decay curve. In the present work, the error obtained in this fashion is doubled to be on the safe side.

The decay data will, then be presented in the form of double plots—the usual plot of the delayed coincidence curve, and below it, the plot of the lifetimes computed using only the data from a fixed number of channels starting at the indicated channels.

EXPERIMENTAL RESULTS AND DISCUSSION

Prompt Spectrum

Since the three 4+ level lifetimes are measured under almost identical energy conditions, only one prompt spectrum is needed for the 4+ levels, and, due to the fact that the corresponding 2+ level lifetimes are larger than about 0.35 nsec, a prompt spectrum was found to be unnecessary for their interpretation. The prompt spectrum shown in Fig. 2 results from the coincidences between the Bi^{212} alpha to the 5+ second excited level of Tl^{208} at 328 keV and the conversion electrons from this level. The larger energy of these electrons was degraded with Al foils until the average energy ($\sim$100 keV) was about that used for the 4+ lifetimes measured. The energy windows for both prompt and timing spectra were the same, of course. The observed limiting-slope effective half-life of 60 psec proved adequate for present purposes.

All the remaining data will be given in the aforementioned double-plot form. In the decay curve plot, open circles are uncorrected data and darkened circles are background corrected data. Only the corrected data are given error brackets. In the plots of successive lifetimes, the heavy black curve represents the lifetime results for the adopted amount of stripping (if any), and the dotted curves represent the cases where there is "obviously" too much or too little background removed, and have no particular connection with the error brackets shown (which belong to the heavy black curve).

$Th^{228}(2+)$

U^{232}, as well as U^{234}, was electroplated at the Oak Ridge National Laboratory onto the evaporated silver light-reflector coating of the $\frac{1}{16}$-in. thick Pilot B scintillator discs to minimize the energy loss of the approximately 40 keV conversion electrons. The U^{232} activity was about 0.5 μCi and the system alpha-energy resolution was about 45 keV FWHM. The alphas to the 2+ level of Th^{228} were plainly visible though the alphas to the 4+ level were just barely so. The 2+ level electrons were quite obvious but the 4+ level electrons could not be seen without gating with the alphas to the 4+ level. Using the L electrons from the 2+ level of Th^{228}, a decay spectrum as shown in Fig. 3 is accumulated. It should be noted that this decay extends over almost five full decades, and the plot of successive lifetimes below indicates that much the same lifetime is obtained regardless of which portion of the decay is used, though data before about channel 56 or 58 is still being influenced by the tail of the prompt spectrum, and thus should not be used for accurate results. The value of the 2+ level half-life is found to be $(4.02\pm0.10)\times10^{-10}$ sec.

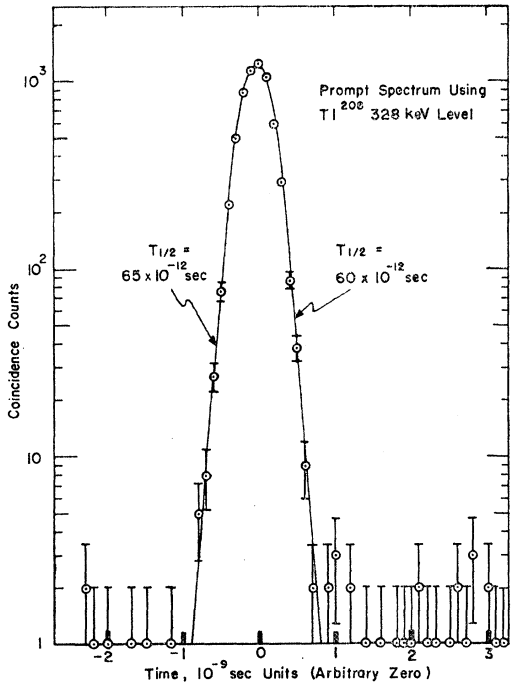

FIG. 2. Prompt coincidence spectrum of the alphas to, and the electrons from, the 328 keV level of Tl^{208}. The electrons' energy was degraded with Al foils until the average energy was approximately that used in the measurement of 4+ level lifetimes.

$Th^{228}(4+)$

Using the L electrons from the 4+ level of Th^{228} several decay spectra were recorded and the sum appears in Fig. 4. It was found that stripping away an extrapolated main curve left a background spectrum with about a 0.8 nsec decay, much like that of the 2+ level of Ra^{224} (due to a small amount of Th^{228} activity which had grown into the source). Using the Ra^{224} spectrum as the background to be stripped out, it may be seen from the plot of successive lifetimes that although no amount of stripping is overly convincing, no appreciable uncertainty is introduced into the earlier values (using the high-count data) of the lifetime. The value of the 4+ level half-life is taken as $(1.61\pm0.05)\times10^{-10}$ sec.

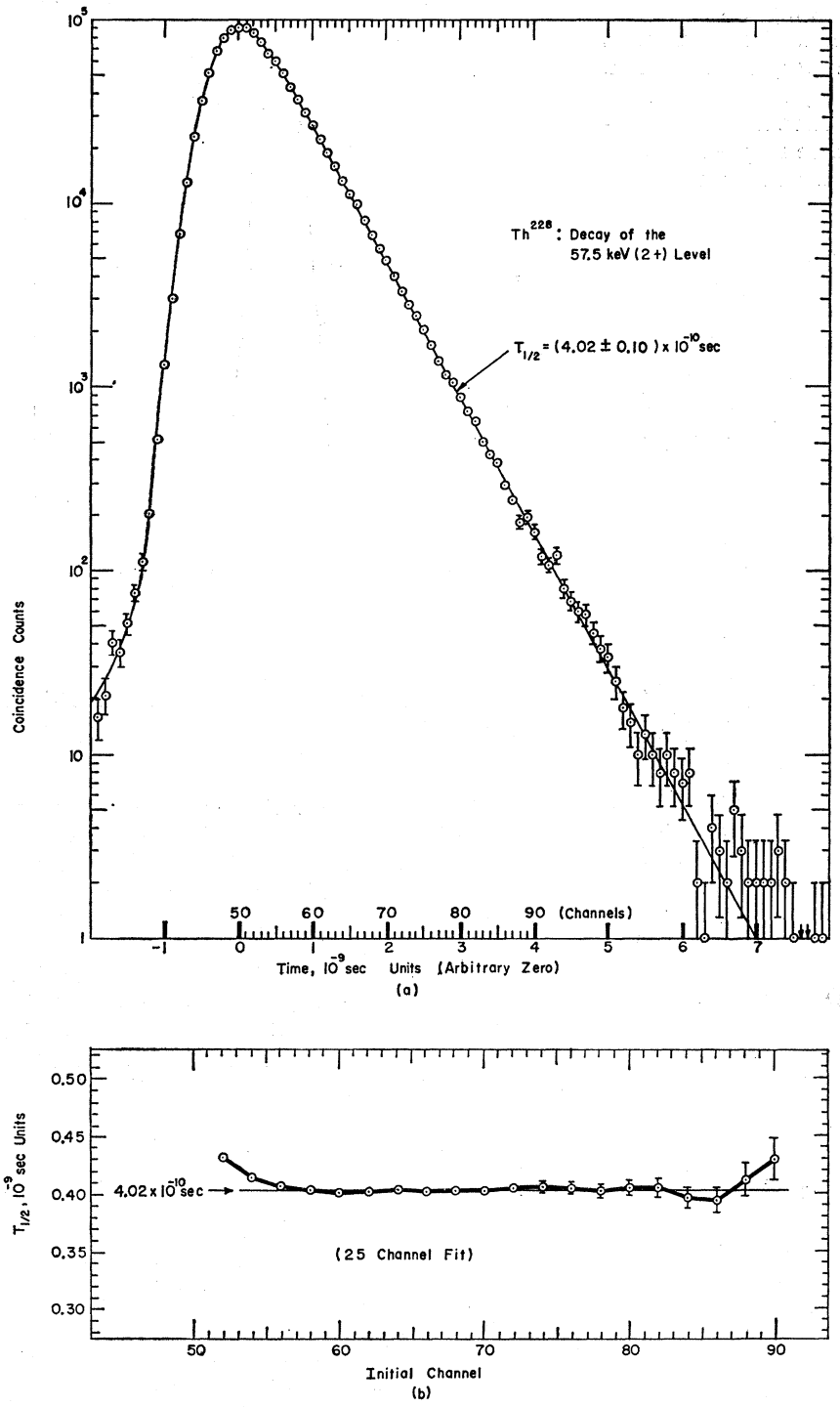

FIG. 3. Delayed coincidence spectrum of the decay of the 57.5 keV (2+) level of Th^{228}.

$Th^{230}(2+)$

The U^{234} source was necessarily thicker and thus of poorer energy resolution due to the 2.5×10^5-year half-life of U^{234} and an activity of 0.17 μCi was thus put on a $\frac{1}{2}$-in. diam Pilot *B* disc. Even then the energy resolution was such that the alphas to the 2+ level of Th^{230} were just barely discernible. Using the *L* electrons of the Th^{230} 2+ level the spectrum shown in Fig. 5 was obtained. The value of the half-life of the 2+ level was found to be $(3.54\pm0.09)\times10^{-10}$ sec.

$Th^{230}(4+)$

Using the *L* electrons of the Th^{230} 4+ level, several 4+ level time spectra were accumulated and the sum

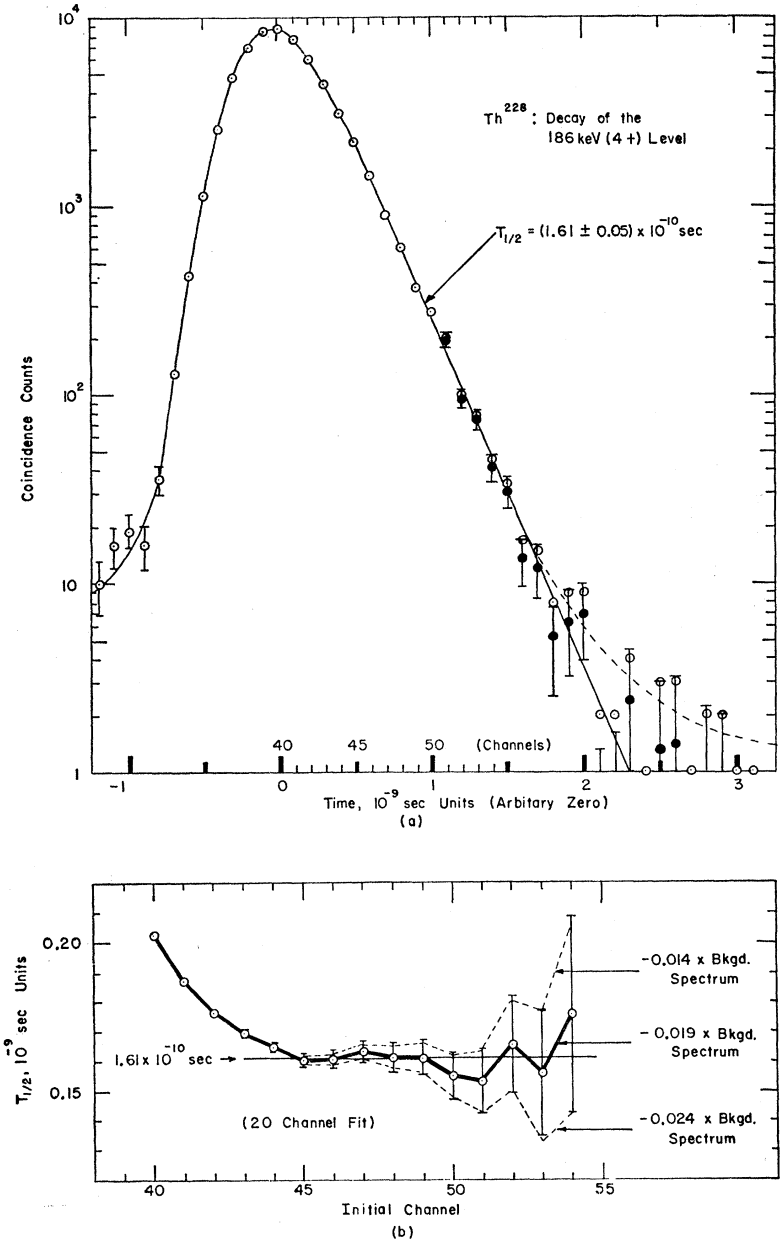

FIG. 4. Delayed coincidence spectrum of the decay of the 186 keV (4+) level of Th^{228}.

of them appears in Fig. 6. The unfortunate occurrence of the statistical fluctuations at the bottom of the decay curve causes the plot of successive lifetimes to ultimately diverge for any reasonable value of background stripping. Again, however, the earlier lifetime values are much more uncertain owing to the number of counts involved than to the background subtraction uncertainty. The value for the half-life of the 4+ level is taken as $(1.66\pm0.05)\times10^{-10}$ sec.

$Ra^{224}(2+)$

The Th^{228} source was made by vacuum evaporating Th^{228} onto $\frac{1}{4}$-mil Mylar foil. Within a week the thorium family had built in, and there were then five additional alpha groups and several β decays to contend with. The alphas to the 2+ level of Ra^{224} could be resolved but the alphas to the 4+ level were outnumbered 10 to 1 by spurious alphas, and the electrons from the 4+ level of Ra^{224} were outnumbered by several hundred to one. Using the L electrons of the 2+ level of Ra^{224}, the spectrum shown in Fig. 7 was obtained. From the plot of successive lifetimes it would appear that there is a prompt component present although it is not evident from the decay spectrum. This slight dip in the plot was found in about 10 similar runs but not in any other nuclide's 2+ level spectrum. The value of the half-life of the 2+ level is found to be $(7.44\pm0.19)\times10^{-10}$ sec.

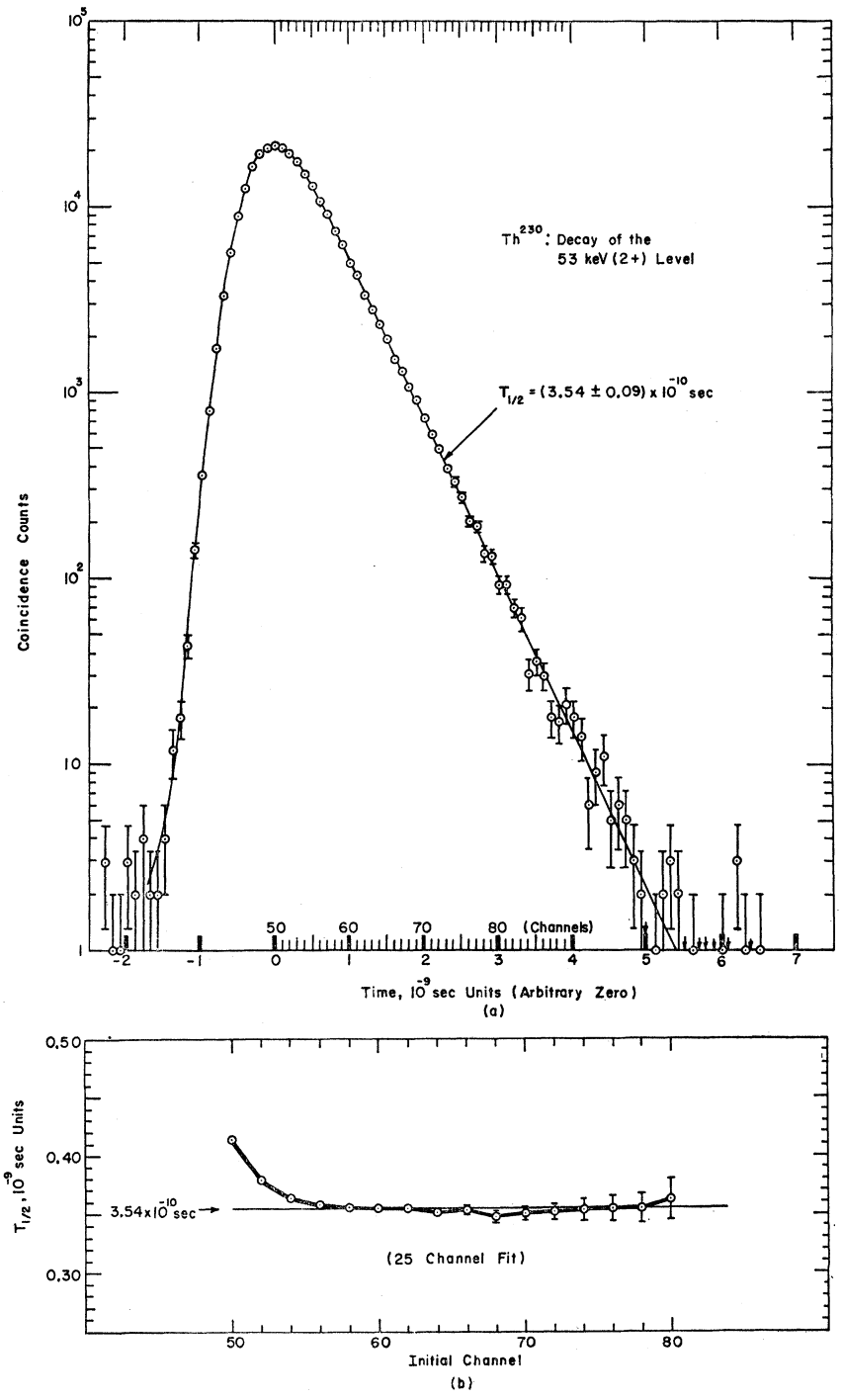

FIG. 5. Delayed coincidence spectrum of the decay of the 53 keV (2+) level of Th^{230}.

Ra^{224}(4+)

Referring to Fig. 8, it may be seen that this is the worst case of background contamination, owing to the previously mentioned unfavorable count rate ratios in the energy discrimination channels. This decay, as were the other 4+ level decays, is a sum of a number of spectra. The contaminating spectrum is clearly that of the Ra^{224} 2+ level decay and this may be stripped out in a fairly convincing fashion. The value adopted for the 4+ level is $(1.81\pm0.09)\times10^{-10}$ sec.

Rn^{220}(2+)

The vibrational nuclide Rn^{220} is the daughter of Ra^{224}, and was in equilibrium with it. The K and L electrons

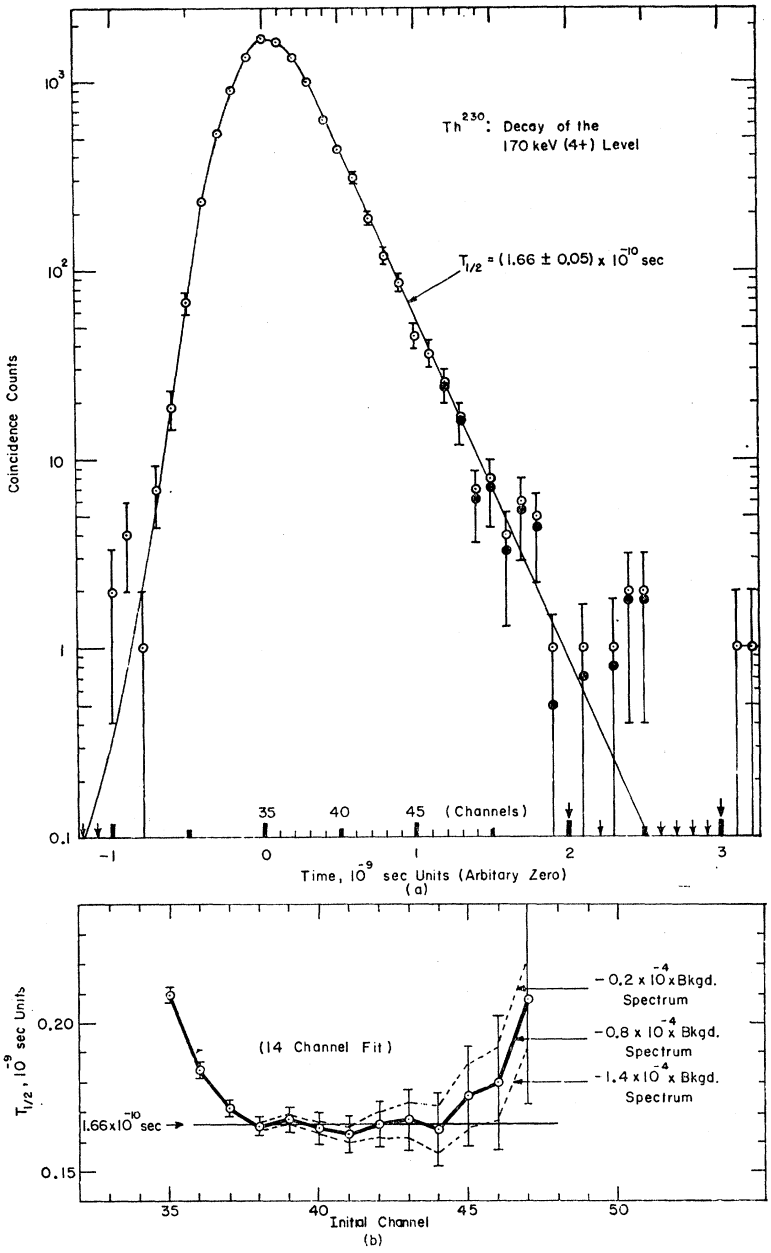

FIG. 6. Delayed coincidence spectrum of the decay of the 170 keV (4+) level of Th^{230}.

of the 2+ level of Rn^{220} were gated with Ra^{224} alphas and then the K electrons were used to obtain the spectrum shown in Fig. 9. The value of the half-life of the 2+ level is found to be $(1.45\pm0.05)\times10^{-10}$ sec.

From Eq. (1) it may be seen that the $B(E2; i\rightarrow f)$ value depends on the energy-dependent factor $f(E)=[1+\alpha_T(E)]E^5$ as well as the half-life $T_{1/2}$. The conversion coefficients were obtained by interpolation of the values of Sliv and Band[18] who claim their absolute values to be accurate to 1 or 2%. Although experimental conversion coefficients exceeding the Sliv and Band values by as much as 10% have been measured, their uncertainties do not preclude the correctness of the theoretical values and in lieu of more definitive data, both the values and suggested error of Sliv and Band will be employed. The $M+N+O$ shells' contribution to the total conversion coefficient was estimated by use of the empirically derived rule[19] of taking it to be one

[18] L. M. Sliv and I. M. Band, *Coefficients of Internal Conversion of Gamma Radiation* (USSR Academy of Sciences, Moscow-Leningrad, 1956 and 1958). [English transl. University of Illinois Reports 57ICCK1 and 58ICCK1 (unpublished)].

[19] E. H. Rester, M. S. Moore, F. E. Durham, and C. M. Class, Nucl. Phys. **22**, 104 (1961); J. O. Rasmussen, H. Slätis, and T. O. Passel, Phys. Rev. **99**, 42 (1955); S. Rosenblum, M. Valadares, and M. Guillot, Compt. Rend. **235**, 238 (1952); S. Rosenblum, M. Valadares, and R. Bernas, *ibid.* **239**, 759 (1954).

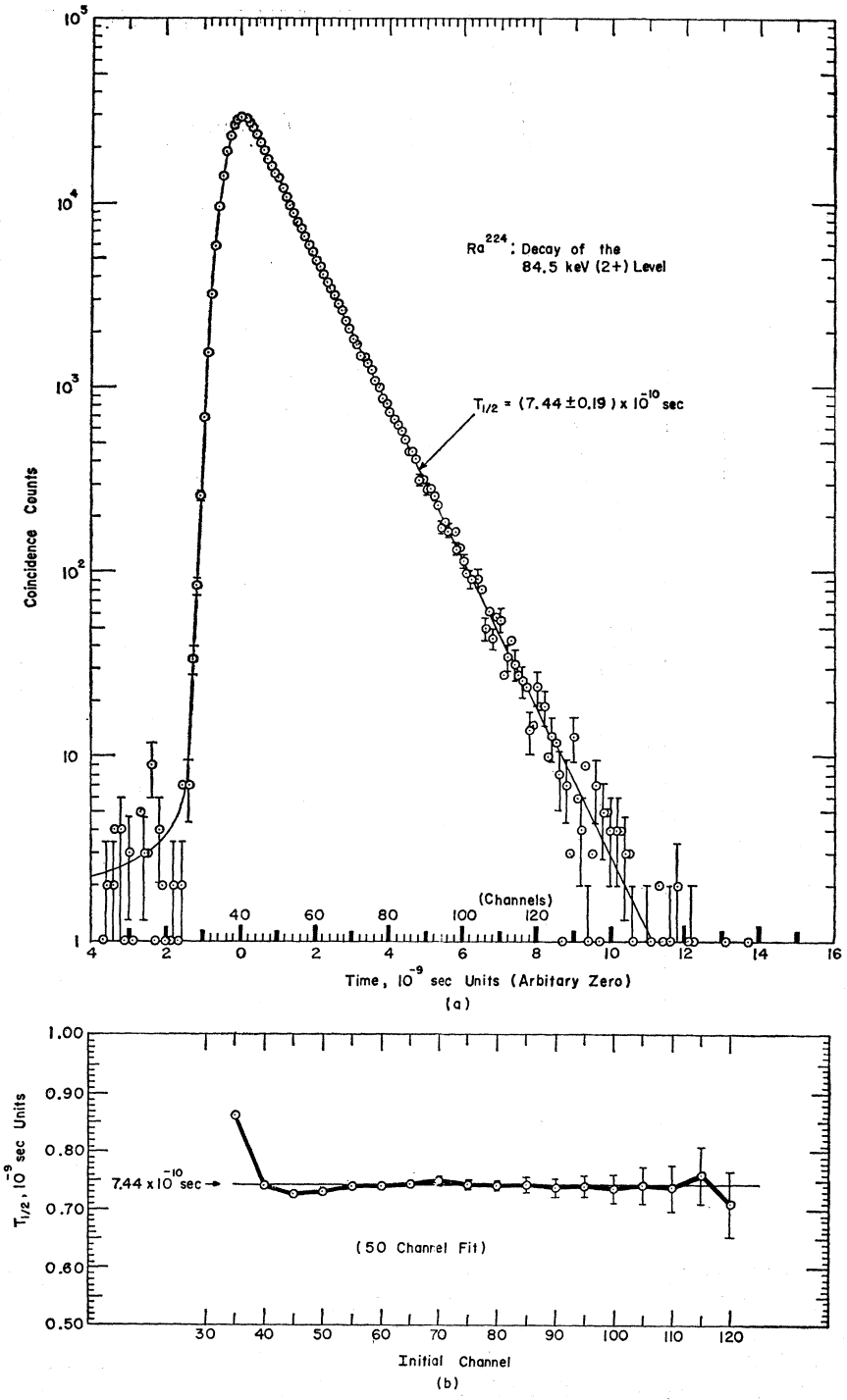

FIG. 7. Delayed coincidence spectrum of the decay of the 84.5 keV (2+) level of Ra^{224}.

third of the L shell contribution. The energy uncertainty dependence of the factor $f(E)$ is most conveniently given as the percent change in $f(E)$ for a percent change in E:$(\Delta f/f)/(\Delta E/E)$. The error in the energies can be taken to be less than $\frac{1}{2}\%$. In Table II, the interpolated values of the various shells' conversion coefficients are given along with the total coefficient, the factor $f(E)$, and the ratios of the percent changes.

Finally, in Table III, the quantities mentioned in

TABLE II. Interpolated values of the internal-conversion coefficients of Sliv and Band.

Nuclide	E_γ (keV)	α_K	α_L	α_{M+N+O}	α_T	$f(E)$	$\frac{\Delta f/f}{\Delta E/E}$
Rn^{220}	241	0.110	0.128	0.0425	0.281	104 $\times 10^{10}$	4.27
Ra^{224}	84.5	0.000	16.0	5.34	21.4	9.64 $\times 10^{10}$	0.54
	168.5	0.216	0.684	0.228	1.13	28.9 $\times 10^{10}$	2.94
Th^{228}	57.5	0.000	120	40	160	10.1 $\times 10^{10}$	0.17
	128.6	0.249	2.75	0.914	3.91	17.3 $\times 10^{10}$	1.55
Th^{230}	52.4	0.000	189	62.8	252	9.98 $\times 10^{10}$	0.15
	117.6	0.249	4.15	1.38	5.78	15.2 $\times 10^{10}$	1.23

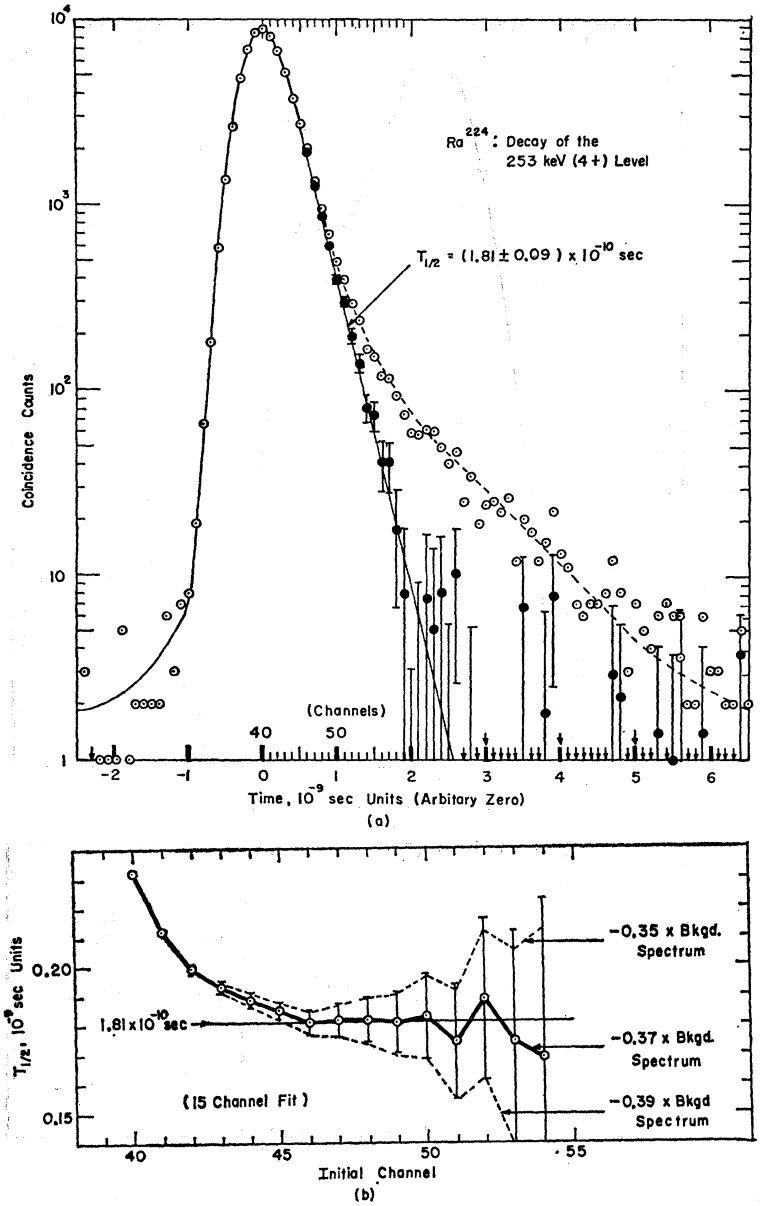

FIG. 8. Delayed coincidence spectrum of the decay of the 253 keV (4+) level of Ra^{224}.

TABLE III. Results of the experiment.

(a) The rotational nuclides Ra^{224}, Th^{228} and Th^{230}

Nuclide	$I\pi$	$\frac{E_{4+}}{E_{2+}}$	E_γ (keV)	α_T	$\frac{f(E)}{(\times 10^{10})} = (1+\alpha_T)\times E^5 \frac{\Delta f/f_0}{\Delta E/E_0}$	Present $T_{1/2}$ (psec)	Bell *et al.* $T_{1/2}$ (psec)	$B(E2; i \to f)$ (10^{-48} e^2-cm^4)	Q_0 (10^{-24} e-cm^2)	β	$\frac{g_0}{g_{rig}}$	$\frac{g_0}{g_{irrot}}$	$\frac{B(E2; 4+ \to 2+)}{B(E2; 2+ \to 0+)}$[a]
Ra^{224}	2+	3.00	84.5	21.4	9.64	0.54	744±19	760±30	0.786±0.021	6.27±0.09	0.166	0.300	12.8	1.37±0.07
	4+		168.5	1.13	28.9	2.94	181± 9		1.08 ±0.06					
Th^{228}	2+	3.23	57.5	160.0	10.1	0.17	402±10	400±30	1.39 ±0.04	8.35±0.12	0.210	0.412	11.1	1.46±0.05
	4+		128.6	3.91	17.3	1.55	161± 5		2.02 ±0.07					
Th^{230}	2+	3.24	52.4	252.0	9.98	0.15	354± 9	370±20	1.60 ±0.04	8.95±0.13	0.222	0.445	10.8	1.40±0.05
	4+		117.6	5.78	15.2	1.23	166± 5		2.24 ±0.08					

(b) The vibrational nuclide Rn^{220}

Nuclide	$I\pi$	E_γ (keV)	α_T	$\frac{f(E)}{(\times 10^{10})} = (1+\alpha_T)\times E^5 \frac{\Delta f/f_0}{\Delta E/E_0}$	Present $T_{1/2}$ (psec)	Bell *et al.* $T_{1/2}$ (psec)	$B(E2; 2+ \to 0+)$ (10^{-48} e^2-cm^4)	$\frac{B_2^{expt}}{B_2^{irrot}}$	C_2^{expt} (MeV)	C_2^{irrot} (MeV)
Rn^{220}	2+	241	0.281	104.0	4.27	145±5	150±10	0.374±0.015	19.3	37.3	37.1

[a] The uncertainties quoted for the $B(E2)$ ratios do not include estimates of the absolute uncertainties in the theoretical internal conversion coefficients employed.

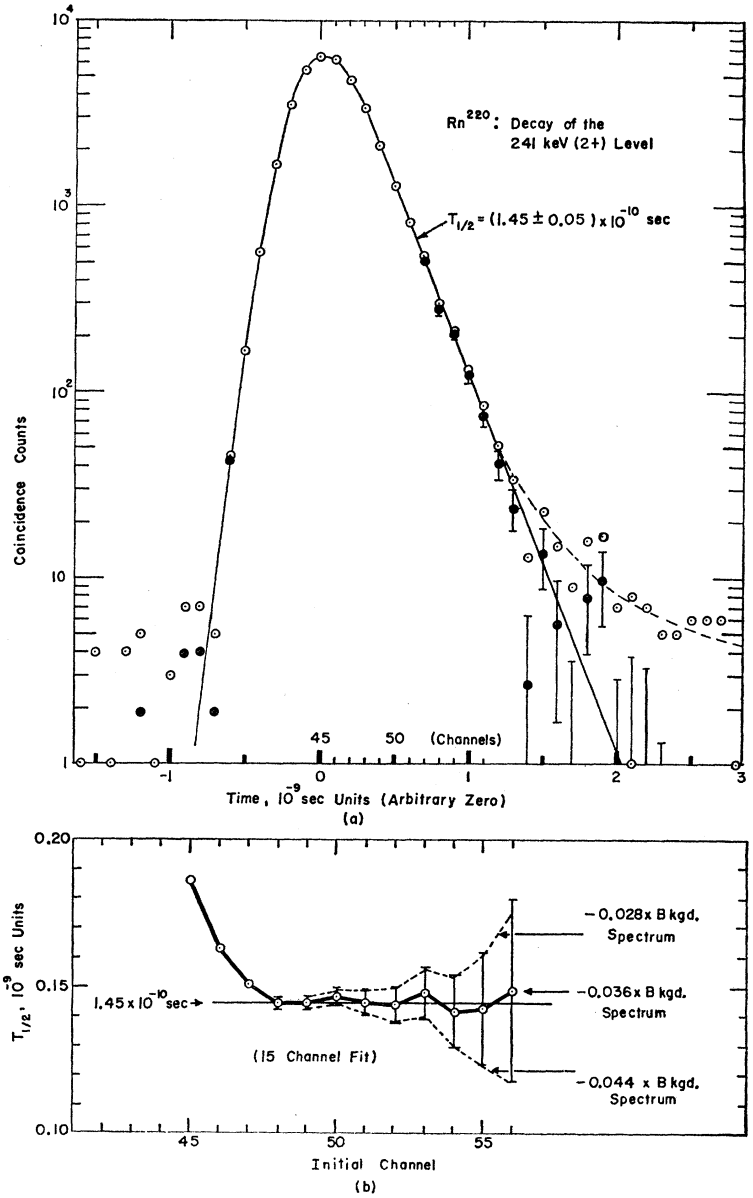

FIG. 9. Delayed coincidence spectrum of the decay of the 241 keV (2+) level of Rn^{220}.

the first section are evaluated for the different nuclides. In treating the errors of quantities involving conversion coefficients, only energy uncertainty errors (and no estimated absolute errors) of the coefficients were included. It may be seen that the experimental ratio of the reduced transition probabilities is rather close to the theoretical value of $10/7(\cong 1.43)$ and that the present values of the half-lives of 2+ levels agree rather well with those of Bell *et al.*[20].

[20] R. E. Bell, S. Bjørnholm, and J. C. Severiens, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **32**, No. 12 (1960).

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