where

$$\Delta E = 2q^2 - 2qk_\alpha t_\alpha + 2qk_j t_j + \Delta_0,$$

and

$$\Delta_0 = \Delta(2M^*/\hbar^2).$$

We perform the integration over t_α,

$$-1 < t_\alpha < T_\alpha, \quad \text{where} \quad T_\alpha = (k_F^2 - k_\alpha^2 - q^2)/2qk_\alpha < 1,$$

to get

$$C(k_\alpha) = \frac{C}{k_\alpha} \int K^2(q) \left(\frac{1}{2q^2 - 2qk_\alpha T_\alpha + 2qk_j t_j + \Delta_0} - \frac{1}{2q^2 + 2qk_\alpha + 2qk_j t_j + \Delta_0} \right) qdqk_j^2 dk_j dt_j. \tag{B4}$$

Next the integration over t_j is performed,

$$T_j < t_j < 1, \quad T_j = (k_F^2 - k_j^2 - q^2)/2qk_j < -1,$$

to get

$$C(k_\alpha) \quad \frac{C}{k_\alpha} \int_{q=k_F-k_\alpha}^{\infty} \int_{k_j=k_F-q}^{k_F} K^2(q) \ln \left| \frac{(2q^2 - 2qk_\alpha T_\alpha + 2qk_j + \Delta_0)(2q^2 + 2qk_\alpha + 2qk_j T_j + \Delta_0)}{(2q^2 - 2qk_\alpha T_\alpha + 2qk_j T_j + \Delta_0)(2q^2 + 2qk_\alpha + 2qk_j + \Delta_0)} \right| qdqk_j dk_j. \tag{B5}$$

If $K(q)$ is given by (A8) statistical factors, etc., now gives $C = (3/2\pi^2)(2M^*/\hbar^2)$. The exchange term was omitted.

The numerical computations were performed on the CDC-1604 computer of the Computer Center at the University of California at San Diego in La Jolla.

The integrations were all made by Simpson's rule, successively dividing the integration intervals by two until the integral changed less than a specified amount.

We calculated, unless otherwise specified, with a Fermi momentum $k_F = 1.4\ \mathrm{F}^{-1}$.

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Spin, Hyperfine Structure, and Nuclear Magnetic Dipole Moment of 23-sec Na^{21}†

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The spin and hyperfine structure of 23-sec Na^{21}, the mirror nucleus to Ne^{21}, have been measured by the atomic-beam magnetic-resonance technique. The Na^{21} was produced by the reaction $Mg^{24}(p,\alpha)\,Na^{21}$. The target material in the form of a powder was bombarded in an oven in the resonance apparatus. If the oven was heated to about 450°C, the Na^{21} escaped from the Mg in sufficient quantity to make a useful atomic beam. Both $\Delta F = 0$ and $\Delta F = 1$ resonances were observed, and the final values are $I = \frac{3}{2}$, $\Delta\nu = 1906.466 \pm 0.021$ Mc/sec. Comparison with Na^{23} yields $\mu_I = 2.386\,12 \pm 0.000\,10$ nm (diamagnetically corrected). The results are discussed in terms of the current nuclear theories.

I. INTRODUCTION

In the last several years much experimental effort has been made both at this laboratory and at others to measure the magnetic moments of radioactive mirror nuclei. The incentive for this effort lies partly with the hope that knowledge of the magnetic dipole moments of both members of a mirror pair will yield information on mesonic currents in these nuclei.[1,2] The success of such a program depends on our being able to use the sum of the moments and other relevant experimental data to obtain a good wave function for the nuclear ground state. By good we mean precise enough so that a discrepancy of the order of 0.1 nm between the individual experimental moments and the theoretical values obtained from this wave function can be attributed to mesonic effects. The magnetic-moment operator used to calculate the moments from the nuclear wave function is

$$\boldsymbol{\mu}_{\mathrm{op}} = \sum_{\substack{\text{all} \\ \text{nucleons}}} (g_l \mathbf{l} + g_s \mathbf{s}),$$

where the g factors are those for the free nucleons. Thus the measurable mesonic effects will include quenching[3] of the g factors in addition to exchange currents. It

† This work was supported by the U. S. Atomic Energy Commission and the Higgins Scientific Trust Fund.

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[1] R. G. Sachs, *Nuclear Theory*, (Addison-Wesley Publishing Company, Inc., Cambridge, Massachusetts, 1953).

[2] R. J. Blin-Stoyle, *Theories of Nuclear Moments* (Oxford University Press, London, 1957).

[3] S. D. Drell and J. D. Walecka, Phys. Rev. **120**, 1069 (1960).

TABLE I. A summary of spin and moment data for all cases where μ_I has been measured for both members of the mirror pair.

Parent	Daughter	$\tau_{1/2}$	Parent and daughter spin and parity	Parent μ_I	Daughter μ_I	Parent Q (barns)	Daughter Q
n	p	13 min.	1/2+	−1.91314(5)	+2.79290(6)		
H^3	He^3	12 yr.	1/2+	+2.97885(1)	−2.12755		
C^{11}	B^{11}	20 min.	3/2−	(−)1.027(10)[a]	+2.68858	(+)0.0308(6)[a]	+0.036
N^{13}	C^{13}	10 min.	1/2−	(−)0.32212(35)[b]	+0.70238		
O^{15}	N^{15}	2 min.	1/2−	(+)0.7189(8)[c]	−0.28309		
Ne^{19}	F^{19}	18.5 sec.	1/2+	−1.887(1)[d]	+2.6287		
Na^{21}	Ne^{21}	23 sec.	3/2+	+2.38612(10)[e]	−0.661757(5)		+0.093(10)

[a] R. A. Haberstroh, W. J. Kossler, O. Ames, and D. R. Hamilton, Phys. Rev. **136**, B932 (1964).
[b] A. M. Bernstein, R. A. Haberstroh, D. R. Hamilton, M. Posner, and J. L. Snider, Phys. Rev. **136**, B27 (1964).
[c] E. D. Commins and H. R. Feldman, Phys. Rev. **131**, 700 (1963).
[d] E. D. Commins and D. A. Dobson, Phys. Rev. Letters **10**, 347 (1963); Lawrence Radiation Laboratory Report No. UCRL 11169 (unpublished).
[e] This work.

may also be important to include a modification of the proton single-particle moment due to the spin-orbit force[4] in both the calculation involving the sum of the moments and that used to obtain the individual moments. A more detailed discussion of these effects can be found in Refs. 1–4 and also in the experimental papers referred to in Table I.[5] In this table we summarize the results for all cases where both moments have been measured.

At this time it appears that the possibility of extracting information on meson currents from the data is still beyond the accuracy of nuclear theory. It is not yet possible, in all but a very few cases,[6] to determine the nuclear wave function well enough so that we can draw conclusions about the form of the magnetic moment operator. Nevertheless, the light mirror nuclei which fall in the nuclear $1p$ and $2s$-$1d$ shells are of particular current interest and many calculations are being carried out in this region.

In this paper we shall report on work in which the nuclear spin and magnetic-dipole moment were measured in 23-sec Na^{21}, the mirror of stable Ne^{21}. This is a particularly interesting region for investigation because there is striking evidence that nuclei in this part of the $2s$-$1d$ shell are very precisely described by the collective model of Bohr and Mottelson.[7]

We have constructed a new atomic-beam magnetic-resonance apparatus for doing experiments on short-half-life nuclei, and in Sec. III we shall describe the machine and the experimental details. In the next section we present the relevant hyperfine structure theory, and in Sec. IV the results are given. Finally we shall discuss the results in the light of the present theory.

[4] J. H. D. Jensen and M. G. Mayer, Phys. Rev. **85**, 1040 (1952).
[5] The entries in this table not otherwise referred to were taken from a table prepared by I. Lindgren and now in E. Karlsson, E. Matthias, and K. Siegbahn, *Perturbed Angular Correlations* (North-Holland Publishing Company, Amsterdam, 1964).
[6] An interesting discussion of this subject is given by A. de-Shalit in *Rendiconti della Scuola Internazionale Di Fisica "Enrico Fermi," XXIII Corso: Fisica Nucleare* (Academic Press Inc., New York, 1963).
[7] See, for example, the review article by H. E. Gove in *Proceedings of the International Conference on Nuclear Structure, Kingston, Ontario, 1960*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1960), p. 438.

II. RELEVANT HYPERFINE STRUCTURE THEORY

The theory is particularly simple for the case of I or $J=1/2$ since then the quadrupole interaction vanishes. For this case we have[8]

$$\mathcal{H}=ha\mathbf{I}\cdot\mathbf{J}-(\mu_J/J)\mathbf{J}\cdot\mathbf{H}_0-(\mu_I/I)\mathbf{I}\cdot\mathbf{H}_0. \quad (1)$$

The first term is the magnetic dipole interaction in the free atom, the second and third terms represent the interaction of the electrons and the nucleus, respectively with an external magnetic field H_0. For very weak external fields the Hamiltonian is very nearly diagonal in the representation $|F,M_F\rangle$, where $\mathbf{F}=\mathbf{I}+\mathbf{J}$ and M_F is the projection of $\mathbf{F}$ on the direction of $\mathbf{H}_0$. For stronger fields the solution of the secular equation leads to the well known Breit-Rabi formula. An energy level diagram for the case of Na^{21} where $I=3/2$, $J=1/2$, and $\mu_I>0$ is shown in Fig. 1. The $\Delta F=0$ transition observed in the early phase of the experiment is shown by the arrow. Observation of this resonance in very weak fields leads immediately to knowledge of the nuclear spin as the resonant frequency is given by

$$\nu\approx\frac{g_J\mu_0H_0}{2Fh}. \quad (2)$$

At higher values of the static field, the resonant frequency depends upon the quantity $\Delta\nu$, where $\Delta\nu\equiv[E(I+1/2)-E(I-1/2)]/h=a(I+1/2)$. Observation of $\Delta F=0$ resonances at increasingly larger values of H_0 will yield a fairly precise value for the hyperfine structure splitting $\Delta\nu$ and will also yield the sign of the nuclear moment. It is then often an easy matter to find

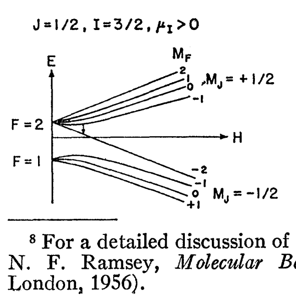

FIG. 1. Hyperfine energy-level diagram for $J=1/2$, $I=3/2$, and $\mu_I>0$. The arrow shows the $\Delta F=0$ transition observed in the earlier part of this work.

[8] For a detailed discussion of the hyperfine structure theory see N. F. Ramsey, *Molecular Beams* (Oxford University Press, London, 1956).

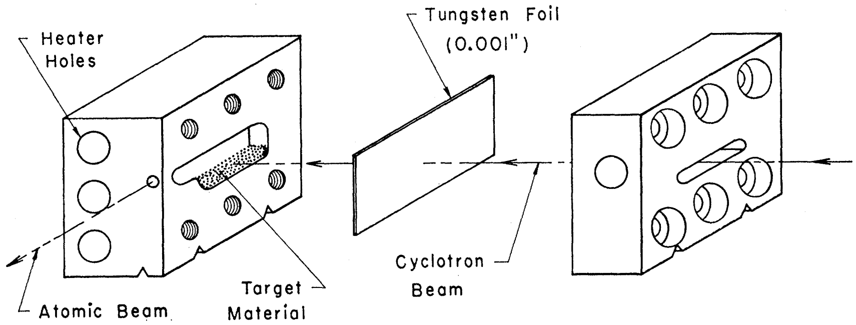

FIG. 2. Exploded view of the oven. The oven is made of stainless steel.

the direct $\Delta F=1$ transitions and obtain highly precise values for $\Delta\nu$.

To extract the nuclear magnetic-dipole moment from $\Delta\nu$ requires a knowledge of the electron wave function at the nucleus. However, if one has $\Delta\nu$ and μ_I for another isotope of the same element the formula of Fermi and Segrè can be used;

$$\frac{\Delta\nu_1}{\Delta\nu_2}=\frac{\mu_{I1}}{\mu_{I2}}\frac{I_2(2I_1+1)}{I_1(2I_2+1)}. \tag{3}$$

The magnetic moment calculated from this expression is generally accurate to about a part in 10^3, the cause of uncertainty being hyperfine-structure anomalies. We shall discuss this further in Sec. IV.

A large part of the data reduction in this experiment was carried out with the aid of the Berkeley programs "Hyperfine 3-9" and "Hyperfine 4."[9]

III. THE EXPERIMENTAL DETAILS

The experiment was carried out using a new atomic beam apparatus specifically designed for doing experiments with short-lived radioactive isotopes. The machine is portable, and can be moved into position on an external beam tube of the Princeton cyclotron. The cyclotron beam enters the oven chamber, passes through a 1-mil tungsten foil, and falls on the target material in the oven. An exploded view of the oven is shown in Fig. 2. In this experiment the target material was magnesium powder, the Na^{21} being produced by the reaction $Mg^{24}(p,\alpha)Na^{21}$. A proton beam energy of about 18 MeV was used and a typical beam intensity was 0.075 μA.

Several yield experiments were carried out by bombarding the Mg powder and then looking at the annihilation radiation from the β^+ decay of Na^{21} with a NaI crystal. These experiments showed that a 0.075 μA beam would produce about 6×10^7 Na^{21} atoms per sec under equilibrium conditions.

The oven was generally run at a temperature of about 450°C. The melting point of Mg is 651°C; however, at temperatures much above 450°C the vapor pressure of Mg becomes too high and we risk depleting the charge in too short a time. Also at these temperatures the Mg tends to grow into crystals. This increases the volume-to-surface ratio and may make it more difficult for the Na to get out. In any case, favorable conditions did not seem to occur much over 450°C. The Na^{21}-beam intensity was measured by inserting a copper "flag" directly in front of the oven hole. The Mg vapor issuing from the hole condensed on the copper and held the Na^{21} on with it. The flag was pulled out after a short exposure and counted. The production rate of 6×10^7 per sec in the oven was associated with a total beam $\approx1.4\times10^7$ Na^{21} atoms per sec leaving the oven hole.

The magnet system in this machine is almost identical to that used in the machine employed for the N^{13} and C^{11} experiments previously done at this laboratory, and a detailed discussion of atom optics is given in the paper on the N^{13} work. A brief discussion will therefore suffice. The A magnet is a 6-pole focusing magnet while the B magnet is a two-pole magnet designed to have a constant gradient across its gap. The A and B magnets are in their own separate vacuum cans. The C magnet is external to the vacuum system and is of the Goodman type.[10]

A novel feature of the machine is the detection technique (Fig. 3). Two collectors are used and will be referred to in what follows as the "flop" and "beam"

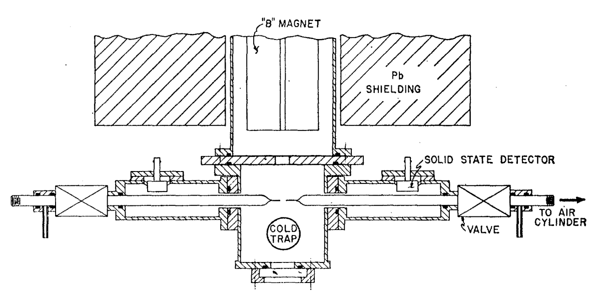

FIG. 3. The detector end of the apparatus, top view. The hot wire detector for stable K and Na beams is located in the B magnet chamber just at the B magnet exit and is not shown in the figure.

[9] We wish to thank Professor Howard Shugart for making the Berkeley programs available to us.

[10] L. S. Goodman, Rev. Sci. Instr. 31, 1351 (1960).

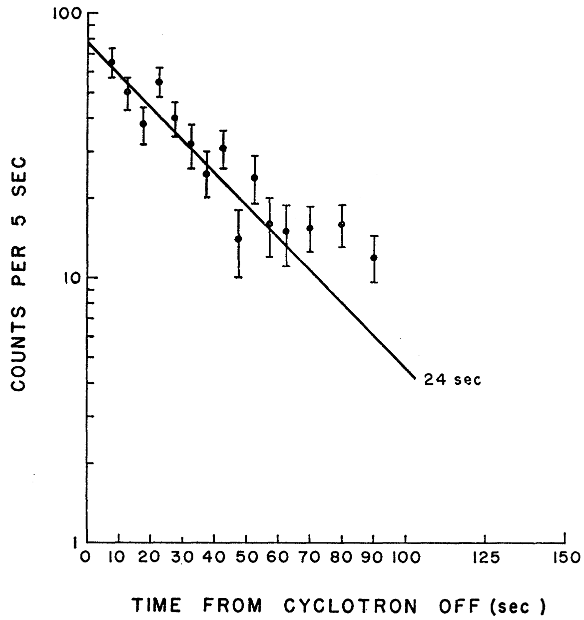

FIG. 4. Decay of beam activity.

collectors. The beam collector is positioned on the weak field side of the B magnet gap and collects a large fraction of the transmitted beam. The flop collector is positioned on the strong field side so that it only should pick up beam when a resonance is induced. The collectors are mounted on the ends of long rods which can be moved by remotely actuated air cylinders. If the rods are retracted 6 in., the collectors are positioned in front of two solid-state detectors mounted in the vacuum system. These are silicon surface-barrier detectors, $\frac{3}{4}$ in. in diam, which are biased to stop 230-keV electrons.

The running procedure is then as follows: The cyclotron beam is turned on for 2 min; during this time the atomic beam is being collected. The cyclotron beam is then turned off, the rods are retracted, and a 1-min count is taken. The rods are then pushed in and the whole cycle repeated.

The 1-min count yields about 140 counts above background on the beam detector and about 20 counts above background on the flop detector. On resonance the flop count increases and the beam count drops so that the ratio flop/beam changes from about 0.14 to 0.50.

The cyclotron beam is off during the counting part of the cycle; however, the background count is still high, amounting to 40 counts per minute in each counter. Part of this comes from the activity in the oven, hence the lead shielding shown in Fig. 3. Part also appears to come from the silicon wafers themselves which are activated by fast neutrons while the beam is on. The reaction may be

$$Si^{28}(n,p)Al^{28} \xrightarrow[\beta^-]{2.3\ \mathrm{min}} Si^{28}.$$

The counters were too close to the target to allow effective shielding against the neutrons.

Various collector surfaces were tried; these included copper which was cleaned in HNO_3, rinsed, and then stored in alcohol until use, and also a copper surface which was prepared by making a Cu-Hg amalgam and then allowing the Hg to pump off in the vacuum. Neither of these surfaces appeared to have an efficiency greater than 10%. A third method of preparing a copper surface, suggested to us by L. S. Goodman, was highly successful. In this method clean iron was dipped in a saturated $CuSO_4$ solution for about 30 sec. The resulting surface appeared quite rough and porous. The surface was rinsed in distilled water, stored in alcohol, and placed into the vacuum system while still wet. This last step appears unnecessary, as several collectors which were allowed to dry in air and then inserted seemed to work perfectly well. The collectors

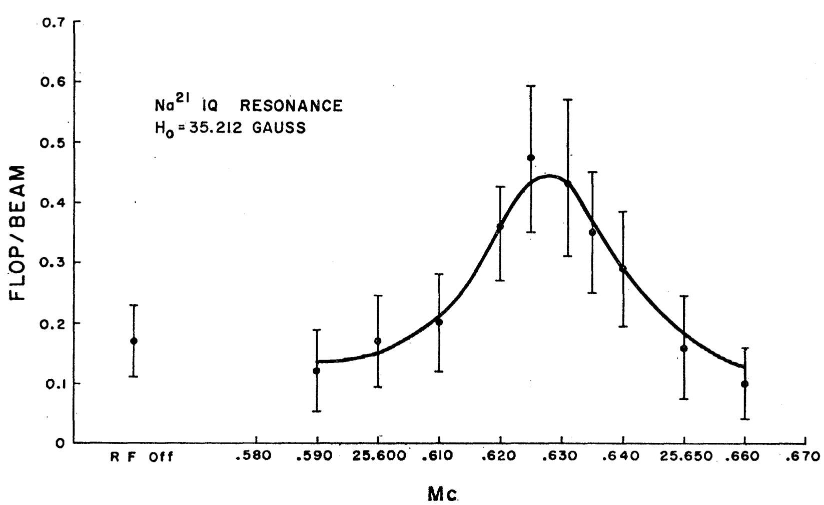

FIG. 5. A typical $\Delta F=0$ resonance taken at $H_0=35.212$ G.

TABLE II. Summary of Na^{21} $^2S_{1/2}$ data.

K^{39} calibration frequency (Mc/sec)	Magnetic field (G)	Na^{21} frequency (Mc/sec)	$F_1M_1F_2M_2$	Residual (kc/sec)
4.700(3)	6.513(4)	4.590(25)	2 −1 2 −2	0
7.572(5)	10.310(7)	7.305(20)		7
7.875(10)	10.703(13)	7.578(15)		−1
10.950(5)	14.615(6)	10.398(10)		4
15.466(7)	20.121(8)	14.404(13)		7
21.295(3)	26.847(3)	19.355(20)		1
29.048(2)	35.212(2)	25.627(5)		4
49.740(4)	54.963(4)	40.891(8)		0
49.756(3)	54.977(3)	40.902(3)		0
0.685(15)	0.974(21)	1905.760(50)	2 −1 1 0 or 2 0 1 −1	−27
0.725(10)	1.030(14)	1905.750(40)		3
0.973(10)	1.380(14)	1905.480(60)		−24
0.973(10)	1.380(14)	1907.460(40)	2 1 1 0 or 2 0 1 1	24
0.973(10)	1.380(14)	1909.370(40)	2 2 1 1	1

Result of least-squares fit: $A=953.2332\pm0.0106$; $\chi^2=2.3$

Comparing isotope	Calibrating isotope
Na^{23}, $^2S_{1/2}$, $I=3/2$	K^{39}, $^2S_{1/2}$, $I=3/2$
$g_J=-2.002309$	$g_J=-2.002310$
$g_I=8.05148\times10^{-4}$	$g_I=1.42111\times10^{-4}$
$A=885.8155$ Mc/sec	$\Delta\nu=461.7197$ Mc/sec

could then be used for many hours without a decrease in collection efficiency. Other surfaces which seemed to work equally well were clean iron and lampblack. Platinum was tried with no success.

IV. RESULTS

Figure 4 shows the decay of the beam activity. A single large collector surface which covered the whole *B* magnet exit aperture was used. After about 90 sec the activity tails off into a long-lived background. The yield experiments described in the previous section showed only two activities with half lives less than many minutes. The predominant activity was 23 sec. A small 7-sec component was seen also; this could have been Al^{25} or Al^{26}.

A. $\Delta F=0$ Data

$\Delta F=0$ resonances $(F,M_F)=(2,-1)\rightarrow(2,-2)$ were observed at many values of the static field from 1 to 55 G. The static field was calibrated using a beam of K^{39} and the amplitude of the rf transition-inducing field was set to be optimum for the above single-quantum transition. This optimum amplitude could be determined by observing the power dependence of the same resonance in K^{39} and by making use of the fact that for this particular pair of isotopes the ratio of optimum transition amplitudes should be just the ratio of the most probable velocities in the *C* magnet.

As described in Sec. III, the data are obtained by subtracting background from the "flop" and "beam" counts and forming the ratio flop/beam; this method makes beam fluctuations of no concern.

The low-frequency runs showed the nuclear spin of Na^{21} to be 3/2, the same as that of its mirror nucleus Ne^{21}.

Figure 5 shows a typical resonance obtained at a higher value of the static field. These higher frequency

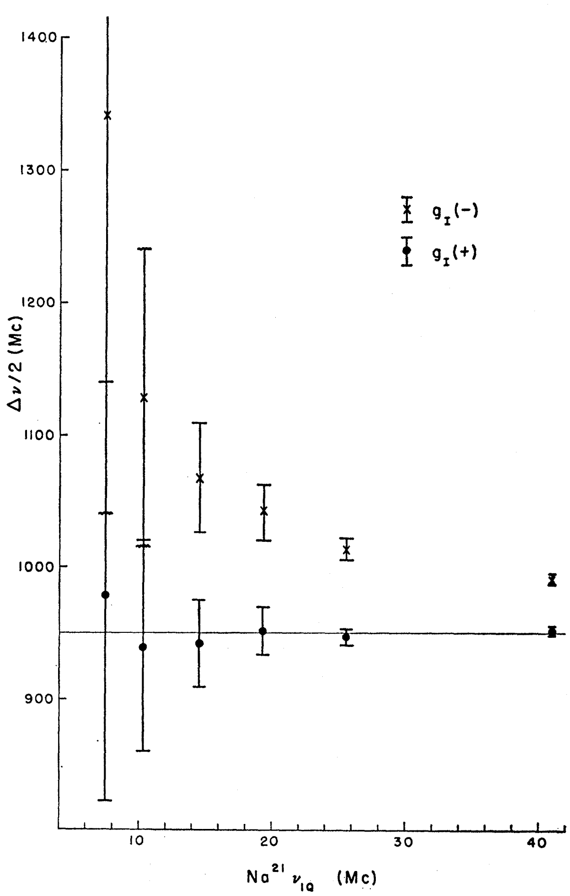

FIG. 6. A summary of the $\Delta F=0$ data. For each frequency $a(=\Delta\nu/2)$ was calculated assuming each sign for g_I.

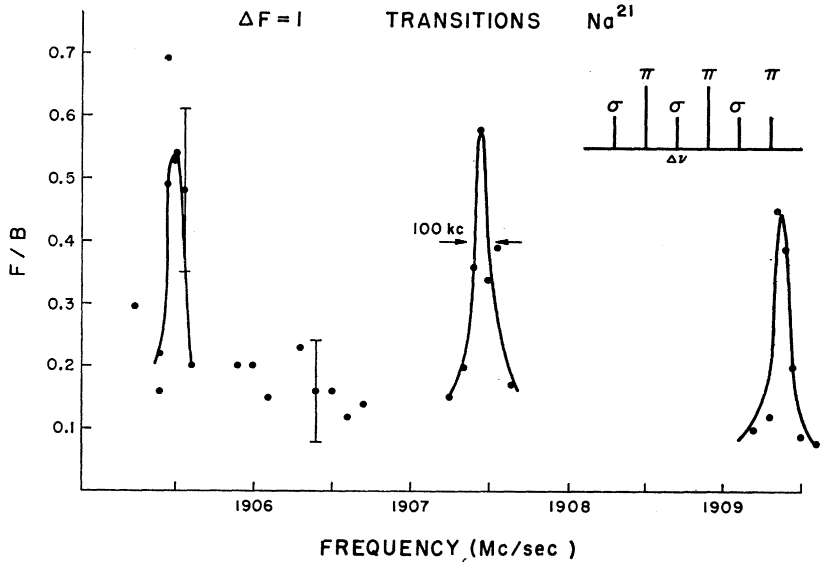

FIG. 7. Three observable π resonances at a field $H_0=1.38$ G.

results are summarized in Table II and Fig. 6. In Fig. 6 we have plotted the values of a $(a=\Delta\nu/2)$ obtained from each resonance as a function of the frequency at which the resonance occurred. For each resonance a value of $\Delta\nu$ is computed for each sign of the moment. We see that the data are internally consistent only if we take $\mu_I>0$. This, of course, is what is expected for Na^{21}. A best fit to all the $\Delta F=0$ data yields the value $\Delta\nu=1905.7\pm2.3$ Mc/sec.

B. $\Delta F=1$ Data

With the above results as a guide a search was carried out for the $\Delta F=1$ transitions in which $\Delta M_F=\pm1$. A hairpin-type loop was used and this gave linewidths of about 120 kc/sec, making the search problem an easy one. The rf equipment and the frequency-measuring procedures were checked by looking at similar transitions in K^{39} $(\Delta\nu=461.7197$ Mc/sec) and in Na^{23} $(\Delta\nu=1771.631$ Mc/sec). Transitions in Na^{23} were also used to determine the optimum rf amplitude for inducing the $\Delta F=1$ transitions in Na^{21}.

The three observable "π" resonances are shown in Fig. 7 and all the data are summarized in Table II. The lowest frequency π resonance was seen at three values of the static field, thus further confirming the identification of these resonances. The best fit to these data yields the value $\Delta\nu=1906.466\pm0.021$ Mc/sec.

C. Calculation of μ_I

To calculate μ_I from $\Delta\nu$ we have used Eq. 3 and have used Na^{23} as the comparison isotope. The best value (diamagnetically corrected) for the Na^{23} magnetic dipole moment is 2.21752 ± 0.00008 nm,[11] while the best value for $\Delta\nu$ is 1771.631 ± 0.002 Mc/sec.[12]

The major uncertainty in converting a value of $\Delta\nu$ into a value for the magnetic moment is often the hyperfine structure anomaly. It is known to be very large for[13] K and[14] Rb; $a/g_I=[2I/(2I+1)](\Delta\nu/\mu)$ may vary as much as 0.4% from one isotope to another.

This anomaly is caused by differences in the distribution of charge and magnetization in the two nuclei. The charge effect (Breit-Rosenthal effect[15]) is due to the nuclear size; assuming $R=1.2\times10^{-13}A^{1/3}$ we compute $\Delta_{BR}=[(a/g_I)_{23}-(a/g_I)_{21}]/(a/g_I)=-4.4\times10^{-5}$. The magnetization effect (Bohr-Weisskopf effect[14]) depends on the structure of the nuclei. To estimate this we must assume a wave function for each nucleus which fits its magnetic moment.

Calculations have been carried out in two ways: (1) Assuming a shell model configuration $(d5/2)^3$ coupled to $I=3/2$ for the protons.[16] Since this configuration has a magnetic moment of 2.875 nm, we have assumed sufficient quenching of the proton g factors to bring the moments for $A=21$ and $A=23$ into agreement with the experimental data. (2) Using Nilsson's model,[17] level 7. The deformation parameter can be

[11] See table referred to in Ref. 5 and also G. Laukien, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1958), Vol. 38/1, p. 338.
[12] R. A. Logan and P. Kusch, Phys. Rev. **81**, 280 (1951).
[13] J. T. Eisinger, B. Bederson, and B. T. Feld, Phys. Rev. **86**, 73 (1952).
[14] A. Bohr and V. Weisskopf, Phys. Rev. **77**, 94 (1950).
[15] M. F. Crawford and A. L. Schawlow, Phys. Rev. **76**, 1310 (1949).
[16] M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley & Sons, Inc., New York, 1955), p. 245.
[17] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **29**, No. 16 (1955).

obtained either from the $A=21$ magnetic moments or from the quadrupole moment of Ne^{21}.

For these cases the predicted correction, Δ_{BW}, falls between -2 and -4×10^{-5}. When a reduced mass correction of $+0.7\times10^{-5}$ is added to the above, we predict a total anomaly of -6 to -8×10^{-5}.

Using this result, we obtain

$$\mu_I(Na^{21})=+2.38612\pm0.000\,10\text{ nm}.$$

The largest source of error is in the moment of Na^{23}. This moment is based on a proton moment equal to 2.79274 nm; recent values for this standard are slightly higher.[11]

V. DISCUSSION

In terms of a simple shell-model picture for the ground state of the $A=21$ system we can consider three $d_{5/2}$ protons coupling to a spin of 3/2 for Na^{21} and the equivalent case for the neutrons in Ne^{21}. This is clearly an approximation, as all 5 nucleons in the shell should be treated on an equal footing. The magnetic dipole moment is easy to calculate for this case and is given by

$$\mu_I=(\mu_S/j)\sum m_j\,, \tag{4}$$

where μ_S is the Schmidt moment for a $d_{5/2}$ nucleon, $j=5/2$, and the sum is carried out over the odd particles.[16] The resulting values which might be called the Schmidt moments for this particular coupling scheme are shown in column 1 of Table III.

As we pointed out earlier, the collective model has been extremely successful in the *sd* shell,[7] and it is therefore interesting to see what predictions the model gives for the $A=21$ pair. Nilsson[17] has shown that for $\Omega\neq1/2$ we may write the magnetic moment as

$$\mu=I/I+1\{(g_s-g_l)\tfrac{1}{2}[\textstyle\sum_l a_{l,\Omega-1/2}{}^2 - \sum_l a_{l,\Omega+1/2}{}^2]+g_lI+g_R\}\,, \tag{5}$$

where the a's are the expansion coefficients of the particle wave function in terms of Nilsson's basic set

$$\chi_\Omega{}^N=\textstyle\sum_{l\Lambda} a_{l\Lambda}|Nl\Lambda\Sigma\rangle.$$

We take g_R to be $Z/A=0.5$ in this calculation. The quantity in the square brackets can now be found in either of two ways. We can use the quadrupole moment[18] of Ne^{21} to give us the deformation parameter of the model, or we can use the sum of the moments in the spirit of the Sachs theorem.[1] The former method requires that we estimate the Nilsson parameter κ which is a measure of the single-particle spin-orbit interaction strength. On the basis of the levels in O^{17}, it seems reasonable to set $\kappa\approx0.1$.[19] This method gives the results shown in column 2 of Table III and a value for the deformation parameter $\eta\approx4.4$. We should point out here that the value of the magnetic moment is rather insensitive to the deformation parameter in the region in question. If we use the sum of the moments to find the quantity in square brackets, then the individual moments deviate from experiment by about 0.2 nm and with different sign. If we assume that the wave function obtained in this way is really the "correct" one, then this result can be interpreted as evidence of mesonic effects. The picture so far is probably considerably oversimplified. In the first place we have not included the spin-orbit correction[4] in our discussion of the Na^{21} moment. Secondly, we have ignored the possibility of admixtures of states from other rotational bands in the ground-state wave function. A recent calculation by Kelson and Levinson[20] which takes the rotation-particle coupling into account gives the results shown in the last column of Table III.

TABLE III. A comparison of the magnetic dipole moments of the $A=21$ mirror pair with some theoretical predictions. A detailed discussion is contained in the text.

		$(d_{5/2})^3{}_{I=3/2}$	χ_Ω from Q	χ_Ω from $\Sigma\mu_I$	Kelson and Levinson
Na^{21}	Theory	+2.875	+2.41	+2.56	+2.09
	Expt.		+2.386		
Ne^{21}	Theory	−1.146	−0.71	−0.84	−0.62
	Expt.		−0.662		

We must conclude that the moments of the $A=21$ system are in good agreement with the collective model, but that the ground-state wave function is not adequately well known for us to assert the presence of mesonic effects.

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[18] G. M. Grosof, P. Buck, W. Lichten, and I. I. Rabi, Phys. Rev. Letters 1, 214 (1958).

[19] This choice is suggested by the splitting between the 5/2+ ground state of O^{17} and the first 3/2+ level at 5.08 MeV which is believed to be a very pure single-particle level.

[20] I. Kelson and C. A. Levinson, Phys. Rev. 134, B269 (1964).