Isomer Ratios for $Y^{87,87m}$ and the Spin Dependence of the Nuclear Level Density*

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Isomeric cross-section ratios have been measured for the following nuclear reactions in which the $Y^{87,87m}$ isomeric pair is produced: $Rb^{85}(\alpha,2n)$, $Sr^{86}(d,n)$, $Sr^{87}(d,2n)$, and $Sr^{88}(d,3n)$. The results are interpreted in terms of the angular momentum dependence of the nuclear level density. Statistical-model calculations based on the predictions of various theoretical formalisms are presented. Ambiguities in the calculation related to level density parameters and the multipolarity of the emitted radiation are discussed. The calculations are in qualitative agreement with experiment, although there are some difficulties in interpreting the level density parameters obtained. The effect of competing proton and alpha-particle channels on the isomer ratios for reactions in which neutrons are emitted has been investigated. The competition does not appreciably alter the isomer ratios in the particular case under discussion.

I. INTRODUCTION

THE angular momentum dependence of the nuclear level density is at present only qualitatively understood. Experimentally, it is a difficult property to determine, and data are rather meagre. Information about the distribution of angular momentum has been obtained in two cases from direct counting of states[1,2] in light nuclei, and some information has also been obtained from angular distributions of particles emitted in nuclear reactions.[3–5] In previous communications[6,7] a formalism for deducing information about the dependence of the level density on angular momentum from isomeric cross-section ratios was described. Although this method is admittedly an indirect one, the results obtained from it are in qualitative agreement with those obtained by other methods. This formalism has been quite successful[6] for calculating the variations in isomer ratios and populations of rotational states in thermal and resonant-energy neutron capture where the initial compound nucleus spins are known or can be inferred. That these calculations for resonant capture are quite insensitive (because of the low spins involved) to the distribution in angular momentum of the nuclear level density gives one some confidence that the model does satisfactorily account for the other factors which govern the final populations of states of different spins. However, in order to investigate the dependence of the level density on angular momentum it is necessary to consider reactions in which the incoming projectile can bring in enough angular momentum so that the spin cutoff factor applied to the de-excitation process can become important.

It is necessary to mention one misconception about theoretical expectations for isomer ratios. In several papers it has been erroneously suggested that for reactions similar to those considered here the statistical model predicts the relative population of the isomeric states to be given simply by the statistical weights of the final states. As discussed in Ref. 6, the isomer ratios as calculated from a statistical model depend on many factors, including the initial angular momentum distribution of the compound nucleus and the number of particles and quanta emitted prior to the final population of the isomeric states. The ratio of statistical weights of the final states does not provide even a limiting value for the isomeric cross section ratio.

In the present paper we report some new experimental results on isomeric cross-section ratios for reactions producing the isomeric pair $Y^{87,87m}$. These results have been compared with calculations utilizing theoretical predictions from various nuclear models. The implications of recent experimental evidence[8] suggesting the contribution of quadrupole radiation to the gamma-ray de-excitation cascade are discussed briefly. The effect of angular momentum on the competition between neutron emission and charged-particle emission is investigated in some detail.

II. EXPERIMENTAL PROCEDURE

The targets were prepared by vacuum volatilization of isotopically enriched RbCl or $Sr(NO_3)_2$ onto thin

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1 C. T. Hibdon, Phys. Rev. **114**, 179 (1959).
2 C. T. Hibdon, Phys. Rev. **122**, 1235 (1961).
3 T. Ericson and V. M. Strutinski, Nucl. Phys. **8**, 284 (1958).
4 A. C. Douglas and N. Macdonald, Nucl. Phys. **13**, 382 (1959).
5 D. Bodansky, R. K. Cole, W. G. Cross, C. R. Gruhn, and I. Halpern, *Proceedings of the International Conference on Nuclear Structure, Kingston, Canada*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, 1960), p. 749.
6 J. R. Huizenga and R. Vandenbosch, Phys. Rev. **120**, 1305 (1960).
7 R. Vandenbosch and J. R. Huizenga, Phys. Rev. **120**, 1313 (1960).
8 J. F. Mollenauer, Phys. Rev. **127**, 867 (1962).

aluminum foils. The target thicknesses were generally on the order of several hundred micrograms per square centimeter. The isotopic compositions of the various target materials are listed in Table I. The target foils were placed between aluminum degrading foils of known thicknesses and exposed to the deflected beam of the Argonne 60-in. cyclotron. The incident beam energy was determined from range measurements using range-energy curves constructed from the proton range-energy measurements of Bichsel.[9] After bombardment, the yttrium was radiochemically purified by a procedure described previously.[10]

The decay scheme[11] of $Y^{87,87m}$ is shown in Fig. 1. The determinations of the isomeric cross section ratios were based on observations of the growth and decay of the 483-keV transition following the decay of Y^{87}. This approach required only the knowledge of the positron emission plus electron capture to isomeric transition branching ratio, $(\beta^+ + \mathrm{EC})/\mathrm{IT}$, of Y^{87m}, and the Y^{87} and Y^{87m} half-lives. Upper limits of 15% for the EC/IT branching ratio[12] and 0.1% for the β^+/IT branching ratio[13] have been reported. We have assumed that Y^{87m} decays exclusively by isomeric transition, although some decay of Y^{87m} directly to the ground state of Sr^{87} might be expected as it is an allowed transition.

TABLE I. Isotopic composition of the rubidium and strontium target materials used in this study, as given by the supplier.[a]

	Isotope	Atomic percent
Rb^{85} (as RbCl)	85	99.69±0.05
	87	0.31±0.05
Sr^{86} [as $Sr(NO_3)_2$]	84	0.21±0.05
	86	88.8 ±0.1
	87	4.55±0.05
	88	6.43±0.05
Sr^{87} [as $Sr(NO_3)_2$]	84	0.1
	86	0.4 ±0.05
	87	95.4 ±0.05
	88	4.2 ±0.05
Sr^{88} [as $Sr(NO_3)_2$]	84	0.02
	86	0.09
	87	0.07±0.05
	88	99.24±0.05

[a] Isotopes Division, Oak Ridge National Laboratory.

The samples were counted on a total absorption counter,[14] in which the sample is placed between two 4- by 4-in. NaI crystals which are surrounded by a large plastic anticoincidence counter. This counter is similar to others which have been described in the literature.[15,16]

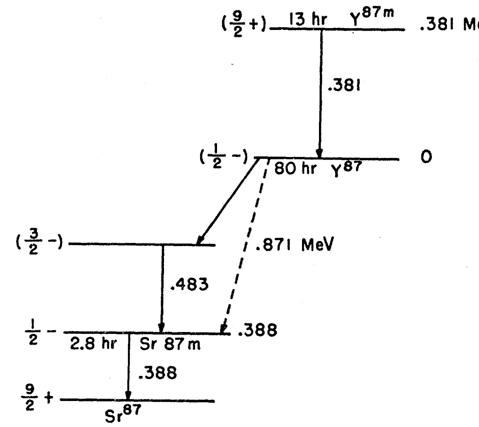

FIG. 1. Decay scheme for Y^{87} and Y^{87m}.

The pulses from the NaI detectors were rejected when coincident with a pulse from the plastic anticoincidence counter. This arrangement gave good discrimination against other activities, particularly Y^{86}, which has many high-energy gamma rays, and also considerably improved the photopeak to Compton-distribution ratio. The pulse-height spectrum for a typical sample recorded at two different times after bombardment is shown in Fig. 2. The intensity of the 483-keV photopeak was usually observed at least four different times after bombardment for each sample, and the resulting growth and decay was analyzed by a least-squares method assuming half-lives of 13 and 80 h for Y^{87m} and Y^{87}, respectively. The half-life of Y^{87m} was determined to be

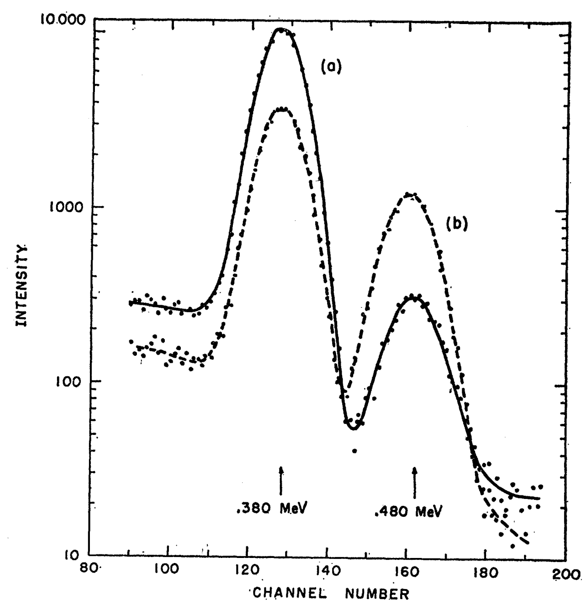

FIG. 2. Gamma-ray pulse-height spectrum from an yttrium sample prepared by the bombardment of Rb^{85} with 22-MeV helium ions. The solid curve (a) corresponds to the spectrum observed approximately an hour after the end of the bombardment and the dashed curve (b) corresponds to the spectrum observed approximately 30 h after the end of bombardment.

[9] H. Bichsel, Phys. Rev. **112**, 1089 (1958).
[10] L. Haskin and R. Vandenbosch, Phys. Rev. **123**, 184 (1961).
[11] *Nuclear Data Sheets*, edited by C. L. McGinnis, National Academy of Sciences, National Research Council (U. S. Government Printing Office, Washington 25, D. C.) NRC 60-3-56.
[12] L. G. Mann and P. Axel, Phys. Rev. **84**, 22 (1951).
[13] E. K. Hyde and G. D. O'Kelley, Phys. Rev. **84**, 944 (1951).
[14] We are indebted to N. Hansen and D. Henderson for the use of this counter.
[15] R. C. David, P. R. Bell, G. G. Kelley, and N. H. Lazar, IRE Trans. Nucl. Sci. **N5-3**, 82 (1961).
[16] W. H. Ellet and G. L. Brownell, Nuclear Instr. Methods **7**, 56 (1960).

TABLE II. Isomeric cross-section ratios for various reactions producing $Y^{87,87m}$.

	Particle energy (MeV) (Laboratory system)	$\sigma_{I=9/2}/\sigma_{I=1/2}$
$Rb^{85}(\alpha,2n)$	13.0	0.35±0.05
	13.8	1.8 ±0.1
	15.3	2.6 ±0.1
	16.8	3.1 ±0.1
	19.2	4.9 ±0.3
	21.6	5.5 ±0.4
	22.2	6.3 ±0.2
	24.4	6.4 ±0.1
	24.9	4.9 ±0.1
	26.2	8.9 ±0.5
	28.6	10.5 ±0.5
$Sr^{86}(d,n)$	4.3	0.55±0.02
	6.6	1.04±0.04
	9.1	1.44±0.06
	11.4	1.44±0.06
	13.4	1.31±0.06
	15.2	1.04±0.15
$Sr^{87}(d,2n)$	4.7	2.2 ±0.3
	7.5	2.6 ±0.1
	10.2	2.8 ±0.1
	12.3	3.4 ±0.2
	15.4	3.8 ±0.1
	17.3	4.4 ±0.1
	19.1	4.9 ±0.2
$Sr^{88}(d,3n)$	17.8	2.3 ±0.1
	20.2	2.7 ±0.1

13±1 h in a separate experiment; this value is not greatly different from that of 14±1 h reported previously.[12]

III. RESULTS

The experimental results are given in Table II and illustrated in Figs. 3 and 4. The errors listed in the table refer only to the standard deviations obtained from the least-squares fits and do not include systematic errors or uncertainties in the decay scheme. The latter errors are believed to be less than 20%. The uncertainties in the average bombarding energies are approximately 0.5 MeV at the higher energies and 1.0 MeV for the lower energies.

Isomeric cross-section ratios for the $Rb^{85}(\alpha,2n)Y^{87,87m}$ reaction have been reported by Iwata.[17] Our results for σ_m/σ_g are approximately 10 times larger than his. Apparently an error was made in his analysis, as a growth-decay curve presented in his paper is more consistent with our ratios than with his.

IV. QUALITATIVE FEATURES OF THE ISOMERIC CROSS-SECTION RATIOS

With the exception of those for the (d,n) reaction, the isomer ratios[18] for particular reactions increase with increasing bombarding energy. This is to be expected since the angular momentum transfer increases with bombarding energy, and interactions with large angular momentum transfer should favor population of the higher spin isomer. The decrease in the isomer ratio for the (d,n) reaction at higher eneries arises from the increased contribution of a direct reaction mechanism (stripping) in which the angular momentum transfer to the residual nucleus is much less than in compound nucleus reactions. For energies a few MeV above the $Sr^{86}(d,2n)$ threshold (indicated by the arrow in Fig. 4) the compound nucleus contribution to the $Sr^{86}(d,n)$ reaction should become small. Low isomer ratios for other direct reactions have been observed previously.[7,19]

The isomer ratio for the $Sr^{87}(d,2n)$ reaction is greater than that of the $Sr^{86}(d,n)$ reaction at the same bombarding energies, primarily because the target Sr^{87} has a spin of $\frac{9}{2}$, whereas the even-even targets have spin zero. The low isomer ratio for the $Sr^{88}(d,3n)$ reaction is partly due to the above-mentioned, target-spin effect, but can also arise from other factors. One of these involves the finite energy difference between the two isomers, such that for certain emitted neutron energies it is possible to populate only the states of lowest energy, in this case, the lower spin isomer. This effect is seen clearly in the isomer ratio for the $(\alpha,2n)$ reaction at its threshold of 13 MeV. As the number of neutrons emitted increases, the effect can persist to energies well above the threshold. Qualitative evaporation calculations suggest that it is important (greater than 10% effect on isomer ratios) for energies of less than 3 MeV above the threshold for the $(d,2n)$ and $(\alpha,2n)$ reactions, and is also important at all energies for which the $(d,3n)$ isomer ratio has been measured. Still another complicating factor may be important for a reaction near its threshold and where compound states with high angular momenta are involved. There may be an angular momentum

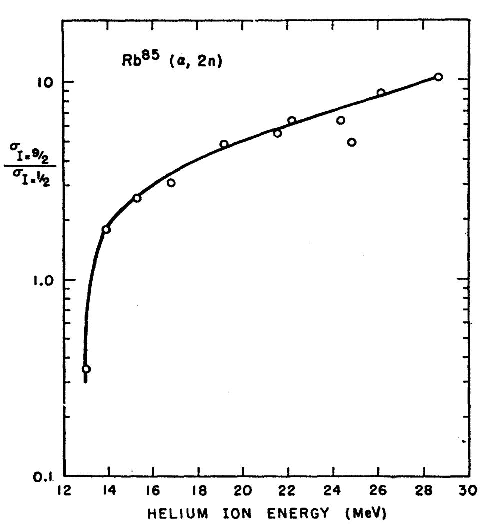

FIG. 3. Experimental results for the $Rb^{85}(\alpha,2n)$ isomer ratios. Energies are in the laboratory system.

[17] S. Iwata, J. Phys. Soc. Japan 17, 1323 (1962).

[18] In this paper we shall use the term isomer ratio to refer to the ratio of the cross section for production of the higher spin isomer to that for production of the lower spin isomer.

[19] T. Matsuo and T. T. Sugihara, Can. J. Chem. 39, 697 (1961).

fractionation of the compound states due to competition between neutron emission and gamma-ray de-excitation. At the low excitation energies following neutron emission, there may be considerably fewer high-angular-momentum states available compared to the higher excitation energy region accessible by gamma-ray de-excitation. Thus, the compound states with high angular momenta would decay primarily by gamma-ray de-excitation, while the compound states with lower angular momenta would have a comparatively greater probability of decaying by neutron emission. Such an effect would favor production of the lower spin isomer for a reaction near its own threshold, and the higher spin isomer for the same reaction when the threshold for emission of an additional particle was exceeded by several MeV. The existence of this effect has not been clearly demonstrated, but may have been observed in the isomer ratio for the $Ag^{107}(\alpha,n)$ reaction.[20] It could be observed best for a reaction near threshold when the spin of the ground state exceeded that of the metastable isomer. In the section that follows, we will ignore experimental data near threshold to minimize these complications. A list of the reaction thresholds is given in Table III.

V. STATISTICAL MODEL CALCULATIONS

The dependence of the nuclear level density on angular momentum is expected to have the functional form

$$\rho(J)=\rho(0)(2J+1)\exp\left(\frac{-J(J+1)}{2\sigma^2}\right), \quad (1)$$

where the factor $\rho(0)$, which is the density of levels with angular momentum zero, contains most of the dependence of the nuclear level density on excitation energy. The spin-cutoff parameter σ which characterizes

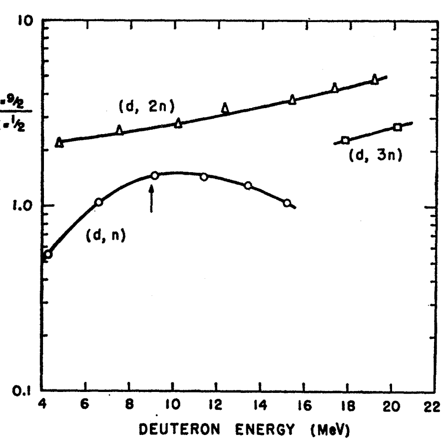

FIG. 4. Experimental results for the $Sr^{86}(d,n)$, $Sr^{87}(d,2n)$ and $Sr^{88}(d,3n)$ isomer ratios. Arrow indicates $Sr^{86}(d,2n)$ threshold. Energies are in the laboratory system.

[20] C. T. Bishop, Argonne National Laboratory Report ANL-6405, 1961 (unpublished).

TABLE III. Reaction thresholds[a] in the laboratory system.

	Reaction	Threshold (MeV)
Rb^{85}	$(\alpha,2n)$	13.4
	$(\alpha,3n)$	26.7
Sr^{86}	(d,n)	−3.7
	$(d,2n)$	9.2
Sr^{87}	$(d,2n)$	4.8
	$(d,3n)$	17.8
Sr^{88}	$(d,3n)$	16.2

[a] These thresholds have been taken from 1960 Nuclear Data Tables, Part 2 (Washington 25, D. C.) 1961).

the distribution function is given by $\sigma^2=ct$, where t is a temperature given by Eq. (5). The quantity $c\hbar^2$ can be interpreted as a moment of inertia,[21,22] or be related to the mean square value of the magnetic quantum number of individual nucleons by[23,24]

$$\langle m^2\rangle g=c. \quad (2)$$

The single-particle level density g can be expressed in terms of the level density parameter a by

$$a=(1/6)\pi^2 g. \quad (3)$$

It has been shown[21,22] that for nucleons moving independently in an infinite square-well potential, $c\hbar^2$ is given by the rigid body moment of inertia

$$\mathcal{I}_r=(\tfrac{2}{5})M_n R^2 A\,, \quad (4)$$

where M_n is a nucleon mass and R is the nuclear radius.

In an earlier report[6,7] on the application of the statistical model to the calculation of isomeric cross-section ratios, calculations were made using the approximation that σ is a constant independent of excitation energy. Analyses of isomeric cross-section ratios for various reactions in which the isomeric pair $Hg^{197,197m}$ is formed indicated a value for σ of approximately 4. This corresponds to a moment of inertia which is appreciably less than the rigid body value.[25] The reduction of the moment of inertia from a rigid body value is usually attributed to the pairing interaction which favors states with particles coupled pairwise to zero angular momentum. These pairs must be broken before the particles involved can recouple to form states of higher angular momentum. In the present section we shall describe calculations based on the predictions of various theo-

[21] H. A. Bethe, Rev. Mod. Phys. 9, 84 (1937).
[22] C. Bloch, Phys. Rev. 93, 1094 (1954).
[23] T. D. Newton, Can. J. Phys. 34, 804 (1956).
[24] T. Ericson, Advan. Phys. 9, 425 (1960).
[25] In Ref. 7 an estimate of $\mathcal{I}/\mathcal{I}_{\rm rigid}\sim 0.1$ was deduced from the $\sigma=4$ result. This estimate was made using what are now considered to be poor choices of the nuclear radius parameter and the nuclear temperature. Although a radius parameter of $r_0=1.5$ F is still thought to be the best estimate for use in calculating transmission coefficients from a square well potential, a value of $r_0=1.2$ F is probably more appropriate for calculating the rigid body moment of inertia. With this radius and a temperature of 0.5 MeV a value of $\sigma=4$ corresponds approximately to $\mathcal{I}/\mathcal{I}_{\rm rigid}=\frac{1}{3}$.

retical models. In these calculations we will allow σ to vary as a function of excitation energy. The models are discussed in the order of their historical development.

A. The Fermi Gas Model

The simplest theoretical model which is appropriate for this problem is the Fermi gas model. The equation of state for this model is[26]

$$U = at^2 - t\,, \tag{5}$$

where U is an excitation energy. Equation (5) contains what Lang and LeCouteur call a thermodynamic temperature. The thermodynamic temperature is slightly smaller than the nuclear temperature which is defined as $1/\tau = d\ln\rho(U)/dU$, where $\rho(U)$ is the nuclear level density. From Eq. (5) and the relation $\sigma^2 = ct$, the angular momentum dependence of the level density at different excitation energies can be determined. Using the formalism described previously,[6,7] isomer ratios have been calculated with parameters based on this simple form of the Fermi gas model. The resulting calculated isomer ratios, for reasonable values of the level density parameter a, are much higher than the experimental values. This is not too surprising, as it has been known for some time that the Fermi gas model must be modified to take into account residual interactions such as the pairing interaction. This has often been done simply by defining an effective excitation energy U which is measured from a fictitious reference surface, usually the odd-odd mass surface. With this choice for the reference surface, one can define the dependence of U on nuclear type by the following relations:

$$\begin{aligned} &U = E^* \text{ for odd-odd nuclei,} \\ &U = E^* - \delta \text{ for odd mass number nuclei,} \\ &U = E^* - 2\delta \text{ for even-even nuclei,} \end{aligned} \tag{6}$$

where E^* is the excitation energy as measured from the actual ground state and δ is a pairing energy. We shall call this modified Fermi gas model a "shifted" Fermi gas model. This formalism leaves σ undefined for even-even nuclei of excitation energy less than 2δ and for odd mass number nuclei of less than δ. Even-even nuclei at such low excitation energy are not considered in this paper, and in any event, would have few levels in this energy interval. For odd-even nuclei $\sigma^2 = \langle m^2 \rangle$ provides a satisfactory estimate for excitation energies below δ. The variation of σ with excitation energy predicted by this model is illustrated in Fig. 5. It can be seen that the σ values are somewhat larger than those given by the more sophisticated superconductor pairing model described in Sec. V(c). Anticipating the fact that even with the superconductor model one must use rather large values of the level density parameter a in order to get small enough σ values to reproduce the isomer ratios,[27] we will not report extensive calculations based on this model. At this point we can remark that the shifted Fermi gas model requires a slightly larger level density parameter a to reproduce the isomer ratios than is the case for the superconductor model.

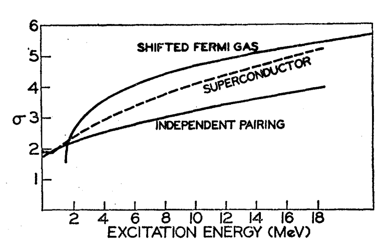

FIG. 5. Dependence of the spin-cutoff parameter σ on excitation energy as given by the various nuclear models with $a = A/8$. These curves are for odd mass number Y^{87} and the abscissa is the excitation energy as measured from the ground state.

B. Independent Pairing Model

Ericson[28] and Lang and LeCouteur[29] have modified the Fermi gas model by including a simple form of pairing interaction. Although this model is most naturally derived for nuclei which are deformed, it is suggested[29] that it should have approximate validity for spherical nuclei. In this model the energy required to break a coupled pair of nucleons 2δ is taken to be independent of the excitation energy. We shall call this model the independent pairing model since it assumes the pairing interaction for a particular pair is independent of other pairs. This model presents an extreme form of the pairing interaction, since it is generally expected that as the excitation energy increases less energy will be required to break additional pairs. With the assumption of a constant pairing energy the following approximate analytical expressions[30] have been derived[29,31]:

$$U' = at^2 - t\,, \tag{7}$$

$$\sigma^2 = c't = ct\exp(-0.874\delta/t)\,. \tag{8}$$

A curious feature of this model is that the effective excitation energy U' is measured from a fictitious reference surface which lies below the even-even mass surface, so that U' is given by

$$\begin{aligned} &U' = E^* + (a\delta^2/4.8) + 2\delta && \text{odd-odd nuclei,} \\ &U' = E^* + (a\delta^2/4.8) + \delta && \text{odd mass number nuclei,} \\ &U = E^* + (a\delta^2/4.8) && \text{even-even nuclei.} \end{aligned} \tag{9}$$

The moment of inertia $c'\hbar^2$ implied by Eq. (8) is considerably less than the rigid body value even at quite high excitation energies ($E^* \sim 20$ MeV).

[26] K. J. LeCouteur and D. W. Lang, Nucl. Phys. **13**, 32 (1959).

[27] In the comparison of the various models with experiment we will match the calculations to experiment by varying the level density parameter a rather than by an arbitrary reduction of the moment of inertia. Thus, the moment of inertia will be that given by the particular model under consideration. For all the models an increase in the parameter a reduces the temperature and hence the spin-cutoff factor σ.

[28] T. Ericson, Nucl. Phys. **6**, 62 (1958).

[29] D. W. Lang and K. J. LeCouteur, Nucl. Phys. **14**, 21 (1959–60).

[30] The quantity termed Δ in Ref. 29 is more commonly designated by 2Δ or 2δ in the literature. The latter convention will be observed in the present paper.

[31] D. W. Lang, Nucl. Phys. **42**, 353 (1963).

Lang and LeCouteur suggest that in addition to odd-even corrections defined by Eq. (6) one should add $\langle m^2\rangle$ to σ^2 [given by Eq. (8)] for odd mass number nuclei and add $2\langle m^2\rangle$ for odd-odd nuclei. This additional term appears to overcorrect for the odd-even effect and results in minor inconsistencies in the model at excitation energies below 5 MeV. Therefore, we have not added $\langle m^2\rangle$ or $2\langle m^2\rangle$, but have required that $\sigma^2=\langle m^2\rangle$ for odd mass number nuclei at excitation energies $E^*<\delta$ and that $\sigma^2=2\langle m^2\rangle$ for odd-odd nuclei at excitation energies $E^*<2\delta$. (The latter condition was of no importance in the present calculation because odd-odd nuclei at low excitation energy were not encountered.)

C. Superconductor Approach to Residual Interactions

Several years ago an analogy between the excitation spectra of nuclei and those of the superconducting metallic state was pointed out by Bohr, Mottelson, and Pines.[32] Since that time considerable progress in the description of low-lying nuclear states has been achieved by application of the superconductor model to nuclei. More recently, Lang[31] examined in more detail the implications of this model for nuclei at higher excitation energies. The application of this model to the calculation of isomer ratios has been discussed elsewhere[33] so we will limit ourselves here to a short summary. Above a certain critical energy the nucleus behaves as a normal Fermi gas, except that the excitation energy is measured from a Fermi energy which lies above the even-even mass surface. Below this critical energy the moment of inertia is reduced from the rigid body value. The condensation energy of the even-even ground state below the Fermi energy amounts to 0.47 at_c^2, where t_c is the critical temperature given by

$$t_c=0.57\theta_0. \tag{10}$$

The correlation parameter θ_0 has been taken 30% larger than the pairing energy parameter δ, as indicated by the results of Ref. 33. The critical energy as measured from the ground state of the even-even system is $U_c=1.473\ at_c^2$. For reasonable pairing energies and a level density parameter $a=A/8$, critical energies of approximately 16 MeV are obtained. The spin-cutoff parameter values from this model approach those given by the shifted Fermi gas model at higher excitation energies, as can be seen by Fig. 5.

In order to apply these models, parameter choices must be made. A pairing energy of $2\delta=2.7$ MeV has been taken from the work of Nemirovsky and Adamchuk.[34] There are various ways to estimate $\langle m^2\rangle$. Although, in principle, $c\hbar^2$ and $\langle m^2\rangle$ are functionally related to each other, they are sometimes evaluated independently. One suggestion[26] has been to use a value of $\langle m^2\rangle=0.146A^{2/3}$ derived[35] from the sequence of states in the shell model. This is perhaps not a very good estimate as it involves an average over all the nucleon states, whereas for the excitation energies of concern an average over the states of the last major shell to be filled is more appropriate. We have taken $\langle m^2\rangle=I_r/(g\hbar^2)$, [see Eq. (2)] which, for acceptable values of g, is in reasonable agreement with the latter average. The parameter $c\hbar^2$ has been taken equal to the rigid body moment of inertia [Eq. (4)] with a nuclear radius parameter $r_0=1.2$ F.

The most difficult parameter to evaluate is the level-density parameter a related to g through Eq. (3). This parameter is usually obtained from evaporation spectra of emitted particles. Unfortunately, the level density parameter a required to reproduce the experimental nuclear temperatures is not the same for the different models discussed. In the calculations the level density parameter has been varied over a range wide enough to determine the value required to account for the experimental isomer ratio data. The significance of the parameters required is discussed at a later point in this paper.

Calculations of the expected isomeric cross-section ratios using the above-mentioned choices of parameters have been carried out using the formalism described previously.[6,7] The barrier transmission coefficients for the incoming projectile have been taken from the optical model calculations of Melkanoff *et al.*[36] for deuterons and from the optical-model calculations of Huizenga and Igo[37] for alpha particles. The transmission coefficients for the outgoing neutrons were taken from the optical-model calculations of Campbell *et al.*[38] The model-dependent quantity $f(E,B,J_c,J_f)$ discussed in Ref. 6 has been taken equal to unity. It has been assumed that the neutrons carry off an average energy of 2τ using $U=a\tau^2-4\tau$. Bishop (Ref. 20) has shown that the calculated isomer ratios are quite insensitive to changes in the average energy carried off by the neutrons (see column 6 of Table 15 in Ref. 20). The average energy carried off by each gamma ray was obtained from $E_\gamma=4(E^*/a-5/a^2)^{1/2}$ (see the Appendix of Ref. 33 for the origin of this expression). Some arbitrariness is introduced into the calculation by the choice of energy at which the final isomer-deciding transition is emitted. The following prescription[33] has been employed:

The cascade is followed until the residual excitation energy E is 2 MeV or less. When E is between 1 and 2 MeV, the probability that the next gamma-ray emission populates the ground or isomeric state is taken to be $(2-E)$ and the probability that two further gammas will be needed is $(E-1)$. For E less than 1 MeV, emis-

[32] A. Bohr, B. R. Mottelson, and D. Pines, Phys. Rev. **110**, 936 (1958).

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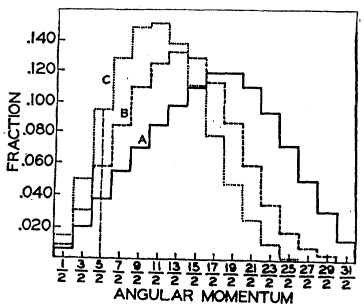

FIG. 6. Illustration showing the distribution in angular momentum at several stages of the calculation for the $Rb^{85}(\alpha,2n)$ reaction at 22 MeV, using the pairing model with the level density parameter choice of $a=A/12$. Curve (A) corresponds to the distribution in angular momentum of the compound nucleus prior to neutron emission. Curve (B) corresponds to the distribution following the emission of the second neutron, and curve (C) is the distribution obtained after the emission of two dipole gamma rays. The vertical bar at $J=\frac{5}{2}$ represents the dividing point; states to the right of the bar populate the $I=\frac{9}{2}$ isomer and states to the left populate the $I=\frac{1}{2}$ isomer.

sion of only one further gamma ray is presumed necessary. Isomer ratios are calculated from the distributions just preceding the final gamma emission. The treatment of the gamma-ray cascade is the most unsatisfactory part of the calculations. The calculations are quite sensitive to the number of gamma rays emitted, as can be seen in Fig. 8. The horizontal bars indicate the change in the calculated isomer ratios obtained when the number of gamma rays is varied by one. The calculations are also quite sensitive to the multipolarity of the radiation, as will be discussed in a later paragraph.

An illustration of the angular momentum distribution at different stages of the de-excitation process is given in Fig. 6. Curve (A) shows the distribution in angular momentum of the compound nucleus prior to neutron emission. This distribution is obtained through use of Eq. (2) of Ref. 7. The next step is the calculation of the angular momentum distribution (not shown in Fig. 6) following emission of the first neutron using Eq. (3) of Ref. 7. The distribution following the emission of the second neutron is shown by the curve (B). This distribution is further modified by the emission of gamma rays, and the distribution obtained after the emission of the second (and in this case next to last) gamma ray is shown by curve (C). The final population is then determined by dividing this distribution at $J=\frac{5}{2}$. States with $J>\frac{5}{2}$ are assumed to populate the higher spin isomer ($I=\frac{9}{2}$) and states with $J<\frac{5}{2}$ to populate the lower spin isomer ($I=\frac{1}{2}$). (This does not mean that a state with, for example, $J=15/2$ will decay directly by a single gamma ray to the $I=\frac{9}{2}$ isomeric state, but rather that the cascade will terminate at the $I=\frac{9}{2}$ state rather than cross over to the $I=\frac{1}{2}$ state.) Half the states with $J=\frac{5}{2}$ are presumed to decay to each isomer.

The results of calculations based on the various models are compared with experiment in Figs. 7 and 8. One of the more encouraging aspects of this comparison is the observation that the $(\alpha,2n)$ and $(d,2n)$ calculations are self-consistent. Approximately the same value of a that is required to fit the $(\alpha,2n)$ data also fits the somewhat different isomer ratios of the $(d,2n)$ data. It was seen in Fig. 5 that the σ values for the shifted Fermi gas model are larger than those for the superconductor model for the same value of the level density parameter a; therefore, the value of a required to obtain a fit with the shifted Fermi gas model will be even larger than $a=A/5.5=16$ as required by the superconductor model and indicated in Fig. 7. This value of a for the Fermi gas model is somewhat larger than that obtained from analysis of nuclear temperature measurements.[39,33] The superconductor model however, as has been discussed elsewhere,[33] requires unusually large a values to fit experimental nuclear temperatures. Thus, the superconductor model is at least consistent in requiring large a values to fit both the experimental nuclear temperatures and the experimental isomer ratios. In principle, the level density parameter a should not be considered a free parameter, as it is related through Eq. (3) to the single-particle level density g. The parameter a therefore can be calculated for free nucleons in a well, and is found[40] to be $a=A/13.5$ MeV^{-1} for a well of radius

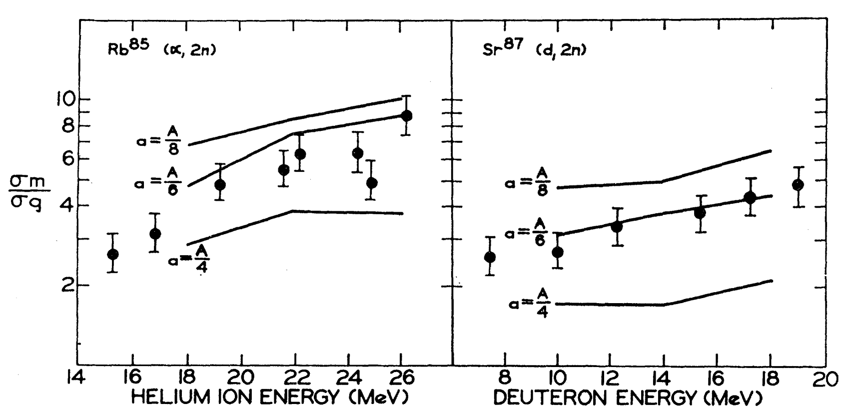

FIG. 7. The experimental data are compared with the calculations (full curves) based on the superconductor model with the indicated values of the level density parameter a.

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[40] D. Bodansky, Ann. Rev. Nucl. Sci. **12**, 84 (1962).

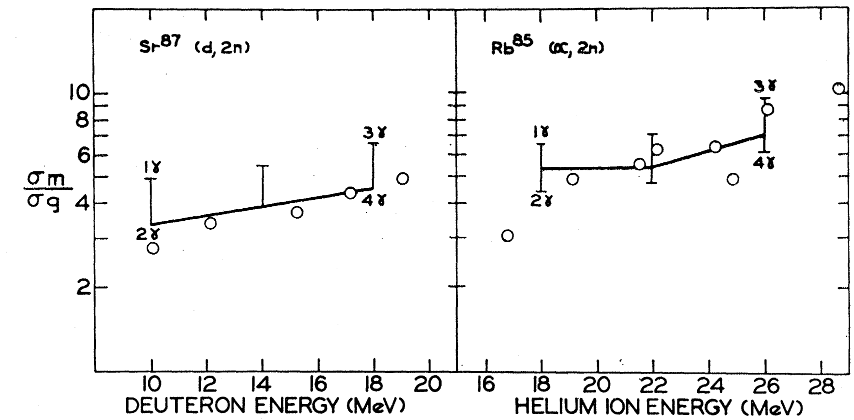

FIG. 8. The experimental data are compared with the calculations based on the independent pairing model of Lang and LeCouteur with $a=A/8$. The sensitivity of the calculation to the number of gamma rays emitted is indicated by the vertical bars. The upper and lower limits show the change in the calculated isomer ratios when the number of gamma rays is allowed to vary by one. The full curve is drawn through the vertical bars in such a way as to give approximately correct values for the average number of gamma rays emitted.

$R=1.2A^{1/3}$ F and $a=A/8.7$ MeV^{-1} for a well of radius $R=1.5A^{1/3}$ F. In this framework an a value of larger than $a=A/8$ is difficult to justify.

If, for the shifted Fermi gas model, one wants to retain a level density parameter of $a=A/8$, one can ask what reduction of the rigid body moment of inertia is required to fit the experimental isomer ratios; a ratio of $\mathcal{I}/\mathcal{I}_{\text{rigid}}$ of approximately 0.65 is indicated. This value is consistent with other measures of this quantity,[38,25,41] and follows an apparent trend of smaller reductions from the rigid body moment of inertia as the mass number is decreased.

The independent pairing model is able to account for the observed isomer ratios with the more generally accepted value of $a=A/8$. This is because this model gives moments of inertia which are considerably less than rigid body values at all excitation energies, whereas, in the superconductor model, the moment of inertia is reduced only below the critical energy. There is one experimental datum which supports the idea of a reduced moment of inertia at high excitation energies. Alexander and Simonoff[42] have concluded from their excitation function data for heavy-ion-induced reactions that the first neutron emitted from a compound nucleus at an excitation energy of approximately 80 MeV carries off 3 units of angular momentum. Calculations based on a rigid body moment of inertia predict that the first neutron should carry off only one unit of angular momentum, so the moment of inertia would have to be reduced considerably from the rigid body value to account for the experimental observation. Although the independent pairing model can reproduce the observed isomer ratios with the generally accepted a value given by $a=A/8$, it requires a values larger than that given by $a=A/8$, to reproduce experimental nuclear temperatures for nuclei in this mass region.

It has been assumed in the calculations that the gamma-ray cascade consists of dipole transitions. A calculation was performed to assess the importance of the multipolarity of the gamma rays on the predicted isomeric cross-section ratios. Spin-cutoff parameters given by the shifted Fermi gas model with $a=A/8$ were used for this calculation. The results for pure quadrupole gamma emission are compared with the results for pure dipole gamma emission in Fig. 9. The apparent rise in the isomer ratios for the quadrupole calculation as the energy decreases reflects the fact that fewer gamma rays are emitted at lower energies. With fewer gamma rays the ability of quadrupole gammas to reach the more abundant states of lower spin, and hence reduce the isomer ratio, is less influential. Whereas, with dipole gamma rays, the calculated values using $a=A/8$ are too high, with quadrupole gamma rays the calculated values are too low and the energy dependence is less satisfactory. A judicious mixture of dipole and quadrupole radiation could probably be found which would reproduce the experimental results. There are at present no decisive experimental results which give the relative contributions of dipole and quadrupole radiation to the gamma cascade. There are several features (radiation widths,[43] gamma-ray energy spectra,[44] and isomeric cross-section ratios[6]) of the gamma-ray cascade following neutron capture which are consistent with predominantly dipole radiation but which might also be con-

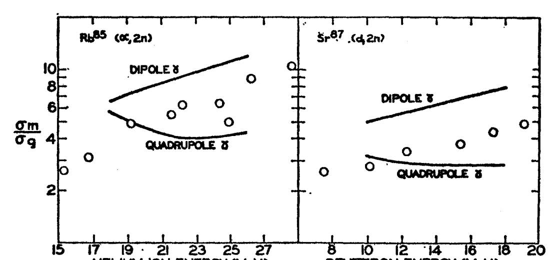

FIG. 9. The sensitivity of the calculations to the multipolarity of the emitted gamma rays is displayed. The number of gamma rays at a given energy was assumed to be the same for dipole and quadrupole emission. The shifted Fermi gas model with $a=A/8$ was used to predict the spin-cutoff parameter values. The isomer ratios for quadrupole emission do not differ as much from the dipole values at low energies because fewer gamma rays are emitted.

[41] J. H. Carver, G. E. Coote, and T. R. Sherwood, Nucl. Phys. **37**, 449 (1962).

[42] J. M. Alexander and G. N. Simonoff, Phys. Rev. **133**, B93 (1964).

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sistent with quadrupole radiation. Photonuclear reactions can be used to investigate the relative contributions of different multipoles. Angular distributions of elastically scattered 7-MeV gamma rays[45] provide evidence that the interaction is primarily dipole; 7-MeV gammas are, however, of considerably higher energy than those encountered in a de-excitation cascade. It is well known that there are many enhanced $E2$ transitions observed between low-lying nuclear states, while the $E1$ transitions at low energies are usually several orders of magnitude slower than single-particle estimates. In fact, examination of a compilation[46] of measured absolute transition probabilities suggests that on the average $E2$ and $M1$ transitions are of comparable speed and are faster than $E1$ transitions. However, these data are strongly influenced by the low-lying collective states with enhanced $E2$ transition probabilities and also by the fact that many of the $E2$ transition probabilities have been measured by Coulomb excitation where only the faster transitions are observed. Another source of information on comparative rates of dipole and quadrupole radiation may be found in a recent literature survey[47] of gamma-ray branching ratios. In this survey only cases in which two or more transitions take place from the same level are considered, yielding relative transition probabilities. These transition probabilities are still influenced by the special properties of the low-lying states, but perhaps to a lesser degree than the absolute transition probabilities. Although the distributions are very broad, it appears from the compilation that on the average both $E1$ and $M1$ transitions are slightly faster (by perhaps a factor of 10) than $E2$ transitions.

It therefore seems reasonable to conclude, that for the complex nuclear states at several MeV excitation energy involved in the gamma-ray cascade, dipole transitions will predominate under normal conditions, but some quadrupole radiation may be expected. However, where there are many states of quite high angular momentum, it may be easier to dispose of the angular momentum by emission of quadrupole radiation. Recent experiments by Mollenauer[8] on the angular distribution of gamma rays from nuclear reactions indicate the presence of quadrupole radiation for helium ion bombarding energies greater than 30 MeV. It seems reasonable to attribute most of this quadrupole radiation to decay of states with high angular momenta which are "forced" to emit quadrupole radiation to get rid of their high angular momenta. Such a quadrupole emission process would not be important for the isomer ratio predictions, as these states would have populated the higher spin isomer anyway (see Fig. 6). If, however, quadrupole emission competes with dipole emission for all initial spin values, then the predicted isomer ratios will be decreased significantly as indicated by the calculation mentioned earlier. On the basis of the presently available evidence we believe that dipole transitions predominate for those states having a low enough angular momentum to be able to populate eventually the ground state or the isomeric state. With this assumption it appears necessary to introduce some type of residual interaction to account for the low values of the experimentally observed isomer ratios.

[45] K. Reibel and A. K. Mann, Phys. Rev. **118**, 701 (1960).

[46] D. H. Wilkinson, in *Nuclear Spectroscopy, Part B*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960).

[47] W. W. Pratt, Nucl. Phys. **28**, 598 (1961).

VI. EFFECTS OF COMPETING CHANNELS

In the calculations described previously, it has been assumed that the initial and intermediate spin distributions at various stages of the de-excitation process have not been distorted by competition from other modes of de-excitation. This problem has been investigated in connection with isomer ratio results by Need[48] and by Sugihara and Dudey.[49] In order for such competition to influence the isomer ratios for reactions in which neutrons are emitted two conditions have to be satisfied: (1) the yield of competing particles must not be negligible compared to the yield of the reaction of interest, and (2) the competition between various types of particles emitted must vary with the angular momentum J of the compound nucleus. We have investigated these points to see whether proton and alpha competition could be distorting the spin distribution and the isomer ratios for the reactions under consideration as compared with neutron emission only. The method of calculation is a modification of that described previously.[6,7,50] The usual computation of isomer ratios is divided into three parts,[50] the calculation of (1) the spin distribution in the initial compound nucleus, (2) the spin distribution following particle emission, and (3) the spin distribution following emission of gamma radiation. The mathematics of the first and third parts of the computation are unchanged when particle competition is included. To obtain the spin distribution following particle emission (part 2) when more than one particle type can be emitted, we begin with Eq. (5) of Need,[49] which adapted to our notation is

$$\Gamma_i(J_c,\epsilon_i,J_f)d\epsilon_i=\frac{1}{2\pi\rho(J_c,U_c)}\times\sum_{S=|J_f-s|}^{J_f+s}\sum_{l=|J_c-S|}^{J_c+S}T_l^i(\epsilon_i)\rho(J_f,U)d\epsilon_i. \quad (11a)$$

Here $\Gamma_i(J_c,\epsilon_i,J_f)$ is the partial width for decay of a state of spin J_c by emission of a particle (i) with energy ϵ_i to give a residual nucleus with spin J_f and energy U; $\rho(J_c,U_c)$ is the level density of the compound nucleus of spin J_c and excitation energy U_c; $T_l^i(\epsilon_i)$ is the barrier transmission coefficient of particle (i) with energy ϵ_i; $\rho(J_f,U)$ is the level density for the residual compound nucleus state of spin J_f and energy U; and s is the intrinsic spin and l the orbital angular momentum of

[48] J. L. Need, Phys. Rev. **129**, 1302 (1963).

[49] T. T. Sugihara and N. D. Dudey (private communication).

[50] W. L. Hafner, Jr., J. R. Huizenga, and R. Vandenbosch, Argonne National Laboratory Report ANL-6662, 1962 (unpublished).

emitted particle (i). Since we are concerned only with competing decay modes from the same compound nucleus state and not with absolute cross sections, the factor $1/2\pi\rho(J_c,U_c)$ will cancel in all calculations. Consequently for isomeric cross-section ratios Eq. (11a) reduces to

$$P_i(J_c,\epsilon_i,J_f)\propto\rho(J_f,U)\sum_{S=|J_f-s|}^{J_f+s}\sum_{l=|J_c-S|}^{J_c+S}T_l{}^i(\epsilon_i). \quad (11b)$$

Here $P_i(J_c,\epsilon_i,J_f)$ is the probability that a particular initial state of spin J_c will decay by emission of particle (i) of energy ϵ_i. The subscript (i) can be taken to denote any particle of any energy, including the same particle type with different energies as well as several different particle types. The level density is given by

$$\rho(J_f,U)=\frac{(2J_f+1)}{\sigma^3a^{1/4}(U+t)^{5/4}}\times\exp\left[2(aU)^{1/2}\frac{-J_f(J_f+1)}{2\sigma^2}\right], \quad (12)$$

where U is the residual excitation energy and t is given by $U=at^2-t$.

The variation of level density with nuclear type was accounted for by using pairing corrections as defined in Eq. (6). Using P_i as defined in Eq. (11b), the fraction $F_j(J_c,\epsilon_j)$ of all de-excitations from an initial spin state J_c which go by mode (j) of the group of modes (i) under consideration is

$$F_j(J_c,\epsilon_j)=\frac{\sum_{J_f}P_j(J_f,\epsilon_j,J_c)}{\sum_i\sum_{\epsilon_i}(\sum_{J_f}P_i(J_f,\epsilon_i,J_c))\Delta\epsilon_i}, \quad (13)$$

where the implied integration has been replaced by a sum over energy using energy intervals of $\Delta\epsilon_i$. This procedure is necessary since transmission coefficients are available only for discrete energies, and is justified as long as the summation extends over the range of probable particle emission energies.

After summing over the normalized distribution $P(J_c)$ of initial spin states J_c, obtained from part 1 of the calculation, the fraction F_j of all the compound nuclear disintegrations by means of mode j is

$$F_j(\epsilon_j)=\sum_{J_c}P(J_c)F_j(J_c,\epsilon_j). \quad (14)$$

Calculations have been made of the interdependence of $F_j(J_c,\epsilon_j)$ and $F_j(\epsilon_j)$ for neutrons, protons, and alpha particles as functions of J_c and of the kinetic energies of the emitted particles. The range of ϵ_j values for neutrons extended from 0.2 to 8 MeV (laboratory coordinates), that for protons from 4 to 10 MeV, and that for alpha particles from 8 to 15 MeV. Input parameters corresponding to $a=A/8$ and spin-cutoff values σ from the independent pairing model of Lang and LeCouteur[29] were used. It was shown in Fig. 8 that these parameters reproduced satisfactorily the observed isomer ratios. In addition to the transmission coefficients required in the previously described computations, transmission coefficients were

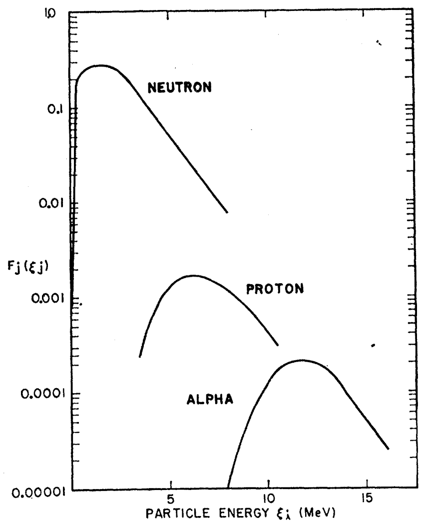

FIG. 10. F_j versus particle energy for neutrons, protons, and alphas emitted from the Y^{89} compound nucleus produced by bombardment of Sr^{87} with 18-MeV deuterons. Note that the most probable energy for emitted neutrons is essentially the nuclear temperature τ (1.5 MeV) and for protons and alpha particles is essentially the Coulomb barrier height (7.2 and 12.8 MeV, respectively).

calculated using optical-model parameters from Glassgold *et al.*[51] Calculations were done for the de-excitation of the compound nucleus Y^{89} resulting from the bombardment of Sr^{87} by 18-MeV deuterons. Some of the results are shown in Fig. 10. The following conclusions have been drawn from these calculations:

(1) After integration over the particle kinetic energies neutrons are found to account for more than 90% of the de-excitations.

(2) Alpha-particle emission contributes less than 1% (and is thus not considered further).

(3) 2τ and the Coulomb barrier energy are reasonable approximations for the average energies of the emitted neutrons and charged particles, respectively, as would be anticipated from evaporation theory.

Having eliminated alpha-particle emission as a serious competitor, we can now proceed to evaluate the effect of proton competition on the isomer ratios. In principle this should be done using Eqs. (13) and (14) and summing over ϵ_j. In practice the complete calculations are very lengthy, and have been approximated for purposes of isomer ratio calculations by using average energies rather than integrating over the particle energy spectra. In this approximation, the total normalized yield of states with spin J_f following emission of a particle of type $i=1$ in competition with a particle of type $i=2$ after weighted summing over all initial states is

$$P_1(J_f)=\sum_{J_c}P(J_c)\frac{P_1(J_f,J_c)}{\sum_{i=1}^{2}\sum_{J_f}P_i(J_f,J_c)}. \quad (15)$$

[51] A. E. Glassgold, W. B. Cheston, M. L. Stein, S. B. Schuldt, and G. W. Erickson, Phys. Rev. 106, 1207 (1957).

TABLE IV. Effects of particle competition on isomer ratios. The first column gives the incident particle kinetic energy (laboratory system). The second gives isomer ratios calculated on the assumption that all compound nuclear decays are by average energy (2τ) neutrons. The third gives the isomer ratios when 7-MeV proton and 2τ neutron emission are considered as competing modes in the initial de-excitation step. The final column gives the fraction of these decays which result in neutron emission.

K.E. (MeV)	σ_m/σ_g	σ_m/σ_g	F_n
	$Sr^{87}(d,2n)$		
10	3.37	3.37	0.984
14	3.98	4.00	0.953
18	4.55	4.61	0.930
	$Sr^{86}(d,n)$		
6	0.94	0.94	0.993
10	1.74	1.75	0.978
14	2.23	2.29	0.964
	$Rb^{85}(\alpha,2n)$		
18	5.37	5.37	0.989
22	5.28	5.13	0.982
26	7.00	7.26	0.947

Here $i=1$ was taken to be a neutron of kinetic energy equal to 2τ, the mean energy for evaporated neutrons. It was shown previously that use of the mean energy gives to a good approximation the same isomer ratios ratios as when the entire evaporation spectrum is considered. Similarly, $i=2$ refers to protons whose kinetic energy is equal to the height of the Coulomb barrier. The limits of the sums are given explicitly in Ref. 50. This spin distribution may now be used as input data for succeeding particle emission calculations, with or without competition, or may be followed by computation of the spin distribution following gamma-ray emission. The final spin distributions may be partitioned to give isomer ratios and then compared with the results of calculations where competition was neglected. Such a comparison is given in Table IV. It is concluded from these comparisons that proton competition does not alter significantly the spin distributions and isomer ratios for the cases considered in this paper. Thus, the calculations presented earlier where only neutron emission was considered are valid.

Figure 11 shows how $\Gamma_n(J_c,\epsilon_n)/[\Gamma_n(J_c,\epsilon_n)+\Gamma_p(J_c,\epsilon_p)]$ varies if protons of several energies are allowed to compete with emission of a 2τ neutron, and how this competition varies with J_c. Although $\Gamma_n(J_c,\epsilon_n)/[\Gamma_n(J_c,\epsilon_n)+\Gamma_p(J_c,\epsilon_p)]$ is not independent of J_c, the spin distributions following neutron emission were not significantly affected by proton competition because

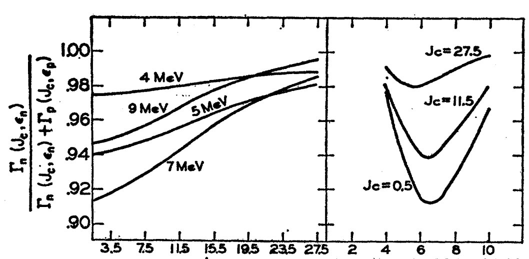

FIG. 11. The fraction of decays of compound nuclear states of spin J_c by average energy (2τ) neutrons when protons of various energies are allowed to compete. The compound nucleus is Y^{89}, produced by bombardment of Sr^{87} with 18-MeV deuterons.

proton emission was a relatively rare event in the cases considered. Indeed, if one were investigating the isomer ratios for the products resulting from proton or alpha-particle emission, one might expect significant effects from neutron competition.

Grover[52,53] has pointed out that gamma-ray emission can also compete with particle emission, particularly when there is a large amount of angular momentum present and when the excitation energy does not greatly exceed the binding energy of the particle to be emitted. We have investigated this problem in a qualitative manner and have concluded that by considering only the experimental data for bombarding energies greater than 4.5 MeV above the threshold of the reaction of interest we have eliminated any difficulties from this effect. More nearly quantitative calculations of the effect of gamma competition in the region near threshold are in progress.[54]

SUMMARY

All of the qualitative features of the isomer ratios for the different nuclear reactions studied are believed to be understood, as discussed in Sec. IV. Attempts to compare statistical model calculations with the data lead to the conclusion that the spin-cutoff parameter σ describing the angular momentum distribution of the level density is smaller than would be expected from a Fermi gas model without residual interactions. If the reduction in σ is interpreted in terms of a reduction from the rigid body moment of inertia, the observed reduction factor of $\mathcal{I}/\mathcal{I}_{\text{rigid}}=0.65$ is qualitatively consistent with what is known from other experiments. Attempts to account for the reduction of the spin-cutoff parameter by residual interactions of the type described by the superconductor model or the independent pairing model have not been very satisfying. The superconductor model requires a level density parameter a which is unreasonably large when related to the density of single particle states of a potential well of reasonable size.[40] The independent pairing model is more successful in describing the reduction in the moment of inertia with a more conventional level density parameter but has difficulties in reproducing other data for nuclear temperatures and nuclear level spacings at the neutron binding energy.[29] Competition from proton or alpha particle emission has been shown to be unimportant in influencing the isomer ratios for the reaction of interest.

ACKNOWLEDGMENTS

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[53] J. R. Grover, Phys. Rev. **127**, 2142 (1962).
[54] J. C. Norman, L. Haskin, and J. R. Huizenga (private communication).