Renormalization Effects of Leptonic Decay Coupling Constants in Broken SU_3 Symmetry*

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By analyzing the tensor $T_{\nu 3}{}^{\mu 3}$ in U_3, the theorem of Ademollo and Gatto on the nonrenormalization of the vector leptonic decay coupling constants to first order in symmetry breaking is reproduced. To second order, by analyzing $T_{\nu 33}{}^{\mu 33}$, it is found that there exists one sum rule among the strangeness-changing vector coupling constants. For the axial vector coupling constants there are in general two sum rules to first order in symmetry breaking. In the φ-ω mixing model one obtains an additional sum rule. With regard to the equal-time commutation relations of weak currents in broken SU_3, a theorem is obtained that the one-particle approximation always leads to results which are consistent only with exact symmetry. A qualitative discussion of the relation between the two results is given.

I. INTRODUCTION

THE three important assumptions in Cabibbo's theory[1] for the leptonic decays of baryons and pseudoscalar mesons are:

(A) Gell-Mann's suggestion[2] that the vector and the axial vector weak interaction currents of strongly interacting particles are particular components of F-spin current transforming like the octet representation of the SU_3 symmetry group.

(B) The strangeness-conserving and strangeness-changing currents, which are described, respectively, by F_1+iF_2 and F_4+iF_5, appear in combinations like

$$J=(\cos\theta)(F_1+iF_2)+(\sin\theta)(F_4+iF_5) \qquad (1)$$

for both vector and axial vector currents.

(C) All renormalization effects of the weak coupling constants due to strong interactions in broken SU_3 symmetry can be neglected.

Assumption (A) is very appealing since it automatically guarantees the rules $|\Delta \mathbf{I}|=\frac{1}{2}$ and $\Delta S=\Delta Q$, which appear to have been fairly well established in recent experiments.[3] Assumption (B), on the other hand, is rather mysterious in that the angle θ is assumed to have nothing to do with strong interactions, and for some unknown reason the weak interaction selects this particular combination. Without this assumption, however, it appears very difficult to accommodate assumption (A) to the experimental fact that the leptonic decay rates of baryons and pseudoscalar mesons for $|\Delta S|=1$ transitions are an order of magnitude smaller than those expected from the universal Fermi interaction theory.

From a theoretical point of view, assumption (C) does not seem to have a firm foundation, even with regard to the vector coupling constants in view of the violation of SU_3 symmetry manifested in mass differences between different isospin multiplets. Nevertheless, under this assumption, and with $\sin\theta_V=\sin\theta_A=0.26$, Cabibbo[1] has obtained a remarkable consistency between the leptonic decay rates of baryons and pseudoscalar mesons and experimental observations so far obtained. With regard to the renormalizations of weak currents due to symmetry-breaking, Sakurai[4] has discussed the effects on $|\Delta S|=1$ vector currents, and has calimed that $\sin\theta_V$ should be somewhat smaller than that of Cabibbo. Under these circumstances, it appears important to investigate the renormalization effects of strong interactions upon weak currents in broken SU_3 in order to justify Cabibbo's approach. This is the first objective of this paper.

From a point of view somewhat different from (C), Oehme[5] has recently argued that the equal-time commutation relations for the vector and the axial vector currents in broken SU_3, together with one-particle approximation, would lead to the result that the ratio of strangeness-changing and strangeness-conserving coupling constants, both vector and axial vector, is equal to m_π/m_K. Thus, the consistent reduction in the $|\Delta S|=1$ coupling strength is due the symmetry-breaking interaction, and therefore there is no need to introduce the angle θ explicitly in an *ad hoc* manner [or $\theta=45°$ in the

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[1] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963); **12**, 62 (1964).
[2] M. Gell-Mann, California Institute of Technology CTSL-20, 1961 (unpublished); Phys. Rev. **125**, 1067 (1962); Physics (to be published).
[3] Proceedings of the Brookhaven Conference on Fundamental Aspects of Weak Interactions, 1963, Brookhaven National Laboratory, Report No. BNL 837 (C-39) (unpublished); *Proceedings of the Sienna International Conference on Elementary Particles* (Societa Italiana di Fisica, Bologna, Italy, 1963).
[4] J. J. Sakurai, Phys. Rev. Letters **12**, 79 (1964).
[5] R. Oehme, Phys. Rev. Letters **12**, 550 (1964); **12**, 604(E) (1964).

form given by (1)]. As has been discussed more recently by Oehme and Segrè,[6] this approach does not seem to be promising as it stands at present. To the present authors, it appears that the correct answer to this question cannot be obtained in one-particle approximation, and it would require higher configurations to be included. This is the second objective of this paper.

II. THE VECTOR COUPLING CONSTANTS IN BROKEN SU_3

A. The Absence of Renormalization to First Order in Symmetry Breaking

Ademollo and Gatto[7] have recently given the theorem that the vector coupling constants for the leptonic decays of baryons and pseudoscalar mesons in broken SU_3 are not renormalized to first order in symmetry breaking at zero-momentum transfer. It is based on the assumptions that (i) the weak vector currents and electromagnetic current belong to the same unitary octet, and (ii) the symmetry-breaking interaction transforms like the eighth component of an octet.

As a preliminary to our discussion, we first reproduce the above theorem by means of tensor analysis in U_3. Following Okubo,[8] let us consider the tensor $T_{\nu 3}{}^{\mu 3}$ (μ, $\nu=1, 2, 3$) representing components of the vector octet current perturbed by the symmetry-breaking interaction which transforms like $T_3{}^3$ to first order. Then, under charge conjugation, it must satisfy the condition

$$CT_{\nu 3}{}^{\mu 3}C^{-1}=-T_{\mu 3}{}^{\nu 3}. \tag{2}$$

The assumption that the tensor is traceless imposes another condition

$$T_{\mu 3}{}^{\mu 3}=0. \tag{3}$$

For the baryon current $T_{\nu 3}{}^{\mu 3}$ must be bilinear in the baryon $N_\beta{}^\alpha$ and antibaryon $M_\beta{}^\alpha$ operators.[8] Under condition (2), the most general form of the tensor component can be written as follows (vector notations suppressed):

$$\begin{aligned}T_{\nu 3}{}^{\mu 3}=&A_1\delta_\nu{}^\mu(MN)+A_2\delta_3{}^\mu\delta_\nu{}^3(MN)+A_3\delta_\nu{}^\mu\{MN\}_3{}^3+A_4\delta_\nu{}^\mu[MN]_3{}^3\\&+A_5[\delta_3{}^\mu M_\lambda{}^3N_\nu{}^\lambda+\delta_\nu{}^3M_\lambda{}^\mu N_3{}^\lambda]+A_6[\delta_3{}^\mu M_\nu{}^\lambda N_\lambda{}^3+\delta_\nu{}^3M_3{}^\lambda N_\lambda{}^\mu]+A_7\{MN\}_\nu{}^\mu+A_8[MN]_\nu{}^\mu\\&+A_9[M_\nu{}^\mu N_3{}^3+M_3{}^3N_\nu{}^\mu]+A_{10}M_3{}^\mu N_\nu{}^3+A_{11}M_\nu{}^3N_3{}^\mu,\end{aligned} \tag{4}$$

where $A_1, \cdots, A_{11}$ are coefficients which in general depend upon momentum transfer, and

$$\begin{aligned}(MN)&=M_\beta{}^\alpha N_\alpha{}^\beta,\\ \{MN\}_\nu{}^\mu&=M_\alpha{}^\mu N_\nu{}^\alpha+M_\nu{}^\alpha N_\alpha{}^\mu,\\ [MN]_\nu{}^\mu&=M_\alpha{}^\mu N_\nu{}^\alpha-M_\nu{}^\alpha N_\alpha{}^\mu.\end{aligned} \tag{5}$$

From condition (3) we then obtain

$$3A_1+A_2+2A_7=0, \tag{6}$$

$$3A_3+3A_4+2A_5+A_{11}=0, \tag{7}$$

$$3A_3-3A_4+2A_6+A_{10}=0. \tag{8}$$

Relations (6), (7), and (8) arise as the vanishing of the coefficients of the linearly-independent terms (MN), $M_\lambda{}^3N_3{}^\lambda$ and $M_3{}^\lambda N_\lambda{}^3$ in (4), respectively.[9]

Under assumption (i) the electromagnetic current must transform like $T_{13}{}^{13}$ in U_3. By introducing $M_\beta{}^\alpha$ and $N_\beta{}^\alpha$ explicitly[8] into (4), one finds that at zero-momentum transfer, the conservation of $T_{13}{}^{13}$ implies

$$A_1=A_3=A_4=A_7=A_9=A_{10}=A_{11}=0. \tag{9}$$

We then obtain from (6), (7), (8), and (9)

$$A_2=A_5=A_6=0.$$

Thus, all of the A's vanish except A_8, and we have

$$T_{13}{}^{13}=A_8[MN]_1{}^1. \tag{10}$$

This being electric current, there exists no renormalization at zero momentum transfer. Since all vector currents belong to the term $[MN]_\nu{}^\mu$ under assumption (ii), there exists no renormalization in the vector coupling constants to first order in symmetry breaking at zero-momentum transfer. This is the theorem stated earlier.[7,10]

B. The Vector Coupling Constant Sum Rule to Second Order in Symmetry Breaking

The octet[11] of vector current perturbed to second order by the symmetry-breaking interaction must transform like the components of a tensor $T_{\nu 33}{}^{\mu 33}$ in U_3. Imposing the condition under charge conjugation similar to (2),

$$CT_{\nu 33}{}^{\mu 33}C^{-1}=-T_{\mu 33}{}^{\nu 33},$$

the most general expansion bilinear in $M_\beta{}^\alpha$ and $N_\beta{}^\alpha$ is

[6] R. Oehme and G. Segrè, Phys. Letters 11, 94 (1964).

[7] M. Ademollo and R. Gatto, Phys. Letters 13, 264 (1964).

[8] S. Okubo, Progr. Theoret. Phys. (Kyoto) 7, 949 (1962); 28, 24 (1962); Phys. Letters 4, 14 (1963); J. Phys. Soc. Japan (to be published).

[9] Since there are three relations (6), (7), and (8) among the eleven coefficients that occur in (4), there should be eight independent terms in (4) at this point. In the equivalent expansion of Ademollo and Gatto (Ref. 7), the term $\mathrm{Tr}(\bar{B}B)$ corresponding to the baryon current in their notation does not appear. However, of course, this does not invalidate their argument.

[10] After completion of this work the authors were shown another independent proof of the theorem by J. Nuyts and H. Ruegg (to be published). They wish to thank Professor L. M. Brown for calling their attention to this work.

[11] A similar sum rule for the pseudoscalar mesons can be obtained. However, it would involve experimentally unobservable $\eta\pi$ and ηK—leptonic decay coupling constants, and, therefore, is not explicitly given here.

given by

$$
\begin{aligned}
T_{\nu 33}{}^{\mu 33} = {} & B_1\delta_\nu{}^\mu(MN)+B_2\delta_\nu{}^\mu[MN]_3{}^3+B_3\delta_\nu{}^\mu\{MN\}_3{}^3+B_4\delta_\nu{}^\mu M_3{}^3N_3{}^3+B_5\delta_3{}^\mu\delta_\nu{}^3(MN)+B_6\delta_3{}^\mu\delta_\nu{}^3[MN]_3{}^3 \\
& +B_7\delta_3{}^\mu\delta_\nu{}^3\{MN\}_3{}^3+B_8[\delta_3{}^\mu M_\lambda{}^3N_\nu{}^\lambda+\delta_\nu{}^3M_\lambda{}^\mu N_3{}^\lambda]+B_9[\delta_3{}^\mu M_\nu{}^\lambda N_\lambda{}^3+\delta_\nu{}^3M_3{}^\lambda N_\lambda{}^\mu] \\
& +B_{10}[\delta_3{}^\mu M_\nu{}^3N_3{}^3+\delta_\nu{}^3M_3{}^3N_3{}^\mu]+B_{11}[\delta_3{}^\mu M_3{}^3N_\nu{}^3+\delta_\nu{}^3M_3{}^\mu N_3{}^3] \\
& +B_{12}[M_\nu{}^\mu N_3{}^3+M_3{}^3N_\nu{}^\mu]+B_{13}M_3{}^\mu N_\nu{}^3+B_{14}M_\nu{}^3N_3{}^\mu+B_{15}\{MN\}_\nu{}^\mu+B_{16}[MN]_\nu{}^\mu, \qquad (11)
\end{aligned}
$$

where $B_1, \cdots, B_{16}$ are coefficients which in general depend upon momentum transfer.

The traceless conditions (3) then requires

$$
\begin{aligned}
3B_1+B_5+2B_{15}&=0, \\
3B_2+B_6+B_8-B_9-\tfrac{1}{2}B_{13}+\tfrac{1}{2}B_{14}&=0, \\
3B_3+B_7+B_8+B_9+\tfrac{1}{2}B_{13}+\tfrac{1}{2}B_{14}&=0, \\
3B_4+2B_{10}+2B_{11}&=0.
\end{aligned} \qquad (12)
$$

Since there are four relations (12) among 16 parameters, $B_1, \cdots, B_{16}$, there should be 12 independent parameters. However, electric current ($T_{133}{}^{133}$) conservation implies that at zero momentum transfer we have

$$B_1=B_2=B_3=B_4=B_{12}=B_{13}=B_{14}=B_{15}=0.$$

Combining with (12), we further get

$$B_5=B_{10}+B_{11}=0.$$

Introducing these into (11), we obtain

$$T_{133}{}^{133}=B_{16}[MN]_1{}^1, \qquad (13)$$

$$T_{233}{}^{133}=B_{16}[MN]_2{}^1, \qquad (14)$$

and

$$
\begin{aligned}
T_{333}{}^{133}= {} & (B_8+B_{16})M_\lambda{}^1N_3{}^\lambda+(B_9-B_{16})M_3{}^\lambda N_\lambda{}^1 \\
& +B_{10}(M_3{}^3N_3{}^1-M_3{}^1N_3{}^3). \qquad (15)
\end{aligned}
$$

$T_{133}{}^{133}$ is the conserved electric current similar to (10). $T_{233}{}^{133}$ is the $\Delta S=0$, $|\Delta \mathbf{I}|=1$ weak vector current. Thus, in relation to (13), (14) is nothing more than the conserved vector current hypothesis to second order in symmetry-breaking. $T_{333}{}^{133}$ is the $\Delta S=1$, $|\Delta\mathbf{I}|=\frac{1}{2}$ weak vector current. Introducing $M_\beta{}^\alpha$ and $N_\beta{}^\alpha$ explicitly,[8] we obtain, in addition to the results of the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule,

$$g_{\Sigma^-n}{}^V=\sqrt{2}g_{\Sigma^0p}{}^V, \qquad (16)$$

$$g_{\Xi^0\Sigma^+}{}^V=\sqrt{2}g_{\Xi^-\Sigma^0}{}^V, \qquad (17)$$

a sum rule to second order in symmetry-breaking at zero momentum transfer

$$(\sqrt{6})(g_{\Lambda p}{}^V+g_{\Xi^-\Lambda}{}^V)+\sqrt{2}(g_{\Sigma^0p}{}^V+g_{\Xi^-\Sigma^0}{}^V)=0, \qquad (18)$$

or

$$(\sqrt{6})(g_{\Lambda p}{}^V+g_{\Xi^-\Lambda}{}^V)+g_{\Sigma^-n}{}^V+g_{\Xi^0\Sigma^+}{}^V=0. \qquad (19)$$

III. THE AXIAL VECTOR COUPLING CONSTANTS IN BROKEN SU_3

A. The Coupling Constant Sum Rules to First Order in Symmetry-Breaking

For the axial vector currents we have, in place of (2), the condition

$$CT_{\nu 3}{}^{\mu 3}C^{-1}=T_{\mu 3}{}^{\nu 3}. \qquad (20)$$

The general expansion (4) and the relations (6), (7), and (8) still hold under (20) and (3). The $\Delta S=0$, $|\Delta\mathbf{I}|=1$ current and $\Delta S=1$, $|\Delta\mathbf{I}|=\frac{1}{2}$ current are now to be identified with the components $T_{23}{}^{13}$ and $T_{33}{}^{13}$, respectively. We then obtain

$$
\begin{aligned}
T_{23}{}^{13}(\Delta S=0)= {} & (C_7+C_8+C_{10})\bar{p}n+\sqrt{2}C_8\bar{\Sigma}^0\Sigma^--\sqrt{2}C_8\bar{\Sigma}^+\Sigma^0+(2/\sqrt{6})(C_7-C_9)\bar{\Lambda}\Sigma^- \\
& +(2/\sqrt{6})(C_7-C_9)\bar{\Sigma}^+\Lambda+(C_7-C_8+C_{11})\bar{\Xi}^0\Xi^-, \qquad (21)
\end{aligned}
$$

$$
\begin{aligned}
T_{33}{}^{13}(\Delta S=1)= {} & (C_6+C_7-C_8)\bar{n}\Sigma^-+(1/\sqrt{2})(C_6+C_7-C_8)\bar{p}\Sigma^0+(1/\sqrt{2})(C_5+C_7+C_8)\bar{\Sigma}^0\Xi^-+(C_5+C_7+C_8)\bar{\Sigma}^+\Xi^0 \\
& +(1/\sqrt{6})[-2(C_5+C_7+C_8)+(C_6+C_7-C_8)-2(C_9+C_{10})]\bar{p}\Lambda \\
& +(1/\sqrt{6})[(C_5+C_7+C_8)-2(C_6+C_7-C_8)-2(C_9+C_{11})]\bar{\Lambda}\Xi^-, \qquad (22)
\end{aligned}
$$

where $C_5, \cdots, C_{11}$ are constants. We have again omitted $\gamma_\mu\gamma_5$ for simplicity.

From (21) we get immediately the relations

$$g_{\Sigma^-\Sigma^0}{}^A=-g_{\Sigma^0\Sigma^+}{}^A, \qquad (23)$$

$$g_{\Sigma^-\Lambda}{}^A=g_{\Lambda\Sigma^+}{}^A. \qquad (24)$$

These relations for the first class weak interaction follow from isospin rotation invariance of strong interactions and must hold to all orders in symmetry-breaking. The relation (23), arising from the terms with C_8 in (21), has its origin in the F-type current in (4). Relation (24), arising from the terms C_7-C_9, has its origin in the D-type current in (4).

From (22) we note immediately the relations

$$g_{\Sigma^-n}{}^A=\sqrt{2}g_{\Sigma^0p}{}^A, \qquad (25)$$

$$g_{\Xi^0\Sigma^+}{}^A=\sqrt{2}g_{\Xi^-\Sigma^0}{}^A. \qquad (26)$$

These relations are simply the results of the $|\Delta\mathbf{I}|=\frac{1}{2}$ rule, and are analogous to (16) and (17).

Since the twelve coupling constants are expressed linearly in terms of the seven parameters $C_5, \cdots, C_{11}$, one would expect five relations (sum rules) among the coupling constants, of which four are given by (23), (24), (25), and (26). However, because of the particular combinations of constants in (21) and (22), we get the following two independent sum rules:

$$g_{np}{}^A+g_{\Xi^0\Sigma^+}{}^A-(1/\sqrt{2})g_{\Sigma^-\Sigma^0}{}^A-\tfrac{1}{2}g_{\Sigma^0 n}{}^A -(\sqrt{\tfrac{3}{2}})(g_{\Sigma^-\Lambda}{}^A-g_{\Lambda p}{}^A)=0\,, \quad (27)$$

$$g_{\Xi^-\Xi^0}{}^A+g_{\Sigma^- n}{}^A+(1/\sqrt{2})g_{\Sigma^-\Sigma^0}{}^A-\tfrac{1}{2}g_{\Xi^0\Sigma^+}{}^A -(\sqrt{\tfrac{3}{2}})(g_{\Sigma^-\Lambda}{}^A-g_{\Xi^-\Lambda}{}^A)=0\,. \quad (28)$$

Thus, besides the four relations (23), (24), (25), (26) which hold to all orders in symmetry-breaking, there are, in general, two sum rules for eight of the twelve axial vector coupling constants to first order in symmetry-breaking at zero-momentum transfer. In a particular model of $\varphi-\omega$ mixing for symmetry-breaking, there exists one additional sum rule (appendix).

B. To Second Order in Symmetry-Breaking

There would be relations analogous to (12) to second order in symmetry-breaking, which again lead to twelve independent coefficients in the expansion of the axial vector current $T_{\nu 33}{}^{\mu 33}$. However, in contrast to the vector current, the lack of conservation law does not allow additional restrictions among the coefficients, and therefore there exists no sum rule analogous to (18) or (19).

IV. USE OF EQUAL-TIME COMMUTATION RELATIONS FOR WEAK CURRENTS IN BROKEN SU_3 SYMMETRY

A. Weak Currents for Baryons

In this section, we state and prove the theorem: Within one-particle approximation, the equal-time commutation relations for weak currents in broken SU_3 symmetry are consistent only in the limit of exact symmetry.

This theorem was suggested by Oehme and Segrè[6] with respect to the form factors of the weak vector currents for pseudoscalar mesons. Here we wish to discuss first the weak currents for the octet baryons in some detail and then briefly take up the case of the pseudoscalar mesons which has been discussed by the above authors. The following set of equal-time commutation relations among currents[2] is sufficient for our purpose for the case of the baryons:

$$\left[\int d\mathbf{x}V_4{}^{1+i2}(x),\partial_\beta{}'A_\beta{}^{4-i5}(x')\right]_{t=t'} =-i[v_6(\mathbf{x}',t)-iv_7(\mathbf{x}',t)](\sqrt{\tfrac{2}{3}}-c\sqrt{\tfrac{1}{12}})\,, \quad (29)$$

$$\left[\int d\mathbf{x}V_4{}^{4-i5}(x),\partial_\beta{}'A_\beta{}^{1+i2}(x')\right]_{t=t'} =i[v_6(\mathbf{x}',t)-iv_7(\mathbf{x}',t)](\sqrt{\tfrac{2}{3}}+c\sqrt{\tfrac{1}{3}})\,, \quad (30)$$

where

$$V_4{}^{1+i2}(x)=F_{14}(x)+iF_{24}(x)\,,$$
$$A_\beta{}^{4-i5}(x)=F_{4\beta}{}^5(x)-iF_{5\beta}{}^5(x)\,,\text{ etc.}$$

in Gell-Mann's notation.[2] v_6 and v_7 are pseudoscalar densities, and c is a constant parameter for the strength of the symmetry-breaking interaction.[2] The F-spin component of the system is given by

$$F_i=-i\int F_{i4}(\mathbf{x},t)d\mathbf{x}. \quad (31)$$

We begin our discussion by taking the matrix element of (29) between the neutron and the Σ^0 states with one-particle intermediate states.

$$\sum\left[\langle\Sigma^0|\int d\mathbf{x}\;V_4{}^{1+i2}(x)|\Sigma^-\rangle\langle\Sigma^-|\partial_\beta{}'A_{\beta 4}{}^{4-i5}(x')|n\rangle-\langle\Sigma^0|\partial_\beta{}'A_{\beta 4}{}^{4-i5}(x')|p\rangle\langle p|\int d\mathbf{x}\;V_4{}^{1+i2}(x)|n\rangle\right]_{t=t'} =-i(\sqrt{\tfrac{2}{3}}-c\sqrt{\tfrac{1}{12}})\langle\Sigma^0|v_6(\mathbf{x}',t)-iv_7(\mathbf{x}',t)|n\rangle\,, \quad (32)$$

where the summation is over the spin and momentum states of the intermediate particle. Since F_1+iF_2 from (31) is conserved, the only intermediate states in (32) are those elastic one-particle states indicated. Thus, (32) is an exact equation even in broken $SU_3(c\neq 0)$. It can be shown easily that the following results from (32):

$$\sum[g_{\Sigma^-\Sigma^0}{}^V(0)g_{\Sigma^- n}{}^A(t_{\Sigma^- n})(m_\Sigma+m_N)(\bar{\Sigma}^0\gamma_4\Sigma^-)(\bar{\Sigma}^-\gamma_5 n)-g_{\Sigma^0 p}{}^A(t_{\Sigma^0 p})g_{np}{}^V(0)(m_\Sigma+m_N)(\bar{\Sigma}^0\gamma_5 p)(\bar{p}\gamma_4 n) =i(\sqrt{\tfrac{2}{3}}-c\sqrt{\tfrac{1}{12}})\langle\Sigma^0|v_6(0)-iv_7(0)|n\rangle\,, \quad (33)$$

where the indicated summation is over the spin states of intermediate particles, and $t_{\Sigma^- n}$ and $t_{\Sigma^0 p}$ are appropriate momentum transfers.

Proceeding similarly with (30), we obtain

$$\sum[g_{\Sigma^0 p}{}^V(0)g_{np}{}^A(t_{np})2m_N(\bar{\Sigma}^0\gamma_4 p)(\bar{p}\gamma_5 n)-g_{\Sigma^-\Sigma^0}{}^A(t_{\Sigma^-\Sigma^0})g_{\Sigma^- n}{}^V(0)2m_\Sigma(\bar{\Sigma}^0\gamma_5\Sigma^-)(\bar{\Sigma}^-\gamma_4 n)] =-i(\sqrt{\tfrac{2}{3}}+c\sqrt{\tfrac{1}{3}})\langle\Sigma^0|v_6(0)-iv_7(0)|n\rangle\,. \quad (34)$$

Since the (F_4-iF_5) that occurs in the above is not conserved in broken SU_3, in contrast to (33), (34) is a result of the approximation in which only elastic one-particle intermediate states are taken into account.

For our purpose it suffices to compare terms linear in momenta $\mathbf{p}_n$ and $\mathbf{p}_{\Sigma^0}$ in the expansions of the left-hand sides of (33) and (34). By eliminating coefficients dependent upon c from the two relations obtained, we get

$$(g_{\Sigma^0\Sigma^-}{}^V g_{\Sigma^-n}{}^A - g_{\Sigma^0 p}{}^A g_{np}{}^V)(m_\Sigma - m_N) = 0\,, \qquad (35)$$

where the coupling constants are those at zero momentum transfer. According to the conserved vector current hypothesis generalized to SU_3 symmetry and the $|\Delta\mathbf{I}| = \frac{1}{2}$ rule, the following relations hold to all orders in symmetry-breaking:

$$g_{\Sigma^-\Sigma^0}{}^V = -\sqrt{2}\, g_{np}{}^V\,, \qquad (36)$$

$$g_{\Sigma^-n}{}^A = \sqrt{2}\, g_{\Sigma^0 p}{}^A\,. \qquad (37)$$

Hence we obtain from (35) the mass relation in exact SU_3 symmetry, i.e., $m_\Sigma = m_N$, provided $g_{\Sigma^-n}{}^A = \sqrt{2} g_{\Sigma^0 p}{}^A \neq 0$.[12]

Taking the matrix element of (29) and (30) between the Σ^0 and the Ξ^0 states and proceeding in the same manner, we find $m_\Sigma = m_\Xi$.

For the purpose of showing that $m_\Lambda = m_N$ results from the approximation we take the matrix elements of (29) and (30) between the neutron state and the Λ state. Since the $\bar{\Lambda}\Sigma$ current is entirely D type, the unrenormalized $g_{\Sigma^-\Lambda}{}^V$ vanishes at zero momentum transfer. Thus, the corresponding renormalized coupling constant at zero momentum transfer must likewise vanish. This fills the lack of relations in the present case analogous to (36) and (37), and we obtain

$$g_{\Lambda p}{}^V g_{np}{}^A (m_\Lambda - m_N) = \tfrac{1}{2} g_{\Sigma^-\Lambda}{}^A g_{\Sigma^-n}{}^V (m_\Lambda + m_\Sigma)(1 - m_N/m_\Sigma)\,.$$

Combined with the previous result ($m_\Sigma = m_N$), we get $m_\Lambda = m_N$.

In this manner we find that in the elastic one-particle approximation the equal-time commutation relations of vector and axial vector baryon currents in broken SU_3 symmetry lead to results which are consistent only with exact SU_3 symmetry. Thus, any deviations of the vector and axial vector coupling constants from those of exact SU_3 symmetry must at least be due to inelastic matrix elements of the currents which are necessarily present in broken SU_3 symmetry. Needless to say, in the limit of exact SU_3 symmetry, elastic one-particle matrix elements satisfy trivially the equal-time commutation relations.

B. Weak Currents for Pseudoscalar Mesons

As was mentioned earlier, the case of the pseudoscalar mesons in one-particle approximation has already been discussed by Oehme *et al.*[5,6] However, these authors did not keep the energy dependence of the form factors. Maintaining the energy dependence, the part of their results relevant to our discussion can be written as follows[13]:

$$\frac{E_\pi + E_K + (E_K - E_\pi)\xi(t)}{2E_K}\,\frac{F_{K\pi}(t)}{F_{\pi\pi}(0)} \approx \frac{B_\pi m_\pi^2}{B_K m_K^2}\cdot\frac{(1 - c/2\sqrt{2})}{(1 + c/\sqrt{2})}\,, \qquad (38)$$

$$\frac{E_\pi + E_K + (E_K - E_\pi)\xi(t)}{2E_\pi}\,\frac{F_{K\pi}(t)}{F_{KK}(0)} \approx \frac{B_K m_K^2}{B_\pi m_\pi^2}\cdot\frac{(1 + c/\sqrt{2})}{(1 - c/2\sqrt{2})}\,, \qquad (39)$$

where $\mathbf{p}_\pi = \mathbf{p}_K$ in each of the relations independently, and $t = (E_K - E_\pi)^2$. According to the conserved vector current hypothesis, we may set $F_{\pi\pi}(0) = F_{KK}(0) = 1$. Since the right-hand sides of (38) and (39) are independent of the momentum, by comparing the two relations, we obtain, as the only consistent results,

$$m_\pi = m_K\,, \qquad (40)$$

$$F_{K\pi}(0) = 1\,. \qquad (41)$$

It is to be recalled that in obtaining (40) and (41) no assumption was introduced about the magnitude of symmetry-breaking coupling constant c. We cannot conclude, from the two relations (38) and (39) alone, that $B_\pi = B_K$ and $c = 0$. This situation is similar to the results in Secs. II and III that for vector coupling constants we have the theorem on the absence of renormalization effects to first order in symmetry-breaking but not for axial vector coupling constants.

V. SUMMARY AND CONCLUSIONS

We have attempted to investigate the problem of the renormalization of the weak currents which may exist because of symmetry breaking from three different points of view. These are (1) analysis of the octet current in U_3, (2) the specific model of φ-ω mixing, and (3) the equal-time commutation relations of the weak currents.

Under the usual assumptions for the electromagnetic current and the weak vector currents and for the symmetry-breaking interaction in SU_3, the theorem of Ademollo and Gatto has been reproduced by analyzing the tensor $T_{\nu 3}{}^{\mu 3}$ which represents the octet current perturbed by the symmetry breaking to first order. By analyzing the octet current perturbed to second order, $T_{\nu 33}{}^{\mu 33}$, one sum rule for the strangeness-changing vector coupling constants was obtained.

The nonexistence of renormalization effects in the vector coupling constants to first order in symmetry

[12] Similar assumptions are to be implicitly understood for the following argument.

[13] Equations (38) and (39) can be reduced to the first relation of Eqs. (9) and (10) [$-\xi$ should be replaced with $+\xi$ in Eq. (10)] in Ref. 6, if we assume $E_K \approx M_K$, $E_\pi \approx M_\pi$ and neglect higher powers of M_π/M_K. The second relation of Eq. (9) in Ref. 6 is not essential for our discussions.

breaking may indicate small corrections, if any, for $|\Delta S|=1$ vector currents. This would provide a support for Cabibbo's estimate of θ_V, and is likely to make Sakurai's estimate[4] of the $|\Delta S|=1$ vector coupling renormalization effects less reliable.

For the axial vector baryon currents to first order in symmetry breaking, $T_{\nu 3}{}^{\mu 3}$ allows us to obtain in general two sum rules for eight of the twelve coupling constants at zero momentum transfer. In a particular model of φ-ω mixing which treats symmetry-breaking in the first order, there exists one more sum rule among the eight coupling constants as is shown in the appendix. The lack of conservation law for the axial vector current $T_{\nu 33}{}^{\mu 33}$ does not allow sum rules to second order.

With regard to the equal-time commutation relations of weak currents in broken symmetry, we have analyzed weaker commutation relations (29) and (30), essentially equivalent to (5.19) in Ref. 2, and obtained a theorem that elastic one particle approximation always leads to results which are consistent only with exact SU_3 symmetry. Thus, any deviations of the weak coupling constants, both vector and axial vector, from those of exact SU_3 must be related to inelastic matrix elements of the currents. The relation between this theorem and the results of the earlier part of this work is not of a quantitative nature at this time. Only qualitatively can it be said that the absence of renormalization in vector coupling constants in the lowest order in symmetry breaking is consistent with the above theorem.

It should be noted also that by analyzing the stronger equal-time commutation relations among weak current densities like

$$[V_4{}^{1+i2}(\mathbf{x},t),\partial_\beta' A_\beta{}^{4-i5}(\mathbf{x}',t)] = -i\delta(\mathbf{x}-\mathbf{x}')[v_6(\mathbf{x},t)-iv_7(\mathbf{x},t)](\sqrt{\tfrac{2}{3}}-c\sqrt{\tfrac{1}{12}})$$

one may obtain more information about the momentum-transfer dependence of form factors of weak currents in broken SU_3 symmetry. This is already suggested by Gell-Mann in connection with charge form factor of the pion.[2]

Within one-particle approximation, Oehme and Segrè[6] have argued that, under the conditions specified by them, the equal-time commutation relations could lead to a relation for the K and π decay coupling constants like

$$m_K g(\Delta S\neq 0)\approx m_\pi g(\Delta S=0).$$

When one looks at the axial vector coupling constant sum rules (27) and (28), it seems rather unlikely that these would be a simple relation like the above since the sum rules are of the form

$$\sum_i a_i g_i(\Delta S\neq 0)=\sum_j a_j' g_j(\Delta S=0)$$

with a_i and a_j' being numerical constants both of the same order of magnitude. Several experimental evidences in support of the absence of renormalization effects in vector currents have already been pointed out by Ademollo and Gatto.[7] Assuming that effects of the symmetry-breaking interaction are small in the dynamics of strong interactions based on SU_3 symmetry, in the event that the sum rules (18), (19), (27), and (28) are verified experimentally, one may safely conclude that the Cabibbo angles θ_V and θ_A have nothing to do with symmetry breaking in SU_3 as they were originally postulated.

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APPENDIX: RENORMALIZATION EFFECTS IN THE φ-ω MIXING MODEL

We wish to show that the results obtained in Secs. II and III can also be obtained in a dynamical model for symmetry breaking arising from φ-ω mixing.[14] For instance, by coupling the lepton current to every F_1+iF_2 or F_4+iF_5 vector current that occurs in the lowest order self-energy diagram of the model, it can be shown that the mass and wave function renormalization procedures establish at zero momentum transfer the nonrenormalization of all vector coupling constants to first order in symmetry-breaking (Sec. II).

In the case of the axial vector coupling constants, it will be shown at zero-momentum transfer that (i) treating the F and D ratio in the couplings of the φ meson to the baryons as a known parameter, one more sum rule is obtained in addition to those in Sec. (III), and (ii) the f_A and d_A ratio in the baryon axial vector current can be expressed in terms of the renormalized axial vector coupling constants.

We shall exhibit our procedure by taking the leptonic decay $\Lambda\to p+e^-+\bar\nu$ as an example. By coupling the lepton current to all F_4+iF_5 and D_4+iD_5 axial vector currents that occur in the lowest order self-energy diagram of the model, it can be shown that the renormalized coupling constant at zero momentum transfer is given by

$$(G/\sqrt{2})\{[-(\sqrt{\tfrac{3}{2}})f_A-(\sqrt{\tfrac{1}{6}})d_A] +\sqrt{3}(g_F-g_D)[-(\sqrt{\tfrac{3}{2}})f_A-(\sqrt{\tfrac{1}{6}})d_A](I_1+2I_2) -\sqrt{3}(g_F+\tfrac{1}{3}g_D)[-\sqrt{3}f_A'-(\sqrt{\tfrac{1}{3}})d_A']I_3\}, \quad (A1)$$

where g_F and g_D are the coupling constants between φ

[14] J. J. Sakurai, Phys. Rev. **132**, 434 (1963).

and the F- and the D-type baryon currents; $(G/\sqrt{2})f_A$ and $(G/\sqrt{2})d_A$ are the F- and D-type axial vector baryon current and lepton current coupling constants; and $(G/\sqrt{2})f_A'$ and $(G/\sqrt{2})d_A'$ are the F-type and the D-type axial-vector currents of the octet of vector mesons and lepton current coupling constants. I_1 and I_2 are the integrals for the self-energy diagrams in which the lepton current is coupled to baryon currents internally and externally, respectively. I_3 is the integral for the diagram in which the lepton current is coupled internally to currents of the vector mesons.

Proceeding in the above manner we obtain the following expressions for the twelve axial vector coupling constants

$$\Delta S=0$$

$$\begin{aligned}
g_{np}{}^A&=(G/\sqrt{2})[(f_A+d_A)(1+2(1-\tfrac{1}{3}r)a)+(2\sqrt{2}/3)(1+r)b],\\
g_{\Sigma^0\Sigma^+}{}^A&=(G/\sqrt{2})[-\sqrt{2}f_A(1+\tfrac{4}{3}ra)-\tfrac{4}{3}b],\\
g_{\Lambda\Sigma^+}{}^A&=(G/\sqrt{2})[\sqrt{\tfrac{2}{3}}d_A+(4/3\sqrt{3})rb],\\
g_{\Sigma^-\Sigma^0}{}^A&=(G/\sqrt{2})[\sqrt{2}f_A(1+\tfrac{4}{3}ra)+\tfrac{4}{3}b],\\
g_{\Sigma^-\Lambda}{}^A&=G/\sqrt{2}[\sqrt{\tfrac{2}{3}}d_A+(4/3\sqrt{3})rb],\\
g_{\Xi^-\Xi^0}{}^A&=(G/\sqrt{2})[(-f_A+d_A)(1-2(1+\tfrac{1}{3}r)a)-(2\sqrt{2}/3)(1-r)b].
\end{aligned}\tag{A2}$$

$$\Delta S=1$$

$$\begin{aligned}
g_{\Sigma^-n}{}^A&=(G/\sqrt{2})[(-f_A+d_A)(1+(1+\tfrac{1}{3}r)a)+\sqrt{2}(1-r)c],\\
g_{\Lambda p}{}^A&=(G/\sqrt{2})[(-\sqrt{\tfrac{3}{2}}f_A-\sqrt{\tfrac{1}{6}}d_A)(1+(1-r)a)+\sqrt{3}(1+\tfrac{1}{3}r)c],\\
g_{\Sigma^0p}{}^A&=(G/\sqrt{2})[(1/\sqrt{2})(-f_A+d_A)(1+(1+\tfrac{1}{3}r)a)+(1-r)c],\\
g_{\Xi^-\Sigma^0}{}^A&=(G/\sqrt{2})[(1/\sqrt{2})(f_A+d_A)(1-(1-\tfrac{1}{3}r)a)-(1+r)c],\\
g_{\Xi^-\Lambda}{}^A&=(G/\sqrt{2})[(\sqrt{\tfrac{3}{2}}f_A-\sqrt{\tfrac{1}{6}}d_A)(1-(1+\tfrac{1}{3}r)a)-\sqrt{3}(1-\tfrac{1}{3}r)c],\\
g_{\Xi^0\Sigma^+}{}^A&=(G/\sqrt{2})(f_A+d_A)(1-(1-\tfrac{1}{3}r)a)-2(1+r)c],
\end{aligned}\tag{A3}$$

where

$$\begin{aligned}
a&=g_F\sqrt{3}(I_1+2I_2),\\
b&=g_F2\sqrt{3}d_A'I_3,\\
c&=g_F\sqrt{3}(f_A'+\tfrac{1}{3}d_A')I_3,\\
r&=g_D/g_F=\frac{3}{2}\,\frac{m_\Sigma-m_\Lambda}{m_N-m_\Xi}.
\end{aligned}\tag{A4}$$

The parameter r is simply related to the baryon masses as shown in the φ-ω mixing model ($r\approx -0.30$, indicating that according to this model the vector meson-baryon coupling is dominantly F type).

We note that there are six independent parameters in (A2) and (A3), i.e., f_A, d_A, r, a, b, and c, for the twelve expressions. Thus, there should be six relations among the twelve coupling constants. It is evident that relations (23), (24), (25), and (26) are trivially satisfied. It can be shown that the sum rules (27) and (28) are also satisfied. If r is treated as a known parameter, one obtains in addition the following sum rule:

$$\begin{aligned}
&[1-(7/3)r^2](1+\tfrac{1}{3}r)g_{np}{}^A+[1-(7/3)r^2](1-\tfrac{1}{3}r)g_{\Xi^-\Xi^0}{}^A-6(\sqrt{\tfrac{3}{2}})(1-r^2)g_{\Sigma^-\Lambda}{}^A-(\sqrt{2}/3)r(5+\tfrac{2}{3}r^2)g_{\Sigma^-\Sigma^0}{}^A\\
&\quad+2(\sqrt{\tfrac{2}{3}})(r)(1-\tfrac{1}{3}r^2)(g_{\Lambda p}{}^A-g_{\Xi^-\Lambda}{}^A)+2(1-\tfrac{1}{3}r^2)(g_{\Sigma^-n}{}^A+g_{\Xi^0\Sigma^+}{}^A)=0.
\end{aligned}\tag{A5}$$

Also one obtains from (A2) and (A3)

$$\frac{f_A}{d_A}=\frac{g_{\Lambda\Sigma^+}{}^A-(\sqrt{\tfrac{1}{6}})(1+\tfrac{1}{3}r)g_{np}{}^A-(\sqrt{\tfrac{1}{6}})(3+\tfrac{1}{3}r)g_{\Xi^-\Xi^0}{}^A-(r/3\sqrt{3})g_{\Sigma^0\Sigma^+}{}^A}{rg_{\Lambda\Sigma^+}{}^A-(\sqrt{\tfrac{1}{6}})(1+r)g_{np}{}^A+(\sqrt{\tfrac{1}{6}})(1-r)g_{\Xi^-\Xi^0}{}^A-(1/\sqrt{3})g_{\Sigma^0\Sigma^+}{}^A}.$$

In the limit of exact SU_3 symmetry, $f_A/d_A=0.30/0.95$ was obtained by Cabibbo.[1,15]

[15] For the more recent value, see W. Willis, H. Courant, H. Filthuth, P. Franzini, A. Minguzzi-Ranzi, *et al.*, Phys. Rev. Letters **13**, 291 (1964); N. Breme, B. Hellesen, and M. Roos, Phys. Letters **11**, 344 (1964).