could be due to changes in field gradient with angle θ of the magnet. Undoubtedly, the impurities might be paramagnetic and of concentration 1 part in 10^8 but it would be hard to understand such electron spin producing a moment locked at all to the lattice. Besides, the effects we report are independent of temperature. The possibility of these magnetic effects being used to study the mechanism of point defects in a quantitative way opens up an interesting research program. For such quantitative studies it will be necessary to know the number of defects. Directional properties of the x radiation, effects of γ radiation, and other questions are being investigated as the work moves along.

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Phenomenological Interpretation of Experiments on the Magnetic Properties of Radiation-Damaged LiF

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The experimental observations in the preceding paper by Adair and Squire have a reasonable phenomenological interpretation. There is a quantitative agreement between theory and experiment. The magnetization due to radiation damage exhibits a residual value in zero external field and is described by a susceptibility tensor with off-diagonal terms in the presence of a magnetic field.

THE phenomena reported by Adair and Squire (A. & S.) in the preceding paper[1] suggest that:

(i) The susceptibility tensor χ describing the magnetism due to damage of the LiF crystal does not have the symmetry properties required by a perfect lattice, so that either $\chi_{xx}-\chi_{yy}$ or $\chi_{xy}+\chi_{yx}$ or both does not vanish as it should. In the following, we write χ_{xy} for a suitable linear combination of these quantities and define the x axis accordingly; the z axis is along the fiber of the torsion pendulum and is also along a crystal axis.

(ii) The magnetization $\mathbf{M}$ of the electrons trapped at vacancies or elsewhere in the crystal does not vanish when the applied external field is removed but takes on some residual value $\mathbf{M}_f$. $\mathbf{M}_f$ will depend in general on the history of the magnetization of the crystal. We assume that $\mathbf{M}_f$ is nearly parallel to the field which has most recently been removed. We neglect the diamagnetism of LiF because it plays no role in this experiment.

(iii) The magnetization exhibits relaxation phenomena. From our Fig. 1, the torque $\mathbf{T}$ acting on the system at equilibrium position in a field $\mathbf{H}$ is

$$T=K(\theta-\theta_0)+\chi_{xy}H^2\cos2\theta+M_fH\sin\theta_f\,, \quad (1)$$

where K is the torsion constant of the fiber, θ is the angle between the applied field $\mathbf{H}$ and a vector defined by the crystal lattice (the x axis), and θ_f is the angle between $\mathbf{H}$ and $\mathbf{M}_f$ ($\mathbf{H}$ is the field presently being applied to the crystal; $\mathbf{M}_f$ is parallel to the field most recently removed from the crystal). θ_0 is the angle between the x axis and the field direction when $H=0$; that is, θ_0 is the root of the equation $\mathbf{T}=0$ when $\mathbf{H}=0$. Let θ_e, θ_{ef} denote the values of θ and θ_f which satisfy the equation $\mathbf{T}=0$ when $\mathbf{H}\neq0$. Define Φ as follows:

$$\theta=\theta_e+\theta-\theta_e\equiv\theta_e+\Phi\,,\qquad \theta_f=\theta_{ef}+\theta_f-\theta_{ef}=\theta_{ef}+\Phi\,. \quad (2)$$

The assumption that Φ is small and the definitions of θ_e and θ_{ef} yield

$$T=K\Phi-2\chi_{xy}H^2(\sin2\theta_e)\Phi+M_fH(\cos\theta_{ef})\Phi.$$

Because of the relaxation phenomena when the crystal

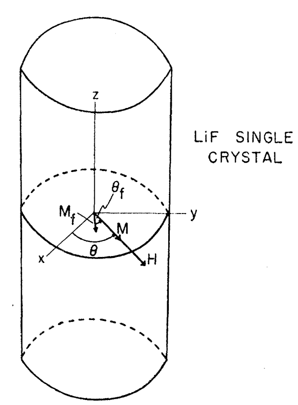

FIG. 1. The LiF crystal at rest in a uniform magnetic field H whose direction makes an angle θ in the x-y plane of the simple cubic axis. The magnetization M_f is the residual magnetization left fixed to the crystal lattice by a previous history of a field along M_f.

[1] T. W. Adair and C. F. Squire, preceding paper, Phys. Rev. **137**, A949 (1964).

is oscillating, **M** may have a component $M_\perp$ perpendicular to **H** which arises from the diagonal part of the susceptibility $\chi_{xx}=\chi_{yy}$, and this fact requires the addition of a term $HM_\perp$ to **T**. Thus

$$\mathbf{T}=K\Phi-2\chi_{xy}H^2\sin 2\theta_e\Phi+M_fH\cos\theta_{ef}\Phi+HM_\perp. \quad (3)$$

$M_\perp$ and $M_{\parallel}$ (the latter is the component of **M** parallel to **H** arising from the diagonal part of χ) satisfy

$$\begin{aligned} dM_{\parallel}/dt&=\alpha(\chi_{xx}H-M_{\parallel})-M_\perp(d\Phi/dt),\\ dM_\perp/dt&=-\alpha M_\perp+M_{\parallel}(d\Phi/dt). \end{aligned} \quad (4)$$

The terms proportional to the relaxation time constant α arise from the usual assumptions about such phenomena, and the terms proportional to $d\Phi/dt$ arise from the motion of the crystal.

The equation governing the oscillation of the torsion pendulum is

$$I(d^2\Phi/dt^2)+\mathbf{T}=0, \quad (5)$$

where I is the moment of inertia of the torsion pendulum. Solution of the coupled nonlinear Eqs. (4) and (5) by straightforward perturbation techniques yields a period for the oscillations (which are not strictly harmonic).[2]

$$\omega^2=\frac{K}{I}-\frac{2\chi_{xy}H^2}{I}\sin 2\theta_e+\frac{M_fH}{I}\cos\theta_{ef}+\frac{K/I}{\alpha^2+K/I}\frac{\chi_{xx}H^2}{I}. \quad (6a)$$

Where we neglect small quantities, the square root of Eq. (6a) becomes

$$\omega=\left(\frac{K}{I}\right)^{1/2}\left(1-\frac{\chi_{xy}H^2}{K}\sin 2\theta_e+\frac{M_fH}{2K}\cos\theta_{ef}+\frac{1/2}{\alpha^2+K/I}\frac{\chi_{xx}H^2}{I}\right). \quad (6b)$$

The phenomena shown in Fig. 2 of the paper by A. & S. can be understood as consequences of Eq. (6b). The term proportional to χ_{xy} explains the modulation of angular frequency with applied field direction. Either of the last two terms in Eq. (6b) could account for ω_1: θ_{ef} was $+30°$ or $-30°$ in the work of A. & S. and did not vary as the magnet was rotated. Since $\cos(+30°)=\cos(-30°)$, two branches were not observed. Since A. & S. find that ω_1^2 is proportional to H, the part of ω_1 depending on relaxation phenomena was not observed in the data reported in their paper.

[2] There is an imaginary part of the angular frequency which gives rise to damping by the magnetic field. Adair and Squire report a 33% change of the attenuation coefficient in a field of 1000 G.

In addition to Eq. (6), in order to interpret the results of A. & S., we need a solution of the equation $\mathbf{T}=0$, $\mathbf{H}\neq 0$; that is

$$K(\theta_e-\theta_0)+\chi_{xy}H^2\cos 2\theta_e+M_fH\sin\theta_{ef}=0 \quad (7)$$

which has the approximate solution

$$\theta_e=\theta_0-\frac{\chi_{xy}H^2}{K}\cos 2\theta_0-\frac{M_fH}{K}\sin\theta_{ef}. \quad (8)$$

θ_{ef} is $+30°$ or $-30°$ depending on the direction of rotation of the magnet in the work of A. & S., giving rise to the two branches in the curve exhibited in their Fig. 4. The term in θ_e proportional to $\cos 2\theta_0$ gives rise to the oscillation in both branches.

Figure 2 by A. & S. suggests that $\mathbf{M}_f$ can be aligned with H by increasing the applied field to some large value.

It should be noted that Eqs. (6) and (8) predict that the oscillations in Figs. 2 and 4 of A. & S. are 90° out of phase, as the experiment confirms [$\theta_e\simeq\theta_0$ in Eq. (6)]. Furthermore, Eqs. (6) and (8) predict for quantities shown in their Figs. 2 and 4.

$$d_2/2D=\omega_2/\omega_0, \quad (9)$$

where d_2 is the amplitude of modulation of the torque experiments in Fig. 4 of A. & S., and where D is the distance from the mirror to the scale in Fig. 1 of A. & S. Their results, taken from Figs. 2 and 4 satisfy this Eq. (9) to within 18%. The factor 2 in Eq. (9) results from the fact that the geometrical arrangement shown in their Fig. 1 gives an angular deflection which is twice the angle of torsion of the wire.

A similar prediction relates the quantities ω_1 and d_1.

$$\frac{2\omega_1/\omega_0}{d_1/2D}=\frac{1}{2}\frac{\cos\theta_{ef}}{\sin\theta_{ef}}, \quad (10)$$

where d_1 is the amplitude shown in Fig. 4 of A. & S., and D is the distance in Fig. 1 of A. & S.

The data of A. & S. satisfy this relation to within a factor of 2 [it should be noted that we have ignored a possible contribution of the last term in Eq. (6) to ω_1].

The fact that the experimental data reported in the preceding paper do not satisfy Eqs. (9) and (10) quantitatively probably results from the fact that the LiF crystal used in obtaining the data from which Figs. 2 and 4 were prepared was not identical because the crystal had been irradiated between measurements.

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