Theory of Giant Quantum Oscillations in Ultrasonic Attenuation in a Longitudinal Magnetic Field

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(Received 27 July 1964)

The effect of collisions on the giant quantum oscillations in the attenuation of both transverse and longitudinal acoustic waves propagating parallel to a dc magnetic field in metals is studied. This is accomplished by using the results of Tosima, Quinn, and Lampert on the theory of collision effects on the magnetoconductivity tensor of a quantum plasma. The experimental conditions under which quantum oscillations should be observable are determined, and the information which they yield about the Fermi surface is discussed.

I. INTRODUCTION

THE magnetic field dependence of the attenuation of acoustic waves in very pure metals at low temperatures has become a very valuable tool in the study of the electronic properties of metals.[1] The most widely studied magnetoacoustic phenomenon, the "geometric resonance," results from a matching of the size of the orbit of the electrons in the dc magnetic field to the wavelength of the sound wave. These geometric resonances yield information[2] about the extremal linear dimensions of the Fermi surface in a direction perpendicular to the dc magnetic field and to the direction of propagation. The object of this paper is to study a different magnetoacoustic phenomenon, the giant quantum oscillations in the attenuation of acoustic waves propagating parallel to the dc magnetic field. The existence of giant quantum oscillations was first predicted by Gurevich, Skobov, and Firsov.[3] These authors studied only the $\Delta n=0$ transitions (that is, transitions in which the Landau-level quantum number n is unchanged). The period of these oscillations is a measure of the cross-sectional area of the Fermi surface at the plane $k_z \simeq ms/\hbar$, where s is the velocity of sound, and the magnetic field is in the z direction. Because s is very small compared to the Fermi velocity, the oscillations essentially give the extremal cross-sectional area of the Fermi surface. The existence of giant oscillation in the $\Delta n=\pm 1$ transitions was pointed out by Quinn[4] and by Miller[5] for the propagation of helicon waves, and by Langenberg, Quinn, and Rodriguez,[6] and Gantsevich and Gurevich[7] for the case of acoustic waves. Langenberg *et al.* have stressed the importance of the quantum oscillations in the $\Delta n=\pm 1$ transitions as a tool for studying Fermi surfaces. It has been shown that the period of these oscillations can be used to determine the cross-sectional area of the Fermi surface not only at an extremum, but at any plane perpendicular to the direction of the dc magnetic field. Further, under suitable experimental conditions, the line shape of the giant oscillations can be used to determine the cyclotron effective mass[8] for the carriers which are absorbing. Thus this effect is capable, in principle, of yielding the cross-sectional area of the Fermi surface and the cyclotron effective mass as a function of v_z, the average velocity (over one orbit) parallel to the direction of propagation of the sound wave. This is considerably more information about the Fermi surface than can be obtained by any of the standard tools used at present. For this reason, it seems desirable to make a careful analysis of the experimential conditions necessary for the observation of giant quantum oscillations. Both the $\Delta n=0$ transitions and the $\Delta n=\pm 1$ transitions will be studied. The model used in the present work is that of a degenerate electron gas embedded in an isotropic lattice of positive ions, the latter being capable of sustaining both longitudinal and shear waves. For more realistic models[9] in which the constant energy surfaces for the electrons in $\mathbf{k}$ space are other than spherical, the analysis is somewhat more complicated. For example, the selection rules $\Delta n=0$ for the absorption of longitudinal waves and $\Delta n=\pm 1$ for the absorption of transverse waves, no longer hold in anisotropic systems; however, many of the qualitative features of the simple isotropic model will hold. In particular, if the averaged semiclassical attenuation due to a given absorption process is known, the experimental conditions on the frequency, magnetic-field strength, relaxation time, temperature, etc., necessary for the observation of quantum oscillations in the attenuation should be readily attainable by application of the concepts of the

[1] See, for example, *The Fermi Surface*, edited by W. A. Harrison and M. B. Webb (John Wiley & Sons, London and New York, 1960), Sec. VI, pp. 214–263.
[2] A thorough analysis of the geometric resonances can be found in M. H. Cohen, M. J. Harrison, and W. A. Harrison, Phys. Rev. **117**, 937 (1960).
[3] V. L. Gurevich, V. G. Skobov, and Yu A. Firsov, Zh. Eksperim. i Teor. Fiz. **40**, 786 (1961) [English transl.: Soviet Phys.—JETP **13**, 552 (1961)]. See also J. J. Quinn and S. Rodriguez, Phys. Rev. **128**, 2487 (1962) for an alternative treatment of giant quantum oscillations.
[4] J. J. Quinn, Phys. Letters **7**, 235 (1963).
[5] P. B. Miller, Phys. Rev. Letters **11**, 537 (1963).
[6] D. N. Langenberg, J. J. Quinn, and S. Rodriguez, Phys. Rev. Letters **12**, 104 (1964).
[7] S. V. Gantsevich and V. L. Gurevich, Zh. Eksperim. i Teor. Fiz. **45**, 587 (1963) [English transl.: Soviet Phys.—JETP **18**, 403 (1964)]. These authors mention, but do not discuss in detail, the effect of collisions on their calculations. Their Eq. (27) is not in agreement with the conditions determined in the present pages.
[8] Y. Shapira and B. Lax, Phys. Rev. Letters **12**, 166 (1964).
[9] See, for example, J. J. Quinn, Phys. Rev. Letters **11**, 506 (1963) and Phys. Rev. **135**, A181 (1964) for a treatment of a system with ellipsoidal energy surfaces.

present theory. Gurevich *et al.*[3] have already considered the effect of a finite relaxation time on the observability of quantum oscillations in the $\Delta n=0$ transitions. However, their method consists of simply replacing the delta function, which implies exact conservation of energy, by a Lorentzian in the expression for the transition rate. The validity of this procedure is not immediately obvious, and a more thorough treatment of collision effects seems desirable. Furthermore, there are some minor errors in the work of Gurevich *et al.*, although the experimental conditions determined by them as being necessary for the observation of giant quantum oscillations in the $\Delta n=0$ transitions are correct. The effect of collisions on the $\Delta n=\pm 1$ transitions has not been studied carefully before. The experimental conditions for the observation of quantum oscillations in the attenuation due to these transitions which we find in this work agree with the conditions obtained by rather qualitative arguments in Ref. 6.

In the present paper, the effect of a finite relaxation time for the electrons is taken into account by making use of the results of Tosima, Quinn, and Lampert[10] for the quantum magnetoconductivity tensor in the presence of collisions. Also, for simplicity, we limit our consideration to propagation of acoustic waves parallel to the dc magnetic field, although giant oscillations can occur for other directions of propagation.

II. ATTENUATION

The motion of the positive ions of a metal acts as the source or driving force which is responsible for the establishment of the self-consistent electromagnetic field. The ionic current density is given by $-ne\mathbf{u}$, where $\mathbf{u}(\mathbf{r},t)$ is the velocity field of the ions, e the electronic charge, and n the mean electron density. The total current density $\mathbf{J}$ is the sum of the ionic current density and the induced electronic current density $\mathbf{j}$. From Maxwell's equations one can obtain the relation

$$\mathbf{J}=\mathbf{\Gamma}\cdot\mathbf{E}, \tag{1}$$

where $\mathbf{E}$ is the self-consistent electric field. For propagation parallel to the dc magnetic field, the tensor $\mathbf{\Gamma}$ is diagonal and has the following nonvanishing components:

$$\Gamma_{xx}=\Gamma_{yy}=i\beta\sigma_0, \tag{2}$$

and

$$\Gamma_{zz}=-i\omega/4\pi. \tag{3}$$

In Eq. (2), $\sigma_0=ne^2\tau/m$ is the dc conductivity and $\beta=(c^2q^2/\omega_p^2\omega\tau)(1-\omega^2/c^2q^2)$, where ω_p is the electron-plasma frequency. An additional relation between $\mathbf{j}$ and $\mathbf{E}$ is obtained from the equation of motion of the density matrix. When one takes into account the motion of the ions on the random scattering of the electrons, this relation becomes[2]

$$\mathbf{j}(\mathbf{q},\omega)=\boldsymbol{\sigma}(\mathbf{q},\omega)\cdot\left[\mathbf{E}(\mathbf{q},\omega)-\frac{m}{e\tau}\mathbf{u}(\mathbf{q},\omega)\right]. \tag{4}$$

The definition of $\mathbf{J}$ together with Eqs. (1)–(4) allows one to calculate the self-consistent electric field and current density. The power absorbed from the sound wave by the electrons can be evaluated from a knowledge of the self-consistent field. When one includes the contribution of collision drag,[2,11] the power absorbed per unit volume can be expressed as

$$Q=\tfrac{1}{2}\,\mathrm{Re}\{\mathbf{j}^*\cdot[\mathbf{E}-(m\mathbf{u}/e\tau)]+(nm/\tau)|\mathbf{u}|^2\}. \tag{5}$$

By definition, the coefficient of attenuation of the sound wave is the ratio of Q to the incident power per unit volume. Explicit expressions for the attenuation coefficients of both longitudinal and transverse acoustic waves in a longitudinal magnetic field have been given by Kjeldaas[12] and by Cohen *et al.*[2] For a longitudinal wave, the attenuation coefficient is

$$\gamma_l=(zm/Ms_l\tau)\,\mathrm{Re}[(\sigma_0/\sigma_{zz})-1], \tag{6}$$

where z is the number of conduction electrons per atom, s_l is the velocity of the longitudinal sound wave, and M is the mass of an ion. For transverse waves, it is convenient to use circularly polarized waves. The attenuation coefficients in this case are

$$\gamma_\pm=(zm/Ms_t\tau)\,\mathrm{Re}\{[(1+i\beta)^2\sigma_0/(\sigma_\pm+i\beta\sigma_0)]-1\}, \tag{7}$$

where s_t is the velocity of transverse sound waves and $\sigma_\pm=\sigma_{xx}\mp i\sigma_{xy}$.

The difference between the present analysis and that of Kjeldaas or Cohen *et al.* is that here we are interested in quantum effects which the previous authors were not able to treat with the semiclassical Boltzmann formalism. Both the real and imaginary parts of the conductivity display quantum oscillations. The oscillations in the real part of $\boldsymbol{\sigma}$ are much more dramatic than those in the imaginary part. In fact, in the absence of collisions and at zero temperature the real parts of $\sigma_\pm$ and σ_{zz} display (as a function of the strength of the dc magnetic field $\mathbf{B}_0$) discontinuous jumps between the value zero and a value which can be orders of magnitude larger than the semiclassical mean value. These giant quantum oscillations are precisely what we wish to study.

III. QUANTUM OSCILLATIONS

In the preceding paper,[10] a quantum mechanical derivation of the magnetoconductivity tensor of a degenerate electron gas, including the effect of collisions, was presented. By using the results of this theory, we

[10] S. Tosima, J. J. Quinn, and M. A. Lampert, preceding paper, Phys. Rev. 137, A883 (1965).

[11] T. Holstein, Phys. Rev. 113, 479 (1959).
[12] T. Kjeldaas, Phys. Rev. 113, 1473 (1959).

can easily show that[13]

$$\mathrm{Re}[\sigma]=-(m\hbar^2\omega_p^2/4\pi N\tau)\sum_{\nu\nu'}\langle\nu'|\mathbf{V}(\mathbf{q})|\nu\rangle \times\langle\nu'|\mathbf{V}(\mathbf{q})|\nu\rangle^*\{[\rho_0(E_{\nu'})-\rho_0(E_\nu)]/(E_{\nu'}-E_\nu)\} \times[(E_{\nu'}-E_\nu-\hbar\omega)^2+\hbar^2\tau^{-2}]^{-1}. \quad (8)$$

We shall study first the quantum oscillations in the real part of σ_{zz}. Because the matrix elements $\langle\nu'|V_z(\mathbf{q})|\nu\rangle$ are given by[14]

$$\langle n', k_y, k_z+q|V_z(q)|nk_yk_z\rangle=(\hbar/m)(k_z+q/2)\delta_{n'n}, \quad (9)$$

where $\delta_{n'n}$ is the Kronecker delta function, the attenuation in this case should correspond to the $\Delta n=0$ transitions studied by Gurevich *et al.* Substituting the matrix element, Eq. (9), into Eq. (8) gives the result

$$\mathrm{Re}\sigma_{zz}=\frac{\hbar^2\omega_p^2}{4\pi Nm\tau}\sum_{nk_yk_z}(k_z+q/2)^2\frac{\partial\rho_0[E_n(k_z)]}{\partial\mu} \times\left\{\left[\frac{\hbar}{m}q(k_z+q/2)-\omega\right]^2+\tau^{-2}\right\}^{-1}. \quad (10)$$

In writing down Eq. (10), we have made use of the approximation

$$\frac{\rho_0(E_{\nu'})-\rho_0(E_\nu)}{E_{\nu'}-E_\nu}\approx\frac{\partial\rho_0(E_\nu)}{\partial E_\nu}=-\frac{\partial\rho_0(E_\nu)}{\partial\mu}, \quad (11)$$

where μ is the chemical potential. The derivative of the Fermi distribution function $\rho_0(E_\nu)$ with respect to μ is equal to $(2kT)^{-1}\cosh^{-2}[(E_\nu-\mu)/2kT]$. We replace the summation over k_y and k_z by an integration, and introduce the following simplifying notation:

$$y=\hbar k_z(2mkT)^{-1/2},$$
$$A_n=[\mu-\hbar\omega_c(n+\tfrac{1}{2})]/kT,$$
$$a_1=\hbar q(8mkT)^{-1/2},$$
$$a_2=(\omega/q)(m/2kT)^{1/2},$$

and

$$B=q\tau(2kT/m)^{1/2}. \quad (12)$$

The expression for the real part of σ_{zz} then takes the form

$$\mathrm{Re}\sigma_{zz}=\frac{\omega_c}{4\pi qa_0}\int_{-\infty}^{\infty}dy\sum_{n=0}^{\infty}\cosh^{-2}[\tfrac{1}{2}(y^2-A_n)] \times(y+a_1)^2\pi^{-1}B[1+B^2(y+a_1-a_2)^2]^{-1}, \quad (13)$$

where v_F is the Fermi velocity and a_0 is the radius of the first Bohr orbit in hydrogen. The integrand in Eq. (13) is a product of two rapidly varying functions. The first function $\sum_n\cosh^{-2}[\frac{1}{2}(y^2-A_n)]$ consists of a series of $\mu/\hbar\omega_c$ peaks of unit height. The width of the peak located at $y^2=A_m$ is of the order $\{(A_m+1)^{1/2}-A_m^{1/2}\}$ and the distance to neighboring peaks is of the order of $\{[(\hbar\omega_c/kT)+A_m]^{1/2}-A_m^{1/2}\}$. Far from a peak the function decreases exponentially. The actual positions of the peaks vary as a function of magnetic field strength. If $Ba_2\gg 1$, the second function

$$(y+a_1)^2\frac{B/\pi}{1+B^2(y+a_1-a_2)^2}$$

has a sharp maximum at $y=a_2-a_1$ of width B^{-1}. Far from its maximum, the function approaches the constant value $(\pi B)^{-1}$. In the limit as B tends to infinity, this function approaches the value $(y+a_1)^2\delta(y+a_1-a_2)$, where $\delta(x)$ is the Dirac delta function. In this case, giant quantum oscillations should be clearly observable; only when a peak of the first function coincides with the position of the delta function is $\mathrm{Re}\sigma_{zz}$ large. When B is finite, we must satisfy the following conditions in order to have clearly observable giant oscillations in $\mathrm{Re}\sigma_{zz}$ as a function of magnetic field: First, the distance between peaks of the first function must be larger than the width of the maximum of the second function; second, the contribution to the integral from a coincidence of a peak of the first function with the maximum of the second must be large compared to the sum of the contributions from all the other peaks of the first function. These two conditions can be approximately expressed as

$$(\hbar\omega_c/kT)^{1/2}>B^{-1} \quad (14)$$

and

$$Ba_2^2/\pi>(\pi B)^{-1}\sum_{n=0}^{n_0}A_n^{-1/2}, \quad (15)$$

where n_0 is defined by $\hbar\omega_c(n_0+\frac{1}{2})=\mu$. Equations (14) and (15) lead to the conditions

$$ql>(\mu/\hbar\omega_c)^{1/2}, \quad (16)$$

and

$$ql>\left(\frac{\mu}{\hbar\omega_c}\right)^{1/2}\frac{v_F}{s}\left(\frac{2kT}{ms^2}\right)^{1/2}. \quad (17)$$

In these equations, v_F and s are the Fermi velocity and sound velocity, respectively; μ is the chemical potential or Fermi energy, and $l=v_F\tau$ is the electron mean free path. In writing down the inequalities (14) and (15), we have assumed that the width of the peaks of the oscillatory function is much smaller than the separation between peaks. This is true if $\hbar\omega_c\gg kT$. This requirement, together with the inequality (16), are the conditions necessary for the existence of readily observable quantum oscillations in the real part of σ_{zz}. Equation (17) is the requirement necessary in order that the amplitude of the oscillations be gigantic compared to the background attenuation. These conditions do not differ essentially from those of Gurevich *et al.*, who have also considered this problem. These authors[3] appear to have made a few minor errors, but these do not invalidate

[13] The notation used in this work will follow that of Ref. 10.

[14] See the paper of Quinn and Rodriguez cited in footnote 3.

their conclusions. One error is the replacement of $(\hbar\omega_{a'a})^2$ by $(\hbar\omega)^2$ in Eq. (14) of their paper; this error would result in the factor $(y+a_1)^2$, appearing in the integrand of Eq. (13) of the present paper, being replaced by a_2^2. The second error is the omission of the square of y in the argument of the hyperbolic cosine appearing in Eq. (13) of their paper. These two errors completely change the behavior of the integrand from that of Eq. (13) of the present paper.

The matrix elements of $V_x(q)$ and of $V_x(q)\pm iV_y(q)$ are given by

$$\langle n', k_y, k_z+q|V_x|nk_yk_z\rangle = i(\hbar\omega_c/2m)^{1/2} \times[(n+1)^{1/2}\delta_{n',n+1}-n^{1/2}\delta_{n',n-1}],$$

and

$$\langle n', k_y, k_z+q|V_x\pm iV_y|nk_yk_z\rangle = \pm 2i(\hbar\omega_c/2m)^{1/2}(n+\tfrac{1}{2}\pm\tfrac{1}{2})\delta_{n',n\pm 1}. \quad (18)$$

With the aid of Eq. (18) one can easily show that

$$\mathrm{Re}\sigma_\pm = \frac{m\omega_p^2\omega_c^2\Omega}{(2\pi)^3\tau N}\int dk_z \sum_{n=0}^{\infty}\frac{\partial\rho_0(E_n[k_z])}{\partial\mu}(n+\tfrac{1}{2}\mp\tfrac{1}{2}) \times\left\{\left[\frac{\hbar q}{m}\left(k_z+\frac{q}{2}\right)-\omega\mp\omega_c\right]^2+\tau^{-2}\right\}^{-1}. \quad (19)$$

In writing down Eq. (19), we have again made use of Eq. (11). Rewriting the integrand of Eq. (19) in terms of the dimensionless variables defined by Eq. (12) gives the result

$$\mathrm{Re}\sigma_\pm = \frac{\hbar\omega_c}{kT}\frac{\omega_c}{4\pi q a_0}\int_{-\infty}^{\infty}dy\sum_n(n+\tfrac{1}{2}\mp\tfrac{1}{2}) \times\cosh^{-2}\left(\frac{y^2-A_n}{2}\right)\pi^{-1}B[1+B^2(y+a_1-a_3)^2]^{-1}, \quad (20)$$

where the new parameter a_3 equals $q^{-1}(\omega\pm\omega_c)(m/2kT)^{1/2}$. As with the integrand in Eq. (13), the integrand of Eq. (20) consists of two rapidly varying functions. The first function $\sum_n(n+\frac{1}{2}\mp\frac{1}{2})\cosh^{-2}[\frac{1}{2}(y^2-A_n)]$ consists of a series of $\mu/\hbar\omega_c$ peaks. The height and width of the peak at $y^2=A_m$ are approximately $[(\mu-A_mkT)/\hbar\omega_c]\mp\frac{1}{2}$ and $A_m^{-1/2}$, respectively; the distance to neighboring peaks is of the order of $\hbar\omega_c/kTA_m^{1/2}$.

For $B\gg 1$, the second function $(B/\pi)[1+B^2(y+a_1-a_3)^2]^{-1}$ has a sharp peak of width B^{-1} at $y=a_3-a_1$. Far from the maximum, the second function decreases as $(\pi By^2)^{-1}$. In order to have clearly observable gigantic oscillations in $\mathrm{Re}\sigma_\pm$ as a function of magnetic field, we must again satisfy the conditions that the distance between neighboring peaks of the first function in the vicinity of the maximum of the second function must be greater than the width of the maximum of the second function, and the contribution to the integral from the coincidence of a peak of the first function with the maximum of the second must exceed the remainder of the integral. These two conditions can be written approximately as

$$\hbar\omega_c/2kT|a_3-a_1|>B^{-1}, \quad (21)$$

and

$$\frac{B\{\mu-(a_3-a_1)^2kT\}}{(a_3-a_1)\pi\hbar\omega_c}>\sum_{n=0}^{n_0'}nA_n^{-3/2}/\pi B. \quad (22)$$

In the inequality (22), $n_0'=(\mu-\alpha kT)/\hbar\omega_c$, where α is a number of the order of but greater than unity. The inequalities (21) and (22) result in the requirements

$$q^2>m/\hbar\tau, \quad (23)$$

and

$$ql>[(\mu/\hbar\omega_c)\omega_c\tau]^{1/2}\{[1-(\omega_c/qv_F)^2]\}^{-1/2}. \quad (24)$$

In writing down the inequality (24) we have assumed $a_3\gg a_1$ and $\omega_c\gg\omega$. The inequality (23), together with the requirement $\hbar\omega_c\gg kT$, are the conditions necessary for the existence of observable quantum oscillations in the real part of $\sigma_\pm$, and hence in the attenuation due to $\Delta n=\pm 1$ transitions. The inequality (24) is the condition necessary for oscillations of gigantic amplitude. The condition (23) was given in Ref. 6 on the basis of rather intuitive arguments. The effect of collisions in both $\Delta n=0$ and $\Delta n=\pm 1$ transitions is relatively unimportant if the width B^{-1} of the maximum of the second function is much smaller than the width of the peaks of the oscillatory functions in the vicinity of that maximum. This requirement leads to the conditions $ql>\mu^{1/2}[(\frac{1}{2}ms^2+kT)^{1/2}-(\frac{1}{2}ms^2)^{1/2}]^{-1}$ for $\Delta n=0$ transitions and $ql>(\omega_c/qv_F)(\mu/kT)$ for $\Delta n=\pm 1$ transitions. When these conditions are satisfied, the function $\pi^{-1}B/[1+B^2(y+a)^2]$ can essentially be replaced by $\delta(y+a)$.

In actual experiments, at least in metals, $\hbar\omega_c$ is usually of the same order of magnitude as kT. In this situation, it is convenient for some purposes to write the expressions for the real parts of σ_{zz} and $\sigma_\pm$ in a different form which explicitly displays their oscillatory behavior. If one returns to Eq. (8) and performs some algebraic manipulation, one can obtain the following equation:

$$\mathrm{Re}\sigma_{zz} = -\frac{\omega_p^2m\omega_c}{(2\pi)^3\hbar q\tau}\frac{\Omega}{N} \times\int_{-\infty}^{\infty}d\kappa F(\kappa)\frac{\kappa}{[(\hbar q/m)\kappa-\omega]^2+\tau^{-2}}, \quad (25)$$

where

$$F(\kappa)=\sum_{n=0}^{\infty}\left\{\rho_0\left[E_n\left(\kappa+\frac{q}{2}\right)\right]-\rho_0\left[E_n\left(\kappa-\frac{q}{2}\right)\right]\right\}. \quad (26)$$

By means of the Poisson sum formula,[15] one can express

[15] See, for example, R. Courant and D. Hilbert, *Methods of Mathematical Physics* (Interscience Publishers, Inc., New York, 1953), Vol. 1, p. 76.

$F(\kappa)$ in the form

$$F(\kappa)=-\left\{\frac{\hbar\kappa q}{m\omega_c}+\frac{4\pi kT}{\hbar\omega_c}\sum_{r=1}^{\infty}(-1)^r \times\cos\left(\frac{2\pi r}{\hbar\omega_c}\left[\mu-\epsilon(\kappa)-\epsilon\left(\frac{q}{2}\right)\right]\right)\sin\left(\frac{\pi r\hbar q\kappa}{m\omega_c}\right) \times[\sinh(2\pi^2 rkT/\hbar\omega_c)]^{-1}\right\}, \quad (27)$$

where $\epsilon(\kappa)=\hbar^2\kappa^2/2m$. For $\hbar\omega_c\gtrsim kT$, only the first oscillatory term in $F(\kappa)$ (that is, the $r=1$ term) has appreciable amplitude; neglecting the small higher-order terms gives the following expression for the difference between $\mathrm{Re}\sigma_{zz}$ and its mean value $\mathrm{Re}\bar{\sigma}_{zz}$:

$$\mathrm{Re}(\sigma_{zz}-\bar{\sigma}_{zz})=\frac{kT}{\epsilon(q)}\frac{(\pi a_0 q\tau)^{-1}}{\sinh(2\pi^2kT/\hbar\omega_c)} \times\int_{-\infty}^{\infty}dz\frac{z+\omega\tau}{z^2+1}\cos\left\{\frac{2\pi\mu}{\hbar\omega_c}\left[1-\frac{(z+\omega\tau)^2}{q^2l^2}\right]\right\} \times\sin\left\{\frac{\pi(z+\omega\tau)}{\omega_c\tau}\right\}. \quad (28)$$

In Eq. (28), we have introduced the dimensionless variable $z=(\hbar q\tau\kappa/m)-\omega\tau$, and we have neglected $\epsilon(q/2)$ compared to μ in the argument of the cosine. For simplicity, we assume that $\omega\tau$ is small compared to unity, although this requirement is not essential. The main contribution to the integral appearing in Eq. (28) is from the region $|z|<1$. If $ql\gg(\mu/\hbar\omega_c)^{1/2}$, as required by Eq. (16), we can obtain a rough approximation to the behavior of the integral by neglecting the z dependence of $\cos((2\pi\mu/\hbar\omega_c)\{1-[(z+\omega\tau)^2/q^2l^2]\})$. When this is done, one obtains

$$\mathrm{Re}(\sigma_{zz}-\bar{\sigma}_{zz})\approx\frac{kT}{\hbar\omega_c}\frac{(a_0q\tau)^{-1}e^{-\pi/\omega_c\tau}}{\sinh(2\pi^2kT/\hbar\omega_c)} \times\cos\left(\frac{2\pi\mu}{\hbar\omega_c}\left[1-\frac{s^2}{v_F^2}\right]\right). \quad (29)$$

The attenuation displays quantum oscillations of the de Haas–van Alphen type; that is, it is a periodic function of ω_c^{-1}. Because of the factor $[1-(s^2/v_F^2)]$ in the argument of the cosine function, the period of oscillations is slightly longer than the de Haas–van Alphen period. The period of the present oscillations is proportional to the cross-sectional area of the Fermi surface at the plane $k_z\approx ms/\hbar$, while the de Haas–van Alphen period is proportional to the cross section at the plane $k_z=0$. The factor $e^{-\pi/\omega_c\tau}$ requires $\omega_c\tau\gg1$ in order that the oscillations be of appreciable amplitude. In this situation where $\hbar\omega_c\gtrsim kT$, it is not really appropriate to refer to the oscillations as giant oscillations. When $\hbar\omega_c\gg kT$, more than one of the oscillatory terms in the summation appearing in Eq. (28) must be taken into account. In this case, it is probably of little value to use the Poisson sum formula.

The same procedure can be applied to $\sigma_\pm$ when $\hbar\omega_c\gtrsim kT$. One can show that if $q^2\gg m/\hbar\tau$, then $\mathrm{Re}(\sigma_\pm-\bar{\sigma}_\pm)$ can be approximated by an expression of the form $A\cos((2\pi/\hbar\omega_c)\mu\{1-[(\omega_c\pm\omega)^2/q^2v_F^2]\})$. We do not bother to write down the approximate but complicated expression for the amplitude A. The main point is that because of the factor $\{1-[(\omega_c\pm\omega)^2/q^2v_F^2]\}$ appearing in the argument of the cosine, the attenuation is not a periodic function of ω_c^{-1} as in the de Haas–van Alphen effect. However, in metals, even at the highest magnetic fields, $(\mu/\hbar\omega_c)\gg1$; therefore, the factor $\{1-[(\omega_c\pm\omega)^2/q^2v_F^2]\}$ changes by an extremely small amount when one alters the intensity of the dc magnetic field sufficiently to cause $\mu/\hbar\omega_c$ to change by unity. Thus the oscillations are approximately periodic over any small range of magnetic field. The period of the oscillations is proportional to the cross-sectional area of the Fermi surface at the plane $k_z=(m/\hbar q)\times(\omega\mp\omega_c)$. The position of the plane is itself a function of ω_c, but it is a slowly varying function compared to the period of the oscillations. Thus the quantum oscillations in the attenuation due to $\Delta n=\pm1$ transitions are, in principle, capable of yielding the cross-sectional area of the Fermi surface as a function of k_z.

IV. SUMMARY

The present work is based upon the free-electron gas model of a metal; however, with a slight generalization, many of the conclusions should be applicable to systems with more complicated constant energy surfaces than spheres. To demonstrate this point one need only compare the present theory with the treatment by Gurevich *et al.*[3] of the quantum oscillations due to $\Delta n=0$ transitions. If one is careful to include the factor $(\omega_{a'a}/\omega)^2$ in the generalization of Eq. (14) of Ref. 3, in which the delta function $\delta(\omega_{a'a}-\omega)$ is replaced by a Lorentzian, one obtains the same result as Eq. (13) of the present paper. Thus the present paper offers some justification for this rather simple treatment of collision effects. For systems with complicated energy surfaces, the calculation of the conductivity tensor becomes very difficult, but the procedure used by Gurevich *et al.* can probably be carried out.

The physical origin of the quantum oscillation has been discussed in detail before.[3,4,6] In absorbing a phonon of wave vector q and energy $\hbar\omega$, an electron makes a transition from the initial state $|nk_yk_z\rangle$ to the final state $|n+\alpha, k_y, k_z+q\rangle$. Conservation of energy and wave vector require that $k_z=\kappa_\alpha$, where κ_α is given by $\kappa_\alpha=(m/\hbar q)\times(\omega-\alpha\omega_c)-(q/2)$. $\alpha=\Delta n$ is the change in the Landau-level quantum number in the transition. In order to have absorption the initial state must be occupied and the final state empty. Thus at zero temperature absorption occurs only if the Fermi energy lies between these two states. Since these states are separated in energy by $\hbar\omega$, only over ω/ω_c of the full period can absorption occur. The period of the oscillations is obtained by determining the change in magnetic field necessary to "push" one Landau level through the Fermi surface at the plane $k_z=\kappa_\alpha$. The period $\Delta(B_0^{-1})$ is inversely proportional to the cross-sectional area of the Fermi surface at the plane $k_z=\kappa_\alpha$; in fact, $\Delta(B_0^{-1})=2\pi e/\hbar cS(\mu,\kappa_\alpha)$, where $S(\mu,\kappa_\alpha)$ is the cross-sectional area of the energy surface $E=\mu$ (Fermi surface) at the plane $k_z=\kappa_\alpha$. Actually, for $\alpha\neq 0$, the value of κ_α changes very slightly over one period; this results in the attenuation being a not exactly periodic function of B_0^{-1}. It should be pointed out that in this paper we have neglected all effects due to the spin of the electron. The generalization needed to include spin is rather trivial.

The effect of finite temperature and collisions can be understood qualitatively in a simple way. If $kT>\hbar\omega$, one can have absorption as long as either the initial or final state lie within kT of the Fermi energy. Owing to collisions, the law of conservation of energy has an uncertainty $\hbar\tau^{-1}$ associated with it. This results in a spread in the allowed values in the plane $k_z=\kappa_\alpha$. If this spread becomes too large, the quantum oscillations are washed out.

For the purpose of illustration, we shall apply the conditions obtained in this paper to an idealized metal and a semimetal. We assume that the metal is characterized by the following parameters: $m=m_0$, the free electron mass, $v_F=10^8$ cm/sec, and $s=2\times10^5$ cm/sec. At an operating temperature of 2°K, the dc magnetic field must satisfy the condition $B_0\gg 15\,000$ G in order to see quantum effects. In order to see giant oscillations due to $\Delta n=0$ transitions, the conditions (16) and (17) must be satisfied. Actually, the condition (17) is the requirement necessary for large amplitude oscillations; that is, oscillations in the attenuation which are of the order of the attenuation itself. Small amplitude oscillations may still be observable if only the inequality (16) is satisfied. For the idealized metal under discussion here, these inequalities result in the following conditions on the frequency of the sound wave: $\omega>10^{-1}\tau^{-1}$ from (16) and $\omega>10^2\tau^{-1}$ from (17). Thus, for $\tau=10^{-9}$ sec, it is necessary that $\omega>10^8$ sec^{-1}, in order to see the oscillations; and it is necessary that $\omega>10^{11}$ sec^{-1}, in order that the amplitude of the oscillations be gigantic. The two conditions necessary for the observation of giant quantum oscillations due to $\Delta n\neq 0$ transitions, namely (23) and (24), are almost identical if one is not too close to the absorption edge where $qv_F=\omega_c$. For our idealized metal they result in the condition $\omega>2\times10^5\tau^{-1/2}$; thus, for $\tau=10^{-9}$ sec, the quantum oscillations should be observable, if $\omega>6\times10^9$ sec^{-1}.

We shall use the parameters $m=10^{-2}m_0$, $v_F=5\times10^7$ cm/sec, and $s=2\times10^5$ cm/sec to characterize a semimetal, or a small pocket of electrons or holes in a metal. At a temperature of 2°K, $B_0\gg 150$ G, in order to see quantum effects in such a material. The condition necessary for the observation of quantum oscillations due to $\Delta n=0$ transitions is $\omega>10^{-2}\tau^{-1}$; the condition necessary for the oscillations to be of gigantic amplitude is $\omega>10^2\tau^{-1}$. The condition necessary for the observation of quantum oscillations due to $\Delta n\neq 0$ transitions becomes $\omega>2\times10^4\tau^{-1/2}$. It should be mentioned that the $\Delta n=\pm1$ transitions can occur only if $\omega>\omega_c s/v_F$. The oscillations due to $\Delta n=0$ transitions have been observed by a number of authors[16] at frequencies as low as 11 Mc/sec. Thus far, the more interesting quantum oscillations due to $\Delta n\neq 0$ transition have not been observed.

Note added in proof. It has been pointed out to the author that the treatment of the attenuation of longitudinal waves is not correct because the diffusion current, arising from the nonuniform electron density associated with the wave, was not included. Equation (4) gives only the conduction current. When the diffusion current is included, the quantity σ_{zz} appearing in Eq. (6) of this paper is replaced by

$$\sigma_{zz}+\sigma_0(m/\hbar q\tau)^2(1+i\omega\tau)[g(\omega-i/\tau)]^2 \times[g(\omega-i/\tau)+i\omega\tau g(0)]^{-1},$$

where the function $g(x)$ is defined by

$$g(x)=N^{-1}\sum_{nk_yk_z}\frac{\rho_0[E_n(k_z+q)]-\rho_0[E_n(k_z)]}{k_z+\frac{1}{2}q-(mx/\hbar q)}.$$

Including this diffusion term does not alter the condition (16) necessary for the observation of the quantum oscillations.

It is a pleasure to acknowledge the very helpful comments of Professor Hubertus Stolz concerning this point, and to thank Professor Sergio Rodriguez for many helpful discussions.

ACKNOWLEDGMENT

The author would like to thank Dr. Y. Shapira for pointing out an arithmetical error appearing in a preprint of this work.

[16] A. P. Korolyck and T. A. Pruschak, Zh. Eksperim. i Teor. Fiz. **41**, 1689 (1961) [English transl.: Soviet Phys.—JETP **14**, 1201 (1962)]; Y. Shapira and B. Lax, Phys. Letters **7**, 133 (1963); Y. Shapira, Phys. Rev. Letters **13**, 162 (1964).