these equations each contain a parameter which may be chosen in an effort to improve the higher density solutions. For this paper we have chosen these parameters by examination of the early terms of the density series for $g \exp(\beta\phi)$ and the pressure. We have compared these new equations, with the MC method as the standard, against the PY and CHNC equations. These comparisons were made for the Gaussian, Lennard-Jones, and hard-sphere models. If we assume that MC is reasonably accurate, then we may conclude that the new equations show definite improvement over the CHNC equation in the hard-sphere and Lennard-Jones models, and improvement over PY for the Gaussian model, for the cases studied here. Because of small differences in the results and the uncertainty in the accuracy of all the solutions, it is not clear whether the new equations are any improvement over the PY equation in the hard-sphere and Lennard-Jones cases or the CHNC equation for the Gaussian cases.

At worst, we feel that the equations presented here will have the property of showing close agreement with either the PY or CHNC equations when one of them provides a good answer. We hope to be able to show definite improvement over both of these equations by selecting a case where neither the PY nor the CHNC equation provides an accurate answer. We feel that, relatively, in the case of the Lennard-Jones potential, the PY approximation should worsen and the CHNC approximation should improve as T^* is lowered.[24] If this is the situation there should exist a range in T^* where the new equations show definite improvement over both PY and CHNC. Also, a calculation at lower T^* should begin to show up the differences in Eqs. (A) and (B) and the differences in methods of choosing the parameters a and m. Unfortunately, it is unlikely that MC results are available in these regions and so a detailed study must await such a calculation.

The methods of choosing the parameters a and m used for this paper have the advantage of simplicity but are not necessarily the optimum. Further studies concerning the selection of a and m might lead to even better representations of the radial distribution functions over a wide range of temperatures, densities, and types of potentials.

ACKNOWLEDGMENTS

We wish to thank A. A. Broyles for several helpful suggestions during the course of this research and R. A. Smith for useful discussions concerning the numerical solutions of these equations. The computations were performed on the IBM-709 computer of the University of Florida Computing Center.

[24] See also Khan (Ref. 4) concerning this point.

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Kompaneets Model for Radio Emission from a Nuclear Explosion

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The electromagnetic signal from a nuclear explosion is computed using the same method as presented by Kompaneets. It is shown that some of Kompaneets' approximations are incorrect and lead to the wrong shape for the radiated signal. His work neglects the important first half-cycle of the signal and hence predicts an initial deflection in the wrong direction. A more accurate solution is presented.

I. INTRODUCTION

In a 1958 article in the Soviet literature, Kompaneets[1] described the basic mechanism for radio emission from a nuclear explosion. This description, however, is incorrect at several points. The purpose of this paper is to show that a correct solution for the same model differs substantially from the solution presented by Kompaneets. In particular, he leaves out the important first half-cycle of the signal so that the initial deflection is in the wrong direction.

We shall use essentially the same method of calculation: We numerically integrate Maxwell's equations in dipole approximation, but with different conductivities and currents. We retain the electronic conductivity and neglect the ionic conductivity (he does the opposite) and we retain the Compton current in the field equations where he chooses to drop it (and therefore loses the first half-cycle).

II. RADIATION MECHANISM

The radiation mechanism used by Kompaneets is essentially the following: A nuclear explosion emits a small fraction (say 0.1%) of its energy in the form of prompt gammas with a mean energy of one, or perhaps several, MeV. Kompaneets takes the time dependence

$$dN/dt=N_0be^{-bt} \tag{2.1}$$

[1] A. S. Kompaneets, Zh. Eksperim. i Teor. Fiz **35**, 1538 (1958). [English transl.: Soviet Phys.—JETP **8**, 1076 (1959)].

with $N_0 \approx 10^{22}$ quanta and $b \approx 10^6$ sec^{-1}. In taking such a simple expression one hopes that the radiated signal is not very sensitive to the form of the time dependence.

The gammas travel for appreciable distances at sea level (mean free path $\lambda \approx 3\times10^4$ cm) and Compton scatter electrons in air molecules. The scattered electrons acquire energies of about 1 MeV and travel predominantly radially producing large numbers of secondaries ($\nu = 3\times10^4$ secondaries per MeV). The Compton electrons travel about a meter in air (range $l \approx 10^2$ cm).

A substantial electric field (several hundred V/cm) is set up by the resultant charge separation. The field immediately causes an electric current to flow back through the ionized region. At the same time electrons begin to attach to oxygen molecules with a time constant of about 10^{-8} sec.[2] The electric current, which is initially electronic, gradually obtains a significant ionic contribution and after several microseconds the electron and ion currents are comparable. The calculation of the conductivity will be performed in the next Section, where we will also criticize Kompaneets' approximation.

If the current is asymmetrical then the system will radiate electromagnetic waves. We will assume (with Kompaneets) that the asymmetry arises because the gammas emerge preferentially in one direction.

After some time (say, tens of microseconds), the current and the electric field are reduced to a low level so that the system effectively stops radiating. Eventually the positive and negative ions recombine.

In an axially symmetric explosion with a Compton current which has only a radial component, the resultant electromagnetic field will have only the components E_r, E_θ, and B_φ. Maxwell's equations in spherical coordinates become

$$\frac{1}{r}\frac{\partial}{\partial r}(rE_\theta)-\frac{1}{r}\frac{\partial}{\partial\theta}E_r=-\frac{1}{c}\frac{\partial}{\partial t}B_\varphi, \tag{2.2}$$

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta B_\varphi) = \frac{1}{c}\frac{\partial}{\partial t}E_r+\frac{4\pi}{c}[\sigma(r,\theta,t)E_r+j_r(r,\theta,t)], \tag{2.3}$$

$$-\frac{1}{r}\frac{\partial}{\partial r}(rB_\varphi)=\frac{1}{c}\frac{\partial}{\partial t}E_\theta+\frac{4\pi}{c}\sigma(r,\theta,t)E_\theta, \tag{2.4}$$

where $j_r(r,\theta,t)$ is the radial Compton current and $\sigma(r,\theta,t)$ is the total conductivity of the air. We shall take the current and conductivity to be of the form

$$j_r(r,\theta,t)=j_0(r,t)+j_1(r,t)\cos\theta, \tag{2.5}$$

$$\sigma(r,\theta,t)=\sigma_0(r,t)+\sigma_1(r,t)\cos\theta, \tag{2.6}$$

[2] Kompaneets takes 4×10^{-7} sec which results in a low value for the saturation field. The more recent value is found in L. M. Chanin, A. V. Phelps, and M. A. Biondi, Phys. Rev. Letters **2**, 344 (1959).

and we shall only keep the dipole part of the electromagnetic fields:

$$E_r(r,\theta,t)=E_0(r,t)+E_1(r,t)\cos\theta, \tag{2.7}$$

$$E_\theta(r,\theta,t)=E_2(r,t)\sin\theta, \tag{2.8}$$

$$B_\varphi(r,\theta,t)=B(r,t)\sin\theta. \tag{2.9}$$

Then, assuming $\sigma_1 \ll \sigma_0$, we obtain the following four differential equations:

$$\frac{1}{c}\frac{\partial}{\partial t}E_0=-\frac{4\pi}{c}(\sigma_0E_0+j_0), \tag{2.10}$$

$$\frac{1}{c}\frac{\partial}{\partial t}E_1=\frac{2}{r}B-\frac{4\pi}{c}(\sigma_0E_1+\sigma_1E_0+j_1), \tag{2.11}$$

$$\frac{1}{c}\frac{\partial}{\partial t}E_2=-\frac{1}{r}\frac{\partial}{\partial r}(rB)-\frac{4\pi}{c}\sigma_0E_2, \tag{2.12}$$

$$\frac{1}{c}\frac{\partial}{\partial t}B=\frac{1}{r}E_1-\frac{1}{r}\frac{\partial}{\partial r}(rE_2). \tag{2.13}$$

Before we can solve these equations we must have expressions for the Compton current and the conductivities. The current is a relatively simple quantity but the conductivity is obtained by solving the differential equations which describe the air chemistry.

III. CURRENTS AND CONDUCTIVITIES

The equations governing the behavior of the electron density n, the negative ion density N_-, and the positive ion density $N_+ \approx N_- + n$ are

$$dn/dt=S(r,t)-\alpha n-\epsilon n N_+, \tag{3.1}$$

$$dN_-/dt=\alpha n-\beta N_+N_-, \tag{3.2}$$

$$dN_+/dt=dn/dt+dN_-/dt, \tag{3.3}$$

where $S(r,t)$ is the secondary electron-source function. The coefficient for electron attachment to O_2 is $\alpha\approx10^8$ sec^{-1} and typically the recombination coefficients are $\beta\approx\epsilon\approx2\times10^{-6}$ cm^3 sec^{-1}.[3,4]

Source of Secondary Electrons

For simplicity let us replace the gamma source with an equivalent monoenergetic source of 1-MeV gammas which emits radiation at a rate $Nf(t)$ where $\int dt f(t)=1$ and N is the gamma yield in MeV ($N\approx2.6\times10^{22}\,Y$ where Y is the gamma yield in tons). At 1 MeV, the main attenuation process is Compton scattering. Let us ignore multiple scattering (so that all the energy of the gamma is deposited at the first scattering). The rate per unit area at which gammas are stopped in an inter-

[3] W. H. Kasner, W. A. Rogers, and M. A. Biondi, Phys. Rev. Letters **7**, 321 (1961).

[4] J. Sawyer, Proc. Roy. Soc. (London) **A169**, 83 (1938).

val dr at a radius r is

$$Nf(t-r/c)(e^{-r/\lambda}/4\pi r^2)(dr/\lambda) \tag{3.4}$$

where λ is the gamma mean free path ($\lambda \approx 3\times 10^4$ cm at sea level). The Compton electron slows down by ionizing atoms (ν ion electron pairs per primary electron, $\nu \approx 3\times 10^4$ for a 1-MeV electron) so the rate of production of secondary electrons, per unit volume, is

$$S(r,t)=Nf(t-r/c)\nu(e^{-r/\lambda}/4\pi r^2\lambda). \tag{3.5}$$

We neglect the electron mean free path compared with the gamma mean free path.

Compton Current

In passing, we can use Eq. (3.5) to obtain an approximate expression for the Compton current. It is convenient (and conventional) to make some rather drastic simplifications here. One assumes that the Compton electrons move with the speed of light c, and that the electrons moving past a given point r are roughly the electrons that were produced between r-l and r where l is the electron range ($l \approx 10^2$ cm). From Eq. (3.5) we get

$$j_r(r,t)=eNf(t-r/c)\frac{e^{-r/\lambda}}{4\pi r^2}\frac{l}{\lambda}. \tag{3.6}$$

It may be more accurate to use a somewhat different time dependence in Eq. (3.6) than in Eq. (3.5) to reproduce the effect of a velocity spectrum.

Conductivity

The conductivity is conveniently written in the form

$$\sigma=e(n\mu+N_+\mu_++N_-\mu_-) \tag{3.7}$$

where μ, μ_+, and μ_- are the electron and ion mobilities. Strictly speaking the mobilities depend on the electric field but for the present we assume they are constant. The values of the mobilities are somewhat uncertain but we can take $\mu \approx 10^6$ cgs[5] and $\mu_+=\mu_- \approx 8\times 10^2$ cgs.[6] In these units $e \approx 4.8\times 10^{-10}$.

From now on we shall use a set of units in which distance is measured in units of λ (we let $x=r/\lambda$), time is measured in units of $\lambda/c \approx 10^{-6}$ sec.

The gamma output is taken to be of the form $f(t) \sim e^{-bt}$ with $b \approx 10^6$ sec^{-1}. The steeply increasing initial part is disregarded.[7]

In the new units the electron density equation with $\epsilon=0$ becomes

$$\frac{dn}{dt}+\alpha n=\frac{\nu N}{\lambda^3}\frac{e^{-x}}{4\pi x^2}f(t-x). \tag{3.8}$$

This equation is easily integrated:

$$n=\frac{\nu N}{\lambda^3}\frac{e^{-x}}{4\pi x^2}\frac{b}{\alpha-b}[e^{-b(t-x)}-e^{-\alpha(t-x)}] \tag{3.9}$$

and for $(t-x) \gg \alpha^{-1}$ we have

$$n \approx \frac{\nu N}{\lambda^3}\frac{e^{-x}}{4\pi x^2}\frac{b}{\alpha}e^{-b(t-x)}. \tag{3.10}$$

In the initial phase the ions can be neglected so the conductivity is given by $\sigma=e\mu n$. In fact, it is not a bad approximation to neglect the ion conductivity entirely and to keep only the electronic conductivity.[8] This is exactly opposite of what Kompaneets does.

Radial Electric Field

The radial electric field E_0 is now obtained by integrating Eq. (2.10) with

$$j_0=(Nel/\lambda^3)(e^{-x}/4\pi x^2)f(t-x). \tag{3.11}$$

Then

$$E_0=\int_x^t dt' \exp\left[-4\pi\int_{t'}^t dt''\sigma(t'')\right] \times 4\pi\frac{Nel}{\lambda^3}\frac{e^{-x}}{4\pi x^2}be^{-b(t'-x)}. \tag{3.12}$$

Since $\alpha \gg b$ we have

$$-4\pi\int_{t'}^t dt''\sigma(t'') \approx -\frac{\mu\nu Ne}{\alpha\lambda^3}\frac{e^{-x}}{x^2}[e^{-b(t'-x)}-e^{-b(t-x)}] \tag{3.13}$$

and

$$E_0 \approx E_a\left[1-\exp\left\{-\frac{E_b}{E_a}\frac{e^{-x}}{x^2}[1-e^{-b(t-x)}]\right\}\right] \tag{3.14}$$

where $E_a=\alpha l/\mu\nu$ and $E_b=Nel/\lambda^3$. Note that the field is essentially independent of yield and typically $\alpha l/\mu\nu \approx 0.5$ esu. For $t-x \gg b^{-1}$ we have

$$E_0 \approx E_a\left[1-\exp\left(-\frac{E_b}{E_a}\frac{e^{-x}}{x^2}\right)\right]. \tag{3.15}$$

At relatively close distances the time to reach $E_0 \approx \alpha l/\mu\nu$ is about $(t-x) \approx (b\mu\nu Ne/\alpha\lambda^3)^{-1}$. At greater distances the

[5] R. A. Nielsen and N. E. Bradbury, Phys. Rev. **51**, 69 (1379)

[6] *American Institute of Physics Handbook* (McGraw-Hill Book Company, Inc., New York, 1963), 2nd ed., pp. 7–228.

[7] In the paper by Kompaneets the time behavior of the system is artificially separated into two phases: a short electronic phase lasting about 10^{-6} sec during which a strong radial electric field is established, and a later ionic phase during which the system radiates if the currents are asymmetrical. Electron recombination is neglected ($\epsilon=0$). This is reasonable where $\epsilon N_+ \ll \alpha$, or $N_+ \ll 10^{13}$ cm^{-3}.

[8] A numerical solution of Eqs. (3.1) and (3.2) indicates that the electronic and ionic conductivities become comparable at about $t-x \approx 10^{-5}$ sec.

time to reach the asymptotic value is $(t-x)\approx b^{-1}$. The electric field is fairly constant with $E_0\approx\alpha l/\mu\nu$ and then drops off rapidly. The effect of higher yield is to extend the region in which there is an electric field and to decrease the time constant for approach to the asymptotic field.[9]

If the gamma output is just a very short pulse which can be approximated by $f(t)=\delta(t)$, we obtain

$$n=\frac{\nu N}{\lambda^3}\frac{e^{-x}}{4\pi x^2}e^{-\alpha(t-x)} \quad (3.16)$$

and

$$E_0=E_b\frac{e^{-x}}{x^2}\exp\left\{-\frac{E_b}{E_a}\frac{e^{-x}}{x^2}[1-e^{-\alpha(t-x)}]\right\}. \quad (3.17)$$

At $t=x$ the field has its maximum value $E_0=E_be^{-x}/x^2$ and then it decays, initially at a rate $\mu\nu Ne/\lambda^3$. For large $t-x$ it reaches the value

$$E_0=E_b\frac{e^{-x}}{x^2}\exp\left(-\frac{E_b}{E_a}\frac{e^{-x}}{x^2}\right). \quad (3.18)$$

This function is zero at $x=0$, rapidly increases and reaches a maximum at $(E_b/E_a)e^{-x}/x^2=1$ and then diminishes. The value at the maximum is $E_0=e^{-1}E_a$. Even though the time behavior of the fields is quite different, the asymptotic results are quite similar. Note that if $E_a\to\infty$ (i.e., $\alpha\to\infty$) then $E_0\to E_be^{-x}/x^2$ for both inputs. It is the finite attachment rate α which makes the residual field depend on the gamma input time behavior. At sufficiently large distances the fields are again identical because there is very little current flow before attachment.

Asymmetry

Let us suppose (with Kompaneets) that the gammas are emitted with a small dipole asymmetry.

$$j_r(r,\theta,t)=j_0(r,t)+\xi j_0(r,t)\cos\theta \quad (3.19)$$

where ξ is a small parameter. Therefore

$$\sigma(r,\theta,t)=\sigma_0(r,t)+\xi\sigma_0(r,t)\cos\theta. \quad (3.20)$$

If we measure the fields in units of ξE_a then the equations we have to solve are

$$\partial E_1/\partial t=(2/x)B-4\pi\sigma_0E_1-4\pi\sigma_1E_0-4\pi j_1, \quad (3.21)$$

$$\partial E_2/\partial t=-(1/x)\partial(xB)/\partial x-4\pi\sigma_0E_2, \quad (3.22)$$

$$\partial B/\partial t=-(1/x)E_1-(1/x)\partial(xE_2)/\partial x, \quad (3.23)$$

with

$$4\pi\sigma_0=yu(x)(\alpha b/\alpha-b)[e^{-b(t-x)}-e^{-\alpha(t-x)}], \quad (3.24)$$

$$4\pi\sigma_1E_0=4\pi\sigma_0[1-\exp\{-yu(x)[1-e^{-b(t-x)}]\}], \quad (3.25)$$

$$4\pi j_1=-yu(x)be^{-b(t-x)}, \quad (3.26)$$

where $u(x)=e^{-x}/x^2$. In our units $\alpha\approx100$, $b\approx1$, and $y=E_b/E_a\approx100Y$ where Y is the gamma yield in tons.

The inhomogeneous term in Eq. (3.21) can be rewritten, in our approximation, as

$$4\pi\sigma_1E_0+4\pi j_1\approx\exp\{-yu(x)[1-e^{-b(t-x)}]\}\times yu(x)be^{-b(t-x)}. \quad (3.27)$$

IV. RADIATED FIELDS

In general, the dipole approximation implies that the size of the source is small compared with the wavelength of the radiation. If this condition is not satisfied higher multipoles make significant contributions, unless they happen to vanish or are very small. In our model we have assumed a dipole asymmetry and the only term dropped in arriving at Eqs. (2.10)–(2.13) is the product $(4\pi/c)\sigma_1E_1\cos^2\theta$ which is proportional to the square of the small asymmetry parameter ξ.

There is another more important question: How accurately will the model describe a real source? One can expect that the dipole approximation will be adequate for a rough description of the signal but will not, in general, give the detailed shape. In particular one can expect that the early part of the signal, which arises from a source which is expanding with almost the speed of light, is not given correctly at all. However, the closer the asymmetry in the Compton current is to the dipole form the better the over-all result will be.

The Distant Fields

In the radiation zone (that is, $x\gg$wavelength $\gg$source radius) we have

$$E_\theta=B_\varphi=[\ddot{Z}(t-x)/x]\sin\theta \quad (4.1)$$

where Z is the dipole moment, $Z=\Sigma\, ex$. It is most useful to present results for the quantity $\ddot{Z}(t-x)$.

At shorter distances, approximately of the order of the wavelength but still much larger than the radius of the source, the fields are somewhat more complicated.

$$B_\varphi=(\ddot{Z}/x+\dot{Z}/x^2)\sin\theta, \quad (4.2)$$

$$E_\theta=(\ddot{Z}/x+\dot{Z}/x^2+Z/x^3)\sin\theta, \quad (4.3)$$

$$E_r=2(\dot{Z}/x^2+Z/x^3)\cos\theta, \quad (4.4)$$

where again $Z=Z(t-x)$. By extending the numerical calculation out to some intermediate distance (say, $x=10$) and comparing with Eqs. (4.2)–(4.4), one can extract the quantity $\ddot{Z}(t-x)$. Note that the above fields are exact solutions to Maxwell's equations in free space.

The solutions are required to satisfy the boundary condition $E_\theta=0$ at $x=a$ for all t, where $a\ll1$. In other words we assume that at the center of the explosion there is a small perfectly conducting sphere.

Numerical Solution

We shall now present a number of solutions. The fields E_1, E_2, and B are obtained by integrating out

[9] Actually, Kompaneets disregards the retardation of the gammas and the decrease of the electric field at large distances.

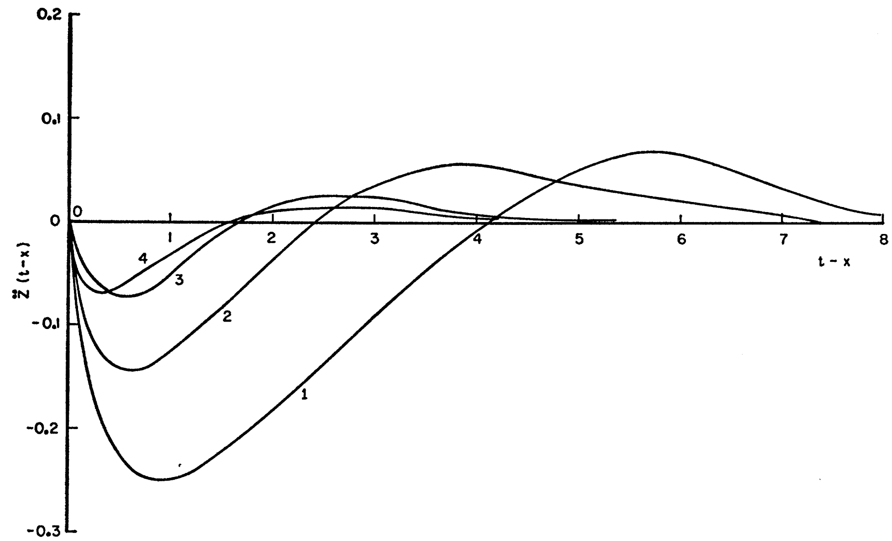

FIG. 1. Second derivative of the dipole moment for several values of E_b/E_a: (1) $E_b/E_a = 100$, (2) $E_b/E_a=10$, (3) $E_b/E_a=1$. Curve (4) is also for $E_b/E_a=1$ but with $a=0.1$. The previous curves were all run with $a=0.5$. Since $E_b/E_a = 1$ corresponds to the smallest ionized sphere the discrepancy would be greater for this case. The unit of time is 10^{-6} sec.

along characteristics. The fields satisfy the boundary condition $E_2(a)=0$ with $a=0.5$. (The results of several cases with $a=0.1$ were not significantly different.) The center of the burst has in effect been replaced by a perfectly conducting sphere.

To compare with Kompaneets we shall take $b=10^6$ $\sec^{-1}$ and the attachment coefficient $\alpha \approx 10^8$ $\sec^{-1}$. Since ξE_a is simply a scale factor we are left with only one parameter, E_b/E_a, to vary. In Fig. 1 we present $\ddot{Z}(t-x)$ for several values of E_b/E_a (essentially the yield).

Kompaneets entirely neglects the direct effects of the Compton current, and he therefore obtains an initial downward deflection (positive E_θ) for the case where the initial asymmetry results in an excess of electrons moving upward. One would expect to get the opposite result as in Fig. 1.

ACKNOWLEDGMENTS

I am indebted to Glenn Peebles and Don MacNeilage for the numerical solutions presented in Figs. 1 and 3.

APPENDIX: REVIEW OF KOMPANEETS' SOLUTION

For the purpose of solving Eqs. (2.11)–(2.13) Kompaneets disregards j_1, the asymmetric part of the Compton current, and he neglects the electronic conductivity. That is, he imagines that the Compton current has died out and all the electrons have attached to oxygen molecules before the system starts radiating so that for the purpose of computing the radiation source we need only use the ionic conductivity. This last approximation is poor but the first is even more serious. It causes Kompaneets to miss entirely the first main half-cycle of the signal. His initial half-cycle is therefore in the wrong direction. We shall, however, make the same approximation in order to check his numerical results.

Ionic Conductivity

When all the electrons have attached, there are equal numbers of positive and negative ions which disappear through recombination at the rate β:

$$dN_+/dt = -\beta N_+^2. \tag{A1}$$

One has immediately

$$N_+ = N_0/[N_0\beta(t-t_0)+1]. \tag{A2}$$

Kompaneets treats the case of small dipole asymmetry in the gamma source which results in a small asymmetry in the initial ion density: $N_0=\bar{N}_0(1+\xi\cos\theta)$. Expanding

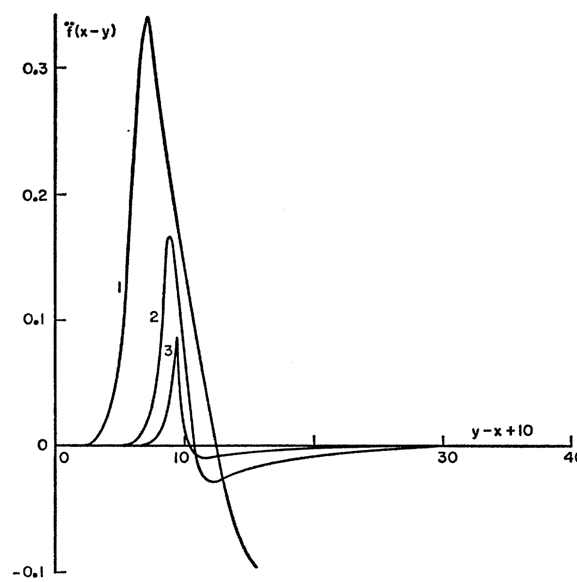

FIG. 2. Second derivative of the dipole moment as obtained in Ref. 1. Values of parameters: (1) $\kappa=4$, $m=200$; (2) $\kappa=1$, $m=10$; (3) $\kappa=1$, $m=1$.

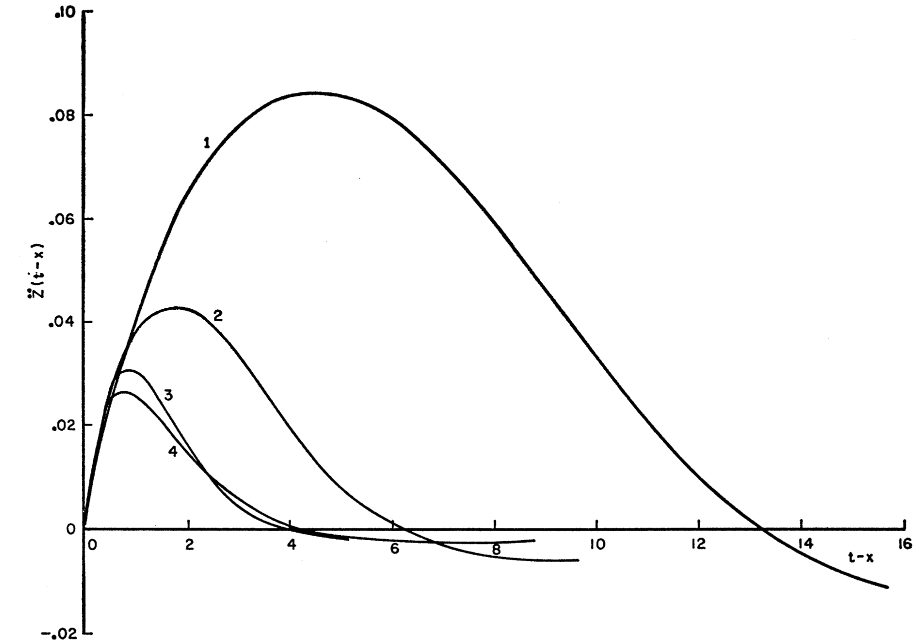

FIG. 3. $\ddot{Z}(t-x)$ recomputed for the same cases as in Fig. 2. Curve (4) has the same values of κ and m as curve (3) but was computed with $a=0.1$.

Eq. (A.2) in ξ yields for the conductivity $\sigma_i=e\mu_i n$

$$\sigma_i=\frac{\bar{N}_0 e\mu_i}{\bar{N}_0\beta(t-t_0)+1}+\xi\frac{\bar{N}_0 e\mu_i}{[\bar{N}_0\beta(t-t_0)+1]^2}\cos\theta \quad \text{(A3)}$$
$$=\sigma_0+\sigma_1\cos\theta.$$

Kompaneets puts $t_0=0$ and

$$\bar{N}_0=(\nu N/\lambda^3)(e^{-x}/4\pi x^2). \quad \text{(A4)}$$

The ionic current leads to a gradual decrease of the large radial electric field. Since the Compton current is absent we have

$$\partial E_0/\partial t+4\pi\sigma_0 E_0=0 \quad \text{(A5)}$$

and

$$E_0(t)\approx E_a\exp\left[-4\pi\int_0^t dt'\sigma_0(t')\right] \quad \text{(A6)}$$
$$\approx E_a/(\bar{N}_0\beta t+1)^\kappa$$

where $E_a=\alpha l/\mu\nu$ and $\kappa=4\pi e\mu_i/\beta$.

Note that at the beginning of the radiation phase we have a spherically symmetrical radial electric field and an asymmetrical conductivity resulting in a net current which is the source of the radiation.

Numerical Integration

Kompaneets then integrates the following equations:

$$\partial E_1/\partial t=(2/x)B-4\pi\sigma_0 E_1-4\pi\sigma_1 E_0, \quad \text{(A7)}$$
$$\partial E_2/\partial t=-(1/x)\partial(xB)/\partial x-4\pi\sigma_0 E_2, \quad \text{(A8)}$$
$$\partial B/\partial t=-(1/x)E_1-(1/x)\partial(xE_2)/\partial x, \quad \text{(A9)}$$

where

$$4\pi\sigma_0=\frac{m\kappa u(x)}{[mu(x)t+1]}, \quad \text{(A10)}$$
$$4\pi\sigma_1 E_0=\frac{m\kappa u(x)}{[mu(x)t+1]^{2+\kappa}}, \quad \text{(A11)}$$

with $u(x)=e^{-x}/x^2$ and $m\,u(x)=\bar{N}_0\beta$. The fields E_1, E_2, and B are now measured in units of ξE_a. Kompaneets computes the radiated field for three sets of values of the parameters m and κ: (1) $m=1$, $\kappa=1$; (2) $m=10$, $\kappa=1$; and (3) $m=200$, $\kappa=4$. The actual numerical process is not described.

In Fig. 2 we reproduced the figure from Kompaneets' article. He presents the second derivative of the dipole moment [which he calls $\ddot{f}(x-y)$, where $y=ct$] for three values of the parameters κ and m.

We have also computed $\ddot{Z}(t-x)$ for the same values of the parameters and the results are presented in Fig. 3. The results differ from those in Fig. 2 in several respects. First, our results are smaller by a factor of about four. Second, the shape of the signals is different in that our signals are relatively steeper at the leading edge. And finally, the two sets of curves do not have the same relative positioning. The discrepancy is hard to explain.