ionic character and would therefore be applicable to the III-V semiconductors. Since the effective charge on each ion in InSb is ≈ 0.3 of an electron charge, the Varley mechanism may be electrostatically possible for a single ionization, whereas a double ionization is required in the case of an alkali halide. Modifications have also been proposed to the Varley mechanism,[14] but further work would be required to consider such details of the mechanisms.

The annealing data show that the damage introduced by x rays has a large annealing stage near 102°K. This stage is to be distinguished from Stage II recovery[3] in electron-irradiated p-type InSb which would anneal near 95°K in p-type InSb having the same carrier concentration as our sample had after irradiation.[8] The defects annealing in Stage II in an electron-irradiated sample are characterized by a threshold electron energy of 268 keV. Electron irradiations with energies below 268 keV produce defects which also anneal near 102°K[15] similar to those produced by x rays. In addition the annealing temperatue for these defects is independent of carrier concentration in contrast to Stage-II annealing, and preliminary results also indicate that the production rate is higher in p-type InSb than it is in n type.[15] Whereas Eisen has identified the defects which annihilate in Stage II as being produced by the displacement of indium atoms,[3] the identity of the defects produced by ionization and annihilating near 102°K is unknown and requires further investigation.

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ACKNOWLEDGMENTS

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Effect of Spin-Orbit Interaction on the Magnetoresistance of Single-Crystal Nickel and Nickel-Iron Thin Films*

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In a previous paper the author has reported the results of transverse magnetoresistance measurements performed on single-crystal nickel and nickel-iron thin films obtained by vacuum deposition. The anisotropies observed are in general agreement with the observations of Döring and Kaya for bulk single-crystal nickel. Smit has analyzed the anisotropies in the magnetoresistance of ferromagnetic materials in terms of the influence of the spin-orbit interaction and of Mott's theory of conduction in ferromagnetic materials. However, these results are suitable only for explaining the anisotropy between the resistances measured parallel to and transverse to the direction of the magnetization in the specimen. It is demonstrated herein that the previous work may be extended in a manner that describes the anisotropies observed in the transverse magnetoresistance of the ferromagnetic thin films as a function of the orientation of the magnetization.

INTRODUCTION

IN general, it is possible to obtain large amounts of information concerning the electrical conductivity of metals and semiconductors by measuring the galvanomagnetic[1–6] effects which are present in specimens of the material under investigation. This is especially true for measurements on single-crystal specimens wherein the knowledge of the orientation of the crystalline structure of the material makes possible a more direct verification of the various phenomenological theories relating to the aforementioned properties. However, the experimental data which are presently available from magnetoresistance measurements on single-crystal ferromagnetic specimens are relatively incomplete.[5,7–10] The work reported herein is concerned with the anisotropies observed in the transverse magnetoresistance measurements performed on single-crystal nickel and nickel-iron thin films.

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EXPERIMENTAL PROCEDURE

The films were obtained by epitaxial vacuum deposition of the nickel and nickel-iron alloy on cleaved and polished sodium chloride substrates which were temperature cycled in an appropriate manner[11,12] in a vacuum of 10^{-5} Torr. The structure, the orientation and the composition of the films were determined by electron transmission diffraction studies, and the thickness of the films was measured by multiple-beam interferometry.

The magnetoresistance data were taken at room and at liquid-nitrogen temperatures in a saturating magnetic-field intensity of 12 kG; the resistance of the films was measured using standard four-point potentiometric techniques. The transverse magnetoresistance was then computed from $\Delta\rho/\rho_0=[\rho(B)-\rho_0]/\rho_0$ where $\rho(B)$ is the value of the resistivity in the presence of the external magnetic field B and ρ_0 is the zero-field value of the resistivity. In all the measurements ρ_0 was less than $\rho(B)$ and $\Delta\rho/\rho_0$ was always negative. The results of these measurements have been reported in a previous paper[13] by the author.

The general shape of the anisotropies observed in the data measured at room temperature for the nickel films is in qualitative agreement with the results of Kaya[14] and of Döring[15] for bulk nickel single-crystal specimens, and essentially the same type of anisotropies were observed in the measurements for the nickel-iron films. These results are typified in Fig. 1 which represents the measured values of $\Delta\rho/\rho_0$ for a nickel film. The data are taken from Fig. 1 of Ref. 13, and were measured with the film oriented such that the current was in a [100] direction and the magnetic field rotated in a plane perpendicular to that direction. In order to avoid confusion when comparing with the theoretical results obtained later, the aforementioned experimental data are plotted in the present Fig. 1 versus Θ, which is the complement of the polar coordinate used in Ref. 13.

The differences between the results shown in Fig. 1 and the results of previous investigators mentioned are as follows: For measurements with the current in a [100] direction, Döring's results showed an exact four-pole symmetry, with the magnetoresistance going positive for orientations of the magnetic near the [100] directions in each quadrant of the plane of rotation. The measurements taken on the ferromagnetic thin films in the present research showed negative values of magnetoresistance for all orientations of the magnetic field in its plane of rotation. Further, the results obtained by Döring indicated maxima near $\Theta=45°$ in each quadrant, whereas the present data show maxima

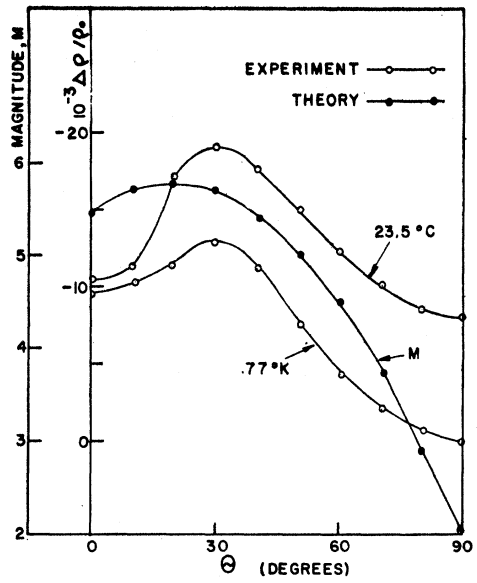

FIG. 1. M, the sum of the magnitudes of the pertinent coefficients in Eqs. (14), and $\Delta\rho/\rho_0$ for a nickel film, plotted versus Θ. Θ is the angle between the direction of the magnetic field and the z axis of the atomic orbitals.

near $\Theta=30°$. This displacement in the maxima may be due to the effects of the sample thickness as is discussed in the following paragraph. However, that the anisotropies observed are not due to demagnetization effects caused by the thin-film geometry, has been shown[13] by comparing the data for single-crystal films with that taken for a polycrystalline film.

Measurements taken at the liquid-nitrogen temperature show additional marked anisotropies in the case of the nickel film. The magnetoresistance, for orientations wherein the magnetic field is parallel to the plane of the film, apparently disappears or at least becomes very small. When the magnetic field is oriented so that it is in a direction parallel to the plane of the film, then the deflection of the electron velocities is in a direction toward the surfaces of the film, and for cases where the mean free path of the electrons is comparable to the thickness of the film (i.e., at low temperatures), the influence of the surface scattering may result in a relaxation time which is relatively independent of velocity causing, in turn, a reduction of the magnetoresistance effect. MacDonald[4] reported a similar effect in measurements on sodium and on rubidium plates. That this diminution of the magnetoresistance originates from the gross geometry of the film may, at first, be inferred from the fact that it also appears in the results of measurements on the polycrystalline film.[13] However, it will be shown that the anisotropies may, in part, also be explained in terms of the spin-orbit interaction. The apparent anomalous decrease in $\Delta\rho/\rho_0$ with a decrease in temperature, as seen in Fig. 1, has been discussed elsewhere.[13]

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SPIN-ORBIT INTERACTION

Smit[16] has accounted for the anisotropy in the resistance of a ferromagnetic metal where the resistance measured parallel to an imposed magnetic field is greater than the resistance measured in a direction perpendicular to the magnetic field, by considering the perturbation due to the spin-orbit interaction of the $3d$ electrons. The considerations are based on the theory of Mott[17,18] which assumes that the electrical current in these materials is ascribable mainly to the $4s$ electrons. The theory, which will be outlined here for convenience, assumes further that the resistance is due to the scattering of the $4s$ electrons into other $4s$ states and into the $3d$ states; and that the direction of spin does not change during the transition. Thus, ignoring the spin exchange between the s electrons, the current may be assumed to be divided into two parts, one with the spin parallel to the direction of the magnetization, and the other with opposite spin. The $3d$ states have a very large density and as such can account for a large part of the resistance. If the temperature is low, then the $3d$ states with parallel spin are assumed to be almost completely occupied and the s-d transitions for the parallel-spin electrons make essentially little contribution to the resistance. Thus, the parallel-spin $4s$ electrons scatter mainly into other 4-s states. If a magnetic field is imposed, the density of the parallel $3d$ states at the Fermi surface, and therefore the resistance, are decreased with an increase in the field (an increase in the magnetization); this rough model also serves to describe the negative magnetoresistance of ferromagnetic samples. Smit assumed the spin-orbit interaction operator to be a small perturbation of the form

$$H^1=K(\mathbf{L}\cdot\mathbf{s})=K[L_zs_z+\tfrac{1}{2}(L_x-L_y)(s_x+is_y)+\tfrac{1}{2}(L_x+iL_y)(s_x-is_y)], \quad (1)$$

where $\mathbf{L}$ is the orbital angular momentum operator, and s_x, s_y, and s_z are the Pauli matrices.

The development given by Smit shows that the effect of the perturbation operator (1) results in a mixing of the parallel and antiparallel states. This mixing of parallel-spin states with antiparallel-spin states represents an effective loss of a fraction of these orbits to the former type of state, and Smit attributed the anisotropy in the magnetoresistance to the unsymmetrical mixing of these states. The zero-order approximations for the $3d$ states in the metal were taken to be the atomic d functions in a cubic field. The perturbation due to the cubic field, which splits the fivefold degeneracy of these states into a twofold and a threefold degeneracy, was ignored in this development under the assumption that the energy difference between these two degenerate levels is very small. It was assumed, however, that the parallel-spin and the antiparallel-spin states are separated owing to the exchange interaction, by an energy difference 2δ, where δ is the exchange energy. Thus, the unperturbed wave functions are

$$\varphi_1=yzf(r), \quad \varphi_2=xzf(r), \quad \varphi_3=xyf(r)$$
$$\varphi_4=\tfrac{1}{2}(x^2-y^2)f(r), \quad \varphi_5=(1/2\sqrt{3})(r^2-3z^2)f(r), \quad (2)$$

where, for a ferromagnetic metal with an fcc structure, the orbitals given by Eqs. (2) are oriented so that the x, y, and z axes are aligned parallel with the cube edges.[19] The perturbed wave functions are then computed, using the usual first-order perturbation theory and Eqs. (1) and (2), and the results are

$$\begin{aligned}
\psi_1^-&=\varphi_1^-+(A/2\delta)(\varphi_3^+-i\varphi_4^++i\sqrt{3}\varphi_5^+),\\
\psi_2^-&=\varphi_2^-+(A/2\delta)(-i\varphi_3^+-\varphi_4^+-\sqrt{3}\varphi_5^+),\\
\psi_3^-&=\varphi_3^-+(A/2\delta)(-\varphi_1^+-i\varphi_2^+),\\
\psi_4^-&=\varphi_4^-+(A/2\delta)(-i\varphi_1^++\varphi_2^+),\\
\psi_5^-&=\varphi_5^-+(A/2\delta)(i\sqrt{3}\varphi_1^++\sqrt{3}\varphi_2^+).
\end{aligned} \quad (3)$$

The above equations are in agreement with the equations given by Smit except for the signs of some of the coefficients. The plus and minus exponents refer to the parallel-spin and antiparallel-spin states, respectively.

After establishing the results given by Eqs. (3), Smit showed that the mixing of the states is not symmetrical, but in fact, the mixing of the φ_1^+, φ_2^+, and φ_5^+ states predominates over the mixing of the φ_3^+ and φ_4^+ states.

Representing the $4s$ electrons by free-electron wave functions, Smit approximated the transition probability from an s-electron state to a φ_n state by

$$P_{sd}\sim\left|\int_{-\infty}^{+\infty}\iiint\exp(-\mathbf{k}\cdot\mathbf{r})V\varphi_n d\tau\right|^2, \quad (4)$$

where $\mathbf{k}$ is the momentum vector, $\mathbf{r}$ the position vector, V the perturbing potential which is assumed to have spherical symmetry, and $d\tau=dxdydz$. In Eq. (4) the function $f(r)$ decays very rapidly as r increases from the center of the atom and V decays as a screened potential function or faster. Thus, the contribution to the value of the integral comes from only a very small range of the spatial dimensions x, y, and z, i.e., x, y, and z the order of interatomic distances. The function $f(r)$ is also assumed to have spherical symmetry.

In the case of a z-directed electron (an electron with its velocity in the direction of the magnetization) the transition probability P_{sd} is found to be zero for φ_1, φ_2, and φ_4. However, in the case of the x-directed electrons, or in the case of the y-directed electrons, the transition probabilities are zero only for φ_1, φ_2, and φ_3. Therefore, a greater number of orbits (with a greater total effective transition probability) can capture electrons moving perpendicular to the magnetic field than those which are moving parallel to the field.

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The model of the d bands used by Smit consists of two sub-bands, one for the parallel-spin energy levels and one for the antiparallel-spin energy levels. The exchange interaction shifts these two levels with respect to each other by the amount of energy 2δ, as previously described. In the case of nickel, for $T=0°K$, the Fermi level is above the top of the filled parallel-spin d-electron levels. However, at temperatures above absolute zero there is a small, nonzero, density of states near the Fermi level which are unfilled. If a magnetic field is applied, a greater number of these states is filled and, as the magnetic field strength is increased it polarizes more of the electrons in the d bands into the parallel-spin condition. Thus, there is less opportunity for scattering into the parallel-spin levels and the resistance due to this scattering is reduced. The model assumed by Smit also considers that the direction of spin is maintained during the scattering, and places the resistance due to the scattering of electrons into the antiparallel-spin s or d states in parallel with the resistance due to scattering of electrons into the parallel-spin s or d states. Since the antiparallel-spin states are more numerous near the Fermi level than are the parallel-spin states, the resistance due to scattering into the parallel-spin states is the smaller of the two and is thus more effective in determining the total effective resistance of the sample. Smit concluded that the anisotropies in the resistance are due to the resistance component produced by the s-d scattering in the parallel-spin states, which represents the lowest component of resistance due to the low density of parallel-spin d states as previously discussed. The above conclusion was explained by Smit using the discussion which follows in the next paragraph. It should be pointed out that Mott's theory assumes that the density of parallel-spin d-electron states is low near the Fermi surface, in order to obtain the conclusions which are made. As mentioned previously, the position of the Fermi surface for nickel is such that the density of states is extremely low, and the model would not seem to account by itself for the magnitude of the effects observed, in ferromagnetic samples, by Smit and the other investigators.

In Eq. (3), the presence of a φ_n^+ state in the expressions for the ψ_n^- can be interpreted as meaning that the antiparallel-spin levels are sharing the φ_n^+ states with the parallel-spin levels, and the greater the amount of mixing of a φ_n^+ state in the ψ_n^- levels, the smaller is the proportionate share of that state available in the parallel-spin levels. The effective number of states which scatter parallel-spin electrons with velocities directed in the x-y plane (perpendicular to the direction of the magnetic field), and which are mixed in Eqs. (3), is greater than those states which scatter parallel-spin electrons with z-directed velocities. However, the former states are less available in the parallel-spin levels than are the latter states. Thus, the resistance of the sample is greater when the measuring current is parallel to the magnetic field than for the case when the current is perpendicular to the magnetic field. This is the result at which Smit arrived, and the result is borne out by his experiments. However, the analysis presented by Smit does not describe the anisotropies observed in the transverse magnetoresistance as a function of the azimuth of the magnetic field, for a ferromagnetic material, but may be modified to do so.

AN EXTENSION OF SMIT'S WORK

In order to describe, in terms of spin-orbit interaction, the anisotropies observed in the transverse magnetoresistance measured in the thin films, it is necessary to modify the perturbation operator (1). The matrices which are used to describe the electronic spin are defined in terms of the value of the z component of the spin which is taken to be in the direction of the magnetic field. Thus, if the magnetic field is rotated away from the z axis of the coordinate system of the atomic orbitals, the z axis of the spin system will also be rotated along with the field. However, the orbital angular momentum is locked into position by its interaction with the crystalline potential field. Thus the electronic spin will sense the symmetry of the crystal via the spin-orbit interaction. If it is assumed that the magnetic field (and therefore the coordinate system of the spin operator) is rotated away from the z axis by an angle Θ, in a plane displaced from the x axis by an angle Φ, then the spin operator may be described in terms of the new coordinate system. The components of the new spin operators may be determined by[20]

$$s_n'=\frac{1}{2}\begin{pmatrix}\cos\gamma & \cos\alpha-i\cos\beta\\ \cos\alpha+i\cos\beta & -\cos\gamma\end{pmatrix}, \tag{5}$$

where α, β, and γ are the angles between the n direction and the x, y, and z axes, respectively. Thus for the case where the current is along a [100] direction (i.e., along the x axis) and the magnetic field is rotated in a perpendicular (100) plane (i.e., in the y-z plane), Eq. (5) generates the new spin operators[20]

$$s_x'=\frac{1}{2}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \tag{6}$$

$$s_y'=\frac{1}{2}\begin{pmatrix}-\sin\Theta & -i\cos\Theta\\ i\cos\Theta & \sin\Theta\end{pmatrix}, \tag{7}$$

$$s_z'=\frac{1}{2}\begin{pmatrix}\cos\Theta & -i\sin\Theta\\ i\sin\Theta & -\cos\Theta\end{pmatrix}, \tag{8}$$

and the new eigenfunctions, because of the rotation,

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become

$$\sigma_+' = [\cos(\Theta/2)]\sigma_+ + [i\sin(\Theta/2)]\sigma_-, \quad (9)$$

$$\sigma_-' = [\sin(\Theta/2)]\sigma_+ + [i\cos(\Theta/2)]\sigma_-, \quad (10)$$

where σ_+ and σ_- are the eigenfunctions for the case where the spin coordinate system had its z axis aligned with the direction of the magnetic field. If the new perturbation operator, corresponding to Eq. (1), is applied, then

$$(\varphi_m\sigma_+', L_-s_+'\varphi_n\sigma_-') = (-i\cos\Theta)(\varphi_m, L_-\varphi_n), \quad (11)$$

$$(\varphi_m\sigma_+', L_+s_-'\varphi_n\sigma_-') = 0, \quad (12)$$

$$(\varphi_m\sigma_+', L_zs_z'\varphi_n\sigma_-') = (-i4\sin\Theta)(\varphi_m, L_z\varphi_n). \quad (13)$$

It should be noted that Θ is not the spherical coordinate θ of the atomic orbitals, and thus the coefficient in Θ may be factored out of the above integrals. The integrals which include $(\varphi_m, L_-\varphi_n)$ are similar to those leading to the coefficients of Eq. (3). The integral $(\varphi_m, L_z\varphi_n)$ introduced an additional mixing of states. Thus, if the coefficients due to (11), (12), and (13) are added for each state, the equation corresponding to Eq. (3) with the modification showing the effect of the rotation of the magnetic field are

$$\begin{aligned}
\psi_1^- &= \varphi_1^- + (A/2\delta)(a_{12}\varphi_2^+ + a_{13}\varphi_3^+ \\
&\qquad + ia_{14}\varphi_4^+ + i\sqrt{3}a_{15}\varphi_5^+), \\
\psi_2^- &= \varphi_2^- + (A/2\delta)(a_{21}\varphi_1^+ + ia_{23}\varphi_3^+ \\
&\qquad + a_{24}\varphi_4^+ + \sqrt{3}a_{25}\varphi_5^+), \\
\psi_3^- &= \varphi_3^- + (A/2\delta)(a_{31}\varphi_1^+ + ia_{32}\varphi_2^+ + a_{34}\varphi_4^+), \\
\psi_4^- &= \varphi_4^- + (A/2\delta)(ia_{41}\varphi_1^+ + a_{42}\varphi_2^+ + a_{43}\varphi_3^+), \\
\psi_5^- &= \varphi_5^- + (A/2\delta)(i\sqrt{3}a_{51}\varphi_1^+ + \sqrt{3}a_{52}\varphi_2^+),
\end{aligned} \quad (14)$$

where

$$\begin{aligned}
&a_{12} = -a_{21} = i2\sin\Theta, \\
&a_{13} = a_{42} = a_{52} = a_{15} = a_{51} = -a_{24} = -a_{25} = -a_{32} \\
&\qquad = -a_{14} = -a_{23} = -a_{41} = -i\cos\Theta, \\
&a_{34} = a_{43} = -i2\sin\Theta.
\end{aligned} \quad (15)$$

Thus, the coefficients in Eqs. (14) can be used as a measure of the share of the φ^+ states which are now in the antiparallel-spin levels, or as a measure of the share of the φ^+ states which are lost to the parallel-spin levels.

In the case of x-directed parallel-spin electrons only the φ_4^+ and the φ_5^+ states have nonzero transition probabilities, so that the variation of the magnitude of the φ_4^+ and the φ_5^+ coefficients in Eq. (14) reflect the change in the resistance due to the scattering of the parallel-spin s electrons into the parallel-spin d levels. This resistance can be taken as being directly proportional to the sum of the magnitudes of all the φ_4^+ and the φ_5^+ terms, since each of the φ_n^+ are themselves individual electron states. If this sum of magnitudes is evaluated for Θ over the range 0° to 90° (the situation repeats itself over all four quadrants), it is found that the variation has an anisotropy with the same general symmetry as is observed in the magnetoresistance measurements depicted in Fig. 1. A plot, versus Θ, of the variation of the sum M of the magnitudes of the φ_4^+ and the φ_5^+ terms is also included in Fig. 1. It may easily be shown that if some factor were to make the coefficient a_{34} stronger, say double its value as computed in Eq. (15), then the shape of the curve for M would come very close to the curves plotted from the experimental results.

The computations, showing the modifications which are necessary in the coefficients of Eqs. (14), for the case where the current is in a [110] direction with the magnetic field rotated in a (110) plane perpendicular to the direction of the current, are somewhat more tedious and will not be presented. However, a rotation of the spin system by an angle Θ from the z axis and an angle Φ from the x-y plane, produces a representation of the spin eigenstates given by[20]

$$\begin{aligned}
\sigma_+'' &= [\cos(\Theta/2)]\sigma_+ + \epsilon^{i\Phi}[\sin(\Theta/2)]\sigma_-, \\
\sigma_-'' &= [\sin(\Theta/2)]\sigma_+ - \epsilon^{i\Phi}[\cos(\Theta/2)]\sigma_-.
\end{aligned} \quad (16)$$

It is thus seen that the half-angle of rotation from the z axis is involved, as before, resulting in a twofold symmetry for the spin-orbit interaction, here, as well as in the previous case. This conclusion would appear to be borne out by the data obtained for the measurements[13] with a [110]-directed current.

CONCLUSION

The transverse magnetoresistance measurements taken during this research showed anisotropies which were in general agreement with measurements performed on bulk samples of single-crystal nickel at room temperature, by previous investigators. The literature[21,22] mentions that the anisotropies in the resistance of ferromagnetic materials is likely to be due to the spin-orbit interaction, which permits the conduction electrons in a solid to sense the symmetry of the crystalline potential in whose environment they exist. The analysis presented herein seems to justify this conclusion, and thus the present data represent the first experimental evidence which has been reported that shows, at least qualitatively, the consequences of the spin-orbit effect, in terms of magnetoresistence measurements, in a manner more general than that presented by Smit.

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