Abrupt Kink Model of Dislocation Motion. III. The Effect of Internal Stresses

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The role of internal stresses in determining the internal friction of cold-worked polycrystalline materials is investigated by means of the abrupt kink model of dislocation motion. An internal friction peak is found which has a characteristic relaxation time equal to the recombination lifetime of kinks. This is identified with the less detailed result of the Seeger-Paré mechanism of the Bordoni peak, as found by elementary rate theory. The results of the theory are shown to be capable of explaining all the features of the Bordoni peak. The need for further systematic experiments is emphasized, particular reference being made to means of differentiating between the above mechanism and that based upon the hypothesis of thermally activated kink motion.

I. INTRODUCTION

In previous publications,[1,2] we have formulated a new kink model of dislocation motion and have explored some of its consequences according to different possible hypotheses regarding the character of the kink motion. The present treatment, which should be viewed as a continuation of this development, is devoted to a study of an additional aspect of the model, namely the effect of internal stresses upon the dislocation motion. Assuming again that the dislocation loops are firmly pinned, we show that in this instance the model yields a refined version of the Seeger[3]-Paré[4] theory of the Bordoni peak.[5] This more detailed description allows a comparison between theory and experiment more extensive than previously possible. It is concluded that this theory, incorporating internal stresses, or our previous model, based upon the hypothesis of thermally activated kink motion, are equally capable of accounting for the basic properties of the Bordoni peak.

The organization of the paper is as follows. The following section contains a brief summary of those earlier results[1] which are pertinent to the subsequent development. The physical origin of the relaxation peak, in terms of our model, is discussed and the mathematical details are presented. Section III is devoted to a comparison between theory and experiment. We consider almost exclusively experiments on polycrystalline Cu. Particular attention is paid to recent work on electron microscopy,[6] strain-aging,[7] and the annealing characteristics of the Bordoni peak.[8] Finally, Sec. IV contains a summary and discussion of this work in relation to our previous theory. The present lack of definite evidence for determining which of the two mechanisms is responsible for the Bordoni peak is discussed and further experimental work is suggested. A model for reconciling both these theories is also tentatively considered.

In the course of this development we shall have need to discuss certain topics which, although relevant to our treatment, are not essential to the main argument. These are kink statistics and the effect of kink-kink interactions in our previous theory.[9] Accordingly, they are discussed in Appendices which are referred to at appropriate points in the text.

II. THEORY

As we have shown earlier, the motion of a dislocation may be determined by the coupled transport equations for the left- and right-kink densities, $n(x,t)$ and $p(x,t)$, respectively. These are

$$\begin{aligned}(\partial n/\partial t)+(\partial I_n/\partial x)-g+rnp&=0,\\(\partial p/\partial t)+(\partial I_p/\partial x)-g+rnp&=0,\end{aligned}\qquad(1)$$

where I_n and I_p are the appropriate currents, g the generation rate per unit length of double kinks, and r is a recombination velocity. Solution of (1) yields then the densities, n and p, from which the dislocation configuration, $y(x,t)$, is found by integration;

$$y(x,t)=a\int_{-L/2}^{x}\{n(x',t)-p(x',t)\}dx'.\qquad(2)$$

Here, a is the normal distance between close-packed rows in the slip plane, and the pinning points have been located at $x=\pm(L/2)$.

The currents I_n and I_p depend both upon the stress σ, acting on the dislocation, and the mutual interaction between the kinks of which it is constituted.[2] That is,

$$I_p=F\mu p-D\partial p/\partial x,\qquad(3)$$

and

$$I_n=-F\mu n-D\partial n/\partial x,\qquad(4)$$

where $F(x,t)$ is the force acting on a right kink, and μ and D are, respectively, the mobility and diffusion coefficient of a kink, it being assumed that the latter satisfy the Einstein relation, $\mu\kappa T=D$.

[1] A. D. Brailsford, Phys. Rev. **122**, 778 (1961).
[2] A. D. Brailsford, Phys. Rev. **128**, 1033 (1962).
[3] A. Seeger, Phil. Mag. **1**, 651 (1956).
[4] V. K. Paré, J. Appl. Phys. **32**, 332 (1961).
[5] D. H. Niblett and J. Wilks, *Advances in Physics*, edited by N. F. Mott (Taylor and Francis, Ltd., London, 1960), Vol. 9, p. 1.
[6] J. E. Bailey, Phil. Mag. **8**, 223 (1963).
[7] H. K. Birnbaum, J. Appl. Phys. **34**, 2175 (1963).
[8] L. J. Bruner and B. M. Mecs, Phys. Rev. **129**, 1525 (1963).
[9] P. D. Southgate and A. E. Attard, Bull. Am. Phys. Soc. **7**, 345 (1962).

The force F is given by

$$F=\int_{-L/2}^{L/2}[n(x')-p(x')]\frac{\partial U(x-x')}{\partial x}dx'+\sigma ab, \quad (5)$$

where $U(x)$ is the interaction energy between kinks of the same type at a separation, x. From elasticity theory,[2,10,11] one finds $U(x)\sim|x|^{-1}$. However, this form is mathematically cumbersome, because the equations become highly nonlocal. Furthermore, it leads to (probably) spurious curvature if one attempts to treat each mobile length of dislocation in isolation from the rest of the dislocation network.[2] Consequently, we believe it is equally realistic to take the model interaction, $U(x)=Sa^2\delta(x)$, which, with the neglect of kink generation, recombination and diffusion, yields the familiar string model of line tension S.[12,13] Thus, we obtain from (5)

$$F=Sa^2[\partial(n-p)/\partial x]+\sigma ab. \quad (6)$$

Without entering into mathematical details, one can understand readily the internal friction effects which are associated with the model. For strain-amplitude independent phenomena, the dislocation response is determined by the relaxation process tending to oppose the dislocation motion induced by a small, harmonically time-varying stress σ_1, say. Now, if the dislocation is otherwise stress-free, there are two relaxation times.[14] One of these, $\tau_D\sim L^2/D_{\text{eff}}$, is a measure of the time for kinks to be redistributed along the dislocation by the stress, σ_1 (see Appendix C). The other, $\tau_E=[r(n_0+p_0)]^{-1}$, determines the relaxation, by recombination, of any kink concentration in excess of the thermal equilibrium value. But for sufficiently small σ_1, the bowing of the dislocation is achieved completely by kink redistribution alone. Hence the only pertinent relaxation time is τ_D, and the relaxation strength is determined by the equilibrium concentration of kinks if interactions are neglected. This is exemplified by our previous theory.

However, if the dislocation is subject, in addition, to a large internal bias stress σ_i, this qualitative description needs substantial modification. For in this instance the dislocations are bowed out. The kink densities are highly nonuniform and may be greatly in excess of their thermal equilibrium values. Such is particularly the case for dislocations which otherwise would be along a close-packed crystal direction. Their response to a small additional stress σ_1, of necessity must be accompanied either by the generation of more kinks, or by the recombination of already existing kinks in opposition to the stress field σ_i, depending upon the relative sense of these two stresses. Thus, in this situation, lifetime effects are of extreme importance and enter directly into the dislocation contribution to the attenuation of sound waves.

To substantiate the above remarks, we will now examine a specific example in more detail. Since the internal stress has most effect upon dislocations with pinning points along the same close-packed row, we will restrict attention to this case for the present. There are then obvious symmetry requirements upon the solutions of (1), namely,

$$n(\sigma,x)=p(\sigma,-x), \quad (7)$$

and

$$n(-\sigma,x)=p(\sigma,x).$$

The former condition is particularly useful in the subsequent analysis.

We will consider next the characteristic time for kink redistribution τ_i, which corresponds to τ_D when $\sigma_i=0$. For large internal stresses, one anticipates $\tau_i\sim(L/\sigma_i ab\mu)$. Its magnitude depends therefore upon the activation energy W for kink diffusion. Experiments[8,15] indicate that a substantial fraction of all dislocations are mobile even at He temperatures. Consequently, for these, $W\approx 0$. Inserting then the relation[16] $D\approx\nu_D b^2$ and using the typical values $L\sim 400$b, $T\sim 100°$K, $\sigma_i\sim 10^{-4}$G, one finds, for an angular frequency ω, that the product $\omega\tau_i$ is small compared to unity even at megacycle frequencies. Thus the lateral redistribution of kinks is effectively instantaneous; the only retarding mechanism involves the generation or recombination of kinks.

The preceding estimate suggests that a reasonable starting point for the solution of (1) is

$$n(x,t)=n_T(1+u),$$
$$p(x,t)=p_T(1+v), \quad (8)$$

where n_T is the *steady-state* value of n for the instantaneous value of the *total* stress, $\sigma_T=\sigma_i+\sigma_1(t)$. Thus, to first order in σ_1,

$$n_T=n_i+\sigma_1(\partial n_i/\partial\sigma_i), \quad (9)$$

n_i, $\{\equiv n(\sigma_i,x)\}$, being the kink density appropriate to the stress σ_i only. The functions u and v, which represent the deviations of the kink densities from the instantaneous values given above, will be directly proportional to the recombination lifetime.

Substitution of (8) into (1), using the fact that in equilibrium the kink currents are zero,[1] leads to the

[10] F. Kroupa and L. M. Brown, Phil. Mag. **6**, 1267 (1961).
[11] J. D. Eshelby, Proc. Roy. Soc. (London) **A226**, 222 (1962).
[12] J. S. Koehler, *Imperfections in Nearly Perfect Crystals*, edited by W. Shockley, J. H. Hollomon, R. Maurer, and F. Seitz (John Wiley & Sons, Inc., New York, 1952), Chap. 7.
[13] A. D. Brailsford (to be published).
[14] See Ref. 1 and Appendix A. Note, however, that due to an oversight τ_E in Eq. (18) is too large by a factor of 2.
[15] G. A. Alers and D. O. Thompson, J. Appl. Phys. **32**, 283 (1961).
[16] J. Lothe and J. P. Hirth, Phys. Rev. **115**, 543 (1959).

following coupled equations:

$$\frac{\partial}{\partial x}(\delta I_n)=-i\omega\left[\sigma_1\frac{\partial n_i}{\partial\sigma_i}+n_iu\right]-g(u+v),$$

$$\frac{\partial}{\partial x}(\delta I_p)=-i\omega\left[\sigma_1\frac{\partial p_i}{\partial\sigma_i}+p_iv\right]-g(u+v), \quad (10)$$

where

$$\delta I_n=-Dn_i\left(\frac{\partial}{\partial x}\right)[u+(Sa^2/kT)(n_iu-p_iv)],$$

and

$$\delta I_p=-Dp_i\left(\frac{\partial}{\partial x}\right)[v-(Sa^2/kT)(n_iu-p_iv)]. \quad (11)$$

It has been assumed that all time-dependent quantities vary as $\exp(i\omega t)$, and only terms of first order in σ_1 have been retained. For fixed pinning points, the boundary conditions appropriate to (10) are

$$\delta I_n=\delta I_p=0 \quad \text{at} \quad x=\pm(L/2). \quad (12)$$

The above system of equations for the unknown functions u and v is complicated. (The densities n_i and p_i will be discussed presently.) However, the preliminary insight into the physical processes involved suggests the following approximation scheme. Suppose for the present that $g=0$. The remaining right-hand members of (10) are proportional to the frequency. Hence, if these terms are treated as small, successive orders in a perturbation expansion of u (and v) must generate a power series solution in $\omega\tau_i$, which, as we have seen, is very small. This may be verified by writing (10) in dimensionless form. For $g\neq 0$, the additional expansion parameter, now regarding the complete right-hand member of (10) as a perturbation, is similarly a function of (τ_i/τ_E). For suppose we write $u=u_0+u_1+\cdots$, where the suffix denotes the order in perturbation theory (to be proper we should multiply the right-hand member of (10) by the factor λ, in which case $u_r\propto\lambda^r$). Then to lowest order the solution of (10) with the boundary condition (12) is $\delta I_{no}=0$, i.e., no current flow. Clearly this will correctly describe the relaxation of the dislocation as long as the lateral redistribution of kinks is much faster than the time for recombination or generation.

In practice, we have only carried this perturbation approach to first order. The zeroth-order solutions are presented below; the corrections to first order are derived in Appendix A. The latter indicate that the mathematical results used in the text are in error by a few percent at most.

To lowest order, therefore, the result $\delta I_{no}=0$, together with (7) and (11), yields the solution

$$u_0(x)=v_0(-x)=A\{1+(2Sa^2p_ikT)\}/\{1+Sa^2(n_i+p_i)/kT\}, \quad (13)$$

where A is a constant of integration. The latter is determined from the equation for u_1, i.e.,

$$-\frac{\partial}{\partial x}(\delta I_{n1})=i\omega\left[\sigma_1\frac{\partial n_i}{\partial\sigma_i}+n_iu_0\right]+g(u_0+v_0), \quad (14)$$

by integrating (14) between $x=\pm(L/2)$, together with the use of (12). We find then

$$A=-\sigma_1(I_2/I_1)i\omega\tau/(1+i\omega\tau), \quad (15)$$

where

$$I_1=\int_{-L/2}^{L/2}n_i(u_0/A)dx, \quad (16)$$

$$I_2=\int_{-L/2}^{L/2}(\partial n_i/\partial\sigma_i)dx, \quad (17)$$

and the relaxation time τ is given by

$$\tau=I_1/2gL. \quad (18)$$

In order to utilize these results for determining the internal friction parameters, we must first determine the densities n_i and p_i, which describe the bowing out of the dislocation in the stress σ_i. Again, since the currents vanish in equilibrium, we have from (1)

$$n_ip_i=g/r. \quad (19)$$

Furthermore, if we ignore any stress dependence of g and r, it follows[1] that

$$g/r=n_0^2, \quad (20)$$

where n_0 is the kink density for $\sigma_i=0$. Thus, (4), (6), (19), and (20) yield the equation

$$\sigma_iab+Sa^2\frac{\partial}{\partial x}\left(n_i-\frac{n_0^2}{n_i}\right)+kT\frac{\partial}{\partial x}\ln n_i=0, \quad (21)$$

which may be integrated to give

$$\sigma_iabx+Sa^2[n_i-(n_0^2/n_i)]+kT\ln(n_i/n_0)=0, \quad (22)$$

the arbitrary constant being determined by (7). These allow us to simplify (16) and (17) by noting the results

$$\partial n_i/\partial\sigma_i=(x/\sigma_i)(\partial n_i/\partial x), \quad (23)$$

and

$$(n_iu_0/A)\simeq-(kT/\sigma_iab)(\partial n_i/\partial x). \quad (24)$$

[The term we have neglected in (13) is always small compared to unity when n_i is large.] Hence we obtain

$$I_1\simeq(n_--n_+)(kT/\sigma_iab), \quad (25)$$

and

$$I_2=(L/2\sigma_i)(n_-+n_+-2\bar{n}), \quad (26)$$

where $n\pm$ are the values of n_i at $x=\pm L/2$, respectively, and $\bar{n}$ denotes the average left kink density on the line.

In addition one can now proceed to express the decrement Δ and modulus defect ΔM_D entirely in

terms of these new parameters. Thus, from (2), (8), (9), and (13) *et seq.*, and the relations

$$\epsilon_D=(b/V)\int_{-L/2}^{L/2} y dx, \tag{27}$$

$$\Delta=(G/2\pi|\sigma_1|^2)\mathrm{Re}\int_0^{2\pi/\omega}\sigma_1^*\dot{\epsilon}_D dt, \tag{28}$$

$$\Delta M_D=G\mathrm{Re}(\epsilon_D/\sigma_1), \tag{29}$$

we find, after some algebra, the following contributions,[17] for a dislocation density Λ_a:

$$\omega\tau(\Delta M_D)=\Delta=\Delta_D\omega\tau/(1+\omega^2\tau^2), \tag{30}$$

where

$$\Delta_D=(Gab\Lambda_aL/2\sigma_i)(n_-+n_+-2\bar{n})^2/(n_--n_+), \tag{31}$$

and τ is given by

$$\tau=(n_--n_+)(kT/2\sigma_iabL)g^{-1}. \tag{32}$$

It will be noted that both Δ_D and τ are even functions of σ_i, as of course they must be if internal stresses are to have any effect at all. Moreover, in the limit of small σ_i (specifically $\sigma_iabL\ll 2kT$) it is easily shown, from a power series solution of (22), that $\tau\rightarrow\tau_E$ and

$$\Delta_D\rightarrow(Ga^2b^2\Lambda_aL^2n_0/18kT)(\sigma_iabL/2kT)^2. \tag{33}$$

These results are of more than academic interest, even though we show in Sec. III that in fact $(\sigma_iabL)\sim 20kT$. For (33) indicates explicitly how the attenuation is increased by the change in kink density resulting from the internal stress. This point is relevant in attempting to relate Λ_a, the "active" density, to the total density Λ in any region of the solid. So far we have only treated dislocations with pinning points in the same close-packed row. However, it is clear that other dislocations, which do not have this precise orientation, must contribute also if they are situated in internal stress fields sufficiently large to increase their kink density substantially. A reasonable criterion, then, is to assume that any dislocation contributes if it resides in an internal stress field greater than the stress[2] σ_c at which it is just forced along a close-packed row. Now if one ignores kink-diffusion $\sigma_c=(2S/bL)\tan\theta$, where θ is the orientation of the pinning points relative to the close-packed direction. Thus, assuming the orientations in the slip plane are random—the only reasonable assumption for prestrained polycrystalline samples—we estimate the fraction f of all dislocations which are "active" to be

$$f\approx 7.5[1-\cos\theta_0], \tag{34}$$

where it has been anticipated that

$$\theta_0=\tan^{-1}(\sigma_ibL/2S)<\pi/6.$$

[17] In addition to the modulus defect (30), there is an additional frequency and relatively temperature-independent contribution which subsequent numerical work shows to be small. It has been neglected for the sake of simplicity.

In contrast to (33), for large internal stresses, (31) and (32) indicate a relatively temperature-independent relaxation strength and characteristic relaxation time proportional to g^{-1}. Since one expects[18,19] $g\sim g_0\exp(-2\epsilon_k/kT)$, where g_0 is some "attempt" frequency per unit length and ϵ_k is the kink self-energy, it follows that the activation energy associated with the relaxation is $2\epsilon_k$. On the basis of rate-theory arguments, a model with this activation energy built in has been proposed by Seeger, later amended by Paré, to account for the Bordoni peak. However, the latter is not very specific in detail and one is forced to use empirical reasoning to account for those properties of the Bordoni peak which distinguish it from the many other peaks observed in cold-worked metals.[20] We believe that (30) *et seq.* constitute a more detailed description of the Seeger-Paré mechanism. The results will be compared extensively with experiment in the next section.

In the course of this comparison, we shall utilize numerical solutions of (22). These have been obtained with the following relations between parameters: $\epsilon_k=\frac{1}{2}Sa^2/w$, where w is the kink width, $w=3b$, and $n_0=b^{-1}\exp(-\epsilon_k/kT)$. Each of these require comment. The first follows from literal application of the string model to its extreme, but it is difficult to see, on purely dimensional grounds, how any other relation could hold. More detailed models[3,11] give essentially the same result. The second is governed purely by the geometrical consistency of the model. For significantly larger widths, the concept of individual kinks becomes meaningless. (However, the possibility of larger widths is also discussed in Sec. IV.) Finally, on the choice of kink statistics, we retain the form given earlier.[1] Several alternative versions appear in the literature.[11,16,21] We believe the latter are only applicable at very low temperatures and should be replaced by the configurational entropy result, given above, at the temperatures of current interest ($\sim$80°K). This subject is discussed in detail in Appendix B.

III. COMPARISON WITH EXPERIMENT

Apart from one prefacing remark (Sec. III.1), we shall consider exclusively experimental observations of the Bordoni peak in cold-worked polycrystalline Cu. This is necessitated by the fact that relevant experimental details, apart from investigations of the peak itself, do not appear to be available for other materials.

The comparison will make use of the recent electron microscope observations of Bailey. He found that with small amounts of cold work ($\sim$0.1% extension) the dislocation structure was highly nonuniform and, with

[18] J. Lothe, Z. Physik **157**, 457 (1960).

[19] T. Jøssang, K. Skylstad, and J. Lothe, in *Proceedings of Conference on The Relation Between the Structure and Mechanical Properties of Metals* (H. M. Stationery Office, London, 1963), Vol. II, p. 528.

[20] For example, S. Okuda and R. R. Hasiguti, Acta Met. **11**, 257 (1963).

[21] A. Seeger and P. Schiller, Acta Met. **10**, 348 (1962).

increasing deformation, consisted of densely populated three-dimensional tangles forming cell boundaries which enclose regions of comparatively low-dislocation density. The densities in the "cell" region, Λ_c, and the boundary region, Λ_b, are approximately in the ratio 1:5. It will be shown in (III.3) and (III.5) that only Λ_c appears to contribute to the Bordoni peak.

We now consider the different experimental aspects in detail.

1. Grain Size

If the Bordoni peak has its origin in the presence of internal stresses, one would expect empirically that a small grain size material would show a well-developed peak for smaller amounts of prestrain than a large grain size sample, since presumably there are more regions of stress concentration in the former case.

Some weak support for this conclusion is provided by the observation[22] that the peak in a single crystal extended 3.25% consists of only a bump on a monotonically increasing background, whereas similar deformation in polycrystals produces well-developed peaks.[5] Unfortunately, no systematic investigation of grain-size effects has been carried out to our knowledge. However, experiments[23] on the behavior of the peak in Al do seem to bear out the qualitative conclusion reached above.

2. Frequency

According to (30) and (32), the internal friction should exhibit a maximum at the frequency ω given by

$$\omega=(\gamma/\eta_i)(4\epsilon_k/kT)g_0 b\exp(-2\epsilon_k/kT)\,, \qquad (35)$$

where $\gamma=(\sigma_i abL/2\epsilon_k)$, $\eta_i=bn_-$, and n_+ has been neglected, since it is negligibly small compared with n_- provided $\gamma\neq 0$.

Thus, a plot of $\ln\omega$ versus T, the peak temperature, should yield the activation energy $2\epsilon_k$, if the temperature range is narrow enough so that the pre-exponential is essentially constant. Such plots[5] do indeed yield a linear relationship (but with significant scatter) and give $2\epsilon_k=0.12$ eV and an attempt frequency of 2.4×10^{12} rad sec^{-1}. However, we again[1] are confronted with the conclusion that, to some degree, the position of the peak should be sensitive to sample preparation. For not only the loop length but also σ_i may be expected to vary from one sample to another. Consequently, there still seems merit in following Niblett and Wilks[5] and considering only data from samples deformed by comparable amounts in the same manner. In this case, they find $2\epsilon_k=0.08$ eV and an attempt frequency $\sim6\times10^8$ rad sec^{-1}. Since this point has still not been completely resolved, we shall compromise and take $\epsilon_k=10\,kT$ in order to examine typical results which are pertinent for the whole range of measurement.

For a given value of γ, the only remaining unknown, g_0, should be determined from (35). Following Paré, we shall take $\gamma=1$, a result we verify independently in Sec. III.3. From (22) we have found $\eta_i=0.05$. Thus, we obtain

$$g_0 b=1.25\times10^{-3}\alpha\omega_D\,, \qquad (36)$$

where α is the experimentally determined attempt frequency in units of the Debye frequency ω_D. Unfortunately, in view of the uncertainties mentioned, we can only place α within the wide range 5×10^{-2} to 1×10^{-6}. The need for further experimental study is clear, as emphasized by the discussion in Sec. IV.

The theoretical problem of calculating a generation rate (but *not* per unit length) has recently been attempted by Jøssang *et al.*[19] They find an attempt frequency ω_A for this process which is essentially

$$\omega_A\sim(2\pi)^{1/2}(b^2/aw)\beta\omega_D\,, \qquad (37)$$

where "β is some number that might be appreciably smaller than one.[19]" If one arbitrarily normalizes ω_A to unit length by setting $g_0=(\omega_A/sb)$, and takes[19] $\beta\sim0.1$, the value of s obtained from (36) is at least $\sim10^3$. This seems unduly large and indicates that ω_A is significantly less than the value $0.1\,\omega_D$ obtained with $\beta=0.1$.

3. Cold Work

Equation (33) indicates that the initial growth of the peak is associated with the accompanying increase in the internal stress. The subsequent insensitivity[5] to further amounts of cold work above $\approx2\%$ elongation, we will show, results from this increase being offset by the decrease in loop length.

The first point to be resolved is whether the dislocations in the "cell" boundaries, or those in the relatively sparsely populated regions, are responsible for the Bordoni peak. On physical grounds, one would guess the latter, for it is reasonable to expect that in the densely populated regions the only response would consist of collective modes of vibration of the whole group rather than the independent motion of individual dislocations. This speculation finds support through the consistency with which experimental observations can be interpreted. For suppose, on the contrary, that the "boundary" dislocations could move freely. Then, since[24] $\sigma_i\propto Gb\sqrt{\Lambda_b}$ and $\Lambda_b\sim L^{-2}$, we find γ is a constant independent of the amount of prestrain. But from (35) it follows that the peak position in temperature, for fixed frequency, depends only upon γ. Thus the observed shift in peak temperature with cold work cannot be accounted for on this basis. Furthermore, the annealing behavior of the peak (see Sec. III.5) cannot be explained. We conclude therefore that only the density Λ_c contributes to the peak.

[22] D. O. Thompson and D. K. Holmes, J. Appl. Phys. **30**, 525 (1959).

[23] T. S. Hutchison and G. J. Hutton, Can. J. Phys. **36**, 82 (1958).

[24] J. E. Bailey and P. B. Hirsch, Phil. Mag. **5**, 485 (1960).

With[25]

$$\sigma_i \simeq 0.1 Gb\Lambda_c^{1/2}, \tag{38}$$

and[13] $S \approx 0.2\, Gb^2$ we estimate that

$$\gamma \simeq 1.3\Lambda_c^{1/2} L. \tag{39}$$

Similarly, with $\Lambda_a = f\Lambda_c$, (31) yields

$$\Delta_D \approx 7.5 f(\eta_i/\gamma)\{1-2(\bar{n}/n_-)\}^2 \Lambda_c L^2, \tag{40}$$

where f is found from (34) to be

$$f \simeq 0.2\Lambda_c L^2. \tag{41}$$

Thus, the only free parameter is γ, or equivalently $\Lambda_c L^2$. For $\gamma = 1$ we have found, from (22), $n_- \simeq 8\bar{n}$. This yields the values $\Lambda_c L^2 = 0.6$, $f = 0.12$ and $\Delta_D = 0.015$. Since the height of the peak is $(1/2)\Delta_D$, the calculated value is 7.5×10^{-3}. This is in very good agreement with the value 6×10^{-3} measured by Bruner and Mecs[8] on polycrystalline samples (grain size ~ 1 mm) after 3% tensile deformation at 4.2°K. Moreover, although the electron microscope observations were performed on smaller grain size material (≈ 0.02 mm), deformed at room temperature, the density estimated from Bailey's work ($\Lambda_c \sim 6 \times 10^9$ cm^{-2}) appears to give an acceptable loop length, namely, $L \simeq 400b$.

It should also be noted that the value of $\Lambda_c L^2$ is reasonably close to the ideal value of 3 (for a simple cubic lattice of dislocations). This suggests a dislocation morphology (in the sparsely populated regions) grossly similar to the ideal one, for the above deformation, and explains the insensitivity of the peak to further cold work.

As we mentioned earlier, (33) provides a qualitative description of the growth of the peak for very small prestrains. However, any attempts to be more quantitative introduce more arbitrary parameters (initial density, morphology, impurity density) than the one phenomenon one is attempting to describe. It seems premature to pursue the matter at the present time.

Since only $\approx 12\%$ of the density Λ_c contributes to the peak, we must still investigate the internal friction effects associated with the majority. Following Bruner and Mecs, we assume they are responsible for the Köster effect which is produced immediately after deformation at 4.2°K. This is easily shown to be

$$(\Delta E/E)_k = -(1-f)\Lambda_c L^2/6, \tag{42}$$

which has the numerical value -8.8×10^{-2}. Again, this is in reasonable agreement with the experimental result $\simeq -5.1 \times 10^{-2}$.

In conclusion we will calculate the associated internal friction. According to the damped string model,[12,13] this gives a contribution Δ_s which, at low frequencies, is given by

$$\Delta_s = \alpha(1-f)(8Gb^2\Lambda_c L^4 \omega B/\pi^6 S^2), \tag{43}$$

[25] H. G. van Bueren, *Imperfections in Crystals* (North-Holland Publishing Company, Amsterdam, 1961), p. 146.

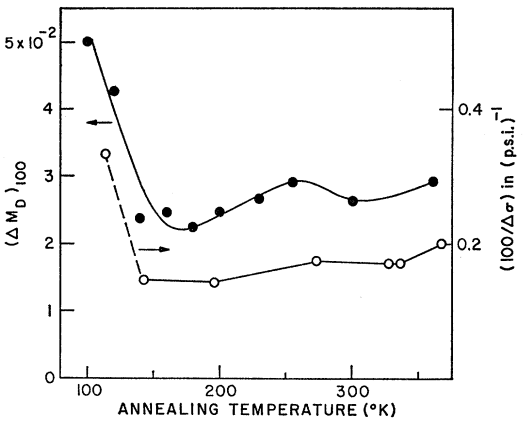

FIG. 1. Variation of the modulus defect ΔM_D associated with the Bordoni peak as a function of annealing temperature, and the variation of the inverse of the yield point as a function of aging temperature. The broken part of the latter curve for temperatures less than ≈ 140°K indicates that this behavior was obtained by linear interpolation of the data given in Ref. 7.

where B is the effective damping constant and α is a numerical parameter which depends upon the loop length distribution. Inserting the experimentally determined value $B \simeq 2 \times 10^{-4}$ dyn-sec-cm^{-2} gives $\Delta_s \sim 4 \times 10^{-8}\alpha$ for the frequency investigated (≈ 0.6 kc/sec). This is negligible compared to Δ_D for any reasonable α.

4. Peak Width

Various factors have been introduced to account for the anomalous half-width, namely distributions in loop lengths,[1] differences in the constitution of kinks,[1] distributions in internal stress[4] and dislocations being "kinked" relative to other than just close-packed directions.[22] The present theory should, in general, incorporate all four. No calculations of half-widths have been performed since there is obviously ample scope to explain almost any experimental result on this topic. However, there is one qualitative result we will note.[1] That is simply that if the half-width decreases, there should be a corresponding increase in peak height. This will be of use in the following section.

5. Annealing

The smooth curve in Fig. 1 describes the data obtained by Bruner and Mecs on the behavior of the modulus defect at 100°K associated with the Bordoni peak, following 16-h anneals at the temperatures shown, after prior deformation at 4.2°K. The other data, taken from Birnbaum's work,[7] shows the variation of the inverse of the yield point after strain-aging at the corresponding temperatures for 10^3 min, following previous deformation at 78°K. Over the whole range, the Köster effect exhibits a monotonic decrease.[8]

To explain these results we assert, first of all, that the Köster effect[26] is a result of loop length changes alone,

[26] W. Köster, Z. Metallkunde 32, 282 (1940).

since this should be insensitive to the internal stress. The gross behavior of ΔM_D then obviously has the same cause. Reference to Fig. 1 shows the close correlation with the variation in the inverse of the yield point. Birnbaum has attributed the *latter* solely to changes in loop length, but we believe this requires some slight modification. The argument is based upon the small increase in ΔM_D subsequent to the minima at ≈ 160 and ≈ 300°K. Bruner and Mecs have already drawn attention to the contrary behavior of ΔM_D and $(\Delta E/E)_k$ in these temperature ranges. In order to account for it, we must postulate that *some* small dislocation rearrangement occurs which tends either to narrow the spectrum of internal stresses, or increase the mean internal stress, or both. The first of these alternatives is necessitated by the observed decrease in peak width and increase in height for all annealing temperatures above 200°K. Any tendency to ordering is expected to promote this effect. The second, which parametrically should be contained in the ratio $(\sigma_i \Lambda_c^{-1/2}/Gb)$ of (38), is contrary to first expectation. However, it cannot be ruled out on the basis of internal energy requirements because of the competing effect of the narrowing in stress distribution. Moreover, the continuity of the flow stress curve[8] (ignoring yield points), which involves gross dislocation movements, is not a valid argument against either possibility; for the flow stress is determined by the regions of highest resistance to dislocations which, by their very existence, must be the densely populated regions.[27,28] As we concluded earlier, the latter are not responsible for the Bordoni peak.

It does not appear possible to go beyond the above qualitative reasoning. However we believe that the theory is not inconsistent with presently available annealing data.

The slight changes in the temperature of the peak with annealing may be interpreted according to the discussion in the next section.

6. Impurities

It is known that impurities reduce the magnitude of the peak and, for fixed frequency, cause a small decrease in the peak temperature.

A semiquantitative description of this effect has previously been derived on the assumption of thermally activated kink motion.[1] We found

$$(\Delta_{\max}/\Lambda)\exp(W/kT) = \text{const.} \tag{44}$$

It is of interest to obtain an analogous expression in the present case. For this purpose, to a first approximation $(\bar{n}/n_-)$ will be assumed constant. We have calculated η_i as a function of γ. To within $\approx 10\%$ the results can be represented by the empirical relation

$$(\eta_i/\gamma) = \lambda\gamma^2/(1+6\lambda\gamma^2), \tag{45}$$

with $\lambda = 0.07$, in the range of interest ($\gamma \approx 1$). In conjunction with (35) and (37) etc., these give the approximate relation

$$\frac{\Delta_{\max}^{1/3}\exp(2\epsilon_k/kT)}{\{1+0.62\Delta_{\max}^{1/3}\}^2} = \text{const.} \tag{46}$$

Hence this model gives the same qualitative variation as (44), namely, any decrease in height of the peak should be accompanied by a shift to lower temperatures. Of course, it is tacitly assumed that the internal stress distribution and loop length distribution are qualitatively the same, but experimentally, as a general characteristic, the behavior described by (46) is a distinguishing feature of the Bordoni peak.

To test (46) quantitatively we have only been able to find detailed work[29] on *cross-rolled single crystals* of Cu containing small amounts of Au (0.065 and 0.25 at.%). Temperature shifts calculated from this data and (46), with $\epsilon_k = 10\ kT$, are 0.7 and 2°K compared with the experimental value of 2 and 6°K. The imperfect agreement could be partly due to uncertainties in the value of ϵ_k since, for example, if we take $2\epsilon_k = 0.08$ eV, the estimates are increased by a factor two. It would be of interest to investigate (44) and (46) by neutron irradiation experiments. As yet, insufficient experimental data are available.

7. Niblett and Wilks Peak

There has been considerable speculation on the origin of the subsidiary maximum first observed by Niblett and Wilks.[5] However, any attempt at a simple classification appears to encounter difficulty. For example, since the peak height is approximately half that of the Bordoni peak after tensile deformation at 4.2°K,[8] it is tempting to attribute the subsidiary maximum to mainly screw dislocations and the main peak to dislocations with Burgers' vector at $\approx 60°$ to the close-packed direction. This point of view is supported by the observation that, after deformation in torsion, the peaks in polycrystalline Au have roughly the same magnitude.[30] Further, it has been found that the main peak is lowered in temperature by impurities, whereas the subsidiary peak remains unaltered. But while these factors conform with expectation, there exists the seemingly conflicting evidence that low temperature anneals, which presumably allow migration of defects to

[27] G. A. Alers and D. O. Thompson, J. Appl. Phys. **32**, 283 (1961).

[28] We are compelled to leave aside the moot point of whether electron micrographs are representative of the bulk. Also we would remind the reader that we are discussing polycrystalline materials and these comments are not a judgment of various work-hardening theories.

[29] H. L. Caswell, J. Appl. Phys. **29**, 1210 (1958). Further details in Tech. Rept. No. 3, Cornell University, 1957, AFSOR-TR-57-69 (unpublished).

[30] S. Okuda, Sci. Papers Inst. Phys. Chem. Res. (Tokyo) **57**, 116 (1963).

the dislocations, cause the subsidiary peak to decrease in magnitude about as rapidly as the main peak, in Cu, and even faster than the main peak, in Au.[30]

In the light of this apparently conflicting experimental evidence, such a simple classification does not appear plausible. An alternative possibility is discussed in the following section.

IV. DISCUSSION

In the preceding treatment we have examined the role of internal stresses in determining the internal friction of a solid. A specific set of assumptions have been made, the most pertinent of which are that the lateral motion of kinks does not require thermal activation and that the generation of kinks is rapid enough to account for the relaxation at the temperature of the Bordoni peak. We have compared the theory with experiment and shown that it can provide an explanation of all the observed properties.

We now wish to compare this theory with an alternative[1] one which was based upon the assumption of thermally activated kink motion. This is usually summarily dismissed by referring to the Köster effect which is observed at 4.2°K immediately after deformation. The implicit assumption made here, that if *some* dislocations may require thermal activation *no* dislocation motion at all is possible at He temperatures, is overlooked. That it may be, in fact, a false conclusion is considered in the discussion given below.

In Appendix C we present some previously unpublished work on the effect of kink-kink interactions on the relaxation peak associated with thermally activated kink motion. This problem has been treated independently by Southgate and Attard using another approach. The results are identical.

We find in this case a maximum decrement given by

$$\Delta_m = (4\Lambda_a L^2 G b^2/\pi^4 S)\,, \tag{47}$$

and a relaxation time

$$\tau_D = L^2/\pi^2 S a^2 n_0 \mu\,, \tag{48}$$

where now we suppose $\mu=\mu_0\exp(-W/kT)$. With $\Lambda_a=f'\Lambda_c$, where f' is the fraction of the total density contributing to the relaxation, and all other parameters the same as used previously, we find $\Delta_m\simeq f'/8$. Thus to obtain the correct magnitude of the Bordoni peak requires $f'\simeq 0.05$. To understand how this might arise we shall consider the following possibilities, none of which can be excluded *a priori*:

(i) The generation of kinks is much smaller than assumed in the earlier sections. This would be the case if, for example, relations of the type $\epsilon_k\sim\frac{1}{2}Sa^2/w$, which apparently ignore distortion in the core region, were a significant underestimate of the kink self-energy. In such an event, the relaxation peak discussed in the previous sections would occur at some higher temperature.

(ii) The Bordoni peak is associated with dislocations in those regions of the material where the internal stress is effectively zero.

(iii) Contrary to our original hypothesis, the kink width is greater than $\sim 3b$. Thus kinks in dislocations with pinning points at appreciable angle to close-packed directions do not move as isolated entities but, through mutual interference, cause the activation energy W to tend to zero when they begin to overlap.

Only (ii) and (iii) are relevant to the value of f'. Neither can be calculated with any precision. However, a crude estimate can be obtained as follows. Suppose the probability distribution $P(\sigma)$ for the internal stress, σ is $P(\sigma)=(2\sigma_0)^{-1}$, $(|\sigma|<\sigma_0)$, and zero otherwise (that is, a square distribution). Then, for a given orientation θ of the pinning points, if we consider only dislocations in stresses less than σ_c, we obtain the fraction $(\sigma_c/\sigma_0)\simeq(2S\theta/\sigma_0 bL)$. The total fractional reduction f' is therefore $f\sim(5S\theta_0^3/\sigma_0 bL)$, where $\theta_0\approx(a/w)$ is the angle at which overlap occurs. Hence, identifying σ_i given by (38) with the mean deviation for the distribution $P(\sigma)$, and using the remaining values given in the text, we find $f\sim 5(b/w)^3$. The width, $w\approx 5b$, which is required to obtain numerical agreement with experiment, probably has little significance other than to illustrate that the above conditions can combine to yield the correct order of magnitude without extremely large values of w.

It is also possible to account for the other experimental properties. In particular, the Köster effect at 4.2°K is a direct consequence of (iii), and its contrary behavior, relative to the modulus defect associated with the Bordoni peak, can be described again in terms of the variation in the stress distribution, etc. For example, that the peak half-width decreases for annealing above 200°K could be interpreted as a narrowing of the loop length distribution possibily accompanied by a decrease in the mean internal stress. Moreover, the insensitivity of the peak height to large prestrains is enhanced because of the additional factor that fewer dislocations are favorably located in the internal stress field. In fact, there appear to be only two properties which differ in a way which is amenable to further experiment. One is the value of the attempt frequency, the other is the sensitivity to impurities. We will consider each in turn.

From (48) the attempt frequency can be estimated using[1] $\mu_0\sim(\omega_D b^2/2\pi kT)$ and inserting a mean value $\langle n_0 a\rangle\sim 5\theta_0^2/2$. We find $\omega_0\sim 10^{-4}\omega_D$. There is a certain amount of flexibility in this number, but it certainly should not be in error by a factor $\sim 10^2$. Consequently this model definitely favors the lower attempt frequency and activation energy quoted earlier. It would be of great interest to investigate by further experiments if the values suggested by Niblett and Wilks are correct. We should re-emphasize that data collected on materials of different purity, deformed by different amounts, in different ways, and subject to different heat treatments, are useless for deciding this question.

The sensitivity to impurity content depends to some extent upon the internal stress distribution. In general, $f' \sim (b/L)P(0)$, and $\Delta_m \propto f\Lambda L^2$. Thus, if $P(0) \propto \sigma_i^{-n}$, we find

$$\{\Delta_m^2/\Lambda^{(2-n)}\}\exp(W/kT) \sim \text{const}. \quad (49)$$

Since this relation depends explicitly upon the density, it should preferably be applied only to changes in the peak induced by neutron irradiation. For the same activation energy, (49) would predict peak shifts ~ 5 to 6 times those obtained from (46). We have been able to locate only one datum in the literature which is relevant to this question, namely, an observation of Niblett and Wilks.[31] They found that neutron doses sufficient to reduce the peak height by ~ 6 relative to unirradiated material ($\Delta_m \sim 2.5\times 10^{-3}$) also produced a peak shift $\sim 5°$K. The value obtained from (46), $\Delta T \sim 2°$K, does seem to favor the generation rate model. However, since the data were obtained from different samples we hesitate to decide on the basis of this one example. Of course, we have ignored the additional complication that, should the peak consist of several components,[22] the net shift could be influenced also by a change in their relative intensity. This might be better assessed if more data were available.

On the basis of the preceding discussion, we believe that, with the available experimental data, to decide upon the mechanism responsible for the Bordoni peak would be premature. While much of the discussion of this section is highly speculative, the need for further experimental and theoretical study is obvious. The accurate determination of the attempt frequency associated with the Bordoni peak is of paramount interest since any value in excess of $\sim 10^{-3}\omega_D$ would exclude the kink-diffusion mechanism from further consideration. Should this be the case in fact, there are two possibilities. Both would imply that (i) above is false. One is that thermal activation for kink motion is never of importance even down to He temperatures. The other is that indeed some kinks do require thermal activation [in the sense of (iii)] and that the associated relaxation is the Niblett and Wilks peak.

In the event of this last possibility there would be in effect three classes of dislocations, the majority, with pinning points not near the same close-packed row, giving rise to a Köster effect, described by the "string" model, for example. The minority would be divided into two types, those in large internal stress fields, which give rise to the Bordoni relaxation, and the remainder, in the regions of small stress, which produce the Niblett and Wilks peak. As a result, the relative magnitude of the two peaks would depend upon the mode of deformation, as observed. In well-annealed single crystals the minority would compromise just one class, presumably a fraction $q \sim 3(b/w)^2$ of the total density. These would still give rise to an internal friction peak. Its estimated magnitude is $\sim 8\times 10^{-3}\, q$, if one uses the value of ΛL^2 determined by Alers and Thompson[15] for an annealed crystal. Since the damping associated with the string model (i.e., kinks requiring no thermal activation), when corrected for resolved shear stress effects,[15] is $\sim 7\times 10^{-3}$ this peak could easily be buried within the experimental scatter without postulating unduly large values of (w/b). At lower frequencies these simple ideas may be insufficient. In this case it might be necessary to consider the effects of distributions in loop length and activation energy in some detail.[32]

Evidently, there is still ample freedom within the model to construct a unified picture of internal friction effects associated with dislocations. Future developments may be awaited with interest.

ACKNOWLEDGMENTS

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APPENDIX A

In the following analysis we derive an upper limit for the first-order corrections to the solutions given in Sec. II.

The first Eq. (10) is of the form

$$-\partial(\delta I_n)/\partial x = \lambda[i\omega\{\sigma_1(\partial n_i/\partial\sigma_i)+n_i u\}+g(u+v)], \quad (A1)$$

where we have introduced the expansion parameter λ. Thus, for $\lambda<1$, we obtain a power series solution

$$u = u_0+u_1+\cdots, \quad u_r \propto \lambda^r. \quad (A2)$$

Equating powers of λ gives $\delta I_{n0}=0$, and

$$-\partial(\delta I_{n1})/\partial x = \lambda[i\omega\{\sigma_1(\partial n_i/\partial\sigma_i) + n_i u_0\}+g(u_0+v_0)], \quad (A3)$$

$$-\partial(\delta I_{n2})/\partial x = \lambda[i\omega n_i u_1+g(u_1+v_1)], \quad (A4)$$

and corresponding equations for higher order. Denoting by $\Phi(x)$ the integral of the right-hand member of (A3) between $-L/2$ and x we obtain from (A3) and (11),

$$u_1+(Sa^2/kT)(n_i u_1-p_i v_1) = F(x)+C, \quad (A5)$$

where

$$F(x) = D^{-1}\int_{-L/2}^{x} \Phi(x')/n_i(x')dx', \quad (A6)$$

and C is a constant of integration. The latter is determined by the boundary condition $\delta I_{n2}=0$, i.e., from (A4),

$$i\omega\int_{-L/2}^{L/2} n_i u_1 dx+g\int_{-L/2}^{L/2}(u_1+v_1)dx = 0. \quad (A7)$$

[31] D. H. Niblett and J. Wilks, Phil. Mag. **2**, 1427 (1957).

[32] R. M. Stern and A. V. Granato, Acta Met. **10**, 358 (1962).

From (A5) and (7), we find

$$u_1=[\{1+2\psi(x)\}C+\{1+\psi(x)\}F(x)+\psi(x)F(-x)]/[1+\psi(x)+\psi(-x)], \quad \text{(A8)}$$

where $\psi(x)=(Sa^2p_i/kT)$. Using (A6), we derive then

$$C=-\left\{L^{-1}\int_{-L/2}^{L/2}F(x)dx+(i\omega/2gL)\times\int_{-L/2}^{L/2}(n_iu_0/A)F(x)dx\right\}(1+i\omega\tau)^{-1}, \quad \text{(A9)}$$

where u_0 and τ are defined in (13) and (18).

A suitable measure of the accuracy of this method is to calculate the fractional change in the total kink density, which we denote by R. After some manipulation we obtain the following relation (for $\lambda=1$),

$$R=(I_1/I_2)(i\omega\tau\sigma_1)^{-1}\left[L^{-1}\int_{-L/2}^{L/2}F(x)dx-\left\{\int_{-L/2}^{L/2}n_iu_0Fdx\Big/\int_{-L/2}^{L/2}n_iu_0dx\right\}\right]. \quad \text{(A10)}$$

To evaluate (A10) we note first that $\Phi(x)$ can be written in the form

$$\Phi(x)=(i\omega\sigma_1L/2\sigma_i)(n_-+n_+-2\bar{n})\chi(x), \quad \text{(A11)}$$

where

$$\chi(x)=\left[I_2(x)/I_2-i\omega\tau I_1(x)/I_1(1+i\omega\tau)-\left(\frac{2x}{L}+1\right)\Big/2(1+i\omega\tau)\right], \quad \text{(A12)}$$

and the argument attached to the integral denotes that in (16) and (17) the upper limit is the variable, x.

The behavior of $\chi(x)$ can be understood by inspection of the form of the density n_i in the two cases where an analytical solution can be found. Thus, from (22), if we ignore diffusion we find

$$n_i=\tfrac{1}{2}[\{(\sigma_ibx/Sa)^2+4n_0^2\}^{1/2}-(\sigma_ibx/Sa)]. \quad \text{(A13)}$$

Alternatively, if we ignore interactions ($S=0$) there results

$$n_i=n_0\exp-(\sigma_iabx/kT). \quad \text{(A14)}$$

In either case, the density, n_i is very small in the region $x>0$, for the range of parameters under consideration. Thus, for $x>0$, to a good approximation we can take $I_2(x)=I_2$, and $I_1(x)=I_1$ so that

$$\chi(x)=\tfrac{1}{2}\{1-(2x/L)\}(1+i\omega\tau)^{-1}, \quad x>0. \quad \text{(A15)}$$

For $x<0$, χ does not have such a tidy form but never exceeds the value of (A15) in order of magnitude. We shall assume (A15) holds in the region $x<0$ for simplicity. This overestimates F slightly since the correct

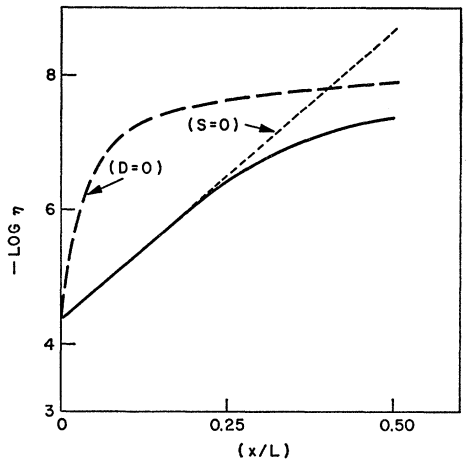

FIG. 2. Comparison of the kink density, $n=b^{-1}\eta$, as obtained by numerical solution of (22) with the approximate analytic expressions quoted in the text.

χ must vanish at $x=-(L/2)$. Thus, from (A6) *et seq.*

$$F\leq\{\Phi/2D\chi(1+i\omega\tau)\}\int_{-L/2}^{x}\{1-(2x/L)\}n_i^{-1}dx. \quad \text{(A16)}$$

For the approximate evaluation of (A16) we shall employ the diffusion result. This gives densities which are larger than the calculated results near $x=-L/2$. This behavior is unimportant, however, since χ should vanish there. At the other extreme, (A14) gives too low a density (see Fig. 2). Thus we again overestimate on this account. Finally, one can easily see, by similar reasoning to the above, that the second member of (A10) is small compared to the first. Consequently after performing the required integrals we find

$$R<\{(n_--n_+)e^{\rho}L^2\}/\{8\rho^4n_0D\tau\}, \quad \text{(A17)}$$

where $\rho=(\gamma\epsilon_k/kT)$. With the values quoted in the text we find $R<3\times10^{-2}$ for the smallest value of τ in the range given in (36). Since we have consistently overestimated (and in particular we note here that the diffusion result is 20 times smaller than the calculated value at $x=L/2$) we believe that the above value is extremely conservative.

APPENDIX B

If it is assumed that kinks can move freely along the dislocation, the increment in energy of a dislocation E associated with a kink having momentum p is

$$E=\epsilon_k+(p^2/2m), \quad \text{(B1)}$$

where m is the effective mass of a kink. Thus, for low concentration, the total density of kinks per unit length is

$$n_0=2\int_0^{\infty}N(E)e^{-\beta E}dE, \quad \text{(B2)}$$

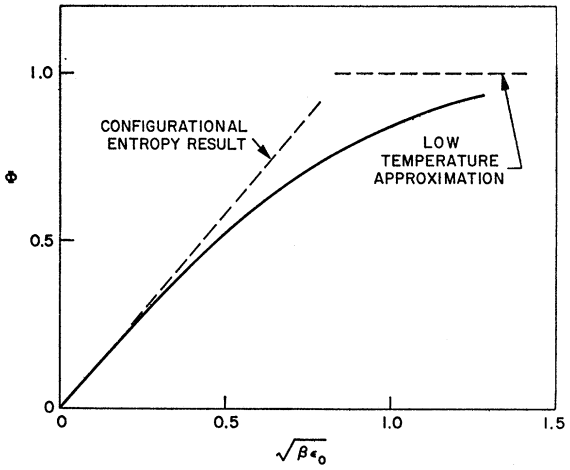

FIG. 3. Variation of $\Phi(\beta\epsilon_0)^{1/2}$ as a function of $(\beta\epsilon_0)^{1/2}$.

where $N(E)$ denotes the density of states per unit energy per unit length and $\beta=(kT)^{-1}$. The differing results in the literature are essentially determined by the differing treatments of (B2).

Applying the usual quantization procedure to this one-dimentional case gives

$$N(E)=\frac{1}{2\pi}\left(\frac{m}{2\hbar^2}\right)^{1/2}(E-\epsilon_k)^{-1/2},\ E\geq\epsilon_k \qquad \text{(B3)}$$
$$=0,\ E<\epsilon_k,$$

and insertion of (B3) into (B2) yields the result given by Eshelby.[11] Unfortunately, this is incorrect. For one overlooks thereby the fact that the dislocation has only a finite number of degrees of freedom per unit length, namely b^{-1}. Thus the integration in (B2) should terminate at $E=\epsilon_k+\epsilon_0$, where $\epsilon_0=(\hbar^2\pi^2/2mb^2)$. After inserting this cutoff the integral becomes

$$n_0=(m\kappa T/2\pi\hbar^2)^{1/2}\Phi((\beta\epsilon_0)^{1/2})\exp(-\beta\epsilon_k), \qquad \text{(B4)}$$

where $\Phi(x)$ is the error integral. The behavior of Φ as a function of $\beta\epsilon_0$ is shown in Fig. 3. As $\beta\epsilon_0\to\infty$, $\Phi\to 1$ and (B4) is then Eshelby's result. But as $\beta\epsilon_0\to 0$, $\Phi\to(2/\sqrt{\pi})(\beta\epsilon_0)^{1/2}$ and one obtains

$$n_0=b^{-1}\exp(-\epsilon_k/\kappa T), \qquad \text{(B5)}$$

which is the configurational entropy result. That is, the one obtained from combinatorial arguments. The reason for the different forms is clear. The combinatorial approach ignores the distribution of states in kinetic energy and amounts to approximating $N(E)$ by a delta function at ϵ_k. This should be reasonable as long as the exponential in (B2) varies slowly over the range ϵ_k to $\epsilon_k+\epsilon_0$, i.e., if $\beta\epsilon_0<1$. On the other hand, it will be poor in the opposite extreme. Then the cutoff is irrelevant and the density is identical with the electron concentration in a one-dimensional intrinsic semiconductor, for example.

To estimate $\beta\epsilon_0$, we take $m=\zeta\epsilon_k/c_t^2$ where c_t is the transverse sound velocity and ζ a numerical constant of proportionality. Hence, with the values typical of Cu, we find $(\beta\epsilon_0)^{1/2}=0.6\zeta^{-1/2}$ for $T=80°\text{K}$. Since specific models give $\zeta\geq 1$, we conclude from Fig. 3 that the result (C5) is the more appropriate, as indicated by the broken line. We should mention that this conclusion is contrary to the statement of Lothe and Hirth who assert that the cutoff in their (different) calculation is not important and who obtain essentially the low-temperature result ($\Phi\equiv 1$) given by (B4).

Finally, one can check the approximation of ignoring the interaction between kinks. Since left and right kinks will on the average be alternately spaced on the line the interaction energy[2] of one kink with all others will be $\sim Ga^2b^2n_0/4\pi(1-\nu)$. This is negligible compared with its self-energy.

APPENDIX C

We will consider here the internal friction associated with the model when $\sigma_i=0$. The stress in Eq. (6) is then $\sigma=\sigma_1\exp(i\omega t)$. We let $n=n_0+n_1$, and $p=p_0+p_1$. By subtraction, we find from (1)

$$[\partial(n_1-p_1)/\partial t]+\frac{\partial}{\partial x}(I_n-I_p)=0. \qquad \text{(C1)}$$

Using Eqs. (2) and (6), together with the boundary conditions $I_n=I_p=0$ at $x=\pm L/2$, this yields, after integration, the following result

$$\frac{\partial y_1}{\partial t}-D_{\text{eff}}\frac{\partial^2 y_1}{\partial x^2}=\sigma_1 a^2 b\mu(n_0+p_0), \qquad \text{(C2)}$$

where y_1 is the displacement and D_{eff} is given by

$$D_{\text{eff}}=D\{1+Sa^2(n_0+p_0)/\kappa T\}. \qquad \text{(C3)}$$

Equation (C3) is identical in form with Eq. (29) of Ref. 1, with D_{eff} replacing D alone. The internal friction is given by Eq. (35) of the latter work but there Eq. (37) should read now

$$\Delta(L,T)=8Ga^2b^2L^3(n_0+p_0)\mu/D_{\text{eff}}V\pi^4, \qquad \text{(C4)}$$

since κT appeared specifically (i.e., $\kappa T\to D_{\text{eff}}/\mu$).

For temperatures $\approx 80°\text{K}$, as long as $n_0a\geq 10^{-2}$, the kink interactions dominate and the ordinary diffusion term in (C3) can be neglected. This and (C4) then give Eqs. (47) and (48) of the text. In conclusion, we note that changes in kink density associated with thermal excitation are unimportant here since at most they will give changes in $(n_0a)\sim 10^{-5}$.