and consequently there is no sudden change and no discontinuity in S. The appearance of superzone energy gaps and the change in the spin wave spectrum at the transition could also contribute to the discontinuity in Dy, but we believe that the primary effect is due to the crystal structure change; the observed sizeable lattice parameter changes[42,43] could have a significant effect on the Seebeck coefficients.

ACKNOWLEDGMENTS

The authors express thanks to F. H. Spedding for his interest and assistance in this work, and to A. R. Mackintosh for many valuable discussions. We also acknowledge the assistance of P. E. Palmer who prepared the arc-melted buttons, of H. E. Nigh who assisted in growing the crystals, and of H. J. Born who assembled part of the apparatus.

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Doppler-Shifted Cyclotron Resonance with Helicon Waves*

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The properties of helicon waves have been studied under the condition that the electron mean free path is larger than the helicon wavelength; this is often called the nonlocal limit. The free-electron theory shows that under this condition there is a threshold magnetic field. Above this threshold, undamped helicon waves can propagate in the metal. Below it, the wave is heavily damped by cylotron resonance of the electrons. This threshold field has been called the Kjeldaas absorption edge or Doppler-shifted cyclotron resonance. At fields just above the edge, the dispersion relation is modified from the simple form appropriate for the local limit. At the edge there is a singularity in the surface impedance of the metal sample. The theory of these effects has been tested by measurements on polycrystalline sodium and potassium, using two experimental techniques. First, the dispersion relation was studied using helicon waves propagating through a metal slab. Second, the surface impedance was studied at radio frequencies. Excellent agreement between theory and experiment was obtained. The values for the radius of the Fermi surface were $(0.92\pm0.01)\times10^{+8}$ cm^{-1} for sodium and $(0.74\pm0.01)\times10^{+8}$ cm^{-1} for potassium. These are to be compared with the free-electron values of $0.923\times10^{+8}$ cm^{-1} and $0.746\times10^{+8}$ cm^{-1}, respectively. The sharpness of the edge as a function of the electron mean free path was also studied. It was found that the fractional width was essentially inversely proportional to $\omega_c\tau$. The edge has also been studied with polycrystalline indium.

I. INTRODUCTION

HELICON waves are transverse electromagnetic waves which, in the presence of an applied magnetic field, can propagate in a metal. For a free-electron metal, with n electrons per unit volume, in a field B, the helicon wave dispersion relation is[1]

$$q^2=\mu_0 ne\omega/B(mks\ \text{units}), \quad (1)$$

where q is the wave vector for a wave of frequency ω and μ_0 is the permittivity of free space.

Helicon waves have been extensively studied, especially in the region where the wavelength is greater than the electron mean free path; both standing wave and transmission experiments have been reported.[2] These experiments can be used to determine such properties of the metal as the Hall coefficient and the magnetoresistivity.

This paper is concerned with the study of helicon waves when the electron mean free path ℓ becomes much larger than the wavelength, that is when $q\ell\gg1$. Under this condition a study of helicon waves can yield information concerning the geometry of the Fermi surface. In contrast to the case when $q\ell\ll1$, it is no longer sufficient to specify the electron's behavior in terms of an average drift velocity. For $q\ell\gg1$ it becomes necessary to consider the details of the electron's orbit and its interaction with the wave. Such considerations lead to the prediction of cyclotron resonance at frequencies very much smaller than the cyclotron frequency. This occurs because an electron at the Fermi surface travels through the slow helicon wave and experiences, as a consequence of the Doppler effect, an electric field having an apparent frequency very much larger than the actual wave frequency. If the apparent frequency equals the electron's cyclotron frequency, Doppler-shifted cyclotron resonance occurs; the electron then absorbs energy from the wave.

The effect was predicted by Kjeldaas[3] for transverse ultrasonic waves propagating in a metal parallel to a

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[1] R. Bowers, C. Legendy, and F. E. Rose, Phys. Rev. Letters **7**, 339 (1961).

[2] R. Bowers, *Symposium on Plasma Effects in Solids, Paris, 1964* (to be published). Also R. Bowers and M. C. Steele, Proc. IEEE **52**, 1105 (1964).

[3] T. Kjeldaas, Phys. Rev. **113**, 1473 (1959).

magnetic field. Later Stern[4] pointed out that it should also occur for helicon waves. Doppler-shifted cyclotron resonance occurs for all magnetic fields smaller than a particular field B_e sometimes known as the Kjeldaas edge. The field B_e is related to the Gaussian curvature of the Fermi surface at the point where the electrons have the largest component of velocity along the magnetic field. The effect of Doppler-shifted cyclotron resonance on the surface impedance of a semi-infinite, free-electron metal, with the applied field normal to the surface, has been calculated by Miller and Haering.[5] They show that there is a singularity in the surface impedance at B_e.

This effect was first observed by Kirsh[6] as a kink in the surface impedance of bismuth as a function of magnetic field at 9 kMc/sec. In bismuth there is an equal number of electrons and holes; thus the waves which can propagate above the edge are Alfvén waves, not helicon waves.[7] The interpretation of Kirsh's experiment is not simple because of the complexity of the band structure of bismuth. At the frequency necessary for this experiment, the edge is only a factor 2 to 3 times larger than the magnetic field for unshifted cyclotron resonance ($\omega=\omega_c$). In contrast to this, the experiments on sodium are performed at much smaller frequencies (a factor 10^4 smaller), with a corresponding increase in B_e. Furthermore, theory and experiment have been correlated to within a few percent.

In metals, effects arising from Doppler-shifted cyclotron resonance have been observed with ultrasonic shear waves[8] and helicon waves.[9,10] The experiments with ultrasonic waves have been performed on polyvalent metals with complex Fermi surfaces. No detailed comparison between theory and experiment has been possible because of this complexity and the difficulty of making experimental corrections for the rotations of the plane of polarization of the ultrasonic wave.

The edge has been observed by studying standing helicon waves[9] in very thin sodium slabs ($\approx 100\,\mu$ thick) at frequencies of several hundred kilocycles and the surface impedance of sodium[10] at frequencies of several megacycles. The measurements were made on very thin sodium slabs in order to make $q\ell$ as large as possible and thus produce a more marked effect at the edge. Unfortunately, to obtain $q\ell\approx 8$, the electron mean free path ($\approx 200\,\mu$) was larger than the thickness of the slabs, complicating the analysis of the results.

This paper describes further measurements on the surface impedance of polycrystalline sodium[10] and the extension of these measurements to polycrystalline potassium and indium. Also discussed are measurements of the dispersion relation of helicon waves at magnetic fields just above the edge. There is a deviation from the simple dispersion relation of Eq. (1), which is found to be in accord with theory. The metals sodium and potassium were studied because their Fermi surfaces are very close to spherical (within 2 parts in 10^3).[11] Thus polycrystalline samples may be used for the measurements and the experimental results can be compared with free-electron theory.

II. THEORY

Helicon waves are circularly polarized electromagnetic waves which can propagate in a metal when $\omega_c\tau\gg 1$ ($\omega_c=eB/m$ is the cyclotron frequency and τ is the electron relaxation time) and $\omega\ll\omega_c$. They can be described as a dynamic manifestation of the Hall effect. To calculate the properties of helicon waves, Maxwell's equations must be solved using the proper constitutive equation to relate the electric field and current. For helicon waves Maxwell's equations can be simplified by neglecting the displacement current compared with the true current. The conductivity tensor, to be used in the constitutive equation, is derived from the Boltzmann transport equation. In the local limit of the conductivity tensor, i.e., $q\ell\ll 1$, the dispersion relation Eq. (1) holds and there is no absorption edge. In the nonlocal limit, i.e., $q\ell\gg 1$, the dispersion relation is modified; also below a particular magnetic field B_e the electrons absorb energy from the wave.

Consider an electron at the Fermi surface with velocity $\mathbf{v}_F$($\sim 10^8$ cm sec^{-1}). It will travel through the slow helicon wave (velocity $\sim 10^3$ cm sec^{-1}) of frequency ω and wave vector $\mathbf{q}$ parallel to $\mathbf{B}$. The electron will experience an electric field with an apparent frequency ω_a given by

$$\omega_a=\omega\pm\mathbf{q}\cdot\mathbf{v}_F\,. \tag{2}$$

There are two magnetic field domains:

(A) At large magnetic fields ($\omega_c\tau>q\ell\gg 1$), the helicon wave can propagate in the metal, but the dispersion relation of Eq. (1) is modified to[12]

$$q^2=(\mu_0 ne/B)\omega f(w,q\ell)\,, \tag{3}$$

where

$$w=qv_F/\omega_c<1\,.$$

In the case $q\ell=\infty$

$$f(w,\infty)=\frac{3}{w^2}\left[\frac{1}{2}-\frac{1-w^2}{4w}\ln\left(\frac{1+w}{1-w}\right)\right].$$

This dispersion relation tends to that of Eq. (1) in the limit $w\rightarrow 0$. From the exact expression for the conduc-

[4] E. A. Stern, Phys. Rev. Letters **10**, 91 (1963).
[5] P. B. Miller and R. R. Haering, Phys. Rev. **128**, 126 (1962).
[6] J. Kirsch and P. B. Miller, Phys. Rev. Letters **9**, 421 (1962) and J. Kirsch, Phys. Rev. **133**, A1390 (1964).
[7] S. J. Buchsbaum and J. K. Galt, Phys. Fluids **4**, 1514 (1961).
[8] A. R. Mackintosh, Phys. Rev. **131**, 2420 (1963).
[9] M. T. Taylor, J. R. Merrill, and R. Bowers, Phys. Letters **6**, 159 (1964).
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[11] D. Schoenberg and P. J. Stiles, IBM Research Paper RC1132 (to be published).
[12] J. J. Quinn and S. Rodriguez, Phys. Rev. **133**, A1590 (1964).

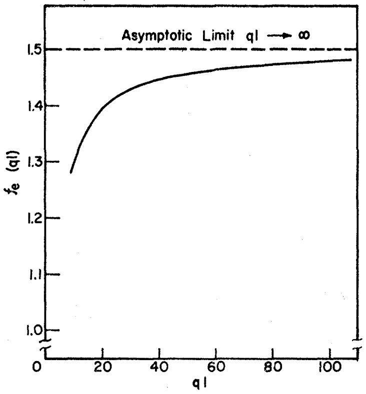

FIG. 1. Variation of the f function at the edge with $q\ell$. The f function can be called either $f_e(q\ell)$ or $f(1,q\ell)$.

tivity tensor with finite $q\ell$, derived by Kjeldaas,[13] it can be shown that for $q\ell>30$ and $w<1$ the above dispersion relation is a very good approximation.

(B) At small magnetic fields ($q\ell>\omega_c\tau\gg1$), there are always some electrons on the Fermi surface which experience an electric field with a frequency equal to their cyclotron frequency. Thus,

$$\omega_a=\omega_c\leqslant qv_F \tag{4}$$

from Eq. (2) and the fact that $\omega\ll\omega_c$ for helicon waves. As long as the inequality in Eq. (4) is true, there always will be some electrons undergoing cyclotron resonance. The Kjeldaas edge is defined by the equality $\omega_c=qv_F$. Miller and Haering[5] have shown, for $q\ell=\infty$, that there is a spike in the resistive component of the surface impedance at the edge. They named this spike "Doppler-shifted cyclotron resonance."

In order to determine the magnetic field at which the edge occurs, the dispersion relation, Eq. (3), must be used to obtain the value of q at the edge. The edge is defined by $w=1$, and in the limit of $q\ell=\infty$, $f(1)=1.5$. The exact value of this function at the edge, f_e, depends on the value of $q\ell$. Figure 1 shows the variation of f_e as a function of $q\ell$.[13] Note that, for $q\ell>30$, f_e is essentially constant and very close to the asymptotic value of 1.50.

The position of the edge is determined by the geometrical properties of a point on the Fermi surface. If the point is an elliptic limiting point then[14]

$$\omega_c=(eB/\hbar)v_FK^{1/2}, \tag{5}$$

where K is the Gaussian curvature at the limiting point. From Eqs. (3), (4), and (5) the positions of the edge is given by

$$B_e=(n\hbar^2\mu_0\omega/eK)^{1/3}fe^{1/3}. \tag{6}$$

For a spherical Fermi surface of radius $k_F=(3\pi^2n)^{1/3}$, Eq. (6) reduces to

$$B_e=\left(\frac{2}{3\pi}\frac{\mu_0\hbar^2}{e}k_F{}^5\right)^{1/3}\nu^{1/3}f_e{}^{1/3}. \tag{7}$$

Note that if f_e is taken to be a constant, which is a good approximation for $q\ell>30$, then the field for the edge is proportional to the cube root of the wave frequency ν. This occurs because, at the frequencies used for the measurements, it is possible to neglect the wave frequency compared with the cyclotron frequency.

III. EXPERIMENTAL INVESTIGATION OF THE DISPERSION RELATION

A convenient method for studying the dispersion relation is the propagation experiment described by Grimes.[15] The experiment (see Fig. 2) is essentially a helicon interferometer. A helicon wave is started at one face of a sodium slab by a coil driven with an oscillator. The wave propagates through the slab parallel to the large magnetic field. A coil at the opposite face picks up both the transmitted helicon wave and the rf magnetic field produced by the driving coil, which acts as a phase reference. The wavelength of the helicon can be changed with the large magnetic field; hence the phase of the helicon wave signal at the pickup coil is also changed with field. Therefore, the detector output will consist of a series of maxima and minima, or "fringes." The maxima and minima occur when the signal due to the helicon wave is in phase or out of phase, respectively, with the direct coupling signal. This condition is satisfied when there are an integral number of half-wavelengths of the helicon wave in the slab.

In order to reduce the direct coupling between the coils, the slab is made much larger than either of the coils, and also the pickup coil can be rotated to reduce the direct coupling between the coils. In order to increase sensitivity, the pickup coil was tuned. After tuning, the voltage produced by the helicon wave signal across the pickup coil had a maximum value of approximately 400 μV. The signal from the pickup coil was then amplified, detected, and displayed on an x−y re-

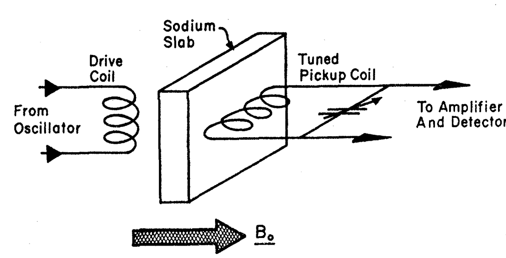

FIG. 2. Schematic diagram of the apparatus used to study helicon wave propagation by the interferometric method. The coils and sample were immersed in liquid helium.

[13] It is easily shown that

$$f(w,q\ell)=\omega_c\tau\,\mathrm{Im}\,(\sigma/\sigma_0),$$

where σ is the appropriate conductivity tensor [see Eq. (13) of Ref. 3] and σ_0 is the dc conductivity.

[14] See p. 230, A. B. Pippard, Rept. Progr. Phys. 23, 176 (1960).

[15] C. C. Grimes and S. J. Buchsbaum, Phys. Rev. Letters 12, 357 (1964).

corder as a function of magnetic field. Such an x—y recorder trace is shown in Fig. 3 for a 1.52-mm-thick sodium slab at 3 Mc/sec.

In Fig. 3, note that as the magnetic field is reduced, the fringes decrease in amplitude. At small fields, the fringes cannot be observed no matter how large the gain of the amplifier. This is to be expected since the edge is approached on decreasing the field. The arrow indicates the calculated position of B_e; note that there is a distinct change in the detector output at this field. This is interpreted as the change in rf drive field, due to the change in the surface impedance of the slab at B_e. Data similar to that in Fig. 3 were obtained for several frequencies between 1 and 3 Mc/sec on three polycrystalline sodium slabs in the thickness range 1 to 2 mm.

To analyze such curves, it was assumed that the Hall coefficient is known (23.57×10^{-11} m^3/C, calculated assuming 1 electron/atom) and that the fringes at the

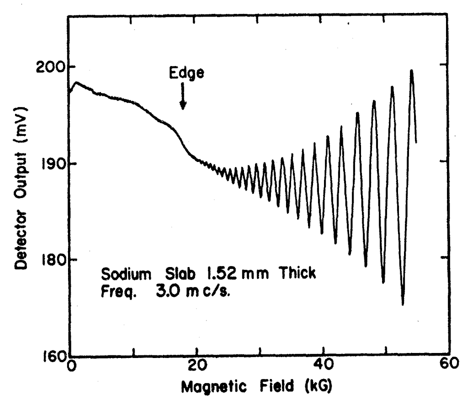

FIG. 3. Tracing of an x-y recorder chart obtained by the interferometric method. The fringes were produced by the interference of the helicon wave signal with the directly coupled signal. The arrow indicates the calculated threshold field or edge.

largest fields obey the simple dispersion relation Eq. (1). The thickness and the number of half-wavelengths N in the slab are determined from the position of the fringes. This was done for all data, obtained at different frequencies, on the same slab; the thicknesses determined for all frequencies should be the same. In order to obtain a unique thickness, it was necessary to use the small w expansion of $f(w,\infty)$[16] for the higher frequencies. The thickness obtained by this means was found to be in good agreement with the measured thickness after making allowance for thermal contraction ($\approx1.5\%$) and the thickness of the oxide coat (≈0.02 mm). The dependence of fringe positions, for both maxima and minima, on the number of half-wavelengths in the slab is shown on a log-log plot in Fig. 4. Note that the data presented in Fig. 4 include the fringes shown in Fig. 3. The simple dispersion relations, $B\propto N^{-2}$, are shown as dashed lines. The data shows clearly there is a devia-

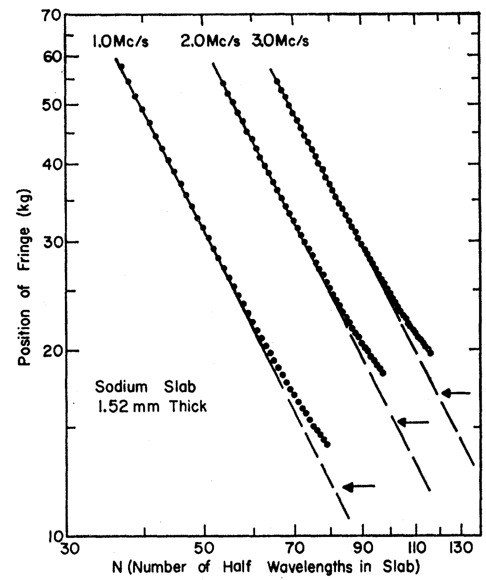

FIG. 4. The dependence of fringe position (both maxima and minima) upon the number of half-wavelengths in the slab, N. The dashed lines are the simple dispersion relation $B\propto N^{-2}$. The arrows indicate the positions of the edge for the frequencies shown. The 3-Mc/sec curve is data from the curve of Fig. 3.

tion from the simple dispersion relation. The arrows indicate the calculated B_e for the three frequencies shown.

If we designate the observed position of a fringe as $B_{\text{obs.}}$, and the position for the same fringe calculated using the simple dispersion relation as $B_{\text{calc.}}$, then from Eq. (3),

$$f(w,q\ell)=B_{\text{obs.}}/B_{\text{calc.}}-1\,,$$

w is proportional to the ratio of q ($=N\pi$/thickness) and B_{obs}. Figure 5 is a plot of $f(w,q\ell)$ against q/B_{obs} for the data in Fig. 4. Only data from the maxima for 1 and 3 Mc/sec are shown to avoid confusion. The upper scale was obtained by assuming $k_F=0.923\times10^{+8}$ cm^{-1} (the free-electron value obtained using the lattice constant[17] of 4.225 Å and one electron per atom). The solid line in

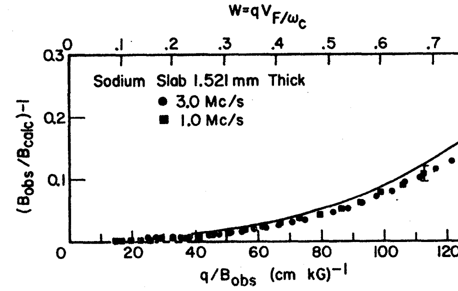

FIG. 5. The fractional deviation from the simple dispersion relation plotted against $q/B_{\text{obs.}}$. Only data from the maxima is presented to prevent confusion.

[16] F. W. Shead, Phys. Rev. 129, 2563 (1963).

[17] C. S. Barrett, Acta Cryst. 9, 671 (1956).

Fig. 5 is the calculated free-electron function $f(w,\infty)$; the data agree within the experimental error. It can be shown that for the conditions under which the experiments were performed the effects of finite $q\ell$ are negligible. Measurements on the other slabs substantiate the above result.

Thus in the region $w<0.8$ there is good agreement between the free-electron theory and experiment. This fact gives more confidence in the extrapolation of the dispersion relation to the edge, as discussed in Sec. II, for the interpretation of the surface impedance measurements.

IV. DOPPLER-SHIFTED CYCLOTRON RESONANCE: DIRECT OBSERVATION OF THE EDGE

The edge, or Doppler-shifted cyclotron resonance, is most effectively studied in the surface impedance. For magnetic fields greater than the edge, the helicon wave propagates in the metal. For all magnetic fields smaller than the edge, some of the electrons undergo Doppler-shifted cyclotron resonance and very heavily damp the wave. Thus at the edge there is an abrupt change in the penetration of the radio-frequency fields into the metal. This manifests itself as a spike in the resistive component and a discontinuity in the reactive component of the surface impedance.[5]

To study the surface impedance at radio frequencies, the samples in the form of slabs (1 mm by 10 mm by 10 mm) were placed inside an rf coil, so that the large surfaces of the slab were normal to the large applied field. The samples were pressed between stainless steel plates and in the case of potassium and sodium were immediately covered with paraffin oil, placed in the coil and cooled to liquid-nitrogen temperature to prevent oxidation of the surfaces. The changes in inductance and the Q of the coil as a function of magnetic field were measured using a Twin-T radio-frequency bridge.[18] This particular bridge has the advantage that by suitably unbalancing the bridge, both variations of the inductance and Q can be studied. A block diagram of the experimental arrangement is shown in Fig. 6. The large magnetic field was produced by a superconducting solenoid; the field could be varied from 0 to 55 kG. The signal from the bridge is amplified and detected. To enhance the observability of the edge, the dc signal from the detector was differentiated. This was a differentiation with respect to time, but, since the field sweep was linear in time, it was equivalent to a field differentiation.

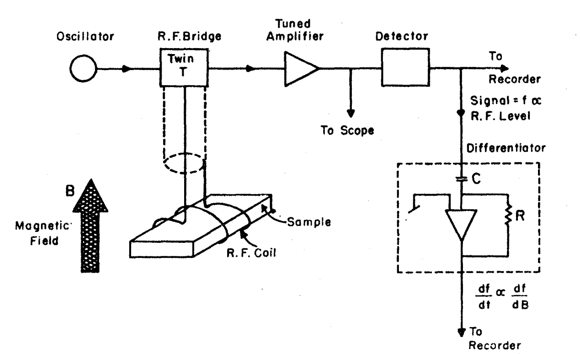

FIG. 6. Block diagram of the experimental arrangement for measuring the surface impedance at radio frequencies. The sample and coil are immersed in liquid helium.

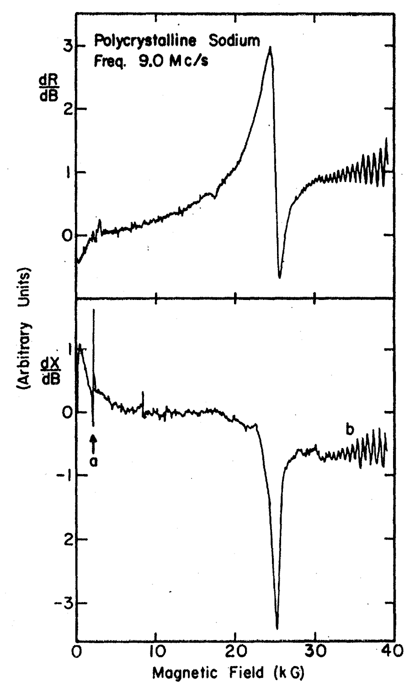

FIG. 7. Doppler shifted cyclotron resonance at 9.0 Mc/sec in polycrystalline sodium at 4°K. R is the resistive component and X the reactive component of the bridge signal. The resonance designated "a" is NMR of the protons in the coil former, and the fringes "b" are due to interference of helicon waves inside the sample.

Figure 7 shows the variation of the field derivative of the resistive (R) and reactive (X) components of the bridge signal. These are proportional to the corresponding components of the surface impedance. This data was obtained on a polycrystalline sodium slab at 9 Mc/sec. At 25 kG there is a very clearly defined edge. It is interesting to note that, for a frequency of 9 Mc/sec, unshifted cyclotron resonance (defined by $\omega=\omega_c$) should occur for a magnetic field of the order of $\frac{1}{2}$ G. The fringes labeled b are due to the interference of helicon waves propagating into the slab from opposite faces. These clearly demonstrate that, in the field region above the edge, helicon waves freely propagate in the metal. The resonance labeled a, seen clearly in the dX/dB curve, is NMR of the protons in the epoxy resin used to make the rf coil free-standing. (Proton NMR has also been seen many times in the dR/dB curves.) This resonance can be used to calibrate the magnet at the low fields although there is some question of the validity of the

[18] H. L. Anderson, Phys. Rev. **76**, 1460 (1949).

extrapolation of this calibration to higher fields for a superconducting magnet.

The edge has been studied in polycrystalline sodium for frequencies between 1 and 50 Mc/sec; B_e varied from 12 to 45 kG.

Figure 8 shows, on an expanded scale, Doppler-shifted cyclotron resonance at 10 Mc/sec in polycrystalline potassium. The edge field B_e is taken as the minimum of the "peak" in the dX/dB curve. This was found to agree very well with the value obtained from the half-height position between the maximum and minimum of the dR/dB curve. The width ΔB of the resonance was taken as the separation in field of the maximum and minimum of the dR/dB curve as indicated in Fig. 8. The edge was studied in potassium over the frequency range 8 to 36 Mc/sec; B_e varied from 17 to 28 kG.

The edge has also been seen in polycrystalline indium; it was very broad and shallow. J. R. Merrill of this laboratory has seen the edge in single-crystal aluminum.[19]

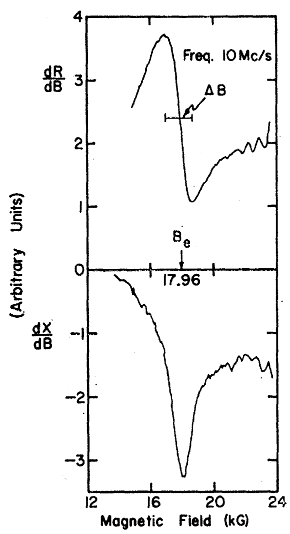

FIG. 8. Doppler shifted cyclotron resonance at 10 Mc/sec in polycrystalline potassium at 4°K. $\Delta B/B_e$ is the fractional width of the edge.

A plot of B_e as a function of the cube root of frequency for all measurements made on sodium, potassium, and indium are shown in Fig. 9. The datum points all lie on straight lines through the origin, as predicted by Eq. (7). The slopes of these lines are related to the radius of the Fermi sphere. The values obtained for k_F from these slopes are shown in Table I. The value of f_e used was estimated from the average value of $\omega_c\tau$ for the data points; since f_e enters as the fifth root, the final answer is somewhat insensitive to the value chosen. The principle error in k_F comes from the lack of precision in the calibration of the superconducting magnet.

The theoretical estimates of k_F shown in Table I were calculated assuming one electron per atom (in the case of In, one hole/atom).

[19] J. R. Merrill (private communication).

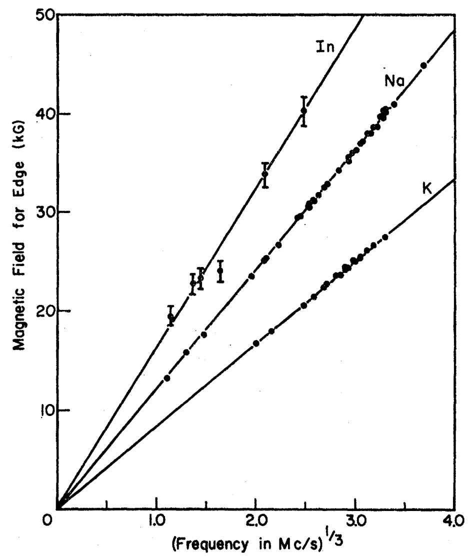

FIG. 9. Variation of the magnetic field for the edge with frequency for polycrystalline potassium, sodium, and indium. The slopes of the lines are related to the radii of the respective Fermi surfaces.

The variation of $q\ell$ obtained by varying the frequency over the range 1–50 Mc/sec for the sodium measurements did not produce a measurable variation in f_e. In an attempt to observe such a variation, measurements were made on less pure sodium. The most noticeable change was a large increase in the fractional width of the edge; a variation of f_e was detected but was well within the experimental error. Sodium samples with resistivity ratios varying from 7300 to 1200 were measured. These

TABLE I. Tabulation of results.

Metal	Slope[a] $[B_e/\nu^{1/3}]$	f_e	k_F[b] measured	k_F[b] theoretical
Na	12.13±0.24	1.475	0.92±0.01	0.923
K	8.39±0.17	1.425	0.74±0.01	0.746
In	16.2 ±0.6	1.39	1.11±0.04	1.05

[a] Slope given in units of kG/(Mc/sec)$^{1/3}$.
[b] Radius of Fermi sphere in units of 10^8 cm^{-1}.

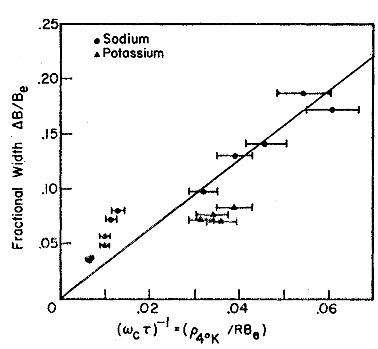

FIG. 10. Variation of the fractional width of the edge with $1/\omega_c\tau$. The solid line is $\Delta B/B_e=\pi/\omega_c\tau$.

samples were prepared by melting together pieces of the very pure sodium with pieces of commercial grade pure sodium. The resulting resistivity ratios were measured using the eddy-current decay method.[20]

Figure 10 shows the results of the measurements on the width of the edge. The fractional width is plotted against the reciprocal of $\omega_c\tau$. $\omega_c\tau = RB_e/\rho_{4^\circ\mathrm{K}}$. The resistivity at 4°K, $\rho_{4^\circ\mathrm{K}}$, was determined using the room temperature resistivity (4.3×10^{-8} Ω-m for sodium and 6.1×10^{-8} Ω-m for potassium) and the measured resistivity ratio. The fractional width appears to be inversely proportional to $\omega_c\tau$. The solid line is $\Delta B/B_e = \pi/\omega_c\tau$.

V. SUMMARY

The properties of helicon waves have been investigated for the case when the helicon wavelength is smaller than the electron mean free path. Two experimental techniques have been used: One was the study of helicon waves propagating through a slab of metal; the other was the measurement of the surface impedance of the metal.

The free-electron theory predicts a deviation from the simple dispersion relation and a threshold field (the Kjeldaas edge) for the propagation of helicon waves. These predictions have been experimentally verified to within a few percent for polycrystalline sodium and potassium at 4°K. The values for the radius of the Fermi sphere deduced from the data are in excellent agreement with the theoretical values.

The measurement of the edge in polycrystalline indium shows that (a) the holes in indium behave very much like free holes with one hole/atom, and (b) an investigation of single crystals should yield valuable information about the indium Fermi surface.

The measurements on the fractional width of the edge demonstrated that it is inversely proportional to $\omega_c\tau$, and the empirical relation $\Delta B/B_e = \pi/\omega_c\tau$ fits the data reasonably well.

ACKNOWLEDGMENTS

The author wishes to thank R. Bowers for his continued encouragement and support of the work. He also acknowledges the help of J. R. Merrill, J. R. Houck, and S. Tallman with the experiments.

[20] C. P. Bean, R. W. DeBlois, and L. B. Nesbitt, J. Appl. Phys. **30**, 1976 (1959).

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Theory of the Thermal Conductivity of Superconducting Alloys with Paramagnetic Impurities

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The electronic thermal conductivity (K_s) of a weakly coupled, isotropic superconductor doped with a small concentration of paramagnetic impurities is computed. The theory of such superconducting alloys has been given by Abrikosov and Gor'kov, and is based on the assumption that the static magnetic impurities are randomly distributed and that their spins are uncorrelated. Starting from a Kubo formula, K_s is calculated by considering the electron-impurity interaction in the ladder approximation. A considerable simplification of the final expression for K_s obtains if the exchange scattering time τ_S is much larger than the total scattering time. Numerical calculations have been made of the ratio of the thermal conductivity in the superconducting and normal states as a function of the reduced temperature ($T/T_c \equiv t$) for different impurity concentrations. Abrikosov and Gor'kov have shown that the energy gap function $\omega_0(T)$ is quite different from the Gor'kov order parameter $\Delta(T)$ in such alloys. It is found, however, that K_s/K_n is less than unity even in the "gapless" region ($\Delta\tau_S < 1$). Moreover, K_s/K_n as a function of t decreases with the paramagnetic-impurity concentration for $t \gtrsim 0.8$ and low concentrations. Some aspects of the Abrikosov-Gor'kov model are reviewed in an Appendix. The numerical values of Δ, ω_0, and the density of states that were used in the evaluation of K_s/K_n are given separately.

1. INTRODUCTION

THE prediction of "gapless" superconductivity in paramagnetic alloys by Abrikosov and Gor'kov,[1] and the confirmation of this prediction by Reif and Woolf[2] are significant recent developments. Although the theory is based on an approximate treatment of a simple model and more detailed experiments are needed,

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[1] A. A. Abrikosov and L. P. Gor'kov, Zh. Eksperim. i Teor. Fiz. **39**, 1781 (1960) [English transl.: Soviet Phys.—JETP, **12**, 1243 (1961)]. This will often be referred to as AG in the text.

[2] F. Reif and M. A. Woolf, Phys. Rev. Letters **9**, 315 (1962); Rev. Mod. Phys. **36**, 238 (1964).