Semiempirical Nuclidic Mass Equation*

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We have developed a semiempirical equation, starting from the liquid-drop model, to account for the systematics of nuclidic masses. The mass excess of a nuclide is expressed as follows (in the scale of $C^{12}=12.000000$ mass units): $M(Z,A)=(0.0089794\,A^2-2.0717\,A+33.448)+\frac{1}{2}[3.258-(60.22/A^{1/2})+431.6/A]\,(Z-Z_A)^2-S+(11.51/A^{1/2})\delta$ MeV, where $Z_A=(A+0.003A^2)/(2+0.01A)$; $\delta=+1,\,0,\,-1$ for odd-Z-even-A, odd-A, even-Z-even-A nuclides, respectively; and S is a term for the shell effect consisting of a series of Cauchy distribution functions in terms of the nucleon numbers. The shell correction is not symmetric with respect to the shell edges, this being the main feature of the present equation. The shell effect on Z_A has been investigated in an alternate approach. Compared with the other nuclidic mass equations, the above equation, with only 34 adjustable constants, has the fewest number of large deviations from the experimental data and very little systematic error. The equation agrees with the 842 experimental masses to within ±0.5 MeV in 57% and within ±1.0 MeV in 91%. Only 11 deviations are between 2.0 and 3.1 MeV.

I. INTRODUCTION

IN the past thirty years, more than twenty mathematical expressions have been formulated to account for the systematics of the nuclidic masses, binding energies, and nucleon separation energies.[1] These expressions, commonly called nuclidic mass equations, represent the mass or the equivalents as a function of the proton number Z and the neutron number N of the nuclide. Some of these expressions are quite inconvenient to use because they involve many complicated functions and adjustable constants. The physical significance of some of the complicated functions is also uncertain. Although many of the quations describe fairly well the general features of the nuclidic mass surface (a three-dimensional plot of nuclidic masses versus N and Z), systematic deviations from the experimental data are observed in every mass equation hitherto published.[1] Most of these systematic deviations originate from the inadequate treatment of the nuclear shell effects and the isobaric mass variation. We have developed a relatively simple semiempirical expression to account for the nuclidic mass systematics starting from an equation based on the liquid-drop model,[2,3] with the shell effects included in a correction term. The numerical values of the constants in the mass equation are evaluated by least-squares fitting of the experimental nuclidic masses[4,5] based on the scale of $C^{12}=12.000000$ mass units. The computations in this work were performed at Argonne with the help of several electronic computers and an abacus.

II. FORMULATION OF THE MASS EQUATION

The basic form of our nuclidic mass equation is

$$M(Z,A)=M_A+\tfrac{1}{2}B_A(Z-Z_A)^2+P_A-S(N,Z)\,,\quad (1)$$

where M is the mass excess (nuclidic mass minus mass number), M_A the mass excess of the stable nuclide ($Z=Z_A$) for mass number A, B_A a measure of the curvature of the isobaric mass section, Z_A the charge (not necessarily an integer) of the most stable isobar, P_A the pairing energy due to the even-odd variation, and S the shell correction term.

M_A is assumed to be a parabolic function of A, and the coefficients were evaluated in a first approximation by a least-squares fit of the experimental masses of the stable, odd-A nuclides not containing closed-shell configurations:

$$M_A=0.0089794\,A^2-2.0717\,A+33.448\ \text{MeV}.\quad (2)$$

For odd-A nuclides, it can be shown that

$$B_A=M(Z,A)-2M(Z+1,A)+M(Z+2,A)\,.\quad (3)$$

With the experimental masses of odd-A nuclides inserted into Eq. (3), we calculated the values of B_A on the basis of which the following expression was obtained as the first approximation of B_A:

$$B_A'=4.68-(86.32/A^{1/2})+(550/A)\text{MeV}.\quad (4)$$

The expression for Z_A was taken from Green's work[6]; his Z_A seems sufficiently satisfactory as a continuous approximation and is adopted without change:

$$Z_A=(A+0.003\,A^2)/(2+0.01\,A)\,.\quad (5)$$

* Based on work performed under the auspices of the U. S. Atomic Energy Commission.
† Supported in part by the National Science Foundation.

[1] J. Wing, Atomic Energy Commission Report, ANL-6814, 1964 (unpublished).
[2] N. Bohr and J. A. Wheeler, Phys. Rev. **56**, 426 (1939).
[3] P. Fong, Phys. Rev. **102**, 434 (1956).
[4] L. A. König, J. H. E. Mattauch, and A. H. Wapstra, Nucl. Phys. **31**, 28 (1962).
[5] V. A. Kravtsov, Nucl. Phys. **41**, 330 (1963).
[6] A. E. S. Green, *Nuclear Physics* (McGraw-Hill Book Company, Inc., New York, 1955).

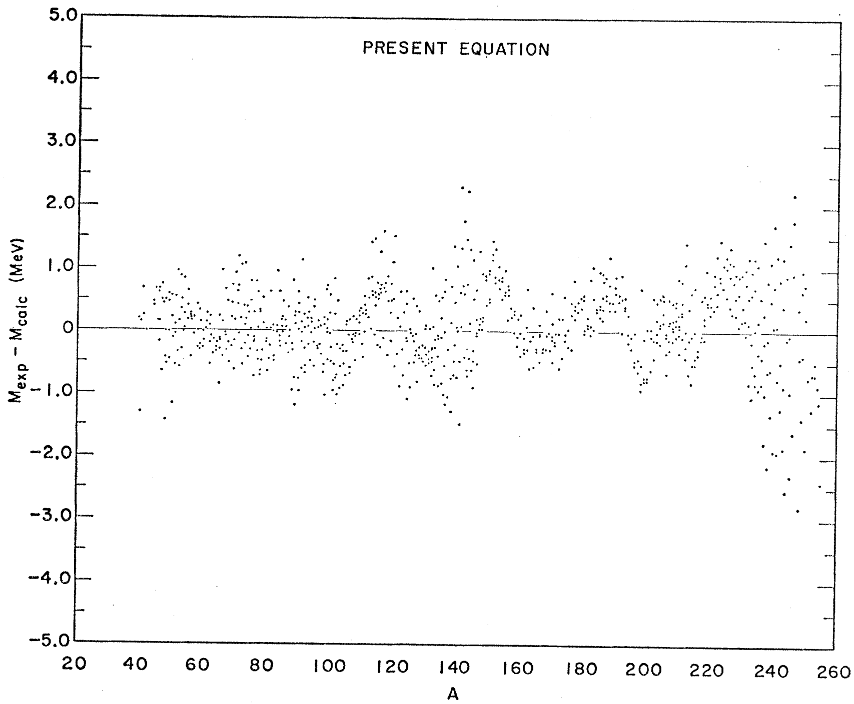

FIG. 1. Deviations of the present mass equation from the experimental values of nuclidic masses.

The pairing energy term was obtained by comparison of Eq. (1), with Eqs. (2), (4), and (5) inserted, with the experimental mass data, disregarding the nuclides with the closed-shell configurations:

$$P_A = 11.51\ \delta/A^{1/2}\ \text{MeV}, \tag{6}$$

where δ equals 0 for odd-A, -1 for even-Z-even-A, and $+1$ for odd-Z-even-A nuclides.

Wapstra[7] suggested a bell-shaped correction curve for the shell effect. We tentatively assumed the shell effect term in our mass equation to take the form of the sum of a series of Cauchy distribution functions in terms of N and Z, with their maxima located at the magic numbers:

$$S'(N,Z) = \sum_i \frac{a_i b_i^2}{(N-N_i^*)^2 + b_i^2} + \sum_j \frac{a_j b_j^2}{(Z-Z_j^*)^2 + b_j^2}, \tag{7}$$

where N_i^* and Z_j^* are, respectively, the magic numbers of neutrons and protons in the closed-shell configurations, a is the maximum magnitude of the shell correction in the ith neutron or jth proton shell, and b is the half-width at $\frac{1}{2}a$. In the neighborhood of a shell edge, only one term in the sum is large; other terms from other shells are small. The advantage of this form of shell correction is that the mass equation remains a continuous function (except for the pairing energy) from one shell region to another. It soon becomes apparent[1] that the correction curve of Eq. (7), which is symmetric with respect to the shell edge, cannot fit experimental data on both sides of the shell edge. On the other hand, each side may be fitted by a Cauchy distribution curve with a width different from that of the other side. Thus we assumed different values of the width $b_\pm$ for N (or Z) greater and smaller than the magic number. The shell correction term thus has the following form:

$$S(N,Z) = \sum_i \frac{a_i b_{i\pm}^2}{(N-N_i^*)^2 + b_{i\pm}^2} + \sum_j \frac{a_j b_{j\pm}^2}{(Z-Z_j^*)^2 + b_{j\pm}^2}, \tag{8}$$

where $b_{i\pm}$ and $b_{j\pm}$ are used when $(N-N_i^*) \gtrless 0$ and $(Z-Z_j^*) \gtrless 0$, respectively. In spite of the discontinuity of b at the shell edge, the equation remains continuous to the first-order derivative. The values of the constants in Eq. (8) were evaluated by a variable metric method for minimization,[8] the input data being the values of S obtained from the combination of Eqs. (1), (2), and (4) to (6) and substitution of experimental masses. The results are listed in Table I.

We then applied the variable metric minimization method for the improvement of the B_A expression, using the experimental masses and all the previous equations and constants except Eqs. (3), (4), and (7). We obtained the following expression for B_A and discarded Eq. (4):

$$B_A = 3.258 - (60.22/A^{1/2}) + (431.6/A)\,\text{MeV}. \tag{9}$$

A similar iteration was performed for M_A. However, the values of the coefficients so obtained were essentially identical with those in Eq. (2) and therefore Eq. (2) is used for M_A without change.

[7] A. H. Wapstra, Physica 18, 83 (1952).

[8] W. C. Davidon, Atomic Energy Commission Report, ANL-5990, 1959 (unpublished).

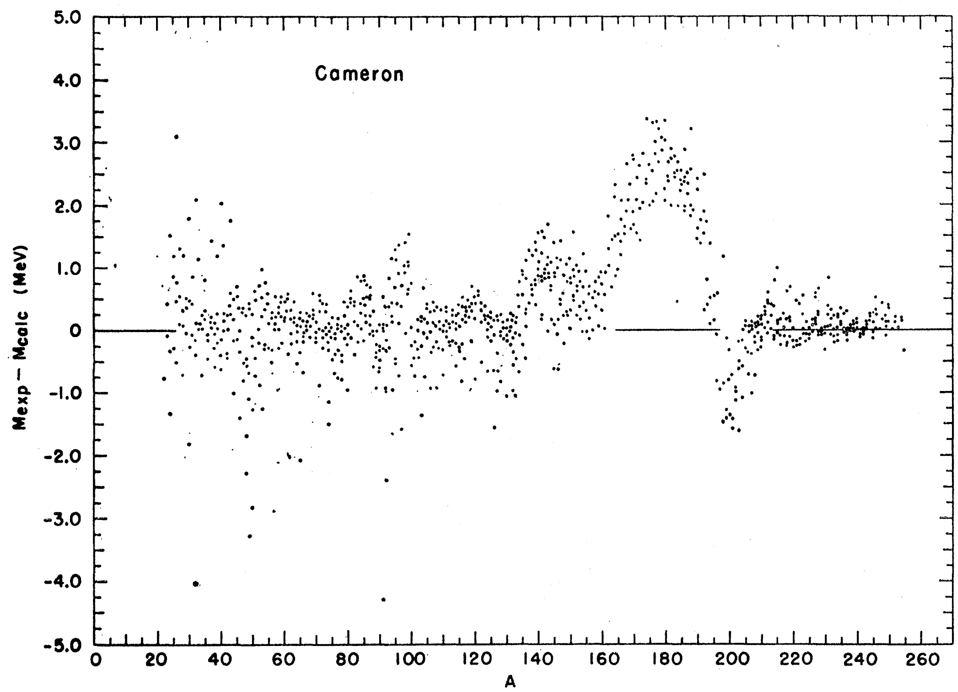

FIG. 2. Deviations of Cameron's mass equation from the experimental values of nuclidic masses.

We have also studied the possibility of developing a mass formula by correcting Z_A for the shell effects. We know that the charge of the beta-stable isotope is not a smooth function of A, but shows fluctuations due to shells.[3] This approach will be discussed in the Appendix. The resulting mass equation, referred to as the alternate equation to avoid confusion, is not as simple and as successful as the present one and is listed only for reference. In addition to the shell correction for Z_A, the terms S remains indispensable and again appears to be asymmetric with respect to the shell edges. The present mass equation includes all shell corrections in the asymmetric S term and thus is simpler; the empirical variation of the charge (integral) of the beta-stable isotope is thus a manifestation of the S term.

III. DISCUSSION

The nuclidic masses predicted by the present equation are compared with the experimental data by plotting the differences in Fig. 1. No large systematic deviations are observed in the mass region of $A=60$ to 220. For $A>230$, there is a wide spread of almost 5 MeV of differences; the neutron-rich nuclides have positive deviations and the neutron-deficient nuclides have negative deviations. This wide spread of differences is a result of inadequate treatment of the isobaric mass variation in this region (B_A and Z_A small compared with the experimental data). Improvement of the present mass equation could be made with additional adjustable constants especially for the B_A and Z_A expressions. However, this was not done because we did not want to add any more adjustable constants into our present equation. A more refined expression for Z_A is given in the Appendix, which may be used for the purpose of determining the stable isobar of a given mass number A.

For comparison, we have also plotted the differences between the experimental and the calculated (masses or binding energies) values, for the mass equations of

TABLE I. Values of the constants in Eq. (8).

N_i^*	a_i	b_{i+} $(N-N_i^*)\geqslant 0$	b_{i-} $(N-N_i^*)\leqslant 0$	Z_j^*	a_j	b_{j+} $(Z-Z_j^*)\geqslant 0$	b_{j-} $(Z-Z_j^*)\leqslant 0$
28	3.49	4.04	1.44	28	3.07	2.27	2.77
50	5.99	5.96	2.88	50	2.74	4.31	3.10
82	5.75	2.49	5.32	82	4.22	1.51	2.35
126	7.76	2.90	5.36				
152	5.02	6.88	5.29				

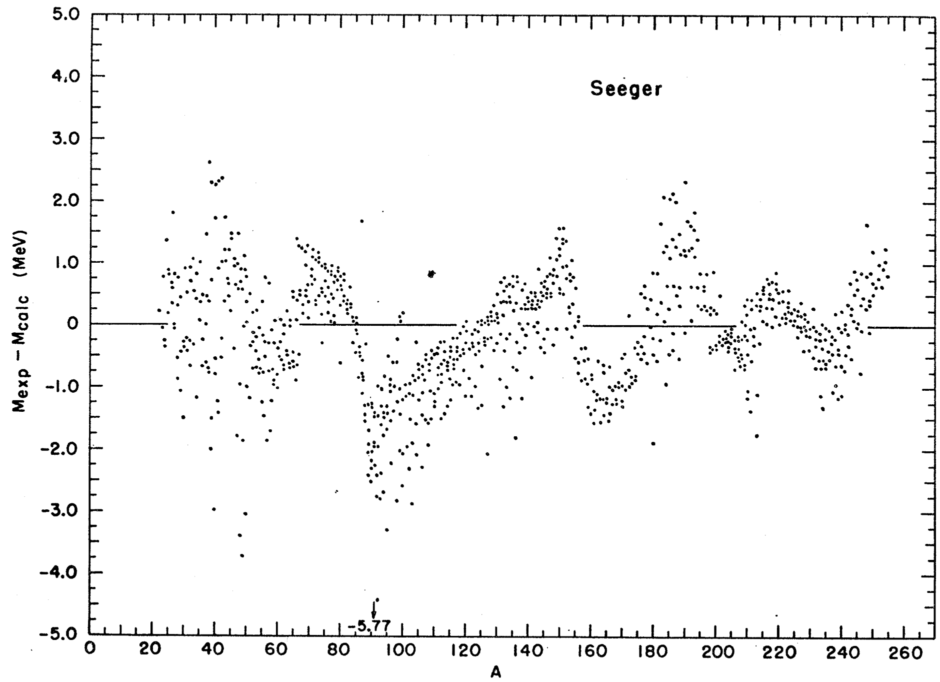

FIG. 3. Deviations of Seeger's mass equation from the experimental values of nuclidic masses.

Cameron,[9] Seeger,[10] Levy,[11] Baker,[12] and Green[6] in Figs. 2 to 6. In the mass region of $A>201$, Cameron and Baker used, for the evaluation of their constants, the experimental masses[13] which are about one MeV lower than those we used.[4] We have, therefore, allowed for a one-MeV correction in the calculations using the equations of Cameron and Baker in this mass region.

It has long been recognized that a liquid-drop-model mass formula, such as Fermi's, deviates from the experimental results by a uniform shift plus systematic fluctuations related to shells.[3] The former may be eliminated by proper changes of the parameters of the formula, but the latter cannot be expressed in terms of simple functions of N and Z. We are thus led to expect a nuclidic mass formula consisting of two parts: a smoothly varying part similar to the liquid-drop model formula plus a rapidly varying shell correction term. How to formulate the shell correction is thus the central problem.

Levy[11] and Green[6] tried to fit the experimental data between the shells with smooth functions, this being obviously the simplest approach because the variation of mass between the shells is the least drastic. The fitted smooth functions expectedly lead to large deviations near the closed shells. Yet the most undesirable feature of this approach is that the formulas are discontinuous over the shell boundaries. Levy divided the mass surface into sections by the magic number lines. The mass surface in each section is approximated by a quadratic surface, the parameters of which change from section to section. Besides large deviations at the shell edge (Fig. 4) there is also a constant deviation of about 1.5 to 2.0 MeV for the heavy elements region. Green's shell correction term consists of a set of parabolic curves with vertices located midway between two magic numbers. The discontinuity at the shell edge is considerable (Fig. 6).

The other alternative is to correct the shell effects in the neighborhood of the shell edges; this approach is mathematically more difficult but physically more reasonable. Seeger[10] expressed the shell effects by a series of sine functions of N and Z with cross product terms. The systematic oscillation of his deviations (Fig. 3) is probably a result of the symmetric nature of the sine functions with respect to the shell edges.

Since this leaves the asymmetric correction over the shell edges as the only alternative, we adopted it. From the point of view of nuclear structure we have no reason to expect a nucleus with extra nucleons outside a closed shell to behave exactly the same as one with an equal number of unfilled levels (holes). The nuclear level spacing behaves differently for these two types of nuclides. So does the nuclear mass which fixes the position of the ground level. Apart from the shell correction term, the rest is empirically fitted into a smooth formula in the present approach. The formula is similar to the liquid-drop model formula in its quadratic dependence on Z, though the expressions of M_A, B_A, and Z_A are purely empirical.

Cameron[9] empirically determined the combined effects of the shell and pairing interactions for each

[9] A. G. W. Cameron, Can. J. Phys. **35**, 1021 (1957).
[10] P. A. Seeger, Nucl. Phys. **25**, 1 (1961).
[11] H. B. Levy, Phys. Rev. **106**, 1265 (1957).
[12] G. A. Baker, Phys. Rev. **112**, 954 (1958).
[13] J. R. Huizenga, Physica **21**, 410 (1955).

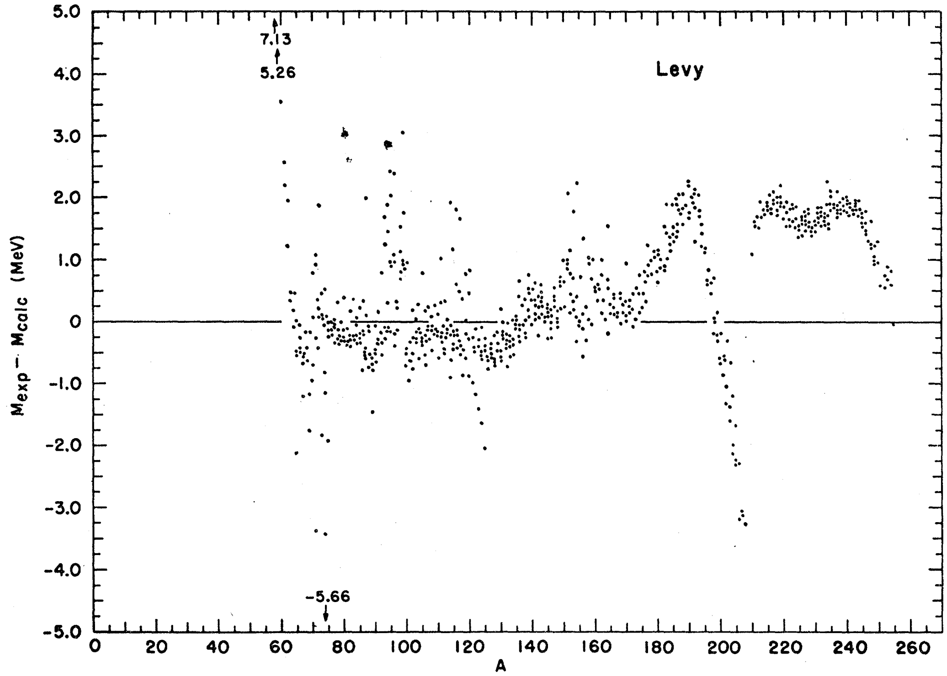

FIG. 4. Deviations of Levy's mass equation from the experimental values of nuclidic masses.

value of N and Z. There is a notable systematic deviation in the rare-earth region (Fig. 2). This may be due to the fact that very few and perhaps poor experimental data were available in this mass region for the evaluation of Cameron's constants. The complication of his formula is brought out by the fact that more than 200 parameters are involved. Many of the deviations in Cameron's, Seeger's, and Levy's mass equations become very large for nuclides far away from the beta stability line, indicating inadequate treatment for the isobaric mass sequences.

Baker[12] expressed the binding energies of nuclides by polynomial functions of neutron excess and mass number. No shell effect was included in his binding energy formula. The spread of his deviations (Fig. 5) is wider than those we have examined so far, except for

TABLE II. Frequency distributions of deviations.

Mass region	Deviation (MeV)	Cameron[a]	Seeger	Levy	Baker[a]	This work	The alternate equation
		(232)	(25)	(81)	(63)	(34)	(47)
$40 \leqslant A \leqslant 70$	<0.5	89	45	16	66	76	57
	0.5–1.0	26	51	8	38	35	38
	1.1–2.0	8	23	5	19	13	27
	>2.0	3	7	6	2	2	4
$70 < A \leqslant 115$	<0.5	146	50	132	118	137	93
	0.5–1.0	44	75	40	70	65	76
	1.1–2.0	11	53	16	15	6	33
	>2.0	2	25	6	6	1	7
$115 < A \leqslant 162$	<0.5	96	101	127	72	92	103
	0.5–1.0	70	68	55	63	78	65
	1.1–2.0	27	23	8	45	21	23
	>2.0	0	1	3	13	2	2
$162 < A \leqslant 208$	<0.5	29	65	40	24	107	46
	0.5–1.0	21	43	33	28	37	60
	1.1–2.0	38	33	46	54	1	34
	>2.0	57	4	14	39	0	5
$208 < A \leqslant 255$	<0.5	158	106	1	154	68	50
	0.5–1.0	11	50	12	12	65	65
	1.1–2.0	0	13	133	2	30	44
	>2.0	0	0	6	1	6	9
$40 \leqslant A \leqslant 255$	<0.5	518	367	316	434	480	349
	0.5–1.0	172	287	148	211	280	304
	1.1–2.0	84	145	208	135	71	161
	>2.0	62	37	35	61	11	27

[a] Corrected for 1-MeV error in these equations for nuclides with $A > 201$. See text.

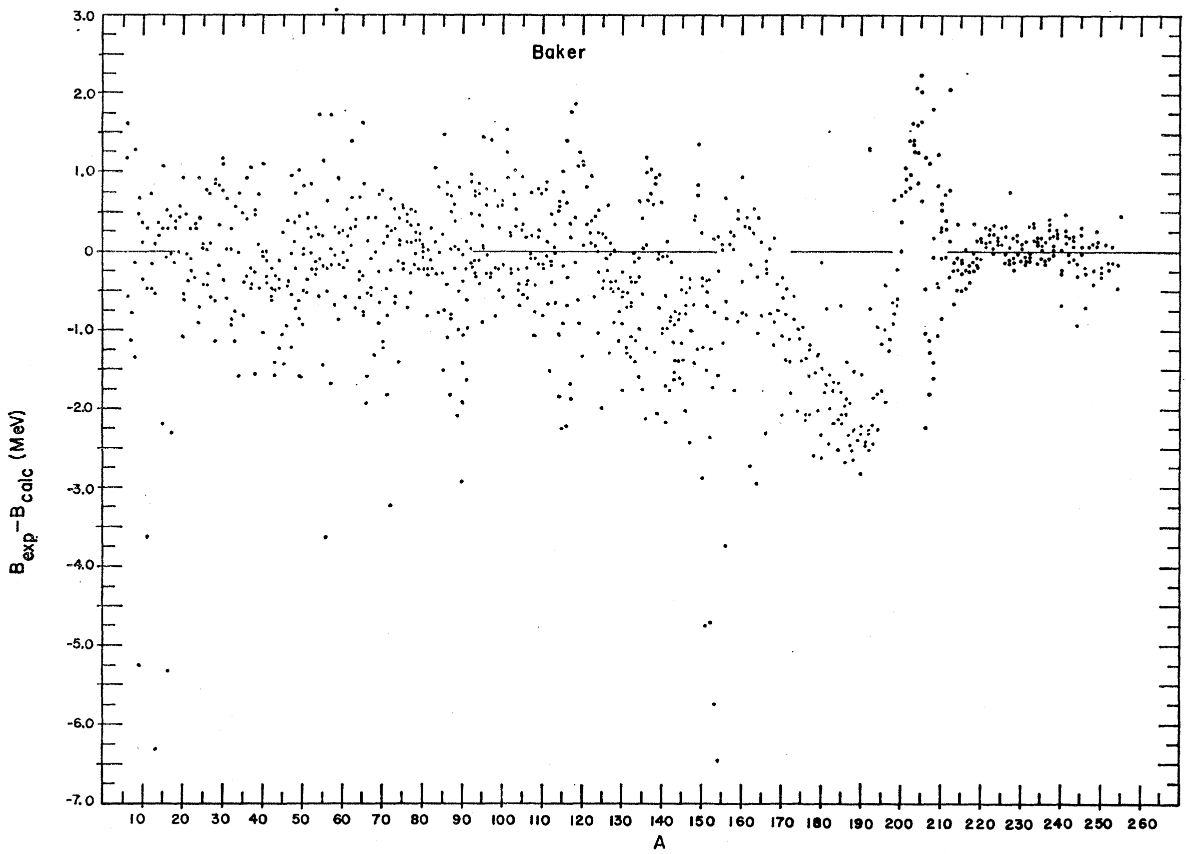

FIG. 5. Deviations of Baker's formula from the experimental values of nuclidic binding energies.

the heavy elements region. These deviations are obviously related to the shell effects which are not adequately accounted for.

Table II lists the frequency distributions of deviations (absolute difference between the experimental and calculated values) found in the nuclidic mass equations which we have considered. The number of the adjustable constants used in a given mass equation is placed in parentheses. The present mass equation has the

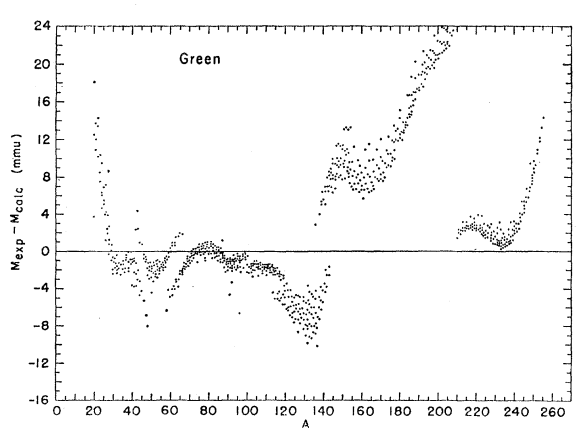

FIG. 6. Deviations of Green's mass equation from the experimental values of nuclidic masses.

fewest deviations greater than 2.0 MeV, and, next to Cameron's, has the highest percent (and number) of deviations smaller than 0.5 MeV. No deviation in the present mass formula is larger than 3.10 MeV, whereas all the other mass equations we have examined so far have one or more deviations larger than 4.0 MeV. In the mass region of $A=60$ to 220, our calculated values are in good agreement with the experimental data. However, in the heavy mass region ($A>210$), Cameron's and Baker's predictions are much better than ours. Finally, it may be mentioned that the present mass equation has only 34 adjustable constants while all the others, except Seeger's, require more than 40 adjustable constants.

The nuclidic mass excesses, neutron and proton binding energies, alpha-particle binding energies, and total beta-decay energies predicted by the present mass equation for $Z=13$ to 110 and $A=22$ to 315 are tabulated in an Argonne National Laboratory report.[14]

APPENDIX: THE ALTERNATE EQUATION

The basic form of the alternate equation is again

$$M(Z,A)=M_A+\tfrac{1}{2}B_A(Z-Z_A)^2+P_A-S, \qquad (10)$$

[14] J. Wing and J. D. Varley, Atomic Energy Commission Report, ANL-6886, 1964 (unpublished).

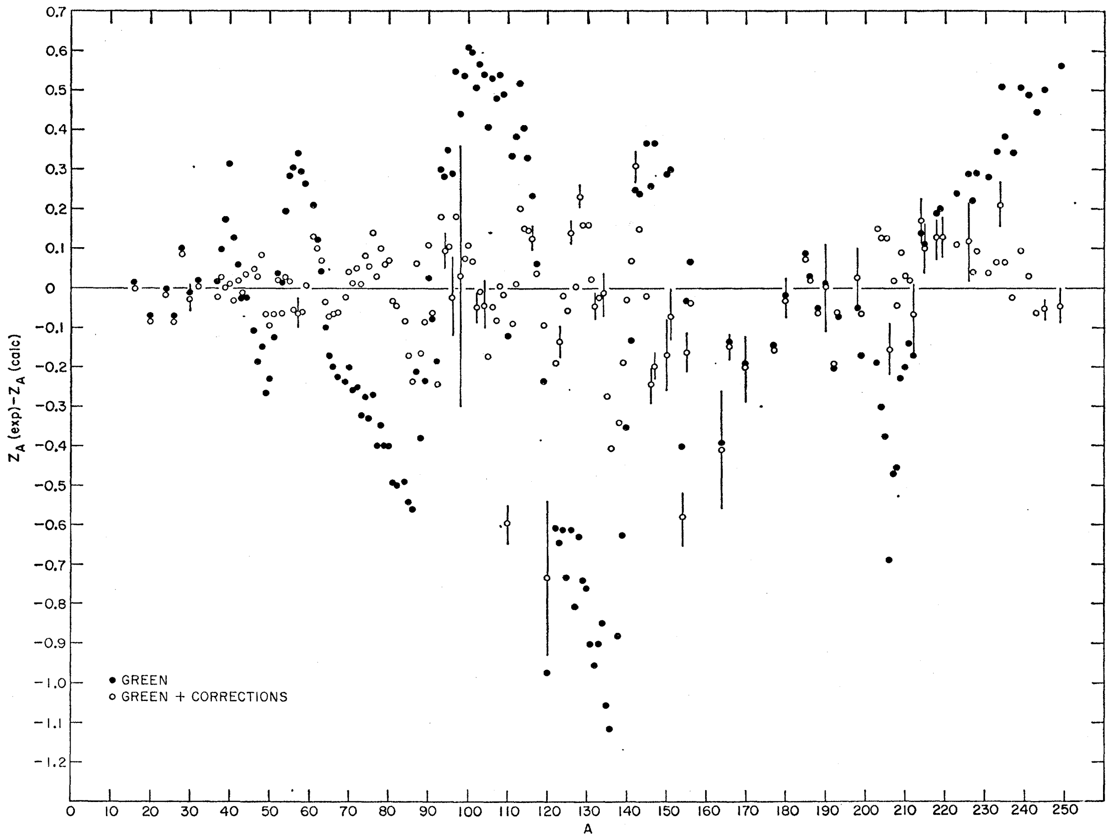

FIG. 7. Deviations of the calculated values of Z_A using Eqs. (5) (solid points) and (12) (open circles) from Dewdney's Z_A values. Experimental errors greater than 0.25 units are indicated by vertical bars.

where the symbols have been defined previously. We took the values of B_A and Z_A derived by Dewdney[15] from experimental total beta-decay energies and tried to find expressions to fit them. A least-squares fit of Dewdney's B_A values for nuclides away from the closed shells gave the following expression:

$$B_A=1.646-(28.41/A^{1/2})+(292/A)\text{MeV}. \quad (11)$$

A plot of the differences between Dewdney's Z_A values and those obtained with Eq. (5) is shown in Fig. 7. The relatively large deviations shown in this figure may be attributed to shell effects on Z_A.[3,15] [Note that, in the heavy mass region, Eq. (5) gives Z_A values which are too small compared with Dewdney's values, and these small Z_A values account for, at least partly, the wide spread of deviations observed in Fig. 1 in this mass region.] We, therefore, modified Eq. (5) to include the shell effects on Z_A by adding a series of Cauchy distribution functions:

$$Z_A=\frac{A+0.003A^2}{2+0.01A}+\sum_i \frac{f_i g_i^2}{(A-A_i^*)^2+g_i^2}, \quad (12)$$

where A^* is the mass number at which maximum deviation of Z_A is observed in Fig. 7. The values of f_i and g_i were obtained by a least-squares fit and are listed in Table III. A plot of Dewdney's Z_A values minus the Z_A values of Eq. (12) is also shown in Fig. 7.

TABLE III. Values of the coefficients in Eq. (12).

A_i^*	f_i	g_i
40	0.32	1.14
48	−0.28	3.06
57	0.50	3.25
83	−1.03	15.74
105	1.32	24.21
130	−1.56	14.97
147	0.97	5.79
206	−0.61	4.28
250	0.61	16.10

TABLE IV. Values of the coefficients in the expression for S (alternate approach).

Z_j^*	a_j	b_j	N_i^*	a_i	b_i
28	2.61	2.62	28	3.30	4.28
50	0.04	3.19	50	4.09	4.23
82	6.70	3.16	82	3.91	4.36
			126	5.96	3.09
			152	3.28	3.52

[15] J. W. Dewdney, Nucl. Phys. 43, 303 (1963).

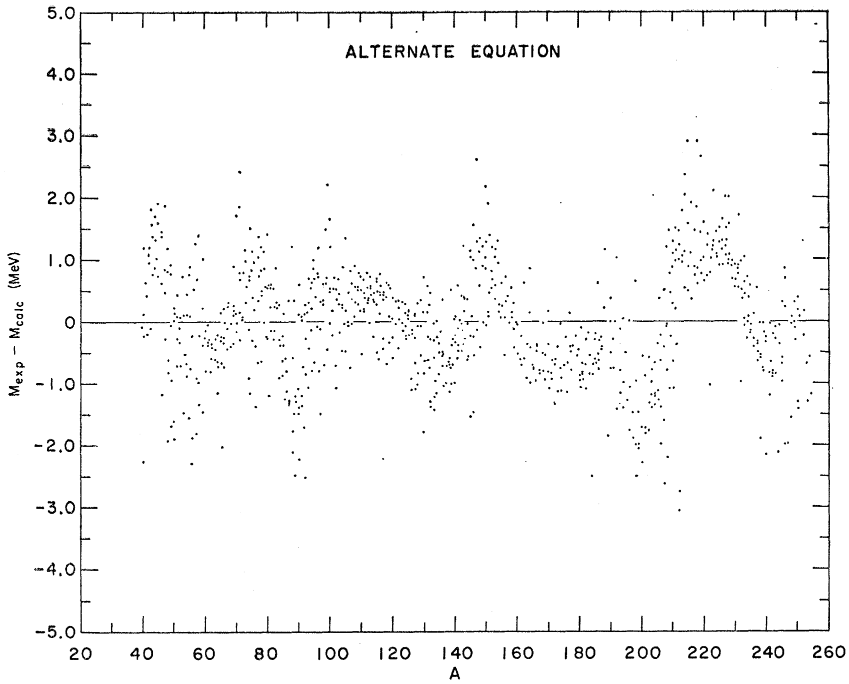

FIG. 8. Deviations of the alternate equation from the experimental values of nuclidic masses.

We used Eq. (6) without change for the pairing energy term here. We have not attempted to correct for the shell effects[1] on B_A and P_A since we intended to include these effects in the shell correction term S.

Having obtained B_A and Z_A, we then calculated the quantity $M_{\text{exp}}-\frac{1}{2}B_A(Z-Z_A)^2$, using Eqs. (10) to (12) and the experimental masses[4,5] of the stable isobars of odd A not containing closed shell configurations. A plot of these calculated quantities versus A exhibited a shape of two slightly different half-parabolas joined at their vertices. By means of least-squares fits, we obtained the following expressions for these two half-parabolas:

$$\text{For } 10<A\leqslant 120,\ M_A=0.009008\,A^2-2.130\,A+36.751\ \text{MeV}, \quad (13)$$

$$\text{For } 120<A<260,\ M_A=0.008189\,A^2-1.763\,A+4.221\ \text{MeV}. \quad (14)$$

We then determined S with the substitution of the experimental mass data in the mass equation specified by Eqs. (6) and (10) to (14). We adopted Eq. (7) for the expression of S instead of Eq. (8) in order to mini-

TABLE V. Empirical expressions of B_A, P_A, and Z_A.

Author(s)	Reference	B_A(MeV)
Wing and Fong	Present work	Eq. (9) $3.258-(60.22/A^{1/2})+431.6/A$
		Eq. (11) $1.646-(28.41/A^{1/2})+292/A$
Fermi	16	$78.064(1.98067+0.0149624A^{2/3})/A$
Ayres *et al.*[a]	17	$2(4a_a+a_cA^{2/3})/A$
		P_A(MeV)
Wing and Fong	Present work	$11.51/A^{1/2}$
Fermi	16	$33.5/A^{3/4}$
Friedlander and Kennedy	18	$132/A$
Green	6	$11.2/A^{1/2}$
Ayres *et al.*[a]	17	$a_\pi/2A$
		Z_A
Wing and Fong	Present work	Eqs. (5) and (12)
Fermi	16	$A/(1.98067+0.0149624A^{2/3})$
Tsen	19	$0.86667AZ_A^{2/3}+147.576Z_A=74.627A$
Green	6	$\frac{1}{2}(200A-0.6A^2)/(A+200)$

[a] a_a, a_c, and a_π are smooth functions of A.

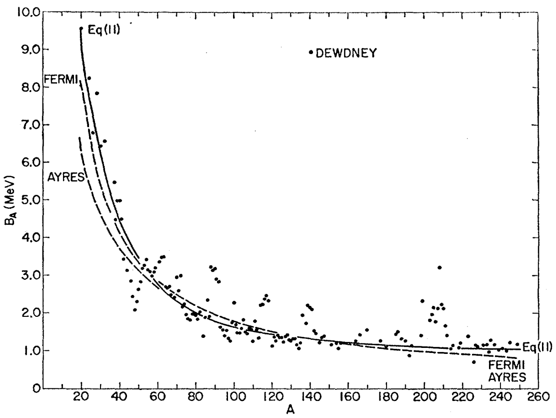

FIG. 9. Comparison of the calculated values of B_A with Dewdney's values.

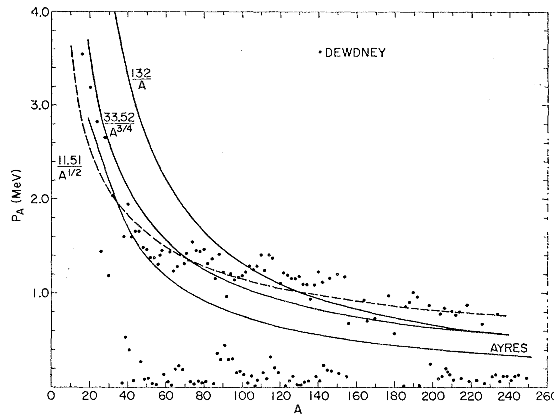

FIG. 10. Comparison of the calculated values of P_A with Dewdney's values.

mize the number of adjustable constants. Applying the variable metric minimization method with the S values determined here as the input data, we obtained the values of the coefficients for S, which are listed in Table IV. Iteration was not necessary here.

Figure 8 shows the deviations of the alternate equation from the experimental data. This equation has more large deviations than the previous equation (see Table II), and has several notable systematic discrepancies. However, this equation has a narrower spread of deviations than the previous one in the heavy mass region, probably a result of better B_A and Z_A values here than in the previous equation. The oscillating variation of the discrepancies of the alternate equation originates from the asymmetric nature of the shell effects on nuclidic masses with respect to N^* and Z^*, which is not taken into account by Eq. (7).

Several empirical expressions for the parameters B_A, P_A, and Z_A in terms of smooth functions of the mass number have been developed,[6,16–19] and these are listed

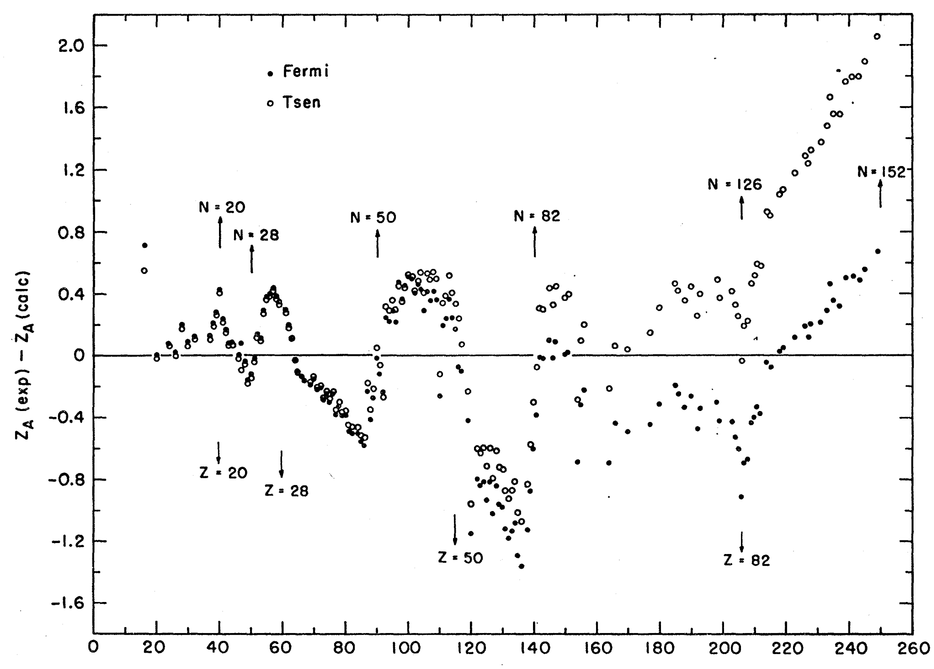

FIG. 11. Deviations of Fermi's and Tsen's calculated Z_A values from Dewdney's values.

[16] E. Fermi, *Nuclear Physics* (University of Chicago Press, Chicago, 1950).
[17] R. Ayres, W. F. Hornyak, L. Chan, and H. Fann, Nucl. Phys. **29**, 212 (1962).
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in Table V. A comparison of these expressions including ours with Dewdney's values[15] is shown in Figs. 7 and 9–11. Except for the closed-shell regions, our expressions are in fair agreement with Dewdney's values. Our empirical expression for Z_A with the shell effect terms included [Eq. (12)] is in much better agreement with Dewdney's values than other Z_A formulas none of which contain shell effects on Z_A.

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Hyperfine Structure and Nuclear Moments of 20.4-Min C^{11}†

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We have measured the hyperfine structure in the $^3P_2$ and $^3P_1$ states of the ground state configuration of C^{11} by the atomic-beam magnetic-resonance technique. The values obtained after corrections for perturbations by nearby fine-structure states are $^3P_2$: $A/h=(-)68.203\pm0.007$ Mc/sec, $B/h=(-)4.949\pm0.028$ Mc/sec; $^3P_1$: $A/h=(-)1.242\pm0.010$ Mc/sec or $(-)1.200\pm0.010$ Mc/sec depending upon the choice of zero-field level ordering, where $B(J=1)=-B(J=2)/2$. From these data it is possible to calculate the nuclear moments of the mirror nucleus, C^{11}, using a theoretical value of $\langle 1/r^3\rangle$ for the p electrons. The results are $\mu_I=(-)1.027\pm0.010$ nm, $Q_{\text{uncorrected}}=(+)(0.0308\pm0.0006)\times10^{-24}$ cm². No signs were measured in these experiments; the indicated signs assume $\mu_I<0$ in C^{11}. A value of 1.5011 ± 0.0006 for g_J was also obtained in the $^3P_2$ state.

I. INTRODUCTION

THE study of the magnetic dipole moments of mirror nuclei should be particularly useful in helping to find good nuclear wave functions. Assuming that these functions are sufficiently well known it may then be possible to check on the form of the magnetic moment operator. Of interest are contributions to this operator from meson currents in the nucleus; these contributions are expected to arise from the exchange of mesons between nucleons (exchange moments) and from the quenching of the anomalous part of the nucleon moment (quenching effects). A theorem due to Sachs[1] states that the exchange moments must be equal and opposite for the members of a mirror pair. A similar theorem should apply to the quenching calculations of Drell and Walecka[2] as they consider only the isotopic vector part of the anomalous magnetic moment. From these considerations it is clear that the sum of the moments of a mirror pair should be more useful in determining the wave function than either of the individual moments. Other effects such as the moment contribution arising from the spin-orbit force must also be taken into account and the reader is referred to Ref. 2 for a further discussion.

Such a program has been carried out for the H^3, He^3 pair resulting in the first direct indication of exchange currents in nuclei,[1] and recently the magnetic moments of the radioactive members of three more mirror pairs have been measured. These nuclei are N^{13},[3,4] O^{15},[5] and Ne^{19};[6] the moments of the stable members are, of course, known. Unfortunately it is not yet possible, for these heavier cases, to do nuclear structure calculations with sufficient accuracy so that the mesonic effects can be detected.

In this paper we report on measurements on the radioactive member of the $A=11$ pair, 20.4-min C^{11}. Previous measurements[7] have determined the spin to be $\frac{3}{2}$. In the next section we shall discuss the necessary hyperfine structure theory. The experimental details are presented in Sec. III, the data and results in Sec. IV, and in Sec. V we discuss the results.

The $A=11$ pair is the first one for which both electric quadrupole moments are now known.

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