

## Elastic and Inelastic Scattering of 18.4-MeV Alpha Particles from $\text{Na}^{23}\dagger\ddagger$

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(Received 27 May 1964)

Absolute differential cross sections for the elastic and inelastic (0.439-MeV and 2.080-MeV states) scattering of 18.4-MeV alpha particles from 100- $\mu\text{g}/\text{cm}^2$  sodium targets have been measured using silicon surface-barrier detectors. The targets were prepared by the vacuum evaporation of sodium metal onto Formvar backings. The elastic and inelastic (2.080-MeV state) cross sections were measured at 2.5° intervals in the laboratory angular range from 10° to 172.5°. The 0.439-MeV state angular distribution was measured from 30° to 172.5°. The elastic cross section exhibits four evenly spaced diffraction maxima between 10° and 110°. At larger angles the pattern becomes irregular, and starts to rise rapidly at about 145°. Both inelastic cross sections exhibit pronounced forward-angle oscillatory structure and backward-angle peaking. The elastic scattering cross section was analyzed using both the simple diffraction model which yielded a "best" interaction radius of 6.14 F, and the five-parameter, smooth-cutoff model of McIntyre, Wang, and Becker. The inelastic cross sections were analyzed using a plane-wave Born-approximation procedure and the direct-interaction theory of McCarthy and Pursey taking  $l=2,4$  and an interaction radius of 6.3 F in each case. A Blair analysis of the inelastic cross sections yielded an average value of 0.17 for the magnitude of the quadrupole deformation parameter. An alpha-particle group corresponding to inelastic scattering from the 2.39-MeV state of  $\text{Na}^{23}$  was observed, and the cross section was estimated to be nominally 0.1 of the  $(\alpha,\alpha_2)$  scattering cross section.

### I. INTRODUCTION

THE spins and parities of the low-lying states of  $\text{Na}^{23}$  are well known,<sup>1</sup> and it has been analyzed in terms of the strong-coupling Nilsson model,<sup>2-6</sup> and the asymmetric-core rotator model.<sup>7</sup> These analyses have met with limited success, but have not been able to uniquely predict the collective properties of the various excited states. Charged-particle scattering affords an independent approach to the study of nuclear levels, especially in connection with their collective and/or single-particle characteristics.

The primary purpose of this investigation was to measure the elastic and inelastic alpha-particle scattering cross sections for  $\text{Na}^{23}$  and to analyze the experimental data in terms of various direct-interaction models. In particular, it was hoped that some of the

information derived from this study could be compared with, and would to some extent complement the results of these collective-model analyses. More generally, since information on alpha-particle scattering from odd- $A$  nuclei is rather incomplete, the present study together with previous investigations at this laboratory<sup>8,9</sup> will provide a somewhat more complete picture of alpha-particle scattering from light nuclei at about 18 MeV.

### II. EXPERIMENTAL

A general description of the cyclotron experimental area including the beam-focusing and analyzing system and the scattering chamber has been presented elsewhere.<sup>8,9</sup> The 18.4-MeV alpha-particle beam used in the present experiment had a nominal rms energy spread of 60 keV. The alpha-particle spectrometer system consisted of silicon surface-barrier counters and an electronics configuration which is identical to that previously described.<sup>8,9</sup> The experimental geometry was defined by an azimuthal detector acceptance angle of 2.3°, a solid angle of 0.001 sr, and a circular beam cross section of 5/64-in. diameter. Measurements were made at 2.5° intervals over an angular range from 10° to 172.5° in the laboratory system.

Most of the targets used in this investigation were prepared by the thermal vacuum evaporation of metallic sodium onto Formvar-film backings. The sodium and Formvar components of the targets had measured nominal thicknesses of 110  $\mu\text{g}/\text{cm}^2$  and 30  $\mu\text{g}/\text{cm}^2$ , respectively. The oxygen and carbon content of the target was appreciable, about 20  $\mu\text{g}/\text{cm}^2$  and 35  $\mu\text{g}/\text{cm}^2$ , respectively. At laboratory angles of less than 20° the

† Work supported in part by the U. S. Atomic Energy Commission.

‡ This report is based on a part of a thesis submitted by B. T. Lucas to the faculty of Purdue University in partial fulfillment of the requirements for the degree of Ph.D. in physics. For a previous report of some of the preliminary results of this investigation see B. T. Lucas, S. W. Cospser, and O. E. Johnson, *Bull. Am. Phys. Soc.* **9**, 439 (1964).

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<sup>1</sup> Unless otherwise specified the level structure and individual level properties proposed for  $\text{Na}^{23}$  in the following reference will be assumed: P. M. Endt and C. Van Der Leun, *Nucl. Phys.* **34**, 1 (1962). In cases where specific quantitative information and/or interpretations are under discussion, a full bibliographical reference to the original report will be made.

<sup>2</sup> G. Rakavy, *Nucl. Phys.* **4**, 375 (1957).

<sup>3</sup> A. E. Litherland, H. M. McManus, E. B. Paul, D. A. Bromley, and H. E. Gove, *Can. J. Phys.* **36**, 378 (1958).

<sup>4</sup> E. B. Paul and J. H. Montague, *Nucl. Phys.* **8**, 61 (1958).

<sup>5</sup> W. D. Braben, L. L. Green, and J. C. Willmott, *Nucl. Phys.* **32**, 584 (1962).

<sup>6</sup> A. B. Clegg and K. J. Foley, *Phil. Mag.* **7**, 247 (1962).

<sup>7</sup> B. E. Chi and J. P. Davidson, *Phys. Rev.* **131**, 366 (1963).

<sup>8</sup> B. T. Lucas, S. W. Cospser, and O. E. Johnson, *Phys. Rev.* **133**, B963 (1964).

<sup>9</sup> B. T. Lucas, S. W. Cospser, and O. E. Johnson, *Phys. Rev.* **135**, B116 (1964).

elastic alpha groups scattered from these contaminants merged with the  $\text{Na}^{23}$  ground-state alpha-particle group. Consequently, a spectral decomposition was necessary. This decomposition was accomplished using the carbon and oxygen elastic cross sections which had been measured at small angles using polyethylene and Formvar targets. Interference among the inelastic groups from sodium and the elastic groups from carbon and oxygen occurred between  $30^\circ$  and  $55^\circ$ . In this angular region targets of NaI deposited on  $500\text{-}\mu\text{g}/\text{cm}^2$  polyethylene backings and sodium metal deposited on  $200\text{-}\mu\text{g}/\text{cm}^2$  gold backings were used in an attempt to minimize the amount of these contaminants present.

The estimated probable systematic error in the absolute differential cross sections due to uncertainties in target thickness, beam integration, and experimental geometry is  $\pm 15\%$ . The experimental differential cross sections have been corrected only to first order for finite geometry.

### III. RESULTS, ANALYSIS, AND DISCUSSION

#### A. Elastic Scattering

No previous measurements of the differential elastic-scattering cross section of alpha particles from  $\text{Na}^{23}$  have been reported.

The experimental cross section is shown in Figs. 1 and 2. A pronounced diffraction structure with rather evenly spaced maxima is quite evident out to about  $110^\circ$ . At larger angles the pattern becomes quite

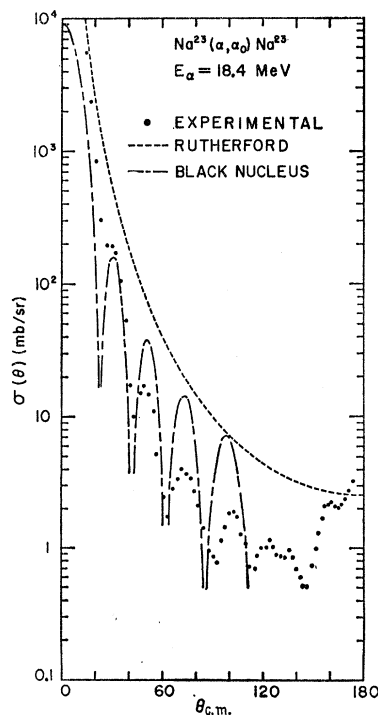


FIG. 1. The Rutherford, black-nucleus, and experimental differential cross sections for the elastic scattering of 18.4-MeV alpha particles from  $\text{Na}^{23}$ . The black-nucleus curve was calculated using an interaction radius of  $R=6.14$  F, a value resulting from a compromise fit to the second, third, and fourth experimental maxima.

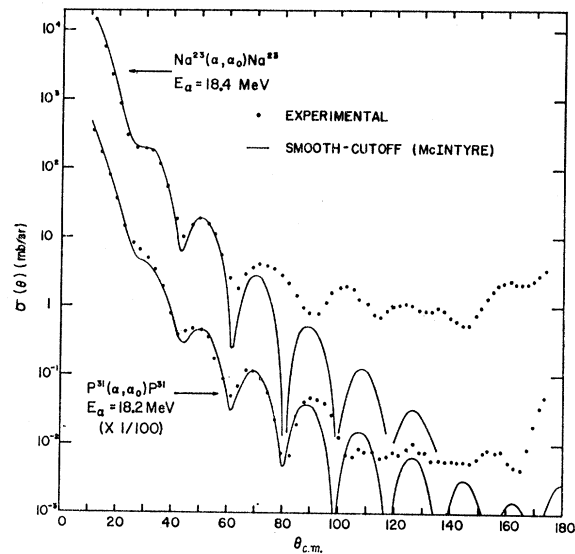


FIG. 2. The experimental differential cross sections for the elastic scattering of alpha particles from  $\text{Na}^{23}$  at 18.4 MeV and  $\text{P}^{31}$  at 18.2 MeV together with the theoretical curves resulting from a smooth-cutoff analysis (see Ref. 11) of the data. The parameters used in calculating the solid curves are listed in Table I.

irregular and rises sharply around  $145^\circ$ . The experimental cross section falls below the Rutherford cross section at all angles less than  $170^\circ$  as seen in Fig. 1. The broken curve in Fig. 1 is the familiar black-nucleus or "simple-diffraction-model"<sup>10</sup> cross section given by

$$\sigma(\theta) = (kR^2)^2 [J_1^2(x)/x^2], \quad (1)$$

where  $k$  and  $\theta$  are the center-of-mass wave number and scattering angle,  $R$  is the interaction radius,  $x \equiv 2kR \times \sin(\theta/2)$ , and  $J_1(x)$  is the ordinary Bessel function of the 1st order. The only adjustable parameter is the interaction radius which is usually determined by fitting the angular positions of the theoretical maxima and minima to the experimental angular distribution. A value of 6.14 F was used in calculating the broken curve in Fig. 1, and was obtained by a compromise fit of Eq. (1) to the second, third, and fourth experimental maxima. The "best-fit" criterion is somewhat subjective and equally satisfactory fits were obtained with  $R$  values in the range between  $\pm 0.10$  F of the value quoted above.

In Fig. 2 are presented the same data shown in Fig. 1 together with the experimental elastic differential cross section of 18.2-MeV alpha particles on  $\text{P}^{31}$  which was originally reported in a previous publication.<sup>9</sup> The solid curve in each case is calculated from the smooth-cutoff model of McIntyre, Wang, and Becker,<sup>11</sup> MWB

<sup>10</sup> J. S. Blair, Phys. Rev. **108**, 827 (1957).

<sup>11</sup> J. A. McIntyre, K. H. Wang, and L. C. Becker, Phys. Rev. **117**, 1337 (1960).

TABLE I. The MWB parameters resulting from analyses of elastic-scattering data for Na<sup>23</sup>, P<sup>31</sup>, and Ag<sup>107,109</sup>.

Reaction	$l_A$	$\Delta l_A$	$\delta$ (rad.)	$l_\delta$	$\Delta l_\delta$	$R$ (F)	$\Delta R$ (F)	$r_0$ (F)
Na <sup>23</sup> ( $\alpha, \alpha$ )Na <sup>23</sup> , 18.4 MeV	8.80±0.05	0.70±0.05	0.40±0.05	8.80	0.70	6.90	0.43	1.55
P <sup>31</sup> ( $\alpha, \alpha$ )P <sup>31a</sup> , 18.2 MeV	9.00±0.05	0.40±0.05	0.30±0.05	9.00	0.40	7.25	0.24	1.51
Ag( $\alpha, \alpha$ )Ag <sup>b</sup> , 22 MeV	10.8	0.75	0	...	...	9.74	0.33	1.52

<sup>a</sup> See Ref. 9.<sup>b</sup> See Ref. 15.

model, which may be written

$$\sigma_E(\theta) = \frac{1}{4}\lambda^2 \left[ -i\eta/\sin^2(\theta/2) \right] \times \exp[-i\eta \ln \sin^2(\theta/2)] \\ - \sum_{l=0}^{\infty} (2l+1) \exp[2i(\sigma_l - \sigma_0)] \\ \times [1 - |A_l| \exp(2i\delta_l)] P_l(\cos\theta)^2, \quad (2)$$

where

$$\sigma_l - \sigma_0 = \sum_{l'=1}^l \arctan(\eta/l'), \quad (3)$$

$$\eta = (Z_\alpha Z_N e^2 / \hbar v), \quad (4)$$

$$|A_l| = \{1 + \exp[(l_A - l)/\Delta l_A]\}^{-1}, \quad (5)$$

$$\delta_l = \delta \{1 + \exp[(l - l_\delta)/\Delta l_\delta]\}^{-1}. \quad (6)$$

There are five adjustable parameters;  $l_A$ ,  $\delta l_A$ ,  $\delta$ ,  $l_\delta$ , and  $\Delta l_\delta$ . Although there is no theoretical basis for Eqs. (5) and (6), explicit calculations for 43-MeV<sup>12</sup> and 60-MeV<sup>13</sup> alpha particles on Ni<sup>58</sup> show that the variation of the amplitude  $|A_l|$  described by Eq. (5) agrees quite well with that predicted by the optical model.

The parameters used to calculate the curves in Fig. 2 are listed in Table I. The "radius"  $R$  and the surface thickness parameter  $\Delta R$  were calculated from  $l_A$  and  $\Delta l_A$  using the semiclassical expressions<sup>14</sup>

$$l_A(l_A + 1) = kR(kR - 2\eta), \quad (7)$$

$$(2l_A + 1)\Delta l_A = 2kR(kR - \eta)\Delta R/R, \quad (8)$$

where  $k$  was defined in connection with Eq. (1) and  $\eta$  was defined by Eq. (4). The quantity  $r_0$  appearing in Table I was calculated from the formula

$$R = R_\alpha + r_0 A^{1/3}, \quad (9)$$

where  $R_\alpha = 2.5$  F. The results of an analysis by McIntyre *et al.*<sup>11</sup> of the Ag( $\alpha, \alpha$ )Ag cross section at 22 MeV measured by Wall *et al.*<sup>15</sup> were included for comparison purposes. The errors indicate the approximate range of parameter values that subjectively yield equally good fits.

<sup>12</sup> E. Rost, Phys. Rev. **128**, 2714 (1962).<sup>13</sup> P. Darriulat, G. Igo, H. G. Pugh, J. M. Meriwether, and S. Yambe, Lawrence Radiation Laboratory Report, UCL-11054, 1963 (unpublished), p. 104.<sup>14</sup> J. A. McIntyre, S. D. Baker, and K. H. Wang, Phys. Rev. **125**, 584 (1962).<sup>15</sup> N. S. Wall, J. R. Rees, and K. W. Ford, Phys. Rev. **97**, 726 (1955).

The Na<sup>23</sup> data could be fit out to about 60°, while a satisfactory fit to the P<sup>31</sup> data extends to about 90°. In contrast to the behavior of the experimental angular distributions, the magnitude of the successive maxima of the theoretical curve decrease steadily with increasing angle and fall well below the experimental points at large angles. The theoretical curves are quite sensitive to the values of  $\Delta l_A$  and  $\delta$  which control the rate of decrease of the cross section with angle and the "peak-to-valley" ratio, respectively. In contrast to the report of McIntyre *et al.*<sup>11</sup> concerning their analysis of the Ag( $\alpha, \alpha$ )Ag data, satisfactory fits to the Na<sup>23</sup> and P<sup>31</sup> angular distributions could not be obtained with  $\delta = 0$ . In the present study it was always assumed that  $l_A = l_\delta$ . However, the variation of  $\Delta l_\delta$  about the quoted value while holding  $\Delta l_A$  constant produced very little change in the angular distribution. It should also be noted that  $\Delta l_A$  and  $\Delta l_\delta$  are considerably smaller for P<sup>31</sup> than for Na<sup>23</sup>, although the other parameters are roughly the same for these two nuclei. No attempt was made to determine whether other significantly different sets of parameters would yield equally good fits.

## B. Inelastic Scattering

The differential cross sections corresponding to the inelastic scattering of 18.4-MeV alpha particles from the excited states of Na<sup>23</sup> at 0.439 MeV and 2.080 MeV are shown in Figs. 3 and 4, respectively. The 0.439-MeV angular distribution could be extended only to 35° (c.m.) because of interference from the intense elastic alpha groups associated with carbon and oxygen contamination in the target. Both experimental angular distributions exhibit pronounced forward-angle oscillations, a somewhat less pronounced structure at intermediate angles, and strong backward peaking. A similar general behavior was also observed for inelastic scattering of 18.2-MeV alpha particles from the first two excited states of P<sup>31</sup>.<sup>9</sup>

The ground and first two excited states of Na<sup>23</sup> are known to have positive parity and spins of  $\frac{3}{2}$ ,  $\frac{5}{2}$ , and  $\frac{7}{2}$ , respectively.<sup>1</sup> The direct-interaction theory for inelastic scattering in the plane-wave Born approximation, Austern, Butler, and McManus (ABM) theory,<sup>16</sup> predicts a theoretical cross section of the form

$$\sigma(\theta) = \sum_l A_l j_l^2(QR), \quad (10)$$

<sup>16</sup> N. Austern, S. T. Butler, and H. McManus, Phys. Rev. **92**, 350 (1953).

where  $j_l^2$  is the square of the spherical Bessel function of order  $l$ ,  $Q$  is the magnitude of the momentum transfer, and  $R$  is the interaction radius. The summation is performed over all  $l$  values consistent with angular momentum conservation. In the case of the  $(\alpha, \alpha')$  reaction on  $\text{Na}^{23}$ , the allowed  $l$  values are 2 and 4 for both the first and second excited states. The solid curves in Figs. 3 and 4 were calculated using Eq. (10) with  $R=6.3$  F. In fitting the  $(\alpha, \alpha_2)$  data the normalization was determined by a compromise fit to the first and second experimental maxima. In the case of the  $(\alpha, \alpha_1)$  data, the curve was normalized to the second and third experimental maxima. It should be mentioned that the inclusion of a rather large contribution of  $l=4$  yields a better fit to the experimental data than  $l=2$  alone only in the sense that the valley around  $30^\circ$  is filled in by the  $l=4$  component. However, a fit to the experimental data using Eq. (10) with only an  $l=2$  component yields equally good agreement among the positions of the theoretical and experimental maxima with the same interaction radius, 6.3 F. In view of the fact that the filling in of the valleys of the differential cross section can also arise from distortion effects,<sup>17</sup> the fits shown in Figs. 3 and 4 allow but do not necessitate the inclusion of a large  $l=4$  component.

An improved direct-interaction theory using more realistic wave functions was developed by McCarthy and Pursey<sup>17</sup> for inelastic alpha-particle scattering. In the quasielastic approximation the theoretical cross

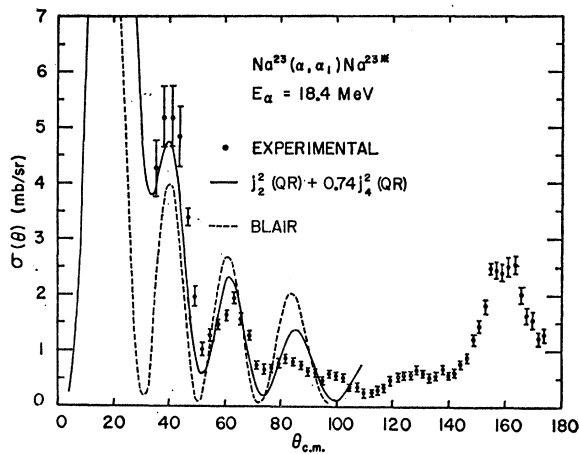


FIG. 3. The inelastic differential cross section for the scattering of 18.4-MeV alpha particles from the 0.439-MeV,  $\frac{5}{2}^+$  state in  $\text{Na}^{23}$ . The indicated probable errors are estimated on the basis of counting statistics and decomposition uncertainties. The solid curve results from a PWBA analysis (ABM theory) with  $l=2, 4$  and  $R=6.3$  F. The Blair-model curve (dashed line) corresponds to an evaluation of Eq. (12) with  $R=6.14$  F and  $|\beta_2|=0.17$ .

<sup>17</sup> I. E. McCarthy and D. L. Pursey, Phys. Rev. 122, 578 (1961).

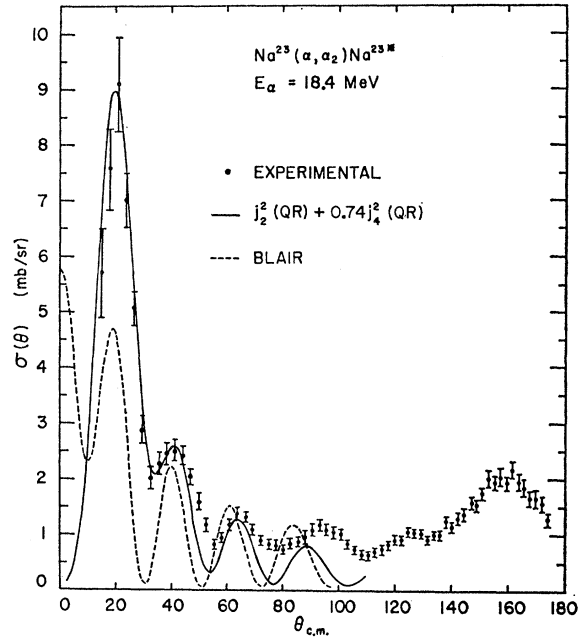


FIG. 4. The inelastic differential cross section for the scattering of 18.4-MeV alpha particles from the 2.080-MeV,  $\frac{7}{2}^+$  state in  $\text{Na}^{23}$ . The indicated probable errors are estimated on the basis of counting statistics and decomposition uncertainties. The ABM curve in this figure is the same as that shown in Fig. 3 except for the normalization. The Blair curve was calculated from Eq. (12) using  $R=6.14$  F and  $|\beta_2|=0.17$ .

section may be written

$$\sigma_l(\theta) \propto \left| \int_0^\infty \exp[-(r-R)^2/\lambda^2] j_l \times [2(kr+i\gamma) \sin(\theta/2)] r^2 dr \right|^2, \quad (11)$$

where  $l$  is the angular momentum transfer,  $k$  is the center-of-mass wave number, and  $R$  is the interaction radius. The anisotropy parameter  $\gamma$  results from an attempt to use wave functions for the incoming and outgoing particles that are consistent with optical-model calculations and is a measure of the angular spread of the wave function over the nuclear surface. The thickness parameter  $\lambda$  controls the degree of radial localization of the interaction.

The cross sections which result from an evaluation of Eq. (11) are shown in Figs. 5 and 6. As in the case of the plane-wave theory, the data were fit using an incoherent sum of  $l=2$  and  $l=4$ . The fit to the  $(\alpha, \alpha_2)$  data shown in Fig. 5 was accomplished using  $R=6.3$  F,  $\gamma=0.8$ ,  $\lambda=0.6$  F, and  $(l=4)/(l=2)=0.4$ . The curve was normalized at  $41^\circ$ . The fit which is much better than that obtained with the plane-wave theory requires a considerably smaller  $l=4$  component, although an equally good fit could not be obtained using only an  $l=2$  component. The  $(\alpha, \alpha_1)$  results are shown in Fig. 6, where  $R=6.3$  F,  $\gamma=0.7$ ,  $\lambda=0.8$  F, and  $(l=4)/(l=2)=0.4$  were used in the evaluation of Eq. (11). Even

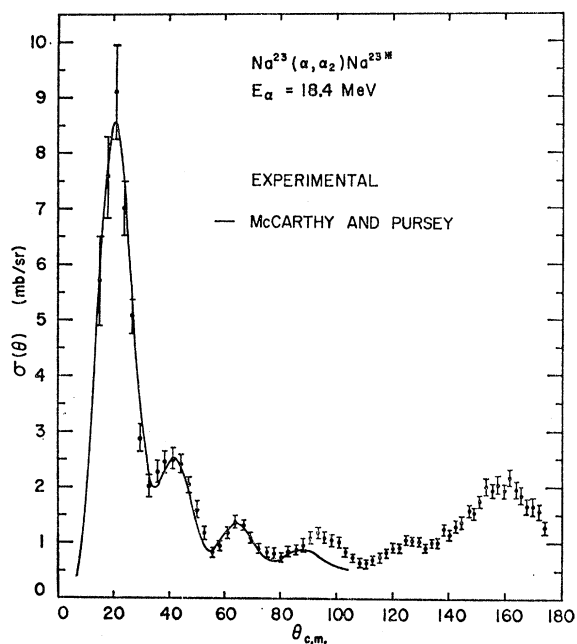


Fig. 5. The inelastic differential cross section for the scattering of 18.4-MeV alpha particles from the 2.080-MeV,  $\frac{3}{2}^+$  state in  $\text{Na}^{23}$ . The data are the same as those presented in Fig. 4. The solid curve results from an evaluation of  $\sigma_2(\theta) + 0.4\sigma_4(\theta)$  with  $\sigma_1(\theta)$  given by Eq. (11). The values of the other parameters were  $R=6.3$  F,  $\gamma=0.8$ , and  $\lambda=0.6$  F.

though the data are incomplete, it is seen that this fit is also quite good. The values of  $\gamma$  and  $\lambda$  determined from this analysis are somewhat smaller than those obtained by McCarthy and Pursey,<sup>17</sup>  $\lambda=0.88$  F and  $\gamma=0.9$ , in their analysis of the inelastic scattering of 41-MeV alpha particles from  $\text{S}^{32}$  and  $\text{Mg}^{24}$ .

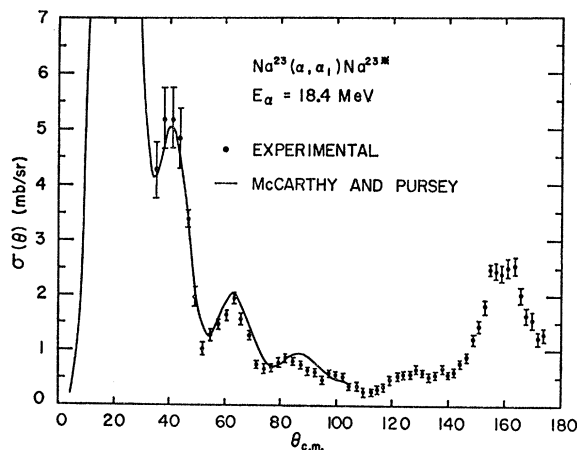


Fig. 6. The inelastic differential cross section for the scattering of 18.4-MeV alpha particles from the 0.439-MeV,  $\frac{5}{2}^+$  state in  $\text{Na}^{23}$ . The data are the same as those presented in Fig. 3. The solid curve results from an evaluation of  $\sigma_2(\theta) + 0.4\sigma_4(\theta)$ , where  $\sigma_1(\theta)$  is given by Eq. (11). The values of the parameters were  $R=6.3$  F,  $\gamma=0.7$ , and  $\lambda=0.8$  F.

The quadrupole moment of  $\text{Na}^{23}$  has been measured<sup>18,19</sup> and was found to be large ( $\sim 0.1 \times 10^{-24}$  cm<sup>2</sup>) and positive. This suggests the nucleus has a large permanent prolate deformation and one would expect the strong-coupling Nilsson model<sup>20</sup> to be relevant in the description of its level structure. Early analyses of the low-lying levels of  $\text{Na}^{23}$  were performed by Rakavy<sup>2</sup> and Litherland *et al.*<sup>3</sup> in terms of the Nilsson model where the ground and first two excited states were taken as the first three members of a  $K=\frac{3}{2}$  rotational band. It is expected that the simple rotational-band level spacings will be modified by the rotation-particle coupling<sup>21</sup> which mixes the  $K=\frac{3}{2}$  band with the lowest  $K=\frac{1}{2}$  and  $K=\frac{5}{2}$  bands. Rakavy<sup>2</sup> considered mixing of only the  $K=\frac{3}{2}$  and  $K=\frac{1}{2}$  bands while Litherland *et al.*<sup>3</sup> considered mixing of only the  $K=\frac{3}{2}$  and  $K=\frac{5}{2}$  bands. Later, Paul and Montague<sup>4</sup> considered mixing of the  $K=\frac{1}{2}$ ,  $\frac{3}{2}$ , and  $\frac{5}{2}$  bands and obtained  $\beta \approx +0.4$  for the deformation parameter. More recently Braben *et al.*<sup>5</sup> made a similar calculation based upon a more complete set of data. These calculations<sup>4,5</sup> yielded far too low an energy for the second excited state at 2.080 MeV. In another recent analysis Clegg and Foley,<sup>6</sup> by mixing the  $K=\frac{3}{2}$  band with a  $K=\frac{1}{2}$  band based upon an intrinsic state different from that used in the other calculations,<sup>4,5</sup> were able to predict the energy of the second excited state quite well. It is worth noting that Litherland *et al.*<sup>3</sup> by mixing the  $K=\frac{3}{2}$  and  $K=\frac{5}{2}$  bands were able to predict the energies of the first three excited states in  $\text{Na}^{23}$  almost exactly. Chi and Davidson<sup>7</sup> analyzed the low-lying levels of several odd- $A$  nuclei in the mass region from  $A=17$  to  $A=35$  in terms of an asymmetric-core rotator model. The energies of the first two excited states of  $\text{Na}^{23}$  are predicted fairly well by this model, and the deformation parameter was found to be 0.085.

The diffraction model of Blair<sup>22</sup> for alpha particles inelastically scattered from a strongly absorbing deformed nucleus yields an expression for the cross section that can be written

$$\sigma(\theta) = (I, 2, K, 0 | I', K)^2 \times [(kR^2)^2 / 16\pi] \beta_2^2 [J_0^2(x) + 3J_2^2(x)], \quad (12)$$

where  $\beta_2$  is the nuclear deformation parameter,  $I$  and  $I'$  are the ground and final-state spins, respectively,  $K$  is the quantum number associated with the projection of the total angular momentum onto the nuclear symmetry axis,  $(I, 2, K, 0 | I', K)$  is a Clebsch-Gordan coefficient, and  $h$ ,  $R$ , and  $x$  were defined in connection with Eq. (1). The interaction radius is determined

<sup>18</sup> P. L. Sagalyn, Phys. Rev. **94**, 885 (1954).

<sup>19</sup> M. L. Perl, I. I. Rabi, and B. Senitzky, Phys. Rev. **98**, 611 (1955).

<sup>20</sup> S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **29**, No. 16 (1955).

<sup>21</sup> A. K. Kerman, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **30**, No. 15 (1956).

<sup>22</sup> J. S. Blair, Phys. Rev. **115**, 928 (1959).

from the elastic-scattering analysis and  $|\beta_2|$  is determined by normalizing Eq. (12) to the experimental inelastic angular distribution at small angles.

In Table II are presented the values of  $|\beta_2|$  obtained by assuming  $K=\frac{3}{2}$  and normalizing Eq. (12) to various experimental maxima in the  $(\alpha, \alpha_1)$  and  $(\alpha, \alpha_2)$  angular distributions. There is reasonable consistency among the values obtained by normalizing to the second and third maxima, but normalization to the first maximum in the  $(\alpha, \alpha_2)$  angular distribution yields a somewhat larger value. All the values shown in Table II are considerably smaller than the value of 0.4 obtained by Paul and Montague,<sup>4</sup> and larger than the value of 0.085 obtained by Chi and Davidson.<sup>7</sup> Since Eq. (12) is derived using the Fraunhofer approximation which is most valid at small angles, the value of 0.23 obtained by normalizing to the  $(\alpha, \alpha_2)$  angular distribution at the first maximum should probably be weighted most heavily. Also, as indicated in the previous discussion, it is generally believed that the first and second excited states of  $\text{Na}^{23}$  are mixed configurations; consequently, transitions to these states from the ground state are not purely collective. If this should be the case the use of Eq. (12) which permits only intraband transitions might tend to underestimate the distortion.

The dashed curve in Figs. 3 and 4 are calculated from Eq. (12) using  $|\beta_2|=0.17$ , an average of the  $|\beta_2|$  values extracted from the  $(\alpha, \alpha_1)$  and  $(\alpha, \alpha_2)$  experimental cross sections at the second and third maxima. Since the first maximum is missing in the  $(\alpha, \alpha_1)$  data, it was thought that a more meaningful comparison could be made between the curves in Fig. 3 and Fig. 4 if the  $|\beta_2|$  value of 0.23 were not included in the average. Since the experimental angular distributions corresponding to the  $(\alpha, \alpha_1)$  and  $(\alpha, \alpha_2)$  reactions are both fit reasonably well with the same  $|\beta_2|$  values, then assuming both states are within the  $K=\frac{3}{2}$  rotational band, Eq. (12) predicts that  $\sigma_{2.080}/\sigma_{0.439}=5/9$ . Figures 3 and 4 show that this condition is experimentally well satisfied at forward angles.

The existence of a state at 2.39 MeV is well established<sup>1</sup> but its spin has not been determined experimentally. The third excited state in the level scheme calculated by Litherland *et al.*<sup>3</sup> has spin  $\frac{5}{2}$ ; while Braben *et al.*<sup>5</sup> predict the spin of this state to be  $\frac{9}{2}$ , although an

TABLE II. Values of the deformation parameter  $|\beta_2|$  obtained by normalizing Eq. (12) to the experimental inelastic differential cross sections for  $\text{Na}^{23}$ .

Group	$ \beta_2 $ from first maximum	$ \beta_2 $ from second maximum	$ \beta_2 $ from third maximum	Relative intensities at $\theta_{\text{lab}}=37.5^\circ$
$\alpha_1$	...	0.19	0.14	1.00
$\alpha_2$	0.23	0.18	0.16	0.51
$\alpha_3$	...	...	...	0.06

argument is presented that it might possibly be  $\frac{5}{2}$ . Clegg and Foley,<sup>6</sup> on the other hand, reported that their level scheme predicts no state at 2.39 MeV.

In the present investigation an alpha group corresponding to this 2.39-MeV state was observed in a few alpha-particle spectra accumulated at forward angles. It was very weakly excited, about one order of magnitude less intense than the  $\alpha_2$  group, as seen from the relative intensities listed in Table II. This might be interpreted to indicate that scattering to this state occurs largely through a single-particle excitation from the ground state, consistent with the current belief<sup>23</sup> that collective excitations are greatly favored over single-particle ones in inelastic-scattering processes. This selection rule has also been reported to operate quite well in the case of inelastic deuteron scattering.<sup>24</sup> On the basis of the present experimental results, the above discussion would suggest that the wave function for the 2.39-MeV state contains little if any admixture of the  $K=\frac{3}{2}$  ground-state rotational band. Certainly more experimental information is needed before more definite conclusions can be drawn.

#### ACKNOWLEDGMENTS

The authors would like to thank Professor D. J. Tendam, Professor E. Bleuler, and the other members of the cyclotron group for their advice and assistance in the operation and maintenance of the cyclotron over the extended periods of time required for this investigation.

<sup>23</sup> B. L. Cohen, Phys. Rev. **116**, 426 (1959).

<sup>24</sup> A. G. Blair and E. W. Hamburger, Phys. Rev. **122**, 566 (1961).