Weak Interactions and Self-Consistent Theories

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It is pointed out that the so-called self-consistent theories encounter obvious contradictions with experimental facts if the weak interactions are introduced into them in a simple manner. By the self-consistent theories we mean the self-consistent quantum electrodynamics with vanishing bare masses, the spontaneous breakdown of strong-interaction symmetries, and the like. It follows at the same time that attempts to realize the μ-e splitting self-consistently are wrong even if the bare mass is finite. Possible modifications are also discussed which make the weak interactions compatible with the self-consistent theories.

1. INTRODUCTION

FOR a given Lagrangian we cannot, in general, construct a unique physical world. In some cases, there is more than one world described by the same Lagrangian. The situation was exemplified by Nambu in his superconductor model of elementary particles,[1] in which there were two solutions, the massless and the massive worlds, to the γ_5-invariant Lagrangian. From a similar standpoint, Johnson, Baker, and Willey[2] reexamined quantum electrodynamics to show that one can obtain a "self-consistent theory" involving only convergent quantities aside from the photon mass. Since they started from the electron with vanishing bare mass, the theory contains no physical quantity having the dimension of length. The physical mass of the electron is, therefore, not determined up to scale transformation.

Haag and Maris[3] developed dilatationally invariant quantum electrodynamics to investigate the μ-e puzzle.

Both theories look closed within the framework of quantum electrodymanics. It appears that attention need not be paid to the strong or the weak interactions.

In the present paper, we shall investigate the weak interactions against the background of such a quantum electrodynamics. It will be shown that simple embedding of the weak interactions directly leads to contradictions with the observations about the $\mu\rightarrow e+\bar{\nu}_e+\nu_\mu$ decay. It implies that "self-consistent" quantum electrodynamics or a μ-e problem should be properly solved after inclusion of the weak interactions. We shall first discuss the self-consistent quantum electrodynamics by Johnson *et al.* and the dilatationally invariant treatment of the μ-e problem by Haag and Maris, and then develop similar discussions of the spontaneous breakdown of higher symmetries in the strong interactions.

2. SELF-CONSISTENT QUANTUM ELECTRODYNAMICS

Johnson *et al.* set the total Lagrangian as

$$L=\bar{\psi}\gamma_\lambda\partial_\lambda\psi+ie_0\bar{\psi}\gamma_\lambda\psi A_\lambda\,, \tag{2.1}$$

where the field operators are all unrenormalized and e_0 is the bare electric charge. We have a trivial solution $\sum(\gamma p)\equiv 0$ to the Schwinger-Dyson equation of the proper self-energy part of the electron, $\sum(\gamma p)$. This solution indicates that we should divide the Lagrangian into the "free" and the "interaction" parts as follows:

$$L_0=\bar{\psi}\gamma_\lambda\partial_\lambda\psi\,, \tag{2.2}$$

$$L_{\rm int}=ie_0\bar{\psi}\gamma_\lambda\psi A_\lambda\,. \tag{2.3}$$

This is, however, not a unique solution. They found a self-consistent solution of $\sum(\gamma p)$ which rapidly falls off as $\gamma p\rightarrow\infty$. In this case, the physical mass m of the electron is finite but not fixed since we have no scale of length in this quantum electrodynamics. We should regard here

$$L_0=\bar{\psi}\gamma_\lambda\partial_\lambda\psi+m\bar{\psi}\psi \tag{2.4}$$

as the "free" Lagrangian and

$$L_{\rm int}=ie_0\bar{\psi}\gamma_\lambda\psi A_\lambda-m\bar{\psi}\psi \tag{2.5}$$

as the "interaction" one.

Up to here, the involved fermion is only the electron. In the actual world there exists the muon which has completely the same properties as the electron except for mass. We shall have to treat the muon in the same manner. The Lagrangian should accordingly be modified as follows:

$$L=\bar{e}\gamma_\lambda\partial_\lambda e+\bar{\mu}\gamma_\lambda\partial_\lambda\mu+ie_0[\bar{e}\gamma_\lambda e+\bar{\mu}\gamma_\lambda\mu]A_\lambda\,, \tag{2.6}$$

where the field operators of the electron and the muon are represented by e and μ, respectively.

Suppose we get a self-consistent solution with the muon and the electron having finite physical masses. Their absolute scale cannot be fixed, although the mass ratio may possibly be determined. In this world or in this representation of the operator ring, the picture is drawn by the Lagrangian divided in the following form:

$$L_0=\bar{e}\gamma_\lambda\partial_\lambda e+m_e\bar{e}e+\bar{\mu}\gamma_\lambda\partial_\lambda\mu+m_\mu\bar{\mu}\mu\,, \tag{2.7}$$

and

$$L_{\rm int}=ie_0(\bar{e}\gamma_\lambda e+\bar{\mu}\gamma_\lambda\mu)A_\lambda-m_e\bar{e}e-m_\mu\bar{\mu}\mu\,. \tag{2.8}$$

The physical masses m_e and m_μ are so determined that the one-particle expectation values of $L_{\rm int}$ may vanish:

$$\langle e|L_{\rm int}|e\rangle=\langle\mu|L_{\rm int}|\mu\rangle=0\,. \tag{2.}$$

[1] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); **124**, 246 (1961).

[2] K. Johnson, M. Baker, and R. Willey, Phys. Rev. Letters **11**, 518 (1963); Th. A. J. Maris, V. E. Herscovitz, and G. Jacob, Phys. Rev. Letters **12**, 313 (1964).

[3] R. Haag and Th. A. J. Maris, Phys. Rev. **132**, 2325 (1963); Th. A. J. Maris, Nuovo Cimento **30**, 378 (1963).

Except for the claims[4–6] predicting a massless boson, everything seems to be going well as long as the theory is concerned with quantum electrodynamics only.

To apply this theory to the actual world, however, we must incorporate the weak interaction process $\mu \to e+\bar{\nu}_e+\nu_\mu$ into the theory. Let us add the responsible interaction term in the form

$$L_{\rm int}^{(\rm w)}=(g/\sqrt{2})[\bar{e}\gamma_\lambda(1+\gamma_5)\nu_e]\times[\bar{\nu}_\mu\gamma_\lambda(1+\gamma_5)\mu]+\text{H. c.}, \quad (2.10)$$

or in its Fierz-transformed form

$$L_{\rm int}^{(\rm w)}=(g/\sqrt{2})[\bar{\nu}_\mu\gamma_\lambda(1+\gamma_5)\nu_e]\times[\bar{e}\gamma_\lambda(1+\gamma_5)\mu]+\text{H. c.} \quad (2.11)$$

We have to add the weak self-energies to L_0 and subtract them from the right-hand side of Eq. (2.11). However, we omit this manipulation, as well as self-currents, if any, since they are irrelevant to the following discussions. Using the equations of motion from the Lagrangian given here, one can easily see

$$\partial_\lambda[\bar{e}\gamma_\lambda(1+\gamma_5)\mu]=0 \quad (2.12)$$

to the zeroth order of the weak interactions. This is not affected at all by whatever particles, such as the nucleon and the mesons, may take part in the Lagrangian. Equations of field operators must not change from one representation to another of an operator ring. Equation (2.12) holds as it stands in both the massless and the massive worlds. In the following, let us show that this directly leads to a violent contradiction to experiment.

To see it we need not explicitly calculate the decay spectrum of the electron. Instead, it suffices to exploit the following theorem.[7,8]

Theorem: The squared matrix element for the $\mu\to e+\bar{\nu}_e+\nu_\mu$ decay, summed over the neutrino and the antineutrino spins, depends only on

$$\partial_\lambda[\bar{e}\gamma_\lambda(1+\gamma_5)\mu] \quad (2.13)$$

in the configuration in which the electron suffers the maximum recoil.

The proof is straightforward. The matrix element is

$$M=i(g/\sqrt{2})[m_{\nu_e}m_{\nu_\mu}/E_{\nu_e}E_{\nu_\mu}]^{1/2}\times\bar{u}_{\nu_\mu}\gamma_\lambda(1+\gamma_5)v_{\nu_e}\langle e|\delta L/\delta j_\lambda|\mu\rangle, \quad (2.14)$$

where $j_\lambda=i\bar{\nu}_\mu\gamma_\lambda(1+\gamma_5)\nu_e$ and $\delta L/\delta j_\lambda$ are the Heisenberg operators, $|e\rangle$ and $|\mu\rangle$ being the physical one-particle states. We have made here the approximation of neglecting the electromagnetic structure of the neutrinos. According to the existing experiments, it is quite a good approximation. The squared matrix element summed over the neutrino spins is

$$\sum_{\nu\ \text{spins}}|M|^2=g^2\langle e|\delta L/\delta j_\lambda|\mu\rangle T_{\lambda\sigma}\langle\mu|\delta L/\delta j_\sigma|e\rangle, \quad (2.15)$$

$$T_{\lambda\sigma}=(1/q_{10}q_{20})[q_{1\sigma}q_{2\lambda}+q_{2\sigma}q_{1\lambda}-(q_1q_2)\delta_{\lambda\sigma}+\epsilon_{\lambda\sigma\alpha\beta}q_{1\alpha}q_{2\beta}], \quad (2.16)$$

where q_1 and q_2 are the energy momenta of the neutrino and the antineutrino, respectively. Both q_1 and q_2 are null vectors: $q_1^2=q_2^2=0$. Therefore, the invariant momentum transfer squared from the neutrino current to the μ-e current is exactly zero in the maximum-recoil configuration, $\mathbf{q}_1\|\mathbf{q}_2$. It should be, however, noted here that the time component of the momentum transfer is nonvanishing and given by

$$q_0=(m_\mu^2-m_e^2)/2m_\mu, \quad (2.17)$$

in the same configuration in the rest system of μ. We may write

$$q_1=(q/q_0)q_{10}, \quad (2.18)$$

$$q_2=(q/q_0)q_{20}, \quad (2.19)$$

$$q=q_1+q_2. \quad (2.20)$$

Substitution of these expressions into Eq. (2.16) shows the third and the fourth terms to be vanishing, giving

$$T_{\lambda\sigma}=2q_\lambda q_\sigma/q_0^2. \quad (2.21)$$

Since $\langle e|\partial_\lambda(\delta L/\delta j_\lambda)|\mu\rangle=-iq_\lambda\langle e|\delta L/\delta j_\lambda|\mu\rangle$, we find

$$\sum_{\nu\ \text{spins}}|M|^2=(2g^2/q_0^2)|\langle e|\partial_\lambda(\delta L/\delta j_\lambda)|\mu\rangle|^2. \quad (2.22)$$

This is just what we want to show. It is a matter of course that the proof is not altered at all for the $\bar{\mu}\to\bar{e}+\nu_e+\bar{\nu}_\mu$ process.

This theorem predicts together with Eq. (2.12) that the high-energy end of the electron spectrum goes to zero if one embeds the weak interaction responsible for the μ-e decay in the simple manner as in Eq. (2.10) or (2.11). The Michel parameter ρ is zero in this case. Experimentally,[9] $\rho=0.78\pm0.02$ and the conventional V-A theory with four-fermion interactions leads to $\rho=\frac{3}{4}$. At any rate, the prediction $\rho=0$ is in violent disagreement with experiment.

Let us consider alternative modifications of the weak interactions in the self-consistent quantum electrodynamics.

(1) *The muon should not be treated on the same basis in the self-consistent theory.* This may be the simplest way of avoiding the above-mentioned difficulty, but the theory must explain why the self-consistent mechanism does not work for the muon, but only for the electron. We know that the muon has entirely the same electromagnetic properties as the electron apart from the mass.

[4] J. Goldstone, Nuovo Cimento **19**, 154 (1961); J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. **127**, 965 (1962).

[5] S. Bludman and A. Klein, Phys. Rev. **131**, 2364 (1963).

[6] See also M. Baker, K. Johnson, and B. W. Lee, Phys. Rev. **133**, B209 (1964); A. Klein and B. W. Lee, Phys. Rev. Letters **12**, 266 (1964); Y. Nambu, Phys. Letters **9**, 214 (1964).

[7] An equivalent theorem has been proved by S. L. Adler for "inelastic" high-energy neutrino reactions (Ref. 8). Here we follow his method of proof.

[8] S. L. Adler, Phys. Rev. **135**, B963 (1964).

[9] R. Plano and A. Lecourtois, Bull. Am. Phys. Soc. 4, 82 (1959).

Of course this modification is incompatible with the dilatational-invariance approach to the muon-electron problem.

(2) *The weak interactions play a substantial role in the self-consistent theories. The lowest approximation to the weak interactions is not valid.* If this is the case, we do not know how to handle the μ-e problem. We can find no reason to expect that the conventional V-A theory, which is in fine agreement with experiment, may be reproduced.

(3) *An original form of the decay Lagrangian is not of the form V-A. The effective interaction after electromagnetic renormalizations appears to be V-A owing to enhancement effects.* This will, however, destroy the underlying elegance associated with the V-A picture of the μ-e decay.

(4) *The decay Lagrangian should not be set as in Eq. (2.10), but as*

$$L_{\rm int}{}^{(w)}=(g/\sqrt{2})[\bar{\nu}_e\gamma_\lambda(1-\gamma_5)e]\times[\bar{\nu}_\mu\gamma_\lambda(1+\gamma_5)\mu]+\text{H. c.} \quad (2.23)$$

Accordingly, the electromagnetic interactions should be

$$L_{\rm int}{}^{(\rm e.m.)}=ie_0(-\bar{e}\gamma_\lambda e+\bar{\mu}\gamma_\lambda\mu)A_\lambda. \quad (2.24)$$

This modification is, however, in vain since it is reduced to the original formulation by the replacement $e^c\rightarrow e'$ and $\nu_e{}^c\rightarrow\nu_e{}'$, where the superscript c means charge conjugation.

(5) *An intermediate boson mediates between the* $(\bar{\nu}_e e)$ *and the* $(\bar{\nu}_\mu\mu)$ *currents.* The nonlocality thus introduced will violate the conservation of the $(\bar{e}\mu)$ current, since it is from the Fierz transform of the original current $\times$current interactions. Although we cannot carry out any reliable estimate, the violation will be too small to explain the fact that $\rho=\frac{3}{4}$, if we consider the very high lower bound on the mass of the weak boson.

Thus, we cannot find any hopeful modification of the weak interactions in the self-consistent quantum electrodynamics. The final way seems to formulate the self-consistent theory with finite bare masses.

3. APPROACHES TO THE μ-e PROBLEM WITH $m_e{}^0=m_\mu{}^0$

We shall extend our arguments to the approaches to the μ-e puzzle, which attempt to explain the observed μ-e mass difference within the framework of the quantum electrodynamics. We showed that the approach assuming the bare masses to be zero cannot satisfactorily accommodate the μ-e decay. Let us show that the circumstances are not changed even if the bare mass, $m_\mu{}^0=m_e{}^0$, is assumed to be finite.

The Lagrangian may be written as

$$L=\bar{e}\gamma_\lambda\partial_\lambda e+m^0\bar{e}e+\bar{\mu}\gamma_\lambda\partial_\lambda\mu+m^0\bar{\mu}\mu+\bar{\nu}_e\gamma_\lambda\partial_\lambda\nu_e+\bar{\nu}_\mu\gamma_\lambda\partial_\lambda\nu_\mu+ie_0(\bar{e}\gamma_\lambda e+\bar{\mu}\gamma_\lambda\mu)A_\lambda+(g/\sqrt{2})[(\bar{\nu}_\mu\gamma_\lambda(1+\gamma_5)\nu_e)(\bar{e}\gamma_\lambda(1+\gamma_5)\mu)+\text{H. c.}]. \quad (3.1)$$

Using the equations of motion one can easily see

$$\partial_\lambda(\bar{e}\gamma_\lambda\mu)=0, \quad (3.2)$$

while

$$\partial_\lambda(\bar{e}\gamma_\lambda\gamma_5\mu)\neq 0. \quad (3.3)$$

Let us again concentrate on the maximum-recoil configuration. The theorem in the preceding section predicts that only the divergence of the axial-vector part $\partial_\lambda(\bar{e}\gamma_\lambda\gamma_5\mu)$ contributes to the μ-e decay in this configuration. This is again in obvious disagreement with experiment. In fact, were it the case, the maximally recoiled electron would have no longitudinal polarization.

Thus, the spontaneous μ-e splitting is not solved, at least within quantum electrodynamics. It does not apply to the real world. We shall not repeat possible exceptions to this conclusion or possible modifications of the way to embed the weak interactions, since they are not substantially different from those given at the end of the preceding section.

Hitherto we have assumed the minimal interactions of electromagnetism. It is, however, evident that all the conclusions are unaffected by introduction of tensor interactions, so long as we introduce them in the form

$$L_{\rm int}=(\kappa_e\bar{e}\sigma_{\lambda\rho}e+\kappa_\mu\bar{\mu}\sigma_{\lambda\rho}\mu)F_{\lambda\rho}, \quad (3.4)$$

with

$$\kappa_e=\kappa_\mu. \quad (3.5)$$

4. SPONTANEOUS BREAKDOWN OF STRONG-INTERACTION SYMMETRIES

Let us further extend our investigations to the spontaneous breakdown of strong-interaction symmetries.[10–12] We shall illustrate by the eightfold way of SU_3.[13] Suppose we set the strong-interaction Lagrangian fully invariant under SU_3. In addition, the weak interaction Lagrangian is assumed to be of the current$\times$current type. Let the vector parts of the hadronic currents be components of the unitary spin current according to recent hypotheses.[14] The conventional manner of spontaneous breakdown requires that some asymmetric solutions to the self-consistency relations exist as well as the symmetric one. Self-consistency is usually required within the strong interactions. The weak interactions are considered to play no role in the

[10] M. Baker and S. L. Glashow, Phys. Rev. **128**, 2462 (1962); S. L. Glashow, *ibid.* **130**, 2132 (1962).

[11] M. Suzuki, Progr. Theoret. Phys. (Kyoto) **30**, 627 (1963); **31**, 222 (1964); **31**, 1073 (1964); K. Kikkawa, *ibid.* **31**, 858 (1964).

[12] R. E. Cutkosky and P. Tarjanne, Phys. Rev. **132**, 1888 (1963); **133**, B1292 (1964). These do not assume any Lagrangian or Hamiltonian.

[13] For SU_3 symmetry refer to M. Gell-Mann, Phys. Rev. **125**, 1067 (1962); Y. Ne'eman, Nucl. Phys. **26**, 222 (1961).

[14] B. d'Espagnat and J. Prentki, Nuovo Cimento **24**, 497 (1962); N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963); M. Gell-Mann, *ibid.* **12**, 155 (1964). The term "hadron," which means a strongly interacting particle, was invented by L. B. Okun in *Proceedings of 1962 International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 845.

breakdown, although the electromagmetic interactions may be possibly relevant to choice of the direction of the spontaneous breakdown.

So long as one assumes the Lagrangian or the Hamiltonian formalism, the vector parts of the weak hadronic currents must be conserved as operator equations to the zeroth order of the weak and the electromagnetic interactions. Whatever the physical mass spectra and the associated coupling constants may be, the asymmetric worlds thus derived are the representations of the single Lagrangian. The Lagrangian shall be divided here as

$$L=L_0+L_{\mathrm{int}}, \tag{4.1}$$

$$L_0=L_0^{(\mathrm{sym})}+L^{(\mathrm{asym})}, \tag{4.2}$$

$$L_{\mathrm{int}}=L_{\mathrm{int}}^{(\mathrm{sym})}-L^{(\mathrm{asym})}+L_{\mathrm{int}}^{(\mathrm{e.m.})}+L_{\mathrm{int}}^{(\mathrm{w})}, \tag{4.3}$$

$$L_{\mathrm{int}}^{(\mathrm{w})}=J_\lambda^+\times J_\lambda, \tag{4.4}$$

$$J_\lambda=J_\lambda+S_\lambda+j_\lambda, \tag{4.5}$$

where $L_0^{(\mathrm{sym})}$ is the free part in which the "mass" terms of the particles belonging to the same multiplets are equal, $L^{(\mathrm{asym})}$ contains the deviations of the masses from the unitary-symmetric values, and $L_{\mathrm{int}}^{(\mathrm{e.m.})}$ and $L_{\mathrm{int}}^{(\mathrm{w})}$ are the electromagnetic and the weak interactions, respectively. We have denoted by J_λ, S_λ, and j_λ the strangeness-conserving, the strangeness-changing, and the leptonic currents, respectively. Since the breakdown of SU_3 occurs in the direction of the eighth axis (λ_8), the vector part of J_λ is evidently conserved to the lowest order of the electromagnetic and the weak interactions if we take it for granted that the vector parts of the hadronic currents are components of the unitary spin current. The conservation of the vector part of S_λ seems to be violated. Nevertheless, it is strictly conserved as an operator equation provided that the breakdown is spontaneous in origin and caused only through the strong interactions.

It seems that this directly leads to contradictions with the observations about the K_{e3} decays.[15] According to analyses by many authors, the strict conservation of the strangeness-changing vector current gives a too small value for the branching ratio $\Gamma(K_{\mu 3})/\Gamma(K_{e3})$ and does not correctly reproduce the shape of the pion spectrum. The present experimental data appear to reject the strict conservation. However, there are some confusions in the existing experimental data on the K_{e3} decays. We must wait a little while to judge it finally.

We can derive some consequences of the strict conservation in the $Y\to N+e^-+\bar{\nu}_e$ decay. If the breakdown is spontaneous in origin, and if the weak interactions take no part in it, we see from the theorem in Sec. 2 that the angular correlation between the momentum of the maximally recoiled nucleon and the polarization of Y is of the form

$$N(\theta)=\text{constant} \tag{4.6}$$

in the rest system of Y in the approximation of $m_e=0$.

If we make routine calculations using the conventional V-A theory, the angular correlation is given by

$$N(\theta)\propto g_V^2+2g_Vg_A\langle P_Y\rangle\cos\theta+g_A^2, \tag{4.7}$$

where $\langle P_Y\rangle$ is the average polarization of Y. If one uses the values for g_V and g_A estimated by the Cabibbo's theory,[16]

$$N(\theta)\propto 1-0.90\langle P_\Lambda\rangle\cos\theta \tag{4.8}$$

for $\Lambda\to p+e^-+\bar{\nu}_e$, and

$$N(\theta)\propto 1+0.92\langle P_\Sigma\rangle\cos\theta \tag{4.9}$$

for $\Sigma^-\to n+e^-+\bar{\nu}_e$.

The hyperon produced in the two-body boson-nucleon reactions is polarized perpendicularly to the production plane. Therefore, the test proposed here will be feasible if a sufficient number of events is accumulated.

Let us enumerate the possible modifications of the spontaneous breakdown of SU_3 symmetry.

(1) *The strangeness-changing weak vector current has nothing to do with the unitary spin current before renormalization. It transforms under SU_3 approximately like a component of the unitary spin current, owing to the strong-interaction renormalizations.*

(2) *The strong interactions cannot be properly described by a Lagrangian or a Hamiltonian formalism.* Then we have no means of showing explicitly the contradictions with the observations on the weak interactions as well as the unpleasant feature associated with the massless scalar boson. If such a theory is shown to be equivalent to the local field theory based on the Lagrangian formalism, this modification will be in vain.

5. SUMMARY

We have shown how difficult it is to include naturally the weak interactions in the various kinds of self-consistent theories. They may be self-consistent within a certain class of interactions. In order that they may apply to the actual world, however, they must accommodate the weak interactions without contradiction to the observations. As for quantum electrodynamics with vanishing bare masses, we have not succeeded in finding any natural way of introducing the decay interactions for the μ-e decay. It was shown that the decay interactions simply introduced lead to a vanishing Michel parameter, $\rho=0$. If a charged intermediate

[15] J. L. Brown, J. Kadyk, G. Trilling, R. Van de Walle, B. Roe, and D. Sinclair, Phys. Rev. Letters **7**, 423 (1961); J. M. Dobbs, K. Landa, A. Mann, K. Reibel, F. Sciulli, H. Uto, D. White, and K. Young, *ibid.* **8**, 295 (1962); J. L. Brown, J. Kadyk, G. Trilling, R. Van de Walle, B. Roe, and D. Sinclair, *ibid.* **8**, 450 (1962); A. M. Boyarski, E. Loh, L. Niemela, D. Ritson, R. Weinstein, and S. Ozaki, Phys. Rev. **128**, 2398 (1962): See also J. L. Brown, J. A. Kadyk, G. H. Trilling, R. T. Van de Walle, B. P. Roe, and D. Sinclair, in *Proceedings of 1962 International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 462.

[16] N. Cabibbo, Phys. Rev. Letters **10**, 531 (1963).

boson mediates the μ-e decay, nonlocality is introduced, which gives a finite value for ρ. However, it is known from the recent high-energy neutrino experiment that the mass of the weak boson, if any, is not smaller than 1.3 BeV. The lower limit seems to tend to increase incessantly. After all, the weak boson will not be able to produce such large values of ρ as observed, although one cannot estimate it in any reliable way.

We have also shown that the μ-e puzzle cannot be solved, at least within the framework of quantum electrodynamics. If it were formally solved, we should encounter evident contradiction with the experiments on the longitudinal polarization of the electron in the μ-e decay. If one constructs a self-consistent theory with $m_e^0 \neq m_\mu^0$, the weak interactions are accommodated without any contradiction.

Outside of quantum electrodynamics, we have two alternative ways of avoiding similar difficulties. One of them is to formulate without a Lagrangian or Hamiltonian a self-consistent deviation theory which cannot be described by the Lagrangian theory in an equivalent way.

The origin of the contradictions pointed out here lies in the strict conservation of the weak currents. If one assumes the weak vertices to be nonvanishing on the light cone, one is led, as is well known, to the massless scalar bosons. The massless bosons are eliminated if the weak vertices are zero on the light cone. However, the present arguments lead to the contradictions independently of the behavior of the weak vertices near the light cone.

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Self-Consistent Calculation of the Scattering Amplitude and the Diffraction Peak

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We investigate the $\pi\pi$ scattering at low-momentum transfers, in order to understand diffraction scattering. A self-consistent calculation of the position $\alpha(s)$ and reduced residue $\gamma(s)$ of the Pomeranchuk-Regge trajectory is carried out using the Balázs method. The result of the calculation under certain simplifying approximations is that the s dependence of $\gamma(s)$ is responsible for the sharp forward peaking in the high-energy scattering and can roughly reproduce the experimental width of the diffraction peak derived from the factorization theorem.

I. INTRODUCTION

RECENT experiments at 5–20 BeV have shown a substantial shrinkage with increasing energy of the forward peak width of p-p and K^+-p elastic scattering, whereas only a slight shrinkage was observed for π-p and K^--p scattering.[1]

It was pointed out[2] that the three Regge pole approximation[3] may still explain the above features of the high-energy scattering if the following assumptions are made:

(i) The slope of the Pomeranchuk trajectory is assumed small in order to understand the absence of strong shrinkage in π-p scattering.

(ii) The s dependence of the residue function is important for the sharp forward peaking in high-energy scattering.

It is very interesting, therefore, to investigate whether one can get (i) and (ii) theoretically, starting from the Mandelstam representation and using unitarity and crossing symmetry. It is the purpose of this paper to investigate the asymptotic behavior of the model of pion-pion scattering to clarify the diffraction mechanism at high energy and low momentum transfer.

Attention is focused on small-momentum-transfer behavior of the position $\alpha(s)$ and residue $\gamma(s)$ of the top-level Pomeranchuk trajectory. This trajectory controls the high-energy scattering at low momentum transfers. There have been several "bootstrap" methods proposed for calculating the π-π amplitude from the requirement of analyticity, unitarity, and crossing symmetry.[4–10] We

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[3] F. Hadjioannou, R. J. N. Phillipps, and W. Rarita, Phys. Rev. Letters **9**, 183 (1962).

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