Nuclear Structure Studies in the Cadmium Isotopes with (d,p) and (d,t) Reactions*

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Many new energy levels up to ~2-MeV excitation are located in the odd-A cadmium isotopes. Spins, parities, as well as spectroscopic factors, are determined with the aid of distorted-wave Born approximation (DWBA) calculations. The distinction between the $d_{5/2}$ and $d_{3/2}$ levels was made according to the pickup-to-stripping cross-section ratios. The systematic behavior of the first excited state for the various spin values enables us to conclude that the first excited states in Cd^{115} and Cd^{117} are $d_{3/2}$ states and the 300-keV state in Cd^{113} is an unresolved $d_{3/2}$-$d_{5/2}$ doublet. The (d,p) spectra for reactions leading to odd cadmium isotopes which differ from each other by as much as four neutron pairs, and to Cd-Pd isotones which differ by one proton pair, are compared. The changes in the observed spectra, as well as in the location of the neutron single-particle states, are rather small in both cases. The various approximations for the evaluation of the pairing-theory parameters, such as the occupation numbers and the single-quasiparticle energies, from the experimental data are discussed. The need for good DWBA calculations for the (d,t) reactions in order to avoid the necessity of renormalizations, and for cross checking of the results, is stressed. The effect of the quadrupole interaction on the single quasiparticles is clearly seen in cadmium from the fact that the $s_{1/2}$ state becomes the ground state in Cd^{111} even though its occupation number is as low as $V_{1/2}^2=0.1$.

INTRODUCTION

THE structure of the low-lying energy levels in odd-A nuclei which have an even number of protons and whose neutrons are in the $50\leqslant N\leqslant 82$ shell are intensively investigated in this laboratory. According to the pairing theory,[1] the location of the neutron single-particle states in this shell, namely the $d_{5/2}$, $g_{7/2}$, $s_{1/2}$, $h_{11/2}$, and $d_{3/2}$ states, can be determined if the energies, the spin and parity assignments, and the spectroscopic factors for all low-lying levels are known. Stripping and pickup reactions are highly suitable for this purpose, as they preferentially excite single-particle and single-hole states. The angular distribution of the reaction products provides the assignments for these levels, and the absolute cross sections determine the spectroscopic factors. In previous papers, the location of the neutron single-particle states were determined for zirconium,[2] molybdenum,[3] palladium,[4] tin,[5] barium, and cerium.[6] It is surprising that despite the large dissimilarity of the observed spectra obtained for these isotopes, the relative positions of the single-particle states are almost unaffected. One exception is the $g_{7/2}$ state. In zirconium it was located at 2.7 MeV above the $d_{5/2}$ state, in tin it was very close to it, and in the palladium isotopes it was found to be almost 1 MeV lower. These large changes in energy were explained by Talmi[7] and by Cohen[8] as being caused by a long-range force that acts between neutrons and protons populating states with the same orbital angular momentum, even though they are in different shells.

The properties of the low-lying levels of the odd-A cadmium isotopes are known mainly from studies of β decay[9] and Coulomb excitation,[10] and even these methods were applied mostly to the lighter isotopes. It seems therefore, that a more complete investigation of the nuclear structure, such as can be obtained from nuclear reaction studies, is very desirable. Making use of a 15-MeV deuteron beam for which the nuclear stripping reactions are expected to go through the direct reaction channel, highly enriched targets, and a detecting system with good resolution, the investigation of the structure of the cadmium isotopes was undertaken.

EXPERIMENTAL

Four even-even Cd isotopes, Cd^{110}, Cd^{112}, Cd^{114}, and Cd^{116}, with thickness ~2 mg/cm² and isotopic purity of about 95%, were prepared as thin targets for high-resolution stripping reactions. Three of them were made by a rolling method and the Cd^{114} target was prepared by vacuum evaporation of a highly enriched isotope on a 0.2-mg/cm² gold backing. The thickness and uniformity of the targets were measured by recording the change in energy in passing through them of a well-collimated alpha-particle beam from a Po source.

The targets were bombarded with 15-MeV deuterons from the University of Pittsburgh cyclotron, and the reaction products were magnetically analyzed with a 60° wedge magnet spectrograph that could be rotated between 0° and 90° with respect to the beam. The detection system consists of photographic plates on which the nuclear tracks are recorded. By this method an

* Work performed at Sarah Mellon Scaife Radiation Laboratory and supported by the National Science Foundation.
[1] S. Yoshida, Nucl. Phys. **38**, 380 (1962).
[2] B. L. Cohen, Phys. Rev. **125**, 1358 (1962); B. L. Cohen and O. V. Chubinsky, *ibid.* **131**, 2184 (1963).
[3] S. A. Hjorth and B. L. Cohen, Phys. Rev. **135**, B920 (1964).
[4] B. Cujec, Phys. Rev. **131**, 735 (1963).
[5] B. L. Cohen and R. E. Price, Phys. Rev. **121**, 176 (1961).
[6] R. H. Fulmer, A. L. McCarthy, and B. L. Cohen, Phys. Rev. **128**, 1302 (1962).
[7] I. Talmi, Rev. Mod. Phys. **34**, 704 (1962).
[8] B. L. Cohen, Phys. Rev. **127**, 597 (1962).
[9] M. Nozawa, Nucl. Phys. **36**, 411 (1962); J. M. Hamilton *et al.*, Arkiv Fysik **18**, 273 (1960); A. Kjelberg *et al.*, Can. J. Phys. **38**, 866 (1960).
[10] P. M. Stelson and F. K. McGowan, Phys. Rev. **109**, 901 (1958).

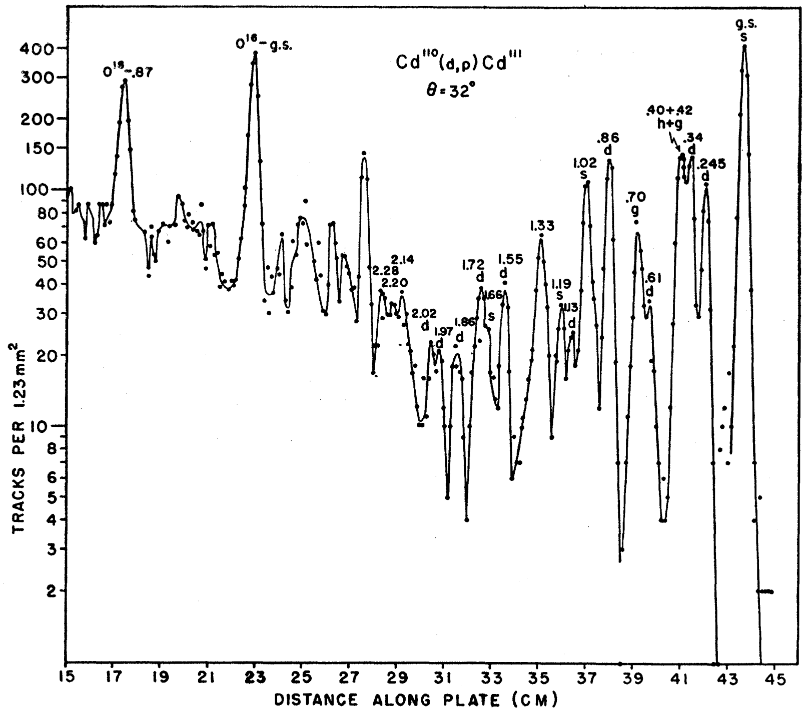

FIG. 1. Energy spectrum of protons from $Cd^{110}(d,p)Cd^{111}$ reaction. The excitation energies of the final nucleus and the angular momentum transfer are also indicated.

energy range of several MeV for protons from typical (d,p) reactions, and tritons with energies greater than 10 MeV from typical (d,t) reactions could be detected and analyzed. The useful energy range for the protons was always sufficient, but the low-energy limitation for the tritons was a serious drawback for reactions having large negative Q values.

The proton spectra were recorded at 8 angles: 9°, 14°, 19°, 25°, 32°, 40°, 50°, and 70°. One typical spectrum, is shown in Fig. 1, and in Fig. 2 the experimental angular distributions of the protons from $Cd^{112}(d,p)Cd^{113}$ are given. The first five angles were taken in order to distinguish clearly among transitions having angular momentum transfer $l_n=0$, 1, 2, 3, and the larger angles to distinguish between $l_n=4$, 5 and to be able to analyze mixtures when levels lie close together. The resolution was ~30 keV and the energies of the peaks were determined to an accuracy of 10 keV.

The triton spectra were recorded at 25° and 45° only, with the exception of $Cd^{110}(d,t)Cd^{109}$ in which a more detailed angular distribution was taken. Some triton spectra are given in Fig. 3.

The errors in the absolute cross section arise from several sources: (1) errors in the geometry; beam direction, and target orientation, (2) change in the beam energy for long runs, (3) errors in the target thickness, and impurities, (4) current integration, (5) plate reading, and (6) statistical errors. The first four error sources did not contribute much in our case as the targets were pure and homogeneous, and all the spectra were recorded with a good, stable beam in two independent runs. The plate-reading errors are estimated to be of the order of 10% and statistical errors are important only for the smallest peaks.

The energy levels of all the isotopes reached by stripping reactions were investigated from the proton spectra up to the energy where carbon and oxygen contamination start to interfere. Although these impurities did not obscure the possibility of determining the energies, they made it very difficult to get a good angular distribution. This put in doubt the value of the angular momentum transfer at high excitation, which becomes more ambiguous any way, especially when levels from the next shell start to come in. The useful range in the triton spectra was quite sufficient for analysis in the pickup reactions leading to Cd^{115}, Cd^{113}, and Cd^{111}. But this useful energy range becomes smaller as the Q values for the reaction become more negative for the lighter isotopes. For $Cd^{110}(d,t)Cd^{109}$ an energy range of less than 1 MeV remains available before the elastic deuterons cover the plates.

RESULTS AND DISCUSSION

Table I represents the energies of the excited states of the five even-odd Cd isotopes up to 2-MeV excitation. Column 2 gives the differential cross section for the stripping reaction calculated at the first maximum after 9° of the angular distribution. The cross sections for pickup reactions at 45° are listed in column 3, and the neutron angular momentum transfers are given in column 4. Since all target nuclei have $J=0^+$ in their g.s. (ground state), this l_n value determines also the parity

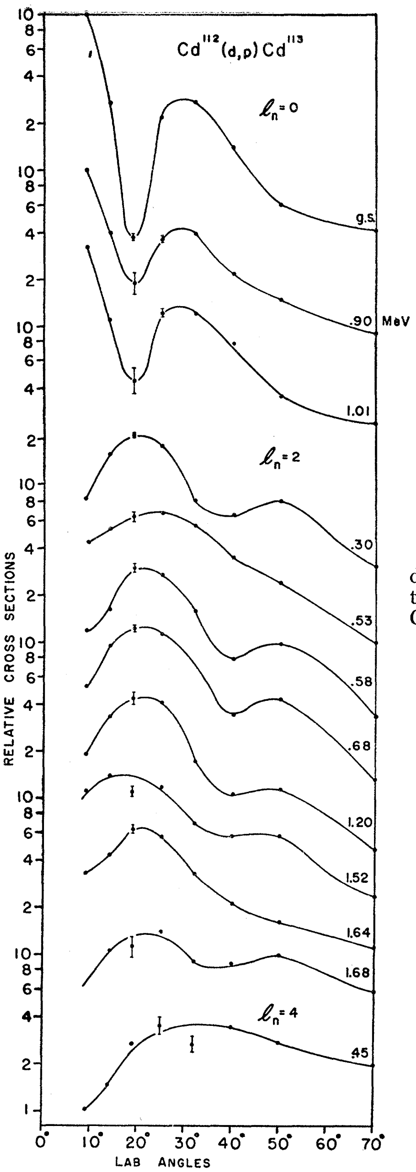

FIG. 2. Angular distribution of protons from $Cd^{112}(d,p)$ Cd^{113} reaction.

of the final states. The spins of these states are readily determined according to the general shell-model theory; $l_n=0$, $l_n=4$ and $l_n=5$ transitions lead to $s_{1/2}$, $g_{7/2}$, and $h_{11/2}$ states, respectively. For $l_n=2$, the distinction between $d_{5/2}$ and $d_{3/2}$ states was made according to the following considerations. The spin-orbit splitting separates the $d_{5/2}$ and the $d_{3/2}$ levels by approximately[11] 2.5 MeV in this region. In the cadmium isotopes the $d_{5/2}$ level, which is the lower level, is almost full whereas the upper $d_{3/2}$ level is almost empty. This difference in population shows up as low cross sections for pickup reactions from the relatively empty $d_{3/2}$ levels in comparison to the larger cross sections leading to the $d_{5/2}$ ones. The situation for the (d,p) reaction is just reversed. Consequently, by forming the ratio $\sigma(d,t)/\sigma(d,p)$ for reactions leading to the same level from two neighboring even-A isotopes, it is possible in most cases to distinguish between the $d_{5/2}$ and the $d_{3/2}$ levels; see Table II and Fig. 4. For levels in Cd^{117} which could be reached only by (d,p) reactions, the assignments for the d states were taken whenever possible from general tendencies in the neighboring isotopes. In Cd^{109} all spin assignments were already known[12] and we give only the cross sections for the relevant (d,t) reactions.

We now discuss some specific states:

The 300-keV State in Cd^{113}

The spin assignment to this state is $\frac{3}{2}^+$ according to the *Nuclear Data Sheets*,[12] based on the angular distribution and polarization of γ rays from Coulomb excitation.[10] A probable $\frac{5}{2}^+$ assignment to this level was earlier suggested by Temmer and Heydenburg.[13] Our conclusion is that an unresolved $\frac{3}{2}^+$, $\frac{5}{2}^+$ doublet state lies at this energy, based on the following observations: In Fig. 5 the very interesting characteristics of the first excited state for different j values are immediately seen. The $h_{11/2}$ and $d_{3/2}$ states decrease in energy with the increase in the number of neutrons, whereas the $g_{7/2}$ and $d_{5/2}$ states are increasing. In particular the first $d_{5/2}$ state in Cd^{111} lies below the first $d_{3/2}$ state, whereas in Cd^{115} their order is inverted. In Cd^{113} only one d state is seen and we suspect that this is therefore an unresolved doublet. Cross-section considerations also support the existence of the doublet at 300 keV in Cd^{113}, as the cross section for the stripping reaction leading to this peak is as large as the sum of the cross sections leading to both the $d_{5/2}$ and $d_{3/2}$ states in the neighboring isotopes. This can be seen in Fig. 6. Another support comes from the pickup-to-stripping cross-section ratios discussed above, which excludes the possibility of having a single $d_{3/2}$ or $d_{5/2}$ state at 300 keV. The last support for the existence of a doublet is the result of a special "best resolution" run as shown in Fig. 7. The first excited state in Cd^{113} is broad compared to the g.s. transition, and the transition to the $h_{11/2}$ state at 265 keV cannot by itself account for this broadening.

The 135-keV State in Cd^{117}

The angular distribution of the protons leading to this level indicates clearly an $l_n=2$ transition. The ratio of the pickup to stripping cross section (in order to distinguish between $j=\frac{5}{2}$ and $j=\frac{3}{2}$ assignments) cannot be formed in this case. Nevertheless, from the systematic tendencies shown in Fig. 5 and the large cross section for the (d,p) reaction, we suggest a $\frac{3}{2}^+$ assignment for it.

[11] B. L. Cohen, Phys. Rev. **130**, 227 (1963).

[12] *Nuclear Data Sheets*, compiled by K. Way *et al.* (Printing and Publishing Office, National Academy at Sciences—National Research Council, Washington, D. C., 1960), NRC 60-3-118.

[13] G. M. Temmer and N. P. Heydenburg, Phys. Rev. **98**, 1308 (1955).

TABLE I. Energy levels and spin assignments for five odd-even Cd isotopes, and cross sections for (d,p) and (d,t) reactions leading to them.

Present work					N.D.S.[a]	
(1) Excitation energy (MeV)	(2) $\sigma(d,p)$[b] (mb/sr)	(3) $\sigma(d,t)$[c] (mb/sr)	(4) Δl	(5) J^π	(6) Excitation energy (MeV)	(7) J^π
			Cd^{109}			
g.s.		0.98			g.s.	$\frac{5}{2}^+$
0.06		0.22			0.058	$\frac{1}{2}^+$
0.20		0.16			0.205	$\frac{7}{2}^+$
0.30		0.10			0.290	$\frac{3}{2}^+$
0.41		0.12				
0.46		0.05				
0.54		<0.05				
0.66		0.07				
0.81		0.06				
			Cd^{111}			
g.s.	2.45	0.81	0	$\frac{1}{2}^+$	g.s.	$\frac{1}{2}^+$
0.245	1.52	1.21	2	$\frac{5}{2}^+$	0.247	$\frac{5}{2}^+$
0.34	2.01	0.21	2	$\frac{3}{2}^+$	0.342	$\frac{3}{2}^+$
0.40	0.72	0.15	5	$11/2^-$	0.397	$11/2^-$
0.42	0.30	0.17	4	$\frac{7}{2}^+$	0.420	$\frac{7}{2}^+$
0.61	0.48	0.19	2	$\frac{5}{2}^+$	0.610	$\frac{5}{2}^+$
0.70	0.42	0.05	(4)	$(\frac{7}{2}^+)$		
0.86	1.56	0.19	2	$\frac{3}{2}^+$		
1.02	0.82	0.25	0	$\frac{1}{2}^+$		
1.13	0.28	0.14	2	$\frac{5}{2}^+$		
1.19	0.18	0.06	0	$\frac{1}{2}^+$		
1.33						
1.55	0.51		2	$(\frac{3}{2}^+)$		
1.66	0.11		0	$(\frac{1}{2}^+)$		
1.72	0.30		2	$(\frac{3}{2}^+)$		
1.86	0.18		2	$(\frac{3}{2}^+)$		
1.97	0.18		2	$(\frac{3}{2}^+)$		
2.02	0.24		2	$(\frac{3}{2}^+)$		
2.14						
2.20						
2.28						
			Cd^{113}			
g.s.	2.25	1.07	0	$\frac{1}{2}^+$	g.s.	$\frac{1}{2}^+$
0.265		<0.05	5	$11/2^-$	0.265	$11/2^-$
0.300	3.77	1.51	2	$\frac{3}{2}^+, \frac{5}{2}^+$	0.300	$\frac{3}{2}^+$
0.45	0.21	0.26	4	$\frac{7}{2}^+$		
0.53	0.41	0.05	2	$\frac{3}{2}^+$		
0.58	0.48	0.22	2	$\frac{5}{2}^+$	0.582	$\frac{5}{2}^+$
0.68	1.46	0.22	2	$\frac{3}{2}^+$		
0.90	0.63	0.40	0	$\frac{1}{2}^+$		
1.01	0.37	0.36	0	$\frac{1}{2}^+$		
1.20	0.70	0.47	2	$\frac{5}{2}^+$		
1.52	0.33	<0.05	2	$\frac{3}{2}^+$		
1.64	0.26	0.05	2	$\frac{3}{2}^+$		
1.68	0.13		2	$(\frac{3}{2}^+)$		
1.82	0.09		0	$(\frac{1}{2}^+)$		
1.90	0.20					
2.05	0.18		2	$(\frac{3}{2}^+)$		
2.19	0.25		2	$(\frac{3}{2}^+)$		
2.26						
2.41						
2.75						
2.80						
			Cd^{115}			
g.s.	2.29	1.56	0	$\frac{1}{2}^+$	g.s.	$\frac{1}{2}^+$
0.18	0.60	0.25	5	$11/2^-$	0.18	$11/2^-$
0.23	2.44	0.48	2	$\frac{3}{2}^+$		
0.38	1.21	1.25	2	$\frac{5}{2}^+$		
0.48	1.42	0.42	(4, 2)	$(\frac{7}{2}^+, \frac{3}{2}^+)$		
0.65	0.60	0.15	0	$\frac{1}{2}^+$		
0.77	1.21	0.36	2	$\frac{3}{2}^+$		
0.89	<0.05	<0.05	(4)	$(\frac{7}{2}^+)$		
0.96	0.13	0.12	0	$\frac{1}{2}^+$		
1.10	0.40	0.25	2	$\frac{5}{2}^+$		
1.19	0.15	0.12	0	$\frac{1}{2}^+$		
1.26	0.18	0.05	2	$\frac{3}{2}^+$		
1.37	0.72	0.27	2	$(\frac{5}{2}^+)$		
1.51						
1.55						
1.62						
1.84						
1.93						
2.03						
2.13						
2.23						
			Cd^{117}			
g.s.	2.42		0	$\frac{1}{2}^+$	g.s.	$\frac{1}{2}$
0.135	3.75		2	$\frac{3}{2}^+, 11/2^+$		
0.30	0.31		2	$(\frac{3}{2}^+)$		
0.42	0.85		2	$\frac{5}{2}^+$		
0.51	0.59		(4, 2)	$(\frac{7}{2}^+, \frac{3}{2}^+)$		
0.67	1.50		2	$(\frac{3}{2}^+)$		
0.82	0.39		2	$(\frac{3}{2}^+)$		
0.98						
1.07	0.20		2	$(\frac{3}{2}^+)$		
1.22	0.36		0	$\frac{1}{2}^+$		
1.36	0.40		2	$(\frac{5}{2}^+)$		
1.48	0.35		2	$(\frac{3}{2}^+)$		
1.67	0.18		2	$(\frac{3}{2}^+)$		
1.84						
1.94						

[a] See Ref. 12. [b] Calculated at the first peak beyond 9°. [c] Calculated at 45°.

The $h_{11/2}$ State in Cd^{117}

The $h_{11/2}$ state which is expected to lie quite low in energy in this nucleus was not detected. It is therefore reasonable to assume that it is covered either by the ground state (g.s.) or the 135-keV state. None of the angular distribution taken could reveal any reasonable $l_n=5$ mixture (Fig. 8). As in $Cd^{112}(d,p)Cd^{113}$, a best available resolution run at 64° was performed (see Fig. 7), and the plates were scanned at smaller than usual intervals, but neither the g.s. nor the 135-keV state could be proved to be a doublet. However, a careful center-of-mass analysis of the relative locations of the g.s. 135-keV and the 670-keV states seems to show that the $h_{11/2}$ lies slightly above the $d_{3/2}$ state.

The Ground State of Cd^{117}

The angular distribution of protons from the Cd^{116}-$(d,p)Cd^{117}$ supports the $\frac{1}{2}^+$ assignment of the g.s. of Cd^{117},[8] which was put into doubt recently by Sharma

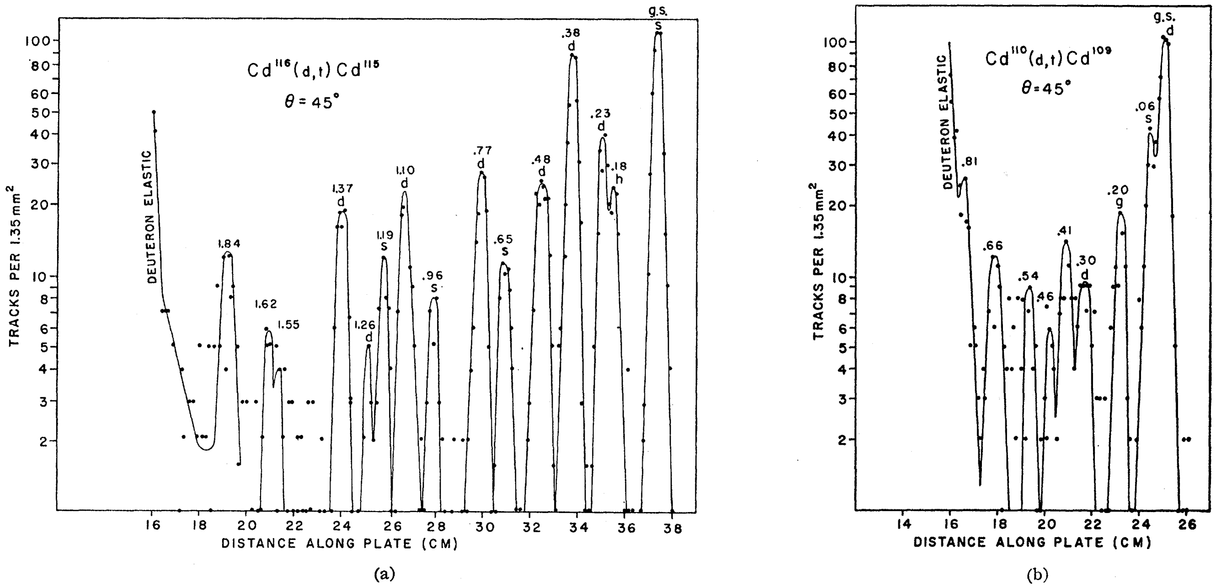

FIG. 3. Energy spectra of tritons from (a) $Cd^{116}(d,t)Cd^{115}$ and (b) $Cd^{110}(d,t)Cd^{109}$. The useful range for triton detection in the lighter isotopes is smaller because of the more negative Q value.

TABLE II. $\sigma(d,t)/\sigma(d,p)$ for $l_n=2$ transitions.

Excitation energy (MeV)	$\sigma(d,p)$	$\sigma(d,t)$	$\sigma(d,t)/\sigma(d,p)$	J
		Cd^{111}		
0.245	1.52	1.21	0.80	$\frac{5}{2}$
0.34	2.01	0.21	0.10	$\frac{3}{2}$
0.61	0.48	0.19	0.40	$\frac{5}{2}$
0.86	1.56	0.19	0.12	$\frac{3}{2}$
1.13	0.28	0.14	0.5	$\frac{5}{2}$
		Cd^{113}		
0.300	3.77	1.51	0.40	$\frac{3}{2}, \frac{5}{2}$
0.53	0.41	0.05	0.12	$\frac{3}{2}$
0.58	0.48	0.22	0.46	$\frac{5}{2}$
0.68	1.46	0.22	0.15	$\frac{3}{2}$
1.20	0.70	0.47	0.67	$\frac{5}{2}$
1.52	0.33	<0.05	<0.15	$\frac{3}{2}$
1.64	0.26	0.05	0.19	$\frac{3}{2}$
		Cd^{115}		
0.23	2.44	0.48	0.20	$\frac{3}{2}$
0.38	1.21	1.25	1.00	$\frac{5}{2}$
0.77	1.21	0.36	0.30	$\frac{3}{2}$
1.10	0.40	0.25	0.62	$\frac{5}{2}$
1.26	0.18	<0.05	<0.28	$\frac{3}{2}$

et al.[14] who suggested the $\frac{3}{2}^+$ configuration based on the β^- decay of this nucleus.

As a subsidiary result, the Q values for the eight reactions performed were also determined and listed in Table III.

THE DEPENDENCE OF NEUTRON STATE ON COUPLING WITH FEW ADDITIONAL NEUTRON AND PROTON PAIRS

Since (d,p) reactions strongly excite neutron states, it is interesting to compare spectra obtained from target nuclei which have the same number of protons but differ by the number of neutron pairs. Looking at Fig. 9 and Table I, it is easy to observe the over-all

[14] R. P. Sharma *et al.* (private communication through Professor Jha).

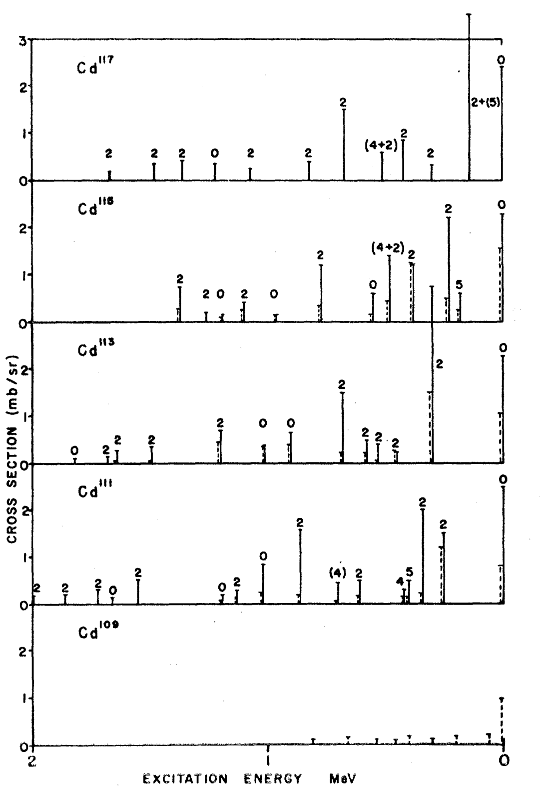

FIG. 4. Schematic representation of the experimental results summarized in Table I. The solid and dashed lines correspond to the (d,p) and (d,t) reactions, respectively.

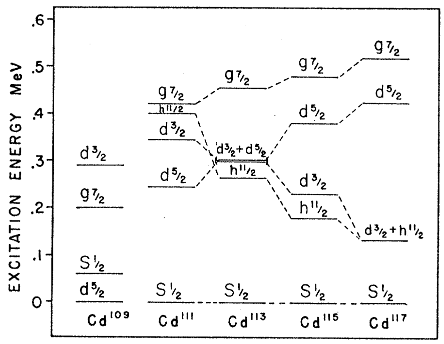

FIG. 5. The location of the lowest excited state of different j in the five odd Cd isotopes.

similarity of the (d,p) spectra obtained from the bombardment of four even-even Cd isotopes. The ground states in all isotopes are the $s_{1/2}$ states and the

TABLE III. Q values (in MeV) for (d,p) and (d,t) reaction in some Cd isotopes.

Isotope	$Q(d,p)$	$Q(d,t)$
Cd^{110}	(4.740±0.03)	−(3.664±0.03)
Cd^{112}	(4.318±0.03)	−(3.129±0.03)
Cd^{114}	(3.923±0.03)	−(2.801±0.03)
Cd^{116}	(3.538±0.03)	−(2.458±0.03)

other low-lying states are those predicted by the shell model, namely $d_{3/2}$, $d_{5/2}$, $g_{7/2}$, and $h_{11/2}$ states, located within 0.6-MeV excitation energy and receiving the major part of the single-particle cross-section intensity. Other transitions, mostly with angular-momentum transfer $l_n=0$ or 2 and lower in intensity, are found up to about 1.7 MeV. At this energy a minimum in the cross sections is observed, and beyond it the proton yield increases again as levels from the next major shell start to appear. A comparison of the (d,p) spectra obtained from five even tin isotopes was already done,[5] and in palladium we have another three cases,[4] namely Pd^{104}, Pd^{106}, and Pd^{108}, for which the (d,p) spectra can be compared. The general similarity among the isotopes with respect to the location of the energy levels, multiplicity, and cross sections, indicates the expected small effect of the additional few neutron pairs on the nuclear structure, at least as long as the neutrons are far from forming a closed shell.

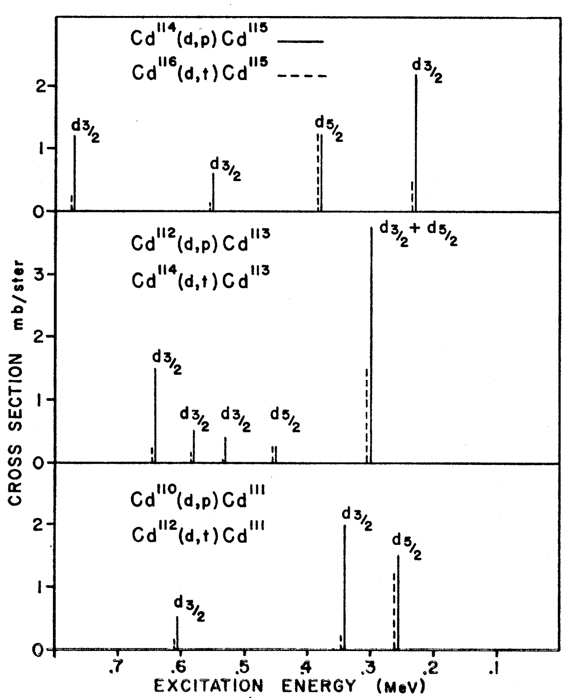

FIG. 6. The realtive cross section for (d,p) and (d,t) reactions with $l_n=2$ used to determine the spin value at the final state.

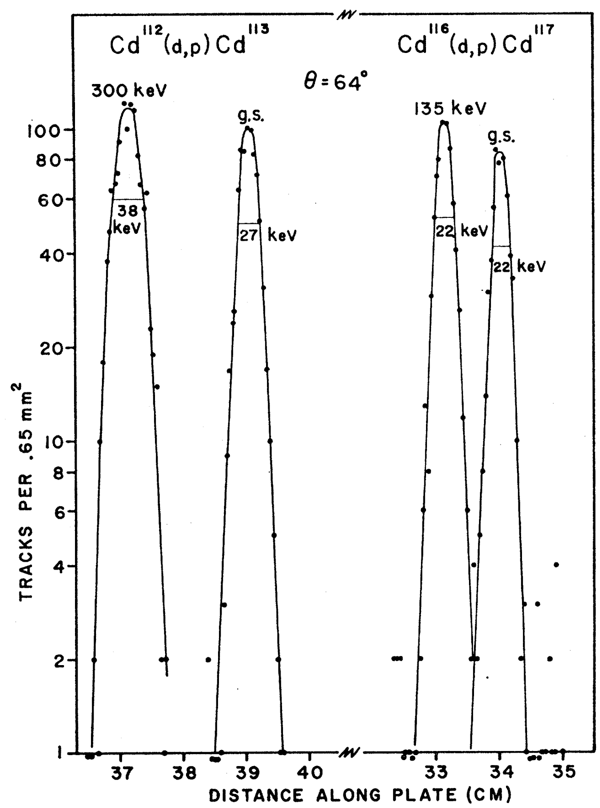

FIG. 7. The energy spectrum of protons leading to the g.s. and first excited state of Cd^{113} and Cd^{117} taken with the best available resolution to investigate the existance of doublets.

The effect of the addition of a proton pair to the nuclear structure of nuclei having the same number of neutrons can be obtained by comparing isotones. Cohen and Price[5] have compared (d,p) spectra obtained from Cd^{114} and Sn^{116}, both having 66 neutrons; and Cd^{116} and Sn^{118} having 68 neutrons. We are able now to compare (d,p) spectra obtained from Pd^{108} and Cd^{110}

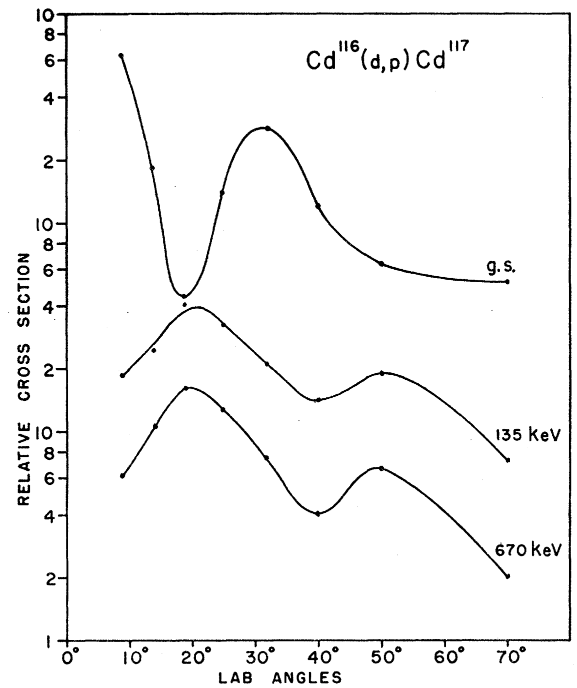

FIG. 8. The angular distribution of protons leading to three states at Cd^{117}. No reasonable $l_n=5$ mixture is found.

both having 62 neutrons. The last case has the advantage that both spectra were obtained with similar good resolution and in both isotones the protons do not form a closed shell. The similarity observed in the $Pd^{108}(d,p)Pd^{109}$ and $Cd^{110}(d,p)Cd^{111}$ spectra, Fig. 10, supports the well-known shell-model assumption that the neutron states are not much affected by the addition of pairs of protons. The poor similarity between the (d,p) and (d,t) spectra reported by Cohen and Price in the even-even Cd–Sn isotones is still found in comparing spectra with good resolution. The single-particle transition intensities are concentrated toward lower excitation energies in Cd compared to Sn, and the multiplicity of the levels in Cd is much larger. Nevertheless, it turns out, as we shall see in the last section, that despite the poor similarity between the (d,p) spectra obtained from Cd and Pd isotopes, on one hand, and from Sn isotopes on the other, the neutron single-particle states do not change their location in an abrupt manner for nuclei in which protons form a closed-shell configuration.[3–5]

LOCATION OF THE SINGLE-PARTICLE NEUTRON STATES

In order to locate the single-particle neutron states in nuclei far from a closed shell, such as the Cd isotopes, one needs a theory that takes into account the residual interaction that exists between all nucleons outside the closed shell. The theory that does this in a simple and elegant way is the pairing theory recently reviewed by Kisslinger and Sorensen.[15] This theory had much success in dealing with many properties of spherical nuclei at low excitation energies, as for example, energy level systematics, electromagnetic transition rates, and many others. We shall apply this theory to investigate the energy level structure at low energies, obtained by stripping and pickup reactions. One important effect of the pairing interaction is to produce strong configuration mixing among the low-lying shell-model wave functions. Consequently, the nucleons outside a closed shell do not fill the lowest possible shell-model orbit, but many orbits become partly occupied simultaneously as one proceeds in adding nucleons. We designate by the occupation number V_j^2 the probability that the orbit j is occupied and by U_j^2 the probability that it is empty. Pairing theory gives us the relations between these occupation numbers and the single-particle

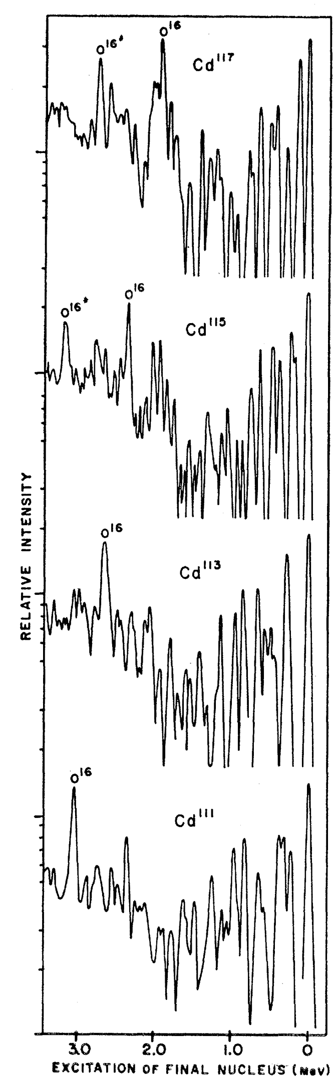

FIG. 9. Comparison of (d,p) spectra obtained by bombarding even Cd isotopes differing from each other by as much as four neutron pairs. (Detection angle 32°.)

[15] L. S. Kisslinger and R. A. Sorenson, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. 32, No. 9 (1960).

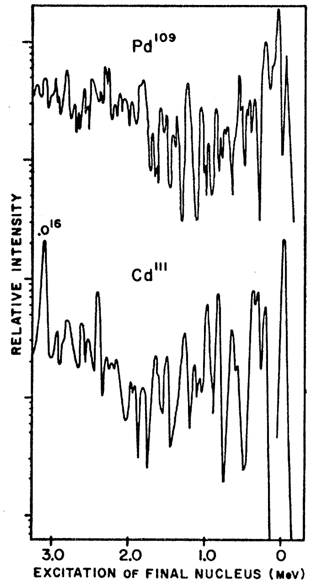

FIG. 10. Comparison of (d,p) spectra obtained by bombarding even Pd and Cd targets differing by one pair of protons. (Detection energy 32°.)

energies ϵ_j of the shell model, as

$$V_j^2=\frac{1}{2}\left\{1-\frac{(\epsilon_j-\lambda)}{[(\epsilon_j-\lambda)^2+\Delta^2]^{1/2}}\right\},\qquad(1)$$

$$U_j^2=\frac{1}{2}\left\{1+\frac{(\epsilon_j-\lambda)}{[(\epsilon_j-\lambda)^2+\Delta^2]^{1/2}}\right\},\qquad(2)$$

where λ is the chemical potential and Δ is approximately half the energy gap. The following sum rule is immediately obvious:

$$V_j^2+U_j^2=1.\qquad(3)$$

By using (1), (2), and (3), the explicit expression for ϵ_j can be obtained in terms of V_j^2 as

$$(\epsilon_j-\lambda)=\Delta\frac{1-2V_j^2}{2[V_j^2(1-V_j^2)]^{1/2}}.\qquad(4)$$

According to pairing theory the ground state of every even-even nucleus is a "vacuum" state, and the energies of elementary excitation from this state are called the single-quasiparticle energies designated by E_j. The relation between these energies and the ϵ_j are

$$E_j=[(\epsilon_j-\lambda)^2+\Delta^2]^{1/2}.\qquad(5)$$

Let us see now to what extent the occupation numbers V_j^2, U_j^2 and single-quasiparticle energies E_j are obtainable from the experimental data.

The relation between the occupation numbers V_j^2 and U_j^2 and the experimental cross sections for stripping and pickup reactions is given by Yoshida.[1] In the cases where the parent nucleus is an even-even nucleus in its $J=0$ ground state, and the final nuclear system is the daughter nucleus with a nucleon captured (or stripped) into a state with the total angular momentum j, the following explicit formulas hold.

$$\frac{\sum_n d\sigma_j^n(d,p)/d\Omega}{[d\sigma_l(d,p)/d\Omega]_{\rm SP}}=U_j^2+\frac{2\lambda+1}{2j+1}\sum_{j'}(\phi_{jj'a})^2V_j^2.\qquad(6)$$

$$\frac{\sum_n d\sigma_j^n(d,t)/d\Omega}{(2j+1)[d\sigma_l(d,t)/d\Omega]_{\rm SP}}=V_j^2+\frac{2\lambda+1}{2j+1}\sum_{j'}(\phi_{jj'a})^2U_j^2.\qquad(7)$$

The second term on the right-hand side of Eqs. (6) and (7) is small compared to the first one for nuclei at or near to a closed shell, and is therefore neglected. ($\phi_{j_1j_2}{}^\lambda$ is the amplitude of the annihilation operator of two quasiparticles j_1j_2 coupled to give the total momentum σ of the creation operator of a phonon, and obtained explicitly in Ref. 1 for all Cd isotopes under discussion.) In order to obtain U_j^2 and V_j^2 from Eqs. (6) and (7), a good estimate of the cross section for the single-particle transition with angular momentum l is needed. Such an estimate for the (d,p) reaction is obtained by using the cross sections predicted by the DWBA calculations at the position of the first maximum of the angular distributions beyond 9°. At this maximum position, the theoretical cross section seems to be least sensitive to changes in many of the optical potential parameters. In addition it was found to give an accurate prediction of the absolute cross sections for the $Zr^{90}(d,p)Zr^{91}$ reaction, where the comparison between experimental and theoretical values is simple, as Zr^{91} has just one nucleon outside a double closed-shell configuration.[2] The V_j^2 calculated by this method are given in Table IV. For

TABLE IV. Occupation numbers V_j^2 obtained by using Eq. (6).

l_j	Cd^{110}	Cd^{112}	Cd^{114}	Cd^{116}
$s_{1/2}$	0.10	0.20	0.27	0.38
$d_{3/2}$	0.19	0.24	0.25	0.30
$d_{5/2}$	0.75	0.75	0.78	0.84
$g_{7/2}$	0.75	0.84	···	···
$h_{11/2}$	0.20	···	0.35	···
$s_{1/2}$[a]	0.17	0.26	0.33	0.43

[a] After renormalization by taking into account the (d,t) data and using the sume rule, Eq. (3).

the (d,t) reaction the single-particle cross section cannot be well estimated by the same method, as for these reactions the DWBA calculations seem to be rather poor at present. The possibility, of taking advantage of the sum rule (3) in order to make use of the (d,t) data also, without relying on the DWBA calculations, can be successfully achieved if the Q-value dependence of the single-particle cross sections for these reactions were known. Assuming the same Q dependence for the (d,t) cross sections as for the (d,p) ones and using the sum rule (3), we find that the occupation numbers change only slightly for $l_n\neq0$ transitions, but to a much larger extent in the $l_n=0$ case. This is not surprising in view of the fact that the DWBA calculations are usually less accurate for $l_n=0$ than for higher angular momentum transfer. The normalized $V_{1/2}^2$ are also listed in Table IV.

The single-quasiparticle energies E_j are related to the experimental energy levels through the following equation:

$$\frac{\sum_n E_n d\sigma_j^n(d,p)/d\Omega}{[d\sigma_l(d,p)/d\Omega]_{SP}} = E_j U_j^2 - \left(\frac{2\lambda+1}{2j+1}\right)^{1/2} \sum_{j'} \langle j|H|j'\lambda,j\rangle \phi_{jj'a}{}^{\lambda} U_j V_j + \frac{2\lambda+1}{2j+1} \sum_{j'} (E_{j'}+\hbar\omega_a)(\phi_{jj'a}{}^{\lambda})^2 V_j^2, \quad (8)$$

and a similar expression for the (d,t) case. The last term on the right-hand side is small compared to the first one because of similar arguments to those given in Eqs. (6) and (7). The second term on the right-hand side is by no means small. The effects of the two phonon interactions on the quasiparticles for odd neutrons was estimated by Kisslinger and Sorensen[16] for nuclei between Sn and Nd. In many cases the shift in the quasiparticle energies is as large as several hundred keV. No such calculations were performed for nuclei between Zr and Sn but it is reasonable to assume that the energy shifts in Cd will be similar to those in Te, as Cd has a pair of holes and Te a pair of protons outside the Sn closed-shell configuration. Nevertheless, a rough estimate of the relative single-quasiparticle energies can be obtained even if the second term is also neglected. In this case the determination of the E_j is straightforward:

$$E_j = \frac{\sum_n E_n d\sigma_j^n(d,p)/d\Omega}{\sum_n d\sigma_j^n(d,p)/d\Omega}. \quad (9)$$

The neutron single-quasiparticle energies thus obtained are listed in Table V. Although "this center of gravity"

TABLE V. Single-quasiparticle energies E_j (in keV) obtained by using Eq. (9).

l_j	Cd^{110}	Cd^{112}	Cd^{114}	Cd^{116}
$s_{1/2}$	353	290	214	160
$d_{3/2}$	860	617	445	346
$d_{5/2}$	430	590	700	720
$g_{7/2}$	420	450	(480)	(530)
$h_{11/2}$	397	265	180	135

formula is not strictly correct, it has the advantage of depending on experimental data only. The calculation of E_j with the aid of the sum rule (10),

$$E_j = \frac{\sum_n E_n d\sigma_j^n(d,p)/d\Omega}{[d\sigma_l(d,p)]d\Omega]_{SP}} + \frac{\sum_n E_n' d\sigma_j^n(d,t)/d\Omega}{(2j+1)[d\sigma_l(d,t)]d\Omega]_{SP}}, \quad (10)$$

can in principle give more accurate results, but only when good DWBA calculations for (d,t) reactions will be available.

An examination at Tables IV and V reveals the following facts. Table IV shows that the $s_{1/2}$ and, to a lesser extent, the $d_{3/2}$ and $h_{11/2}$ levels are filling in the Cd isotopes. Table V shows that the $d_{5/2}$ and $g_{7/2}$ levels lie below the Fermi level as their energies increase with the increase of the neutron number; whereas the $s_{1/2}$, $d_{3/2}$, and $h_{11/2}$ levels are above the Fermi level, as their energies are decreasing, Eq. (5).

An interesting fact seen in nuclei with ground states having the $s_{1/2}$ assignment, is the surprisingly small occupation number obtained from the (d,p) reaction, as small as $V_j^2=0.1$ in the case of $Cd^{110}(d,p)Cd^{111}$.[17] According to the pairing theory the occupation number for the ground-state spin must be close to 0.5 as this state lies close to the Fermi level [$\epsilon_{g.s.}\approx\lambda$ in Eq. (4)]. Similar results were obtained also for Mo^{100} where $V_{1/2}^2=0.15$ and even in the Te isotopes it was reported[18] that these states are not as full as expected. It is not clear whether the DWBA calculations give too low values for the (d,p) cross sections in the ln=0 case (we tried several DWBA calculations with rather different parameters), or the simple pairing theory does not treat this special case accurately enough. A possible explanation of the peculiarity is the effect of the P_2 forces acting especially strongly on $s_{1/2}$ states, pushing them down to be the observed ground state, even though they are still high above the Fermi sea.

The energies ϵ_j of the neutron single-particle states were calculated separately from the occupation numbers with Eq. (4), and from the single-quasiparticle energies with Eq. (5). The results are tabulated in Table VI. As the energies are not expected to change much in a small region their average position is also given. Looking at this table, only a general agreement between location of the single-particle states obtained by the two different methods is seen, and several discrepancies can be noted. First is the width of the shell defined as the difference in energy between the lowest lying level and the highest one. It is larger by 1 MeV for the ϵ_j calculated from the E_j than from those calculated from the V_j^2. The other discrepancy is the relative positions of the $d_{3/2}$ and $s_{1/2}$ states. We believe that the level order predicted from the E_j data is the correct one and the $s_{1/2}$ lies lower. This belief is based on the experimental spin values of the g.s. of the even-odd nuclei in this region. The $d_{3/2}$ state becomes the ground state in nuclei only towards the end of the shell; e.g., the heavier isotopes of Sn^5, Te^{11}, Ba and Ce,[6] whereas the $s_{1/2}$ value for the ground state is found even in some of the Mo isotopes,[3] most of the Cd isotopes, and the light Sn isotopes. The interesting shift to lower energy of the $g_{7/2}$ neutron

[16] L. S. Kisslinger and R. A. Sorenson, Rev. Mod. Phys. 35, 857 (1963).

[17] B. Rosner, Bull. A. Phys. Soc. 9, 484 (1964).

[18] R. K. Jolly and B. L. Cohen, Bull. Am. Phys. Soc. 9, 484 (1964).

TABLE VI. The relative position of the neutron single-particle states in Cd isotopes (in MeV).

	$\Delta\epsilon_j$ calculated from V_j^2				Average energy	Average energy	$\Delta\epsilon_j$ calculated from E_j			
	Cd^{110}	Cd^{112}	Cd^{114}	Cd^{116}			Cd^{110}	Cd^{112}	Cd^{114}	Cd^{116}
$\epsilon_{1/2}-\epsilon_{5/2}$	2.64	1.78	1.77	1.81	2.00	2.25	2.15	2.37	2.09	2.38
$\epsilon_{3/2}-\epsilon_{5/2}$	1.83	1.66	1.83	2.05	1.84	2.85	2.89	2.95	2.82	2.72
$\epsilon_{5/2}-\epsilon_{5/2}$	0	0	0	0	0	0	0	0	0	0
$\epsilon_{7/2}-\epsilon_{5/2}$	0	−0.30	···	···	−0.15	0.21	0.01	0.24	0.32	0.28
$\epsilon_{11/2}-\epsilon_{5/2}$	1.77	···	1.44	···	1.61	2.25	2.23	2.08	2.36	2.32

single-particle level relative to the other neutron levels that stay almost at constant energies between Zr and Sn, is seen also in the Cd isotopes. Here the $g_{7/2}$ level lies within a few hundred keV of the $d_{5/2}$ level at the bottom of the shell, whereas in Zr it is 2.5 MeV higher. This fact stresses once again the strong interaction between the neutrons filling the $g_{7/2}$ level and the protons filling the $g_{9/2}$ level,[7,8] both having the same n and l quantum numbers. The location of the $g_{7/2}$ neutron single-particle level, as given in Table VI, seems to confirm the statement made by Cujec[4] that the n-p interaction responsible for the shift of the $g_{7/2}$ level is most effective when the proton $g_{9/2}$ state is only half full. But a much better theory to treat the vibrational nuclei, and much better data for the transitions leading to $g_{7/2}$ states, must be obtained before such a revolutionary statement can be proved.

One word of caution must be said about the sensitivity of the single-particle energies ϵ_j to the experimental and theoretical errors in the determination of the absolute cross section. The sensitivity is especially large if these energies are at high excitations. Even if we assume an over-all error of only 20% in the evaluation of the occupation number from the absolute cross section obtained in (d,p) reactions, the value of ϵ_j can change by as much as 50%, in cases where $V_j^2 \approx 0.2$. As the occupation numbers V_j^2 become larger this sensitivity decreases rapidly to a minimum at $V_j^2=0.5$.

Finally the strength of the pairing interaction was calculated for the $Cd^{110}(d,p)Cd^{111}$ data according to the formula[15]

$$G=\Delta\frac{2}{\sum_j (2j+1)U_jV_j} \qquad (13)$$

and was found to be $GA=22.3$ MeV. No calculation for the other isotopes was performed as the V_j^2 data is not complete in any other case.

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APPENDIX

The optical-model parameters (in units of MeV and fermis) used for the DWBA calculation were:

Deuterons

$V=82$	$r_0=1.3$	$a=0.79$	$r_{0c}=1.3$
$W=80$	$r_0'=1.34$	$a'=0.65$	

Protons

$V=51$	$r_0=1.25$	$a=0.65$	$r_{0c}=1.25$
$W=60$	$r_0'=1.25$	$a'=0.47$	

(Surface derivative absorption, lower cutoff at 6.7 F.)