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Effect of Velocity-Dependent Forces on the Integrated Cross Section of Photonuclear Reactions

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The effect of correlations between nucleons is discussed for the integrated cross section of photonuclear reactions, considering only the central force. The two-body potential used is well-behaved and velocity-dependent. The calculations performed by first-order perturbation theory give an increase of about 14% for the integrated cross section above the value found by neglecting correlations.

INTRODUCTION

SUM-RULE calculations for nuclear photoeffect using the independent-particle model (IPM), in which knowledge only of the wave function for the ground state is required, are in fairly good agreement with experiment. However, photonuclear reactions at high energy are understandable only if we assume a strong correlation between nucleons. A model that considers such correlations is the quasideuteron model as proposed by Levinger.[1] This author studied the effect of this correlation on the bremsstrahlung-weighted cross section σ_b and found that it decreased by about 10%.[2] Okamoto[3] calculated the same effect on the integrated cross section σ_{int} and bremsstrahlung-weighted cross section σ_b and compared the results obtained using the quasideuteron model with calculations made with IPM. He concludes that the independent-particle model can be regarded as a good approximation for photonuclear reactions. Lately Okamoto[4] has considered the effect of tensor forces and found that there exists a great discrepancy between the two models in the results of the photonuclear reactions.

We shall calculate the effect of correlations between nucleons for σ_{int}, considering the two-body potential to be central with a velocity-dependent part as used by Rojo and Simmons,[5] and replacing the static part of that potential by an equivalent Gaussian potential that satisfies effective-range theory, as done by Levinger *et al.*[6]

In Sec. I we shall study the results of the IPM, disregarding correlations. In Sec. II we shall study the effect of the quasideuteron model by first-order perturbation theory; we find that it increases σ_{int} about 14%.

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[1] J. S. Levinger, Phys. Rev. **84**, 43 (1951).

[2] J. S. Levinger, Bull. Am. Phys. Soc. **1**, 37 (1956).

[3] K. Okamoto, Phys. Rev. **116**, 428 (1959).

[4] K. Okamoto and K. Hasegawa, Prog. Theoret. Phys. (Kyoto) **28**, 137 (1962).

[5] O. Rojo and L. M. Simmons, Phys. Rev. **125**, 273 (1962).

[6] J. S. Levinger, M. Razavy, O. Rojo, and N. Webre, Phys. Rev. **119**, 230 (1960).

I. CALCULATIONS WITH IPM

In the electric-dipole approximation, the integrated cross section is defined as

$$\sigma_{\text{int}}\equiv\int\sigma(\omega)d\omega=\frac{\pi^2e^2\hbar}{Mc}\sum_n f_{0n}\,, \tag{1}$$

where f_{0n} is the oscillator strength which, summed over all the states, gives

$$\sum_n f_{0n}=-\frac{M}{\hbar^2}\langle 0|[[H,z],z]|0\rangle. \tag{2}$$

It is seen that the integrated cross section calculated with the electric-dipole approximation depends only on the wave functions for the ground state of the system.

We assume a Hamiltonian of the form

$$H=\sum_i\frac{p_i^2}{2M}+\sum\frac{p_j^2}{2M}+\sum_{i,j}\bigg[V_0(1+xP_{ij}{}^M)+\frac{\lambda}{M}[p^2W(r)+W(r)p^2]\bigg], \tag{3}$$

where the potential contains a static part with a fraction x of Majorana exchange and a velocity-dependent part as in Ref. 5; thus,

$$\lambda W(r)=5\exp[-3.6r],\quad r \text{ in fermis}.$$

Disregarding correlation between nucleons, the ground-state wave function will be a product of plane waves.

$$\langle\mathbf{r}|0\rangle=\prod_i\prod_j\exp[i(\mathbf{k}_i\cdot\mathbf{r}_i)]\exp[i(\mathbf{k}_j\cdot\mathbf{r}_j)]\quad (i=\text{proton},\ j=\text{neutron}), \tag{4}$$

with values $\mathbf{k}_i$, $\mathbf{k}_j$ chosen to satisfy the Pauli principle. With these assumptions, and $v=V_0(1+xP_{ij})+(\lambda/M)\times[p^2W(r)+W(r)p^2]$,

$$\sigma_{\text{int}}=\frac{2\pi^2e^2\hbar}{Mc}\bigg\{\frac{NZ}{A}-\frac{Mx}{3\hbar^m}\times\langle 0|\sum_{ij}r_{ij}{}^2VP_{ij}|0\rangle+\lambda\langle 0|\sum_{ij}W(r_{ij})|0\rangle \tag{5}$$

leads to

$$\sigma_{\rm int}=15A(1+0.8x+0.37)\ \text{MeV-mb}. \tag{6}$$

It is seen that the velocity-dependent part increases the value of the integrated cross section by about the same amount as it is increased by the effect of Majorana exchange force as treated by Levinger and Bethe.[7]

The value calculated with Eq. (6) agrees fairly well with the experimental values obtained up to now.

II. CALCULATIONS WITH QUASIDEUTERON MODEL

If we assume correlations between nucleons, the wave function of the ground state will be, in first-order perturbation theory,

$$|\ \rangle=|0\rangle+\sum_{n\neq 0}\frac{\langle 0|v|n\rangle}{E_0-E_n}, \tag{7}$$

where $|n\rangle$ represents the excited state of the nucleus without correlations:

$$\begin{aligned}|0\rangle&=|0_i\rangle|0_j\rangle\\ |n\rangle&=|n_i\rangle|n_j\rangle;\end{aligned} \tag{8}$$

where i refers to the proton and j to the neutron. One obtains, after substitution in (2), the following expression:

$$\begin{aligned}\sigma_{\rm int}=60\biggl\{&\frac{NZ}{A}-\frac{Mx}{3\hbar^2}\langle 0|\sum_{ij} r_{ij}^2V_0P_{ij}^M|0\rangle+\lambda\langle 0|\sum_{ij} W(r_{ij})|0\rangle+\lambda\sum_{n\neq 0}\frac{\langle 0|v|n\rangle}{E_0-E_n}(\langle 0|\sum_{ij} W(r_{ij})|n\rangle+\langle n|\sum_{ij}W(r_{ij})|0\rangle)\\ &-\frac{Mx}{3\hbar^2}\sum_{n\neq 0}\frac{\langle 0|v|n\rangle}{E_0-E_n}(\langle 0|\sum_{ij} r_{ij}^2V_0P_{ij}^M|n\rangle+\langle n|\sum_{ij} r_{ij}^2V_0P_{ij}|0_\sigma\rangle)+O[\langle 0|v|n\rangle]^2\biggr\}.\end{aligned} \tag{9}$$

The first two terms are the ones calculated by Levinger and Bethe; the third one has been calculated in Ref. 6. We will calculate the remaining terms following the same steps as those used by Okamoto.[3]

$$\text{(a)}\quad \Delta_1{}^1=\lambda\sum_{n\neq 0}\frac{\langle 0|V_0(1+xP_{ij}^M)|n\rangle}{E_0-E_n}. \tag{10}$$

Taking account of the fact that

$$E_n-E_0=\frac{\hbar^2}{M}\mathbf{s}\cdot(\mathbf{s}+\mathbf{k}_i-\mathbf{k}_j), \tag{11}$$

where

$$\mathbf{s}=\mathbf{k}_i'-\mathbf{k}_i=\mathbf{k}_j-\mathbf{k}_j',$$

$$\mathbf{s}'=\mathbf{k}_j'-\mathbf{k}_i=\mathbf{k}_j-\mathbf{k}_i',$$

with

$$\mathfrak{F}(s)=\int V(r)e^{i(\mathbf{s}\cdot\mathbf{r})}d^3r, \tag{12}$$

one obtains

$$\Delta_1{}^1=-\frac{\lambda M}{\hbar^2\Omega^2}\times\int\sum_{\mathbf{s}}\sum_{\mathbf{k}\,i}\sum_{\mathbf{k}j}\frac{\mathfrak{F}(s)+x\mathfrak{F}(s')}{\mathbf{s}\cdot(\mathbf{s}+\mathbf{k}_i-\mathbf{k}_j)}W(r)e^{i(\mathbf{s}\cdot\mathbf{r})}d^3r \tag{13}$$

with Ω the nuclear volume. The following substitutions are made:

$$\mathbf{k}_i=k_F\mathbf{p},\quad \mathbf{s}=k_F\mathbf{l},\quad k_j=k_F\mathbf{n},$$

where k_F is the wave number corresponding to the Fermi momentum ($k_F=1.52/r_0$), leading to

$$\Delta_1{}^1=-(1+x)\frac{\lambda M}{\hbar^2}\frac{\Omega}{(2\pi)^9}k_F{}^7\int W(r)d^3r\times\int d^3p\int d^3n\int d^3l\frac{\mathfrak{F}(l)e^{ik\ (\mathbf{l}\cdot\mathbf{r})}}{\mathbf{l}\cdot(\mathbf{l}+\mathbf{p}-\mathbf{n})}. \tag{14}$$

The Pauli principle is introduced through the relations

$$|p|,|n|<1\quad\text{and}\quad|p+s'|,|p-s'|>1.$$

The evaluation of the triple "integrals" in the above equation has been done by Euler[8]:

$$\int d^3p\int d^3n\frac{1}{\mathbf{l}\cdot(\mathbf{l}+\mathbf{p}-\mathbf{n})}=\frac{4\pi}{15}\frac{P(l)}{l}.$$

[$P(l)$ is given in the Appendix of Okamoto's work.] We use for V_0 the Gaussian shape satisfying the effective-range theory:

$$V_0=-s_0V_0'e^{-(r/\gamma)^2},\quad V_0'=(229.2/b^2)\ \text{MeV}\times 10^{-26}\ \text{cm}^2;\quad \gamma=[b/(2.06)^{1/2}],$$

where s_0 is the well depth parameter and b is the intrinsic range. We also set

$$\frac{M}{\hbar^2}\mathfrak{F}(l)=\frac{M}{\hbar^2}\int_0^\infty V_0e^{ik_F{}^2(\mathbf{l}\cdot\mathbf{r})}d^3r=-s_0W_0\pi^{3/2}\gamma^3\exp(-\tfrac{1}{4}\gamma^2k_F{}^2l^2),\quad W_0=(M/\hbar^2)V_0{}^1,$$

and $b/r_0=\xi$, r_0 being the nucleon radius, to obtain

$$\Delta_1{}^1=(1+x)\lambda s_0\xi(3.86A)\int_0^\infty\frac{P(u)e^{-\gamma^2k_F{}^2u^2}}{[18.7+(3.04u)^2]^2}udu. \tag{15}$$

[7] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950); hereafter referred as LB.

[8] H. Euler, Z. Physik, **105**, 553 (1937).

Since we are interested in a bound on this integral, we use several approximations to obtain

$$\Delta_1{}^1 \leq (1+x) 0.006 \times 3.86 \lambda S_0 A \xi\,, \tag{16}$$

and with

$$S_0=1;\quad \lambda=0.2;\quad N=Z=A/2$$

this becomes

$$\Delta_1{}^1 \leq 0.156(NZ/A)(1+x)\,.$$

The conjugate term is equal to this last one since it is real. (b) Using the same considerations, we proceed to evaluate

$$\Delta_1{}^2 = \frac{\lambda^2}{M}\sum \frac{\langle 0|p^2W(r)+W(r)p^2|n\rangle}{E_0-E_n}\langle 0|\sum_{ij} W(r_{ij})|n\rangle\,. \tag{17}$$

This term can be written as

$$\Delta_1{}^2 = \frac{\lambda^2\hbar^2}{2M\Omega}\sum_{n\neq 0}\frac{(s^2+s'^2)G(s)}{E_0-E_n}\langle 0|\sum_{ij} W(r_{ij})|n\rangle\,,$$

with

$$G(s)=\int W(r)e^{i(\mathbf{s}\cdot\mathbf{r})}d^3r\,. \tag{18}$$

After calculation this term becomes

$$\Delta_1{}^2 = -k_1\lambda^2\xi^9\int_0^\infty \frac{P(u)u^3du}{[18.7+(3.04u)^2]^4}\,,$$

where

$$k_1 = 18.7\frac{32}{3}\frac{4}{3\pi^2}(1.59)^9A = 184A\,,$$

and with this value one finally obtains

$$\Delta_1{}^2 = -0.022NZ/A\,.$$

The conjugate term is equal to this, so

$$\Delta_1{}^2+\Delta_1{}^{2*} = -0.044NZ/A\,.$$

So far we have calculated the term of the double commutator belonging to the component of displacement in the direction of polarization of the incident radiation. There remains to be calculated the part of the double commutator depending on the exchange potential. This first part was calculated by Okamoto, who obtained

$$\Delta_2{}^1 = 0.058(NZ/A)s_0{}^2x(1+x)\,.$$

The only term left is the exchange term for the velocity-dependent contribution. This term gives

$$\Delta_2{}^2 = -S_0W_0\frac{2\lambda x}{45}\frac{\Omega}{(2\pi)^6}k_F{}^9\int r^2e^{-r^2/\gamma^2}e^{ik_F(\mathbf{l}'\cdot\mathbf{r})}r^2\frac{\sin\theta}{2\pi}$$
$$\times d\theta d\phi dr\int_0^\infty l'P(l')G(l')\exp(-\tfrac{1}{4}\gamma^2k_F{}^2l'^2)dl'\,. \tag{19}$$

The first integral is

$$W_0\int r^2e^{-r^2/\gamma^2}e^{ik_F(\mathbf{l}'\cdot\mathbf{r})}d^3r = \frac{\pi^{3/2}}{4}\gamma^5W_0(6-k_F{}^2\gamma^2l'^2)e^{-\frac{1}{4}\gamma^2k_F{}^2l'^2}\,,$$

and substitution in (19) yields

$$\Delta_2{}^2 = -s_0W_0\frac{\lambda x}{90}\frac{\Omega}{(2\pi)^6}\pi^{3/2}\gamma^5k_F\int_0^\infty 4\pi(6-k_F{}^2\gamma^2l^2)$$
$$\times l^3P(l)G(l)\exp(-\tfrac{1}{4}\gamma^2k_F{}^2l^2)dl\,, \tag{20}$$

where $G(l)$ has been calculated previously.

Then

$$\Delta_2{}^2 = -16s_0x\lambda K_2\xi^3$$
$$\times\int_0^\infty \frac{6-4\alpha u^2}{[18.7+(3.04u)^2]^2}P(u)e^{-\alpha u}u^3du\,. \tag{21}$$

We have taken

$$K_2 = \frac{5.53\pi^{3/2}}{27\pi^3}\frac{(1.52)^9}{(2.06)^{5/2}}4.32A = 1.06A\,,$$

$$\alpha=\gamma^2k_F{}^2 = 1.12\xi^2,\quad\text{and}\quad l'=2u\,.$$

After several approximations, such as neglecting $(3.04u)^2$ for $u<1$ and 18.7 compared with $(3.04u)^2$ for $u>1$, we obtain

$$\Delta_2{}^2 \simeq 16s_0A\xi^3\lambda x\frac{34.3}{349.7}\xi^{-6}\,, \tag{22}$$

and for $\xi=2$, $s_0=1$ this yields

$$\Delta_2{}^2 \leq 0.16(NZ/A)x\,.$$

The conjugate term of the perturbation has the same value, so

$$\Delta_2{}^2+\Delta_2{}^{2*} \leq 0.32(NZ/A)x\,.$$

Adding all the contributions with the appropriate sign, one obtains for the net contribution of the correlation:

$$\Delta = 60(NZ/A)(0.300-0.022x+0.058x^2)\ \text{MeV-mb}\,.$$

For the value of $x=\frac{1}{2}$ that corresponds to the case of a Serber mixture, this correction is about 14% of the value obtained when no correlation is assumed and the IPM is used [Eq.(6)].