Thermal Neutron Capture by Deuterium and Structure of the Three-Body Wave Function*

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The study of three-nucleon wave functions has recently drawn considerable attention owing to experiments on elastic scattering of high-energy electrons from H^3 and He^3. We attempt here to obtain independent information on these wave functions using the available experimental data on slow-neutron capture by deuterium. This reaction goes mainly through magnetic dipole radiation from both exchange and spin magnetic-moment interaction. Three types of three-body wave functions, Gaussian, Irving, and Irving-Gunn, are considered. An upper limit on the probability of the S' state of mixed symmetry ($^2S_{1/2}$ state with $T=\frac{1}{2}$) is deduced from the experimental capture rate.

I. INTRODUCTION

RECENT experiments[1] on the elastic scattering of high-energy electrons from H^3 and He^3 have been interpreted theoretically by Schiff[2] in terms of three wave functions based on different dependence on the internucleon distances. The spatial wave functions used are exponential, Gaussian, and Irving. Two parameters α and P, associated with each wave function, appear in the calculations. The first is a size parameter for which the value obtained from these experimental data is in good agreement with the one obtained from the Coulomb energy of He^3. The other parameter is the probability of the mixed-symmetry S' state which is found to be about 4%.

The purpose of our calculation is to obtain an independent estimate for an upper limit on this probability by a study of the capture rate of thermal neutrons by deuterons.[3] The contribution to this capture rate from the interaction with emission of magnetic dipole radiation by the nucleon spins is independent of the dominant fully symmetric S state, as was shown many years ago by Schiff,[4] and is particularly sensitive to the mixed-symmetry S' state probability. Indeed, it was proposed in Ref. 2 as an independent method of determining this probability. We give also an estimate of the order of magnitude of the exchange moment contribution to this capture rate.

In Sec. II we discuss the various states to be considered for the triton ground-state wave function as well as for the continuum state, and the necessary spin-isotopic spin formalism is developed. Section III deals with the theoretical calculation of the total capture rate for the two cases considered (i.e., emission of magnetic dipole radiation by the nucleon spins and by the exchange magnetic moment). In Sec. IV we discuss in detail the spatial wave functions that have been used. Section V gives a discussion of the numerical results.

II. THEORETICAL FORMALISM

In order to study the symmetry properties of the three-body functions, we make use of the classification of triton wave functions given by Derrick and Blatt[5] and adopt Schiff's[2] notation.

Among the ten possible states in which the ground-state wave function of the three-nucleon system with even parity and $J=T=\frac{1}{2}$ (charge invariance requires the ground state to have $T=\frac{1}{2}$) can be classified, we shall be concerned only with the fully symmetric $^2S_{1/2}$ state and the mixed-symmetry $^2S_{1/2}$ state, denoted, respectively, by S and S'. There are good reasons (given in Refs. 2, 5, and 6) to believe that the antisymmetric $^2S_{1/2}$ state, the three $^2P_{1/2}$ states, and the $^4P_{1/2}$ state give a negligible contribution.

In the case of a transition occurring under the influence of a spin magnetic dipole radiation which, at low energies, is the dominant case, the possibility of D-state contribution is ruled out because of the orthogonality of the S and D spatial functions and because transitions from D to D states give a negligible contribution.

In the case of transition occurring under the influence of exchange currents, Austern[7] has shown that the electric-quadrupole contribution is completely negligible. His calculation included transitions from the continuum fully symmetric S state to the triton $^4D_{1/2}$ state as well as transitions from the ^2D part of the continuum state to the triton $^2S_{1/2}$ state.

Thus we are concerned only with the $^2S_{1/2}$ states S and S' for the triton ground state and with the $^2S_{1/2}$ and and $^4S_{1/2}$ states for the continuum. The doublet spin

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[1] H. Collard, R. Hofstadter, A. Johansson, R. Parks, M. Ryneveld, A. Walker, M. R. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. Letters **11**, 132 (1963).

[2] L. I. Schiff, Phys. Rev. **133**, B802 (1964).

[3] N. T. Meister, T. K. Radha, and L. I. Schiff, Phys. Rev. Letters **12**, 509 (1964).

[4] L. I. Schiff, Phys. Rev. **52**, 242 (1937).

[5] G. Derrick and J. M. Blatt, Nucl. Phys. **8**, 310 (1958).

[6] J. M. Blatt, G. H. Derrick, and J. N. Lyness, Phys. Rev. Letters **8**, 323 (1962).

[7] N. Austern, Phys. Rev. **83**, 672 (1951); **85**, 147 (1951).

functions are

$$\chi_1=6^{-1/2}[(+\,+\,-)+(+\,-\,+)-2(-\,+\,+)],$$
$$\chi_2=2^{-1/2}[(+\,+\,-)-(+\,-\,+)].$$

The quartet spin functions are

$$\chi_3=(+\,+\,+),$$
$$\chi_4=3^{-1/2}[(+\,+\,-)+(+\,-\,+)+(-\,+\,+)].$$

A + (or −) indicates that the nucleon has spin up (or down).

The isospin functions η_1 and η_2 for a system of two neutrons and one proton, in a state of quantum number $T=\frac{1}{2}$, are as follows:

$$\eta_1=6^{-1/2}[2(+\,-\,-)-(-\,-\,+)-(-\,+\,-)],$$
$$\eta_2=2^{-1/2}[(-\,+\,-)-(-\,-\,+)].$$

A + (or −) indicates that the nucleon is a proton (or a neutron).

The combinations of spin and isotopic spin functions that are needed are the following:

$$\begin{aligned}\phi_0&=2^{-1/2}(\chi_2\eta_1-\chi_1\eta_2)\,,\\ \phi_1&=2^{-1/2}(\chi_2\eta_2-\chi_1\eta_1)\,,\\ \phi_2&=2^{-1/2}(\chi_2\eta_1+\chi_1\eta_2)\,,\\ \phi_3&=\chi_3\eta_1\,,\\ \phi_4&=\chi_3\eta_2\,,\\ \phi_5&=\chi_4\eta_1\,,\\ \phi_6&=\chi_4\eta_2\,.\end{aligned}\tag{1}$$

We note that the four χ's, the two η's, and the seven ϕ's form three orthonormal sets of functions, that ϕ_0 is fully antisymmetric, and that the functions (ϕ_1,ϕ_2), (ϕ_3,ϕ_4), and (ϕ_5,ϕ_6) from three sets of functions of mixed symmetry which transform according to the permutation table given in Eqs. (3) of Ref. 2.

The triton function has the following form:

$$\psi_T=u\phi_0+v_2\phi_1-v_1\phi_2\,,\tag{2}$$

where u is a fully symmetric space function and v_1 and v_2 are space functions of mixed symmetry defined as follows:

$$\begin{aligned}v_1&=6^{-1/2}[g(12,3)+g(13,2)-2g(23,1)]\,,\\ v_2&=2^{-1/2}[g(12,3)-g(13,2)].\end{aligned}\tag{3}$$

$g(12,3)$ is a function symmetric in an interchange of particles 1 and 2, but neither symmetric nor antisymmetric in an interchange of 1 and 3 or 2 and 3.

The continuum wave function is given by

$$\begin{aligned}\psi_{\text{cont}}{}^{\text{Doublet}}&=u'\phi_0+v_{2d}'\phi_1-v_{1d}'\phi_2\,,\\ \psi_{\text{cont}}{}^{\text{Quartet}}&=v_{2q}'\phi_3-v_{1q}'\phi_4+v_{2q}'\phi_5-v_{1q}'\phi_6\,.\end{aligned}\tag{4}$$

u', v_{1d}' and v_{1q}', and, v_{2d}' and v_{2q}', are spatial functions which exhibit the same symmetry properties as u, v_1, and v_2, respectively.

The spatial functions u, v_1, and v_2 are orthogonal. Similarly u', v_{1d}', v_{1q}', v_{2d}', and v_{2q}' are orthogonal. Because of the orthogonality of the continuum state and the bound state, we have the additional relation:

$$\langle\psi_T\psi_{\text{cont}}\rangle=0\,,$$

which reduces to

$$\langle uu'\rangle+\langle v_1v_{1d}'\rangle+\langle v_2v_{2d}'\rangle=0\,,\tag{5}$$

where we have used the orthogonality of the spin-isotopic spin functions ϕ. The following additional relations, coming from the symmetry properties of the spatial functions, have been used in our computation:

$$\begin{aligned}\langle uv_{2q}'\rangle=\langle uv_{2d}'\rangle=\langle v_2u'\rangle=\langle v_1v_{2d}'\rangle\\ =\langle v_2v_{1d}'\rangle=\langle v_1v_{2q}'\rangle=\langle v_2v_{1q}'\rangle=0.\end{aligned}\tag{6}$$

III. CAPTURE RATE CALCULATION

A. Direct Magnetic Dipole Transition

At thermal energies, as was already mentioned in the preceding section, the transition occurs mainly through magnetic dipole radiation. The nucleon spin transition operator is

$$\mathbf{G}_{\text{direct}}=\tfrac{1}{2}\sum_{i=1}^{3}[\mu_p\boldsymbol{\sigma}_i(1+\tau_{iz})f_{\text{mag}}{}^{p}(\mathbf{r}'-\mathbf{r}_i)+\mu_n\boldsymbol{\sigma}_i(1-\tau_{iz})f_{\text{mag}}{}^{n}(\mathbf{r}'-\mathbf{r}_i)].\tag{7}$$

The σ's and τ's are unit amplitude Pauli matrices that operate, respectively, on the χ's and η's. μ_p and μ_n are, respectively, the static magnetic moment of a proton and a neutron. $\mathbf{r}_i$ is the position vector of particle i with respect to the center of gravity of the three nucleons. The functions $f_{\text{mag}}{}^{p}(\mathbf{r}'-\mathbf{r}_i)$ and $f_{\text{mag}}{}^{n}(\mathbf{r}'-\mathbf{r}_i)$ can be considered as the spatial distributions of moment densities about the centers of the nucleons.

The transition matrix element is given by

$$\begin{aligned}(\text{M.E.})_{\text{direct}}&=\int\int\psi_T(\mathbf{r}_i)e^{i\mathbf{q}\cdot\mathbf{r}'}\mathbf{G}\psi_{\text{cont}}(\mathbf{r}_i)d^3r_id^3r'\\ &=\mu_pF_{\text{mag}}{}^{p}(q)\sum_{i=1}^{3}\int\psi_T(\mathbf{r}_i)e^{i\mathbf{q}\cdot\mathbf{r}_i}\boldsymbol{\sigma}_i\tfrac{1}{2}(1+\tau_{iz})\\ &\quad\times\psi_{\text{cont}}(\mathbf{r}_i)d^3r_i+\mu_nF_{\text{mag}}{}^{n}(q)\sum_{i=1}^{3}\int\psi_T(\mathbf{r}_i)\\ &\quad\times e^{i\mathbf{q}\cdot\mathbf{r}_i}\boldsymbol{\sigma}_i\tfrac{1}{2}(1-\tau_{iz})\psi_{\text{cont}}(\mathbf{r}_i)d^3r_i\,,\end{aligned}$$

with

$$F_{\text{mag}}{}^{p}(q)=\int e^{i\mathbf{q}\cdot\mathbf{R}}f_{\text{mag}}{}^{p}(\mathbf{R})d^3R\,.$$

A similar expression is obtained for $F_{\text{mag}}{}^{n}(q)$.

In the static limit where $q^2\to0$, which is applicable for slow incoming neutrons, it is reasonable to assume that the functions $f_{\text{mag}}{}^{p}(\mathbf{r}'-\mathbf{r}_i)$ and $f_{\text{mag}}{}^{n}(\mathbf{r}'-\mathbf{r}_i)$ can be taken as δ functions. In that case, $F_{\text{mag}}{}^{p}(q)$ and $F_{\text{mag}}{}^{n}(q)$

being normalized to unity at $q^2 \to 0$, may be set equal to unity.

The mean transition probability is given in terms of

$$\bar{M}_{\text{mag dipole}}^2 = \frac{1}{6}\sum_m \sum_{x,y,z} \{ |\langle \psi_{T,m}| e^{i\mathbf{q}\cdot\mathbf{r}'} G_x |\psi_{\text{cont}}^{\text{Doublet}}\rangle|^2 + |\langle \psi_{T,m}| e^{i\mathbf{q}\cdot\mathbf{r}'} G_x |\psi_{\text{cont}}^{\text{Quartet}}\rangle|^2 \}. \quad (8)$$

The summation over m corresponds to the two possible spin projection $m=\pm\frac{1}{2}$ for the bound state.

After summation on spin and isotopic-spin variables, one finds the following result:

$$\bar{M}_{\text{direct}}^2 = (4/9)(\mu_n-\mu_p)^2[(\langle v_1 v_{1d}'\rangle + \langle v_2 v_{2d}'\rangle)^2 + (\langle v_1 v_{1q}'\rangle + \langle v_2 v_{2q}'\rangle)^2]. \quad (9)$$

It is to be noted that this expression vanishes if there is no state S' of mixed symmetry, as was mentioned in Sec. I. This result, originally obtained in Ref. 4 without the isospin formalism, requires the use of the orthogonality relation (5).

B. Exchange Magnetic Dipole Transition

We assume that the exchange moment arises only when two of the nucleons are close together. We assume also that the exchange-moment spatial distribution is close to the center of mass of this pair. $\mathbf{G}_{\text{exchange}}$ is assumed to be of isovector character, as are the static moments, and hence is given by the following expression:

$$\mathbf{G}_{\text{exchange}} = (\mu_T - \mu_p)\sum_{i<j} u(r_{ij})\sigma_{ij}(\tau_{ij})_z. \quad (10)$$

The τ dependence is chosen to satisfy the requirement that, for mirror nuclei, the expectation values of expression (10) be of opposite signs. Thus we have the two possibilities:

$$\text{(a)} \quad \tau_{ij} = \tau_i - \tau_j,$$
$$\text{(b)} \quad \tau_{ij} = \tau_i \times \tau_j.$$

The σ dependence has also to be antisymmetric in the interchange of particles i and j because the space dependence $u(r_{ij})$ is symmetric. We thus obtain for the axial vector form of σ_{ij}:

$$\text{(a}') \quad \sigma_{ij} = \sigma_i - \sigma_j,$$
$$\text{(b}') \quad \sigma_{ij} = \sigma_i \times \sigma_j.$$

It turns out that the combinations (a)(a′) and (b)(b′) give identical results for the matrix elements of $\mathbf{G}_{\text{exchange}}$. Similarly combinations (a)(b′) and (b)(a′) give identical results. However, the contributions associated with (a)(b′) and (b)(a′) are proportional to the S'-state probability because the expectation value of $\mathbf{G}_{\text{exchange}}$ between the symmetric part of the triton ground-state wave function vanishes. Henceforth we consider only the combination (a)(a′).

For the spatial dependence we choose the following phenomenological model, suggested by Schiff[8]:

$$u(r_{ij}) = g(r_{ij}) f(\mathbf{r}' - \tfrac{1}{2}(\mathbf{r}_i + \mathbf{r}_j)), \quad (11)$$

where $g(r_{ij})$ gives the dependence on the separation of the pair and $f(\mathbf{r}'-\frac{1}{2}(\mathbf{r}_i+\mathbf{r}_j))$ gives the distribution of the exchange moment about the center of mass of the pair ij at a point $\mathbf{r}'$.

Following the same procedure as the one outlined for the case of a transition through spin magnetic dipole radiation, we make the assumption that $f[\mathbf{r}'-\frac{1}{2}(\mathbf{r}_i+\mathbf{r}_j)]$ can be taken as a δ function. Similarly, at the static limit, where $q \to 0$, the transition matrix element for the exchange current is independent of the form factor $F(q)$ which is normalized to unity.

We have retained only the fully symmetric S state for the bound state, because of the small percentage of mixed-symmetry S' state. However, in the case of the continuum state, there is no reason to believe that the mixed-symmetry S' states are present only with a small probability, so that these states have also been included.

After summation on spin and isotopic-spin variables, we find that the continuum doublet S' state does not give any contribution and obtain the following result:

$$\bar{M}_{\text{exchange}}^2 = (\mu_T-\mu_p)^2(16/3) \times \{\tfrac{1}{3}\langle u[g(r_{12})+g(r_{23})+g(r_{31})]u'\rangle^2 + \tfrac{1}{4}[\langle u[g(r_{12})-g(r_{13})]v_{2q}'\rangle + (1/\sqrt{3}) \times \langle u[g(r_{12})+g(r_{13})-2g(r_{23})]v_{1q}'\rangle]^2\}. \quad (11\text{a})$$

By symmetry arguments this result can be simplified to the following form:

$$\bar{M}_{\text{exchange}}^2 = (\mu_T-\mu_p)^2 \times 16\{\langle ug(r_{12})u'\rangle^2 + \tfrac{1}{3}[\langle ug(r_{12})v_{2q}'\rangle + (1/\sqrt{3})\langle u[g(r_{12})-g(r_{23})]v_{1q}'\rangle]^2\}. \quad (11\text{b})$$

The second term of (11) can be further simplified; it reduces to

$$\langle ug(r_{23})v_{1q}'\rangle^2.$$

Thus the final result can be expressed as follows:

$$\bar{M}_{\text{exchange}}^2 = (\mu_T-\mu_p)^2 \times 16[\langle ug(r_{12})u'\rangle^2 + \langle ug(r_{23})v_{1q}'\rangle^2]. \quad (11\text{c})$$

Three alternative forms for $u(r_{ij})$ are discussed by Sachs.[9] Of these, the form (9.38b) is equivalent to attaching a spatial exchange operator $P_{ij\,\text{spatial}}$ to expression (11). However, using the transformation properties of u_1', v_1', and v_2' under the spatial exchange operators given in Eq. (3) of Ref. 2, we find that the result (11c) is unaltered. The two other forms mentioned by Sachs[9] represent spin-orbit coupling. They have not been considered in the present work.

[8] L. I. Schiff (private communication).

[9] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Cambridge, Massachusetts, 1953), p. 253; and also M. Verde, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 144.

C. Total Magnetic Dipole Transition

The square of the total matrix element, including both direct and exchange magnetic dipole transitions, is given by the following expression:

$$\bar{M}_{\text{total}}^2=(16/9)\{[\tfrac{1}{2}(\mu_n-\mu_p)(\langle v_1v_{1d}'\rangle+\langle v_2v_{2d}'\rangle)+3(\mu_T-\mu_p)\langle ug(r_{12})u'\rangle]^2+[\tfrac{1}{2}(\mu_n-\mu_p)(\langle v_1v_{1q}'\rangle+\langle v_2v_{2q}'\rangle)+3(\mu_T-\mu_p)\langle ug(r_{23})v_{1q}'\rangle]^2\}. \quad (12)$$

The capture cross section is

$$\sigma=\frac{1}{v}\left(\frac{m_e}{m_p}\right)^{3/2}\frac{W_r^3e^2\hbar}{3m_p^2e^2}\bar{M}^2, \quad (13)$$

where W_r is the energy, expressed in nuclear units (0.506 MeV), given up to the photon, m_e is the electron mass, m_p is the proton mass, and v is the mean velocity of thermal neutrons.

IV. SPACE FUNCTIONS

As with the bound state, we define, for the continuum state, the fully symmetric spatial function u' and the functions of mixed symmetry v_1' and v_2' as follows:

$$u'=A'(3)^{-1/2}[g'(12,3)+g'(13,2)+g'(23,1)],$$
$$v_1'=B'(6)^{-1/2}[g'(12,3)+g'(13,2)-2g'(23,1)],$$
$$v_2'=B'(2)^{-1/2}[g'(12,3)-g'(13,2)],$$

where $g'(12,3)$ [similar to the function $g(12,3)$ which appears in the definition of the bound state] is a symmetric function with respect to the interchange of particles 1 and 2 but does not exhibit symmetry properties with respect to the interchange of nucleons 1 and 3 or 2 and 3. Proper normalization of ψ_{cont} leads to

$$A'=-1/(2)^{1/2} \quad \text{and} \quad B'=-\tfrac{1}{2}.$$

Three types of bound-state wave functions are considered: Gaussian, Irving, and Irving-Gunn functions. For consistency we have associated a Gaussian continuum wave function to the Gaussian bound-state wave function and an exponential continuum wave function to the Irving and Irving-Gunn bound-state functions. Details of the calculation are given in Appendices A and B.

A. Gaussian Wave Functions

1. Triton Wave Function

We have adopted Schiff's[2] Gaussian wave function for the triton ground state.

The fully symmetric space function is given by Eq. (24) of Ref. 2.

$$u=A\exp[-(\alpha^2/2)(r_{12}^2+r_{13}^2+r_{23}^2)],$$

with

$$A^2=(3^{3/2}\alpha^6)/\pi^3 \quad \text{and} \quad \alpha=0.384\ \text{F}^{-1}. \quad (14)$$

The states of mixed symmetry are given in terms of the following function $g(12,3)$.

$$g(12,3)=B\exp[-(\alpha'^2/2)(r_{13}^2+r_{23}^2)-(\beta^2/2)r_{12}^2],$$

where we allow the size parameter α' occurring in $g(12,3)$ to be different from the parameter α of the fully symmetric state.

We define $\epsilon=\alpha'-\beta$.

For small ϵ, the probability of the state S' is given by

$$P\simeq(\pi^3B^2\epsilon^2)/(3^{3/2}\alpha'^8). \quad (15)$$

2. Continuum Wave Function

Among the various calculations existing on elastic scattering of slow neutrons by deuterons,[10] the only wave function that seems to be in good agreement with experimental results is of Gaussian form. We have used the trial function adopted by Burke and Haas, which is one of the few functions available where the coefficients, fitted by a variational calculation, are given explicitly and which is in fairly good agreement with experimental results.

When polarization of the deuteron by the incident neutron is taken into account, they evaluate the doublet and quartet scattering lengths to be $a_2=2.97$ F and $a_4=5.72$ F, respectively. These values exhibit an insignificant change from the scattering lengths obtained when polarization effects are not included. ($a_2=2.72$ F and $a_4=5.24$ F). Thus, we have neglected these polarization effects in our computation and Eq. (7) of Ref. 11 becomes

$$g'(\bar{1}\bar{2},3)=\chi(\mathbf{R})F(\mathbf{r}),$$

with

$$\chi(\mathbf{R})=[e^{-aR^2}+ce^{-bR^2}]/n,$$

where the constants a and b are given in Ref. 12, and $F(\mathbf{r})=f(\mathbf{r})/(kr)$. Since we are concerned with S wave functions we need only the spherically symmetric term $f_0(r)$ given by

$$f_0(r)=(1-c_1\exp(-\nu_1r^2))\sin kr+(c_0-c_2\exp(-\nu_1r^2))(1-\exp(-\nu_1r^2))\cos kr, \quad (16)$$

where

$$\mathbf{R}=\mathbf{r}_1-\mathbf{r}_2;\quad \mathbf{r}=-\mathbf{r}_3+\tfrac{1}{2}(\mathbf{r}_1+\mathbf{r}_2);$$

n is a normalization factor given by the following expression:

$$n^2=(\pi/2a)^{3/2}+2c(\pi/(a+b))^{3/2}+c^2(\pi/2b)^{3/2}.$$

The only difference between doublet and quartet S' states is in the constants occurring in $f_0(r)$ [Eq. (16)].

[10] T. C. Griffith and E. A. Power, *International Conference on Nuclear Forces and the Few-Nucleon Problems* (Pergamon Press, Inc., New York, 1960), Vol. 2.

[11] P. G. Burke and F. H. Haas, Proc. Roy. Soc. (London) **A252**, 177 (1959).

[12] P. G. Burke and H. H. Robertson, Proc. Phys. Soc. (London) **A70**, 777 (1957).

As was stated in Sec. II, the two wave functions, ψ_{cont} and ψ_T of the continuum and bound states, respectively, have to be orthogonal. But the functions we have at our disposal are approximate ones which do not satisfy the orthogonality relation (5). Thus, in order to realize this property, we have adjusted the coefficient c_1 appearing in $f_0(r)$ for the doublet case. We have reduced the value given by Burke and Haas for c_1 from 3.84 to 2.05. This adjustment leaves the quartet scattering length, which is the one in closest agreement with experiments, as well as the asymptotic behavior of the continuum functions, unchanged.

3. Exchange Moment

To be consistent again, we choose for the spatial part of the exchange moment a Gaussian form given by

$$g(r_{ij})=C_G\exp(-\mu_G r_{ij}^2).$$

Since it is assumed that exchange effects are due to two-body interactions, we take for μ_G the square of the range of nuclear forces ($\mu_G=0.5625\times10^{26}$ cm^{-2}).

The constant C_G is determined by equating the expectation value of the exchange operator (10) between the dominant fully symmetric part of the triton ground state to $(\mu_T-\mu_p)$. It is found to be equal to 1.668.

4. Form-Factor Analysis

An independent estimate of the S'-state probability based on the elastic electron-scattering data[1] was obtained by Schiff.[2] The body form factors $F_1(q)$ and $F_2(q)$, defined, respectively, in Eqs. (13) and (14) of Ref. 2, were used to determine the two unknown parameters α and P. We give below the expressions for $F_1(q)$ and $F_2(q)$ taking into account the possibility of a different size parameter α' for the mixed symmetry state S'.

$$F_1(q)=\exp(-q^2/18\alpha^2),$$

$$F_2(q)=\left(\frac{P}{6}\right)^{1/2}\left(\frac{q^2}{6\alpha^2}\right)\left(\frac{2\alpha\alpha'}{\alpha^2+\alpha'^2}\right)^5\exp(-q^2/[9(\alpha^2+\alpha'^2)]).$$

B. Irving Wave Function

1. Triton Wave Function

The fully symmetric space function is given as follows:

$$u=A\exp[-(\alpha/2)(r_{12}^2+r_{13}^2+r_{23}^2)^{1/2}],$$

with $A^2=(3^{3/2}\alpha^6)/(120\pi^3)$ and $\alpha=1.27$ F^{-1}.

The states of mixed symmetry are given in terms of the following function $g(12,3)$.

$$g(12,3)=B\exp[-(\alpha^2r_{12}^2+\alpha^2r_{23}^2+\beta^2r_{12}^2)^{1/2}+(\alpha/2)(r_{12}^2+r_{13}^2+r_{23}^2)^{1/2}].$$

The probability is given by

$$P=420\pi^3(B\epsilon)^2/(3^{3/2}\alpha'^8).$$

2. Continuum Wave Function

In this case, since there is no reason to adopt a Gaussian form for the continuum wave function, we choose for the deuteron wave function $\chi(\mathbf{R})$ a Hulthen wave function.

$$\chi(\mathbf{R})=(N/(4\pi)^{1/2})[\exp(-aR)-\exp(-bR)]/R,$$

with $a=0.232$ F^{-1}, $b=1.434$ F^{-1} and $N^2=0.766$ F^{-1}.

For the neutron function $F(r)$ we choose the following form:

$$F(r)=(1-c_1\exp(-\nu r))-(a_s/r)(1-\exp(-\nu r)),$$

where a_s represents the neutron-deuteron scattering length. We choose for a_s the accepted set of experimental values:

$$(a_s)_{\text{doublet}}=0.8\text{ F}\quad(a_s)_{\text{quartet}}=6.2\text{ F}.$$

Choosing a reasonable value for the parameter ν, namely $\nu=0.5$ F^{-1}, the coefficient c_1 is determined by the orthogonality of ψ_{cont} and ψ_T. It is to be noted that the orthogonality relation (5) is not sensitive to variations of the parameter ν, while it is very sensitive to variations of the coefficient c_1, just as was found for the Gaussian case. In the present case, we have determined the value of c_1 to be equal to 3.8.

3. Exchange Moment

We choose for the spatial part of the exchange moment the following form:

$$g(r_{ij})=C_I\exp(-\mu_I r_{ij}),$$

where $\mu =0.75$ F^{-1} and C_I is found to be equal to 1.21.

4. Form-Factor Analysis

The two-body form factors $F_1(q)$ and $F_2(q)$ are given by

$$F_1(q)=[1+(2q^2/9\alpha^2)]^{-7/2},$$

$$F_2(q)=(21P)^{1/2}\left(\frac{2\alpha^3}{9\alpha'^2}\right)\left(\frac{\alpha+\alpha'}{2\alpha'}\right)\times q^2\left[\left(\frac{\alpha+\alpha'}{2\alpha'}\right)^2+\frac{2q^2}{9\alpha'^2}\right]^{-9/2}.$$

C. Irving-Gunn Wave Function

1. Triton Wave Function

$$u=A\exp[-(\alpha/2)(r_{12}^2+r_{13}^2+r_{23}^2)^{1/2}]/(r_{12}^2+r_{13}^2+r_{23}^2)^{1/2},$$

with $A^2=3^{1/2}\alpha^4/(2\pi^3)$ and $\alpha=0.769$ F^{-1} as given in Ref. 13.

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[13] B. L. Berman, L. J. Koester, and J. H. Smith, Phys. Rev. **133**, B117 (1964).

The function $g(12,3)$ is given by

$$g(12,3)=B\frac{\exp[-(\alpha^2 r_{13}^2+\alpha^2 r_{23}^2+\beta^2 r_{12}^2)^{1/2}+(\alpha/2)(r_{12}^2+r_{13}^2+r_{23}^2)^{1/2}]}{(r_{12}^2+r_{13}^2+r_{23}^2)^{1/2}}.$$

The probability P is given in terms of $B\epsilon$ and α' by the following relation:

$$P=10\pi^3(B\epsilon)^2/(3^{3/2}\alpha'^6).$$

2. *Continuum Wave Function*

We choose for the continuum wave function the same form, and for the parameter ν the same value, as the ones used in the Irving case. The orthogonality between ψ_{cont} and ψ_T determine the coefficient c_1 to be equal to 4.9.

3. *Exchange Moment*

Again we choose for the spatial part of the exchange moment operator the same form as the one given in the case of Irving wave function.

$$g(r_{ij})=C_{\text{I.G.}}\exp(-\mu_{\text{I.G.}}r_{ij}),$$

where $\mu_{\text{I.G.}}=\mu_{\text{I.}}=0.75\ \text{F}^{-1}$ and $C_{\text{I.G.}}=1.25$.

4. *Form-Factor Analysis*

The form factors $F_1(q)$ and $F_2(q)$ are given by the following expressions:

$$F_1(q)=(\tfrac{4}{3})\frac{[1+2(1+2q^2/9\alpha^2)^{1/2}]}{[1+(1+2q^2/9\alpha^2)^{1/2}]^2}(1+2q^2/9\alpha^2)^{-3/2},$$

$$F_2(q)=\left(\frac{8\alpha^2q^2}{3\alpha'^4}\right)\left(\frac{P}{10}\right)^{1/2}(x^2+2q^2/9\alpha'^2)^{-3/2}(x+(x^2+2q/9\alpha'^2)^{1/2})^{-2}$$
$$\times\left[\frac{(x+2(x^2+2q^2/9\alpha'^2)^{1/2})}{(x^2+2q+9\alpha'^2)}+(\tfrac{2}{3})\frac{1}{(x+(x^2+2q^2/9\alpha'^2)^{1/2})}\right],$$

with

$$x=(\alpha+\alpha')/2\alpha'$$

V. NUMERICAL RESULTS

The latest experimental data on the capture of thermal neutrons of mean velocity $v=2.2\times10^5$ cm/sec by deuterium are due to Jurney and Motz,[14] and give a value of 0.60 ± 0.05 mb for the cross section.

An upper limit for the probability P of the state of mixed symmetry S', obtained by neglecting the contribution from the exchange moment interaction, has already been reported.[3,15] We find that this probability depends sensitively on the parameter α' which occurs in the S'-state wave function. For $\alpha'=\alpha$, which was the case considered by Schiff,[2] this probability is found to be, respectively, 0.09, 0.005, and 0.003% for the Gaussian, Irving, and Irving-Gunn wave function.

If one considers only the capture due to exchange moment interaction, it is found that the cross section σ_{exchange}, which is independent of α', is of the same order of magnitude as the experimental capture-rate cross section in the case of Gaussian wave function. In the cases of Irving and Irving-Gunn wave functions, the order of magnitude of the exchange moment contribution is found to be, respectively, 50 and 100 times the experimental value. In Table I we give the cross section σ_{exchange} for the different types of wave functions.

However, it is found that the spin and exchange moment contributions interfere destructively and thus the value of the S'-state probability is considerably increased, particularly in the case of the Irving-Gunn wave function. This probability is also sensitively dependent on the value of the parameter α' occurring in the S'-state wave function. For each value of α',

TABLE I. The contribution to the capture cross section due to exchange interaction only is given for Gaussian, Irving, and Irving-Gunn wave functions.

	σ_{exchange} (mb)
Gaussian wave function:	0.24
Irving wave function:	29.6
Irving-Gunn wave function:	59.8

[14] F. T. Jurney and H. T. Motz (unpublished).
[15] The values quoted in Ref. 3, were rough estimates; they have been changed by a more precise calculation to those in column (1).

TABLE II. Values for an upper limit of the probability P of the state S' of mixed symmetry obtained from the slow-neutron capture rate and from the H^3 and He^3 form-factor analysis, are given for different values of the size parameter α'.

	P			
α'	From slow-neutron capture rate neglecting exchange moment (%)	From slow-neutron capture rate including exchange moment Solution 1 (%)	From slow-neutron capture rate including exchange moment Solution 2 (%)	From form-factor analysis (%)
	(a) Gaussian wave functions			
α	0.093	0.012	0.23	3.5
1.1α	0.21	0.025	0.53	2.9
1.3α	1.02	0.11	2.74	2.5
2.0α	15	1.9	35	7
	(b) Irving wave functions			
α	0.005	0.044	0.41	4.6
1.1α	0.0065	0.043	0.58	4.3
1.3α	0.011	0.022	1.05	4.26
1.5α	0.016	0.0003	1.57	4.73
2.0α	0.028	0.15	2.53	8.06
	(c) Irving-Gunn wave functions			
α	0.003	0.20	0.31	3.20
1.1α	0.004	0.35	0.41	2.93
1.3α	0.008	0.67	0.965	2.71
1.5α	0.016	1.10	2.09	2.73
2.0α	0.062	0.46	12	3.50

two values for the probability are obtained as solutions of the quadratic equation (12).

We give in Table II this upper limit on P for different values of α'. For a comparative study we also tabulate the S' probability obtained from the H^3 and He^3 form-factor analysis, and the values obtained from the slow neutron capture rate neglecting the exchange moment contribution.

Assuming that the solution (2) [column (4) of Table II], obtained from capture-rate analysis is the one to be compared with the values of P obtained from form-factor analysis [column (5)], it is found that for $\alpha'=\alpha$, there is no agreement between these two values. As was reported in Ref. 3, the inclusion of the D-state contribution being studied by Schiff[16] is expected to lower the value of the S'-state probability needed to account for the electron elastic-scattering data. We have already mentioned that in our case the D-state contribution is negligible. Thus, at present, if one allows the size parameters α and α' to be different, the values of α' which seem to be most suitable in each case are $\alpha'=1.3\alpha$ for the Gaussian wave function and $\alpha'=1.5\alpha$ for the Irving-Gunn and perhaps for the Irving wave functions.

A theoretical analysis of the preliminary inelastic-scattering experimental data[17] by Griffy and Oakes[18] indicate that while the Irving-Gunn wave function gives the best agreement with the experimental results, the Gaussian wave function is in poor agreement. They also find in the case $\alpha'=\alpha$ that they can fit the experimental data with practically no S'-state admixture as shown in Fig. 4 of Ref. 18. The results of Table II(c) support this conclusion. However, an independent estimate of the parameter α' is necessary in order to fix the value of the probability P. Further, it is expected that a value of α' too different from α may not fit the charge form factor or the Coulomb energy of He^3. Thus we conclude from our results that the probability of the S' state of mixed symmetry cannot exceed 2%.

[16] L. I. Schiff, (private communication).
[17] A. Johansson, (private communication).
[18] T. A. Griffy and R. J. Oakes, Phys. Rev. **135**, B1161 (1964).

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APPENDIX A: GAUSSIAN WAVE FUNCTION

Evaluation of the integrals needed in the calculation.

(1)

$$I_1=\int d^3r\frac{d^3R}{R}e^{-aR^2-br^2-c\mathbf{r}\cdot\mathbf{R}}\sin(\tfrac{1}{2}kR)=\frac{4\pi^3k}{(4ab-c^2)^{3/2}}\exp\left[-\frac{k^2b}{4(4ab-c^2)}\right].\tag{A1}$$

(2) The integral I_2, similar to I_1, but with $\sin(\frac{1}{2}kR)$ replaced by $\cos(\frac{1}{2}kR)$, requires some attention. The integration on r can be easily performed. The result is the following:

$$I_2=\int d^3r\frac{d^3R}{R}e^{-aR^2-br^2-c\mathbf{r}\cdot\mathbf{R}}\cos(\tfrac{1}{2}kR)=\frac{4\pi^{5/2}}{b^{3/2}}\int_0^\infty dR\,R\cos(\tfrac{1}{2}kR)e^{-[(4ab-c^2)/(4b)]R^2}.$$

After partial integration one obtains

$$I_2=\frac{8\pi^{5/2}}{(4ab-c^2)b^{1/2}}(1-(\tfrac{1}{2}k)J)\tag{A2}$$

with

$$J=\int_0^\infty e^{-[(4ab-c^2)/(4b)]R^2}\sin(\tfrac{1}{2}kR)dR.$$

Let us replace $\sin(\frac{1}{2}kR)$ in J by its expression in terms of $\exp(\frac{1}{2}ikR)$. With appropriate changes of integration variable in the two terms of J, one obtains:

$$J=2\left(\frac{b}{4ab-c^2}\right)^{1/2}F(x),\tag{A3}$$

with

$$F(x)=e^{-x^2}\int_0^x e^{t^2}dt,$$

and

$$x=\tfrac{1}{2}k(b/(4ab-c^2))^{1/2}.$$

The function $F(x)$ has been tabulated, for real values of x, by Miller and Gordon[19] for values of x ranging from zero to 12.00. The function $F(x)$ can be expanded in the following way:

$$F(x)=x\left(1-\frac{2x^2}{3}+\frac{4x^4}{15}-\frac{8x^6}{105}+\frac{16x^8}{945}-\cdots\right).$$

The values of x, of interest to us, turned out to be always small enough for the approximation $F(x)\sim x$ to be valid.

The final expression of I_2 is given by

$$I_2=\frac{8\pi^{5/2}}{(4ab-c^2)(b^{1/2})}\times\left[1-k\left(\frac{b}{4ab-c^2}\right)^{1/2}F\left(\tfrac{1}{2}k\left(\frac{b}{4ab-c^2}\right)^{1/2}\right)\right]. \quad \text{(A4)}$$

APPENDIX B: IRVING AND IRVING-GUNN WAVE FUNCTIONS

Using the following notation.

$$r_{12}=|\boldsymbol{\varrho}+\tfrac{1}{2}\mathbf{r}|,\quad r_{13}=|\boldsymbol{\varrho}-\tfrac{1}{2}\mathbf{r}|\quad\text{and}\quad r_{23}=r,$$

we obtain

$$u=A\exp[-(\tfrac{1}{2}\alpha)(2\rho^2+3r^2/2)^{1/2}]/(2\rho^2+3r^2/2)^{n/2},$$

$$v_1=(6)^{-1/2}B\epsilon\frac{(2\rho^2-3r^2/2)}{(2\rho^2+3r^2/2)^{(n+1)/2}}\times\exp[-(\tfrac{1}{2}\alpha)(2\rho^2+3r^2/2)^{1/2}],$$

$$v_2=(2)^{1/2}B\epsilon\frac{\boldsymbol{\varrho}\cdot\mathbf{r}}{(2\rho^2+3r^2/2)^{(n+1)/2}}\times\exp[-(\tfrac{1}{2}\alpha)(2\rho^2+3r^2/2)^{1/2}],$$

with n equal to 0 and 1, respectively, for the Irving and Irving-Gunn wave functions.

The integrals required in the calculation are of the form

$$I=\int d^3r d\rho^3\frac{(2\rho^2-3r^2/2)^m}{(2\rho^2+3r^2/2)^{(n+1)/2}}\times\exp[-(\tfrac{1}{2}\alpha)(2\rho^2+3r^2/2)^{1/2}][\exp(-\lambda r)]\times\{1-c_1\exp(-\nu\rho)-(a_s/\rho)[1-\exp(-\nu\rho)]\}/r,$$

where the parameter m takes the values 0 or 1 and λ is a constant. Following the method given by Gunn and Irving,[20] we change the variables r and ρ into the new set R and ψ with

$$\sqrt{2}\rho=R\sin\psi\quad\text{and}\quad(\tfrac{3}{2})^{1/2}r=R\cos\psi.$$

R ranges from 0 to ∞ and ψ from 0 to $\frac{1}{2}\pi$. Although these integrals can be evaluated analytically, because of the complexity of the analytical result, the calculations were done numerically with a computer.

In evaluating the analytical expressions for the body form factors $F_1(q)$ and $F_2(q)$, we transform the two three-dimensional integrals over $\boldsymbol{\varrho}$ and $\mathbf{r}$ into one six-dimensional integral. The angular integration is performed by expanding the plane wave in Gegenbauer polynomials.[21] The final integration is performed using the following relations.[22]

$$\int_0^\infty[\exp(-pR)]R^3J_2(aR)dR=2^3a^2\Gamma(\tfrac{7}{2})\pi^{-1/2}p/(p^2+a^2)^{7/2}$$

and

$$\int_0^\infty[\exp(-pR)]RJ_2(aR)dR=\frac{a^2[p+2(p^2+a^2)^{1/2}]}{(p^2+a^2)^{3/2}[p+(p^2+a^2)^{1/2}]^2}.$$

[19] W. Lash Miller and A. R. Gordon, J. Phys. Chem. **35**, 2785 (1931).

[20] J. C. Gunn and J. Irving, Phil. Mag. **42**, 1353 (1951).

[21] A. Sommerfeld, *Partial Differential Equations in Physics* (Academic Press Inc., New York, 1949).

[22] W. Magnus and F. Oberhettinger, *Formulas and Theorems for the Functions of Mathematical Physics* (Chelsea Publishing Company, New York, 1954).