THE
PHYSICAL REVIEW
A journal of experimental and theoretical physics established by E. L. Nichols in 1893
SECOND SERIES, VOL. 136, NO. 2B 26 OCTOBER 1964

Low-Lying States of F^{19}†

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(Received 8 May 1964)

The core-particle coupling model employed in a previous paper for the interpretation of the low-lying positive-parity states of Ne^{21} is applied to F^{19}. The lowest positive-parity states of F^{19} are investigated in terms of the coupling of a 2*s*-1*d* hole to the Ne^{20} core. Satisfactory agreement with experimental results is obtained for parameters which compare well with the parameters used in Ne^{21}. The negative-parity states of F^{19} require the coupling of both a 2*s*-1*d* and a 1*p* hole. The large number of parameters in this model does not allow any definite conclusions for this case, though the preliminary results yield a reasonable interpretation of the *E*1 and *E*3 transition data.

I. INTRODUCTION

In this paper the calculations for the low-lying positive-parity states of Ne^{21} with a core-coupling model[1] are extended to the low-lying positive- and negative-parity states of F^{19}.

An interpretation of the properties of these states has been given by both the shell model[2–4] and the collective model.[5–9] While the positive-parity states are satisfactorily described by both models (see Table III), one encounters difficulties in explaining the slow $\frac{1}{2}^+ \to \frac{1}{2}^-$ 0.11-MeV and $\frac{1}{2}^+ \to \frac{3}{2}^-$ 1.46-MeV *E*1 transitions and the enhanced $\frac{1}{2}^+ \to \frac{5}{2}^-$ 1.35-MeV *E*3 transition. Litherland *et al.*[10] give values for

$$|M|^2 = B(E\lambda)/B(E\lambda)_{\text{s.p.}} \tag{1.1}$$

of

$$|M|^2 = 10^{-3} \quad \text{for the } E1 \text{ transitions},$$

$$|M|^2 = 12 \pm 4 \text{ for the } E3 \text{ transition}.$$

Here we suggest a structure of the low-lying F^{19} states as the coupled system of a collective Ne^{20} core and single-hole states. For the *positive-parity states* of F^{19} we can then deduce the following composition from the Ne^{20} energy spectrum below 10 MeV[11]:

$$\begin{aligned}\psi^{(+)}(F^{19}) = a_1^{(+)}\psi^{(+)}(Ne^{20},\ \text{ground-state band}) \\ \times\ \psi^{(+)}(2s-1d\ \text{hole}) \\ +a_2^{(+)}\psi^{(-)}(Ne^{20},\ 2\ \text{bands})\ \psi^{(-)}(1p\ \text{hole}) \\ +a_3^{(+)}\psi^{(+)}(Ne^{20},\ \text{higher bands}) \\ \times\psi^{(+)}(2s-1d\ \text{hole}).\end{aligned} \tag{1.2}$$

As in the calculation of the Ne^{21} properties, we neglect the last two contributions for the low-lying states of F^{19}. The negative-parity-core–negative-parity-hole states should lie more than 10 MeV above the first contribution, for we have a core separation of approximately 5 MeV and we need an additional 5–6 MeV to break up the 1*p* shell as is indicated by the position of the first negative-parity states in O^{16} and O^{17} (see Ajzenberg-Selove and Lauritsen[12]). The intrinsic configuration of the higher positive-parity-core states differs from the configuration of the ground-state band. The initial separation of approximately 7 MeV and the poorer overlap of the intrinsic-core wave functions will assure only a very small contribution towards the low-lying positive-parity states of F^{19}.

† Work submitted as a partial fulfillment of the requirements for a Ph.D. in Physics.

[1] R. M. Dreizler, Phys. Rev. **132**, 1166 (1963); referred to in the text as I.
[2] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).
[3] M. G. Redlich, Phys. Rev. **110**, 468 (1958).
[4] M. Harvey, Phys. Letters **3**, 209 (1963).
[5] G. Abraham and C. S. Warke, Nucl. Phys. **8**, 69 (1958).
[6] E. B. Paul, Phil. Mag. **2**, 311 (1957).
[7] K. H. Bhatt, Nucl. Phys. **39**, 375 (1962).
[8] G. Rakavy, Nucl. Phys. **4**, 375 (1957).
[9] B. E. Chi and J. P. Davidson, Phys. Rev. **131**, 366 (1963).
[10] A. E. Litherland, M. A. Clark, and C. Broude, Phys. Letters **3**, 204 (1963).
[11] A. E. Litherland, J. A. Kuehner, H. E. Gove, M. E. Clark, and E. Almqvist, Phys. Rev. Letters **7**, 98 (1961).
[12] F. Ajzenberg-Selove and T. Lauritsen, Nucl. Phys. **11**, 5 (1959).

The Hamiltonian of this system will then be the same as the one used in the case of the Ne^{21} states [see Eq. (I4.13)] and the parameters of the Hamiltonian should be similar to the Ne^{21} parameters [see Eq. (I5.23)] except for the modifications

(a) changes of sign due to the use of holes instead of particles (see Sec. III),

(b) particle parameters change slightly as we deal with an odd proton instead of a neutron (see Sec. IV),

(c) the addition of a "hole" might affect the core more than the addition of a particle (see Sec. IV).

With these assumptions we find that the $d_{3/2}$ contributions are too large. This is probably due to the fact that we neglect antisymmetrization effects in the proposed model, which, if taken properly into account, should make it more difficult to couple a $d_{3/2}$ hole to the Ne^{20} core than a $d_{5/2}$ or $s_{1/2}$ hole. To correct this deficiency we lift the $d_{3/2}$ single-hole part up by the introduction of a parameter $\Delta_{3/2}=\Delta$. This can be interpreted as a change of magnitude in the single-particle parameters for the coupling of a $d_{3/2}$ hole instead of a $d_{3/2}$ particle at the beginning of the $s-d$ shell (see Sec. III).

We find that with these assumptions method (b) of reference I (all $d_{5/2}$ states, 2^4–2^4 pole interaction) gives good agreement with experiment, while method (a) (truncation) does not give satisfactory results.

If we write the equivalent expression to Eq. (1.2) for the *negative-parity states* of F^{19} (only states with $J^\pi=\frac{1}{2}^-$; $\frac{3}{2}^-$ and $\frac{5}{2}^-$ will be considered):

$$\begin{aligned}\psi^{(-)}(F^{19})=&a_1^{(-)}\psi^{(+)}(Ne^{20}, \text{ground-state band})\\&\times\psi^{(-)}(1p \text{ hole})\\&+a_2^{(-)}\psi^{(-)}(Ne^{20}, 2 \text{ bands})\\&\times\psi^{(+)}(2s-1d \text{ hole})\\&+a_3^{(-)}\psi^{(+)}(Ne^{20}, \text{higher bands})\\&\times\psi^{(-)}(1p \text{ hole}),\end{aligned}\quad(1.3)$$

we find that the last term can be neglected by the same argument as before, while we have to take both the first and second terms into account, as the pure hole and core contributions bring these states close together in this case.

The measured negative-parity states of Ne^{20} have the spins[11] $L^\pi=1^-$, 2^-, $(3^-)^2$, 4^-, and 5^-. The 1^- state at 5.80 MeV is likely to be a state of the configuration Ne^{20} positive-parity core and a hole + particle state with negative parity, as a 1^- collective state can not arise from simple surface oscillations, but involves changes in the internal composition or the density of the nucleus, which require large energies (e.g., photonuclear vibrational state with $T=1$).[13] A similar structure is assumed for the "unnatural parity" states with $L^\pi=2^-$, 4^-. If this assumption holds, we would expect only a small contribution of these states coupled with a $2s-1d$ hole in the low-lying negative-parity states of F^{19}.

Furthermore, we will assume that the two 3^- states stem from a single-collective state with $L^\pi=3^-$, which is split by particle-hole terms. The position of the collective state can be taken as the center of gravity of the measured 3^- states in Ne^{20}. Though only one 5^- state is observed, the same argument should hold for this state. The position is given by the center of gravity of the measured state and a second state predicted by the $J(J+1)$ rule (see Ref. 11).

The Hamiltonian of the system of a $1p$ hole coupled to the Ne^{20} core ground-state band and a $2s-1d$ hole coupled to rotational states with $L^\pi=3^-$ and 5^- will then be given by the same Hamiltonian as for the positive-parity states of F^{19} plus two additional terms H_1 and H_3:

$$H=H_c+H_p+\sum_{k=1}^{4}H_k-D'(s\cdot L)\quad(1.4)$$

(see I, Sec. II). H_1,H_3 are dipole-dipole and octupole-octupole interaction terms between core and particle (hole) of the form [see I, Eq. (4.12)].

$$H_k=f_k\alpha_k(r_p)\alpha_k(R_c)\sum_{M_k=-k}^{k}(-)^{M_k}Y_{k,M_k}(c)Y_{k,-M_k}(p).\quad(1.5)$$

These terms give nonzero matrix elements between even-parity-core–odd-parity-hole and odd-parity-core–even-parity-hole states. (They give no contribution in the case of the F^{19} even-parity states, as we neglected the odd-parity-core–odd-parity-hole states.)

The $E3$ transition in F^{19} will then be mainly given by the collective transitions from the odd-parity-core states to the even-parity-core states and so show the measured enhancement. $E1$ transitions between these states should be forbidden, as they are of the type $T=0\rightarrow T'=0$ (T,T' isotopic spin). MacDonald[14] has shown that isotopic impurities introduced by Coulomb forces give a small contribution (impurity smaller than 3.9% for the Ne^{20} ground state in a statistical-model estimate). So the observed slow $E1$ transitions between negative and positive-parity states of F^{19} can be obtained by a partial cancellation of the reduced-core part and the hole part of the transition matrix elements.

It should be noted that the coupling of a $p_{1/2}$ hole to the 0^+ and 2^+ states of Ne^{20} at 0.00 and 1.63 MeV, respectively,[15] gives the required level spacing, but fails to yield the experimental transition rates.

Even with the simplifying assumptions described above the number of parameters of the Hamiltonian for negative-parity states is rather large. If we assume the negative-parity-core states to give the same radial matrix elements as the positive-parity-core states and if we fix the parameters, which do not influence the composition of the low-lying states greatly at reasonable values, we find that an acceptable fit of the energy spectrum and the three measured electric-transition

[13] A. M. Lane and E. D. Pendlebury, Nucl. Phys. **15**, 39 (1960).

[14] W. M. MacDonald, Phys. Rev. **100**, 51 (1955).

[15] R. F. Christy and W. A. Fowler, Phys. Rev. **96**, 851 (1954).

TABLE I. Properties of the low-lying states of F^{19}.

		$\|M^2\|$	Reference
$\mu_{1/2+}$	2.6287 ± 0.0007 nm	...	a
$\mu_{5/2+}$	3.63 ± 0.11 nm	...	see following text
$QM_{5/2+}$	$\pm0.13\times10^{-24}$ cm^2	...	b
$B(E1;\frac{1}{2}^-\rightarrow\frac{1}{2}^+)$	$(0.46\pm0.07)\times10^{-29}$ cm^2	$(1.2\pm0.2)10^{-3}$	see following text
$B(E2;\frac{5}{2}^+\rightarrow\frac{1}{2}^+)$	$(0.224\pm0.015)\times10^{-50}$ cm^4	8.1 ± 0.5	see following text
$B(E3;\frac{1}{2}^+\rightarrow\frac{5}{2}^-)$	$(0.80_{-0.27}^{+0.26})\times10^{-75}$ cm^6	12 ± 4	see following text
$B(E1;\frac{1}{2}^+\rightarrow\frac{3}{2}^-)$	$(0.752\pm?)\times10^{-29}$ cm^2	10^{-3}	see following text
$B(E2;\frac{1}{2}^+\rightarrow\frac{3}{2}^+)$	$(0.50_{-0.17}^{+0.16})\times10^{-50}$ cm^4	9 ± 3	see following text
$T(M1;\frac{3}{2}^-\rightarrow\frac{1}{2}^-)$	$(3.80\pm1.90)\times10^{13}$ sec^{-1}	...	see following text
$\Gamma(\frac{3}{2}^+\rightarrow\frac{1}{2}^+)/\Gamma(\frac{3}{2}^+\rightarrow\frac{5}{2}^+)$	$<4\%$	...	see following text

[a] J. E. Mack, Rev. Mod. Phys. 22, 64 (1950).
[b] K. Sugimoto, A. Mizobuchi, and Y. Yomanoto, J. Phys. Soc. Japan 13, 1548 (1963).

rates can be obtained for a large number of sets of the remaining parameters. If we try to distinguish between these sets by using the available data on the $M1$ transition of the $\frac{3}{2}^-$ state at 1.46 MeV to the $\frac{1}{2}^-$ state at 0.11 MeV, we find a transition probability too large by a factor of approximately 2.5 in comparison with the experimental value and varying very slowly within the sets of reasonable parameters.

In Sec. II the experimental data and previous theoretical results for F^{19} are summarized. The parameter changes for the coupling of a hole instead of a particle are discussed in Sec. III. The results of the energy fit, moments, and transition rates for the positive- and negative-parity states in terms of this model are given in Secs. IV and V, respectively.

II. EXPERIMENTAL AND THEORETICAL RESULTS FOR F^{19}

A. Experimental

F^{19} has been investigated by a large number of reactions. The resulting level scheme can be taken from the Nuclear Data Sheets.[16] The spins and parity of the 1.35- and 1.46-MeV states have been recently assigned by Prentice *et al.*[17] as $\frac{5}{2}^-$ and $\frac{3}{2}^-$, respectively. No assignments are available for the 3–4-MeV region. The measured static moments and the available data on transition rates between the six lowest states are summarized in Table I.

The lifetime of the 0.110-MeV $\frac{1}{2}^-$ state has been determined by Thirrion *et al.*[18] as

$$\tau_{1/2-}=(1\pm0.25)\times10^{-9}\text{ sec}, \qquad (2.1a)$$

corresponding to an $E1$ reduced transition probability of

$$B(E1;\tfrac{1}{2}^-\rightarrow\tfrac{1}{2}^+)=(0.47_{-0.09}^{+0.16})\times10^{-29}\text{ cm}^2. \qquad (2.1b)$$

This value is in rough accordance with the value of

$$B(E1;\tfrac{1}{2}^-\rightarrow\tfrac{1}{2}^+)=(0.63\pm0.16)\times10^{-29}\text{ cm}^2, \qquad (2.1c)$$

obtained from the Coulomb excitation of F^{19} with Ne^{20} ions by Stelson and McGowan[19] and the earlier value of

$$B(E1;\tfrac{1}{2}^-\rightarrow\tfrac{1}{2}^+)=(0.23\times10^{-29}\text{ cm}^2\ \text{(factor 2 uncertainty))}, \qquad (2.1d)$$

obtained by Sherr *et al.*[20] from the Coulomb excitation by α particles. From Eqs. (2.1b)–(2.1d) we can infer a mean value of

$$B(E1;\tfrac{1}{2}^-\rightarrow\tfrac{1}{2}^+)=(0.46\pm0.07)\times10^{-29}\text{ cm}^2, \qquad (2.1e)$$

which corresponds to $(1.22_{-0.18}^{+0.19})\times10^{-3}$ times the single-particle estimate as given by Wilkinson.[21]

For the lifetime of the 0.197-MeV $\frac{5}{2}^+$ state the following values have been given:

$$\begin{aligned}\tau_{5/2+}&=(1\pm0.20)\times10^{-7}\text{ sec},^{22}\\&=0.8\times10^{-7}(\text{factor 2})\text{ sec},^{18}\\&=(1.23\pm0.07)\times10^{-7}\text{ sec},^{23}\\&=(1.25\pm0.03)\times10^{-7}\text{ sec}.^{24}\end{aligned} \qquad (2.2a)$$

From the last two values we obtain for the reduced transition probability

$$B(E2;\tfrac{5}{2}^+\rightarrow\tfrac{1}{2}^+)=(0.222_{-0.006}^{+0.005})\times10^{-50}\text{ cm}^4. \qquad (2.2b)$$

This value can be compared with the direct measurement by Coulomb excitation:

$$\begin{aligned}B(E2;\tfrac{5}{2}^+\rightarrow\tfrac{1}{2}^+)&=(0.167\pm0.033)\times10^{-50}\text{ cm}^4,^{19}\\&=0.113\times10^{-50}\text{ cm}^4(\text{factor 2}).^{20}\end{aligned} \qquad (2.2c)$$

If we do not consider the measurements with a large uncertainty, we obtain a mean value for $B(E2)$ of

$$B(E2;\tfrac{5}{2}^+\rightarrow\tfrac{1}{2}^+)=(0.224\pm0.015)\times10^{-50}\text{ cm}^4, \qquad (2.2d)$$

which yields $|M|^2=(8.1\pm0.5)$.

[16] *Nuclear Data Sheets*, Compiled by K. Way *et al.* (Printing and Publishing Office, National Academy of Science—National Research Council, Washington 25, D. C.) NRC-61-516.
[17] J. D. Prentice, N. W. Gebbie, and N. S. Caplan, Phys. Letters 3, 201 (1963).
[18] J. Thirrion, C. A. Barnes, and C. C. Lauritsen, Phys. Rev. 94, 1076 (1954).
[19] P. H. Stelson and F. K. McGowan, Nucl. Phys. 16, 92 (1960).
[20] R. Sherr, C. W. Li, and R. F. Christy, Phys. Rev. 96, 1258 (1954).
[21] D. H. Wilkinson, in *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove (Academic Press Inc., New York, 1960), Part B, p. 859.
[22] G. A. Jones, W. R. Phillips, and C. M. P. Johnson, Phys. Rev. 96, 547 (1954).
[23] P. Lehmann, A. Lévêque, and M. Fiehrer, Compt. Rend. 241, 700 (1955).
[24] C. M. P. Johnson, Phil. Mag. 1, 573 (1956).

TABLE II. Measurements of the magnetic moment of the $\frac{5}{2}^+$ second excited state of F^{19}.

Reference	(a)	(b)	(c)	(d)	(e)	(f)[g]
target	fluid	fluid	fluid	on film	on film	solid
$g_T \times 10^{-7}$	1.74±0.15	1.84±0.15	1.74±0.08	1.20±0.80	2.23±0.50	···
$\mu_{5/2}$(nm)	3.51±0.42	3.70±0.45	3.51±0.26	2.42±1.69	4.50±1.13	3.69±0.04

[a] M. Martin, R. Szostak, and P. Marmier, Helv. Phys. Acta **31**, 481 (1958).
[b] P. Lehmann, A. Lévêque, and R. Pick, Phys. Rev. **104**, 411 (1956).
[c] W. R. Phillips and G. A. Jones, Phil. Mag. **1**, 576 (1956).
[d] K. Sugimoto and M. Mizobuchi, Phys. Rev. **103**, 739 (1956).
[e] P. B. Treacy, Nucl. Phys. **2**, 239 (1956).
[f] R. M. Freeman, Nucl. Phys. **26**, 446 (1961).
[g] The given error seems somewhat small, as the Larmor frequency is only determined with ±3% accuracy.

The discrepancy between the values of the reduced transition probability from the Coulomb excitation and the value inferred from the lifetime measurement cannot be explained by secondary Coulomb excitation effects, as was pointed out by Beder.[25]

The remaining electric transition rates in Table I have been calculated from the values of $|M|^2$ given by Litherland *et al.*[10] (Coulomb excitation) and the single-particle estimates given by Wilkinson.[21]

The data for the magnetic moment of the $\frac{5}{2}^+$ state are given in Table II. A value of $(1.24\pm3\%)\times10^{-7}$ sec is adopted for the lifetime τ. If we omit the two earlier measurements with solid targets, we obtain a mean value of

$$\mu_{5/2}=(3.63\pm0.11)\text{ nm}. \qquad (2.3)$$

The value for the transition probability of the 1.35-MeV $M1$ transition from the $\frac{3}{2}^-$ to the $\frac{1}{2}^-$ state follows from the lifetime of

$$\tau=(0.25\pm50\%)\times10^{-13}\text{ sec} \qquad (2.4a)$$

given by Booth[26] and the $M1$–$E2$ amplitude mixing ratio of

$$\delta=-0.23\pm0.10 \qquad (2.4b)$$

given by Prentice *et al.*[17]

The branching ratio for the transitions from the 1.56-MeV $\frac{3}{2}^+$ state to the lower positive-parity states can be inferred from the data given in Ref. 12.

The logft value of the ground-state–ground-state β^+ decay of Ne^{19} to F^{19} has been determined by Wallace and Welch[27] as

$$\log ft(\tfrac{1}{2}^+\,Ne^{19}\rightarrow\tfrac{1}{2}^+\,F^{19})=(3.26\pm0.03). \qquad (2.5)$$

B. Theoretical Interpretation

F^{19} was the first nucleus for which the application of the shell model and the unified model gave equally good results.

1. Positive-Parity States

Shell-model calculations in an intermediate-coupling situation including configuration mixing have been carried out by Elliott and Flowers[2] (harmonic-oscillator potential and a residual two-particle central Yukawa interaction with Rosenfeld exchange) and Redlich[3,28] (harmonic-oscillator potential and two-particle central Gaussian interaction with ordinary and space exchange in equal mixtures, also slightly deformed harmonic-oscillator potential).

Collective-model calculations have been presented by Paul[6] [strong-coupling approximation with $\beta=0.3$ ($\eta=2.91$ $\kappa=0.10$); $C=0.30$] Abraham and Warke[5] (weak coupling $P=0.78$), Bhatt[7] (Nillson model $\eta=4$; $\kappa=0.07-0.10$; $C=0.33$), Rakavy[8] [Nilsson model $\epsilon=0.29$ ($\eta=2.58$ $\kappa=0.10$); $C\approx0.55$] and by Chi and Davidson[9] (asymmetric-core model). The best results of these calculations are summarized in Table III.

2. Negative-Parity States

Christy and Fowler[15] have suggested that the three lowest negative-parity states of F^{19} might be explained by the coupling of a $p_{1/2}$ hole to the Ne^{20} ground and first excited state. This could give the observed $\frac{5}{2}^-$, $\frac{3}{2}^-$ doublet at approximately the right energy. A more quantitative calculation was carried out by Harvey[4] using the SU_3 approach for the excitation of a $1p$ particle from the O^{16} core into the $2s$–$1d$ shell. The author finds that, although the fit of the energy spectrum is quite reasonable, the measured enhanced $E3$ transition rate[10] from the $\frac{5}{2}^-$ to the ground state is not given satisfactorily by this model (factor 10 discrepancy).

III. THE COUPLING OF HOLES

The usual procedure for the relation of the matrix elements of particle and hole configurations, which are conjugate with respect to closed shells (see Bell[29] and further references given there), does not seem to be applicable in a straightforward manner to relations between particle and hole matrix elements, which are conjugate with respect to any given configuration. Further complication arises from the fact that the operators in our model are not symmetrical functions of the variables of all the particles involved, as we simpli-

[25] D. Beder, Phys. Letters **3**, 306 (1963).
[26] E. C. Booth, Nucl. Phys. **19**, 426 (1960).
[27] R. W. Wallace and J. A. Welch, Phys. Rev. **117**, 1297 (1960).
[28] M. G. Redlich, Phys. Rev. **98**, 199 (1955); **99**, 1427 (1955).
[29] J. S. Bell, Nucl. Phys. **12**, 117 (1959).

fied the actual many-body problem to essentially a two-body problem.

In the case of atomic and nuclear physics the hole state is to be imagined as the absence of a particle from a positive-energy state. So the energy of a hole will be of opposite sign from the energy of an equivalent particle. As the core is unchanged in our case, we find (neglecting exchange and other effects for the time being) for the Hamiltonian (1.4)

$$H(\text{core and hole})=H_c-H_p-H_{\text{coupl.}}. \qquad (3.1)$$

To investigate the behavior of the particle operators for the static moments and electric transitions we will make use of the description of holes and particles given by Brink and Satchler.[30] If we describe a particle state (q-number theory) by the application of a creation operator to the vacuum (or any other) state

$$|jm\rangle=\eta_{jm}{}^{+}|0\rangle, \qquad (3.2)$$

the equivalent hole state $|\bar{j}\bar{m}\rangle$ will be given by the application of the particle-destruction operator η_{j-m} to a state $|\alpha\rangle$ containing the particle state $|j-m\rangle$ (phase factors neglected)

$$|\bar{j}\bar{m}\rangle=\eta_{j-m}|\alpha\rangle. \qquad (3.3)$$

In a more symmetrical way η_{j-m} can be interpreted as the creation operator of a hole state $|\bar{j}\bar{m}\rangle$ and $|\alpha\rangle$ can be taken as the vacuum state of the world of holes. As the time-reversal operator T changes the sign of both orbital momentum and intrinsic spin (see, e.g., Wick[31]), Eqs. (3.2) and (3.3) imply that a hole state and a particle state are related by (c-number theory)

$$\psi(\text{hole})=T\psi(\text{particle}) \qquad (3.4)$$

besides following the energy behavior (3.1). In particular, this means $\mathbf{l}$ and $\mathbf{s}$ change sign. In addition, the charges are conjugate

$$e(\text{particle})+e(\text{hole})=0. \qquad (3.5)$$

From these properties, it follows that the magnetic moment does not reverse sign

$$\mu(\text{hole})=\mu(\text{particle}) \qquad (3.6)$$

(compare Talmi and Unna[32] p. 362), but the particle parts of the electric moments do change sign

$$(er_p{}^{\lambda}Y_{\lambda}(p))_{\text{hole}}=-(er_p{}^{\lambda}Y_{\lambda}(p))_{\text{particle}}, \qquad (3.7)$$

as the space part is not affected by time reversal.

Besides the changes in sign in Eqs. (3.1), (3.5), and (3.7) the replacement of the coupling of a particle to the core by the coupling of a hole gives rise to the following corrections:

(a) From the general outlines of the shell model, we can assume that in Ne^{20} the four particles outside the

TABLE III. Theoretical results for the low-lying positive-parity States of F^{19}.

	References	(2)	(3),(28) undeformed	deformed	(5)	(6)	(7)	(8)	(9)	This model	Experiment
$\frac{1}{2}^+$	E(MeV)	0	0	0	0	0	0	0	0	0	0
	μ(nm)	2.80–2.87	2.94	2.91	2.76	2.75	2.54	2.76	2.81	2.63	2.629
	$\log ft(\beta^+)$	3.10–3.25	3.20	3.25	...	3.52	...	3.11	4.52	3.13±0.02	3.26±0.03
$\frac{5}{2}^+$	E(MeV)	0.2	0.17	0.16	0.25	0.24	≈0.50	0.20	0.20	0.199	0.197
	μ(nm)	3.30	3.74	3.68	4.30	3.70	3.65	3.80	...	3.59	3.63±0.11
	τ(sec)	5×10^{-7} 1×10^{-7}[a]	2.7×10^{-7}	...	...	0.9×10^{-7}	21×10^{-7}	2.75×10^{-7} 1.31×10^{-7}[b]	3.42×10^{-7}	1.24×10^{-7}	$(1.24\pm0.04)\times10^{-7}$
	QM(cm²)	...	...	...	...	...	...	...	-0.64×10^{-25}	-0.13×10^{-24}	$\pm0.13\times10^{-24}$
$\frac{3}{2}^+$	E(MeV)	1.50	...	...	1.66	1.55	2.70	1.57	1.56	1.559	1.556
	$B(E2;\frac{3}{2}^+\rightarrow\frac{1}{2}^+)$cm⁴	...	...	...	...	...	...	...	...	$(0.275\pm0.042)\times10^{-50}$	$(0.25\pm0.08)\times10^{-50}$
	$\Gamma(\frac{3}{2}^+\rightarrow\frac{1}{2}^+)/\Gamma(\frac{3}{2}^+\rightarrow\frac{5}{2}^+)$	0.6%	...	...	...	0.8%	...	...	...	0.3%	<4%
$\frac{9}{2}^+$	E(MeV)	2–3	...	...	...	2.90	...	2.81	≈2.80	(2.79)	2.797

[a] Corrected for inclusion of core vibrations.
[b] The quoted values are for ϵ=0.31, 0.45.

[30] D. M. Brink and G. R. Satchler, Nuovo Cimento **4**, 549 (1956).
[31] G. C. Wick, Ann. Rev. Nucl. Sci. **8**, 1 (1958).
[32] I. Talmi and I. Unna, Ann. Rev. Nucl. Sci. **10**, 353 (1960).

closed shells have a much smaller probability of occupying the $j=\frac{3}{2}$ single-particle state than the $j=\frac{5}{2}, \frac{1}{2}$ states. So it will require more energy to add a $j=\frac{3}{2}$ hole to the Ne^{20} core than a $j=\frac{5}{2}, \frac{1}{2}$ hole. To account for this we will add a term

$$H_{\text{hole}}=\Delta\delta_{j,3/2} \tag{3.8}$$

to the Hamiltonian (3.1) and treat $\Delta\geq 0$ as an additional free parameter. This amounts to replacing the particle part of the Hamiltonian for the $j=\frac{3}{2}$ hole state $-H_p$ by $-H_p+\Delta$.

(b) It was pointed out by Pandya[33] that the interaction energy of a particle-hole system does not simply reverse sign with respect to a particle-particle system, but shows a more complicated behavior. Though the results of Pandya are not applicable to our core-particle system, we can hope to use them as a rough guide and state that apart from the change in sign we should expect a change of the parameters D' of the $(\mathbf{s}\cdot\mathbf{L})$ term and of the coupling term H_k. As the matrix elements of H_k do not depend on $\mathbf{J}=\mathbf{L}+\mathbf{j}$ [see I, Eqs. (5.5), (5.12)] we should expect a smaller change for the parameters of H_k in comparison to D'.

(c) Although the results for the coupling of a particle to the Ne^{20} core indicate that the assumption of an undisturbed core yields reasonable results, we cannot be sure whether the picture of an undisturbed core can be maintained for the coupling of a hole.

IV. RESULTS OF THIS MODEL FOR THE POSITIVE-PARITY STATES

A. Energy Fit and Parameters of the Hamiltonian

The diagonalization process of the Hamiltonian for F^{19} follows the same pattern set out in the case of Ne^{21} with the exception that we have to choose the parameters to allow for the changes indicated in the last chapter.

It was found that the diagonalization of the energy matrix of Ne^{21} (positive-parity states) gives satisfactory results for two sets of parameters. Case (a) of I followed the Nilsson model description more closely and adopted truncation (see Chi and Davidson[9]), while case (b) allowed for a small admixture of the truncated state $j=\frac{5}{2}$ $K=\frac{1}{2}$ in the low-lying states of Ne^{21} by taking into account the H_4 part of the interaction Hamiltonian.

In the case of F^{19} we find the following results:

Case a. If we use the truncation process consistently, we should assume that in Ne^{20} only the $j=\frac{5}{2}$ $K=\frac{1}{2}$ subshell with four nucleons is filled besides the closed $1s$ and $1p$ shells. So the only single-hole state of the $n=2$ shell that should be coupled to the core is a $j=\frac{5}{2}$ $K=\frac{1}{2}$ hole. The energy pattern can be reproduced in this case, but the magnetic moment of the $J=\frac{1}{2}$ ground state of F^{19} gives the single-particle value $\mu_{1/2}=2.79$ nm, which is not in good agreement with the experimental value. If we undertake to couple a two-hole $(j=\frac{5}{2}, K=\frac{1}{2})$ one-particle state in addition to the single-hole state, the situation can probably be improved, but we would need a larger number of parameters, as we have to take into account besides the 2 hole-particle-core interaction the hole-hole and particle-hole residual terms. The lack of experimental data in the 4–10-MeV region for transitions and static moments does not allow a detailed examination in this parametric description.

Characteristically a coupling of the remaining possible single-hole states besides the truncated state of the $n=2$ shell, does not give a satisfactory fit even of the energy spectrum for a wide range of parameters.

Case b. The parameters of the Hamiltonian for the positive parity states are [see Eq. I (5.12)] using a slightly different notation than in I:

$$Q_{22}(2), Q_{20}(2), Q_{22}(4);\ D_0', D_2';\ D_2, E_{20};\ C_L, \tag{4.1a}$$

where

$$Q_{ll'}(\lambda)=(f_\lambda/\pi)\langle l|\alpha_\lambda(r_p)|l'\rangle\langle\text{intr}|\alpha_\lambda(R_c)|\text{intr}\rangle$$
$$E_{20}=E_0-E_2(E_2 \text{ is fixed as } E_2=0), \tag{4.1b}$$

and the subscript of D and D' denotes the l values of the single-particle states involved.

In addition we have the parameter Δ [see Eq. (3.8)].

A good fit of the low-energy data for the positive-parity states of F^{19} can be obtained for

$$\begin{aligned} &Q_{22}(2)=-9.80 \quad Q_{20}(2)=7.70 \quad Q_{22}(4)=-15.80,\\ &D_2=-3.00 \quad D_2'=-0.80 \quad D_0'=-0.80,\\ &E_{20}=1.00 \quad \Delta=4.00,\\ &C_2=0.30 \quad C_4=0.20 \quad C_6=0.18. \end{aligned} \tag{4.2}$$

(all in MeV).

The core parameters lie between the values obtained from O^{18} and Ne^{20} (see Ajzenberg-Selove and Lauritsen[12]):

	Ne^{20}		O^{18}
C_2	0.27	0.30	0.33
C_4	0.21	0.20	0.18
C_6	0.18	0.18	? .

(4.3)

If we assume similar patterns of change for the single-proton and single-neutron parameters within the major $n=2$ shell, we find that the values of D_2 and E_{20} agree well with the values used in the case of Ne^{21} [see Eq. I (5.23b)]; for we have at the beginning of the shell from the data of O^{17} and F^{17}

	Neutron	Proton
D_2	2.03	1.88
E_{20}	−1.16	−1.38.

(4.4)

The magnitude of the $2^\lambda-2^\lambda$ pole-coupling parameters are unchanged from the Ne^{21} values; there is a change of the parameters D_0' and D_2', but it is not drastic.

[33] S. P. Pandya, Phys. Rev. 103, 956 (1956).

The resulting energy scheme [readjustment to E(ground state)=0] has the sequence

$$0.000: J=\tfrac{1}{2}^+;\ 0.199: J=\tfrac{5}{2}^+;\ 1.559: J=\tfrac{3}{2}^+$$
$$1.810(2.786,\ 2.792): J=\tfrac{9}{2}^+;\ 4.441(5.228,\ 5.266): \quad (4.5)$$
$$J=\tfrac{7}{2},$$

and states above 10 MeV.

The values given in brackets in the case of the $\frac{7}{2}$ and $\frac{9}{2}$ states are for the correction of the $2^\lambda-2^\lambda$ pole-interaction parameters (see I, p. 1173) in the case L and/or $L'=6$. For the first value in brackets $\alpha=0.16$, $C_6=0.18$, for the second value $\alpha=0.185$, $C_6=0.17$ is used, as compared with $\alpha=0.20$, $C_6=0.18$ for Ne^{21}.

Besides one "band" all the remaining levels are more than 10 MeV above the ground state. Therefore, the number of parameters is larger than the number of identified levels and the energy fit alone is not significant. The very close agreement of the parameters with the parameters used for Ne^{21} should be noted, however.

The expansion coefficients $c^J(j,K)$ of the final wave functions $|JM\rangle$ in terms of the "strong-coupling" wave function $|JM,\ jK\rangle$

$$|JM\rangle=\sum_{j,K} c^J(jK)|JM,jK\rangle \qquad (4.6)$$

[see Eq. I (5.25)] for the three lowest states are given in Table IV.

Some of the positive-parity states of F^{19} between the ground-state "band" and the higher levels in this model can probably be imagined as 2-hole–1-particle coupling to Ne^{20}.

B. Transitions and Moments

The operators for electric multipole and magnetic dipole transitions in the core-particle system are

$$Q_q^{(\lambda)}=Q_q^{(\lambda)}(c)+Q_q^{(\lambda)}(p), \qquad (4.7a)$$

with the core part

$$Q_q^{(\lambda)}(c)=eZR_c^\lambda Y_{\lambda,q}(\vartheta_c,\varphi_c) \qquad (4.7b)$$

and the particle part

$$Q_q^{(\lambda)}(p)=e_{\text{eff}}(\lambda)r_p^\lambda Y_{\lambda,q}(\vartheta_p,\varphi_p), \qquad (4.7c)$$

and

$$M_q^{(1)}=\mu_0[3/4\pi]^{1/2}(g_cL_q+g_ll_q+g_ss_q) \qquad (4.8)$$

(μ_0=nuclear magneton).

The quadrupole moment operator is then defined as

$$QM=[16\pi/5]^{1/2}Q_0^{(2)}, \qquad (4.9)$$

and the magnetic-moment operator

$$\boldsymbol{\mu}=\mu_0(g_c\mathbf{L}+g_l\mathbf{l}+g_s\mathbf{s}). \qquad (4.10)$$

For the effective charge of the single particle we will consider the recoil effect between core and particle

$$e_{\text{eff}}^{(\lambda)}=e(1+(-)^\lambda Z/A^\lambda) \quad \text{for a proton}$$
$$=(-)^\lambda Z/A^\lambda \quad \text{for a neutron}, \qquad (4.11)$$

and will not use the quadrupole corrections for the distortion of the closed-core shells by a nonspherical field of the outside particle(s) (see Mottelson[34]). So we stay within the picture of a spherical core potential suggested in I.

TABLE IV. Expansion coefficients $c^J(j,K)$ for the three lowest positive-parity states of F^{19}.

J	j	K	$c^J(j,K)$
$\frac{1}{2}$	$\frac{5}{2}$	$\frac{1}{2}$	−0.65006
	$\frac{3}{2}$	$\frac{1}{2}$	0.69332
	$\frac{1}{2}$	$\frac{1}{2}$	0.31100
$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	−0.01419
		$\frac{1}{2}$	−0.59824
	$\frac{3}{2}$	$\frac{3}{2}$	0.04589
		$\frac{1}{2}$	0.74974
	$\frac{1}{2}$	$\frac{1}{2}$	0.27874
$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	0.01294
		$\frac{3}{2}$	0.04292
		$\frac{1}{2}$	0.68817
	$\frac{3}{2}$	$\frac{3}{2}$	−0.02713
		$\frac{1}{2}$	−0.66048
	$\frac{1}{2}$	$\frac{1}{2}$	−0.29573

Conventional values of the single-particle gyromagnetic factors are

$$g_l=1.000, \quad g_s=5.586 \quad \text{(proton)}. \qquad (4.12)$$

1. Magnetic Moments

Evaluation of the matrix elements of the operator (4.10) in representation (4.6) with the values (4.12) and a value of $g_c=0.43$ for the core contribution as in I gives for the magnetic moment of the lowest $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states:

$$\mu_{1/2}=2.63 \text{ nm}$$
$$\mu_{5/2}=3.59 \text{ nm}. \qquad (4.13)$$

These values are in good agreement with experiment.

2. Transitions Between Positive-Parity States and the Quadrupole Moment of the $\frac{5}{2}^+$ Second Excited State

If we express the core contributions to the quadrupole features in terms of the intrinsic moments

$$Q_{LL'}^{(2)}=Q_{L'L}^{(2)}=\langle \text{intr}(L)|ZR_c^2|\text{intr}(L')\rangle \qquad (4.14)$$

(see I p. 1174) and employ harmonic-oscillator wave functions to evaluate the radial-particle part, we obtain for the $\frac{5}{2}^+\rightarrow\frac{1}{2}^+$ $E2$ transition and the quadrupole moment of the $\frac{5}{2}^+$ state using Eqs. (4.7), (4.9), and (4.11)

$$B(E2;\tfrac{5}{2}^+\rightarrow\tfrac{1}{2}^+)=\frac{1}{\pi}(0.1266Q_{20}+0.1205Q_{22}+0.2515Q_{42}-0.9221b^2)^2,$$
$$\langle\tfrac{5}{2}\,\tfrac{5}{2}|QM|\tfrac{5}{2}\,\tfrac{5}{2}\rangle=-e(0.1935Q_{20}+0.0292Q_{22}+0.1399Q_{42}+0.2076Q_{44}-1.0414b^2). \qquad (4.15)$$

(b="size parameter").

[34] B. M. Mottelson, in *The Many Body Problem*, edited by C. de Witt (Methuen and Company Ltd., London, 1959), p. 283.

Only Q_{20} and Q_{42} can so far be obtained from experiment [see I, Eq. (6.9)]

$$Q_{20}=(3.99_{3.26}{}^{4.50})\times10^{-25}\text{ cm}^2,$$
$$Q_{42}=(2.77_{1.98}{}^{4.92})\times10^{-25}\text{ cm}^2. \quad (4.16)$$

In addition to Eqs. (4.15), we can use the Ne^{21} data (ground-state quadrupole moment and $\frac{3}{2}^+\to\frac{5}{2}^+$ transition)

$$\langle\tfrac{3}{2}\tfrac{3}{2}|QM|\tfrac{3}{2}\tfrac{3}{2}\rangle=e(0.0029Q_{20}+0.2515Q_{22}+0.1743Q_{42}-0.0315Q_{44}+0.0050b^2),$$
$$B(E2;\tfrac{3}{2}^+\to\tfrac{5}{2}^+)=\frac{1}{14\pi}(1.4471Q_{20}-0.0245Q_{22}+1.0625Q_{42}+0.2836Q_{44}+0.0369b^2)^2, \quad (4.17)$$

to determine values of Q_{22},Q_{44}, and b^2 compatible with the measured data

$$B(E2;\tfrac{5}{2}^+\to\tfrac{1}{2}^+)=0.224\times10^{-50}\text{ cm}^4,$$
$$|\langle\tfrac{5}{2}\tfrac{5}{2}|QM|\tfrac{5}{2}\tfrac{5}{2}\rangle|=e0.13\times10^{-24}\text{ cm}^2,$$
$$\langle\tfrac{3}{2}\tfrac{3}{2}|QM|\tfrac{3}{2}\tfrac{3}{2}\rangle=e0.093\times10^{-24}\text{ cm}^2, \quad (4.18)$$
$$B(E2;\tfrac{3}{2}^+\to\tfrac{5}{2}^+)=0.20\times10^{-49}\text{ cm}^4,$$ [34a]

and Eq. (4.16). With the resulting set of values

$$b^2=(0.60\mp0.01)\times10^{-25}\text{ cm}^2,$$
$$Q_{20}=(3.89_{3.37}{}^{4.42})\times10^{-25}\text{ cm}^2,$$
$$Q_{22}=(2.44_{2.19}{}^{2.69})\times10^{-25}\text{ cm}^2, \quad (4.19)$$
$$Q_{42}=(2.40_{2.82}{}^{1.98})\times10^{-25}\text{ cm}^2,$$
$$Q_{44}=(3.68_{3.97}{}^{3.39})\times10^{-25}\text{ cm}^2,$$

we obtain for the $E2$ transition probabilities between the $\frac{3}{2}^+$ and $\frac{1}{2}^+$, $\frac{5}{2}^+$ states

$$T(E2;\tfrac{3}{2}^+\to\tfrac{1}{2}^+)=(3.12_{2.66}{}^{3.63})\times10^{11}\text{ sec}^{-1}, \quad (4.20a)$$
$$T(E2;\tfrac{3}{2}^+\to\tfrac{5}{2}^+)=(0.95_{0.93}{}^{0.97})\times10^{11}\text{ sec}^{-1}. \quad (4.20b)$$

The first value (reduced transition probability)

$$B(E2;\tfrac{3}{2}^+\to\tfrac{1}{2}^+)=(0.275_{0.234}{}^{0.320})\times10^{-50}\text{ cm}^4)$$

shows good agreement with the available experimental value corresponding to $|M|^2=10\pm1.5$.

The value of b^2 is larger by a factor of 2 than the conventional shell-model value for a harmonic-oscillator potential (see Raz[35], Carlson and Talmi[36]), but it gives rough agreement of the matrix element

$$\langle 2s|r_p{}^2|1d\rangle=(18.97\mp0.31)\times10^{-26}\text{ cm}^2,$$

with the value calculated by Barton *et al.*[37] with a realistic potential well for O^{17} (17.28×10^{-26} cm^2). In I the single-particle contribution, which is approximately 0.1% for neutrons, was neglected. The values of Q_{22} and Q_{44} look very reasonable in comparison to Q_{20} and Q_{42}.

Equations (4.20) together with the values of the corresponding $M1$ transition probabilities

$$T(M1;\tfrac{3}{2}^+\to\tfrac{1}{2}^+)=1.20\times10^{11}\text{ sec}^{-1},$$
$$T(M1;\tfrac{3}{2}^+\to\tfrac{5}{2}^+)=1.47\times10^{14}\text{ sec}^{-1}, \quad (4.21)$$

give a value of

$$\Gamma(\tfrac{3}{2}^+\to\tfrac{1}{2}^+)/\Gamma(\tfrac{3}{2}^+\to\tfrac{5}{2}^+)\approx0.3\%, \quad (4.22)$$

which is compatible with experiment.

The $\log ft$ value of the ground-state β^+ transition from Ne^{19} is

$$\log ft=3.13\pm0.02, \quad (4.23)$$

if we use the same neutron wave function for Ne^{19} as the proton wave function (4.6) and values of

$$x=0.560\pm0.012,\quad B_g=(2.783\pm0.07)\times10^{+3}$$

(see Koefod-Hansen and Winther[38]).

V. NEGATIVE-PARITY STATES

The parameters of the Hamiltonian (1.4) for the negative-parity states are

$$Q_{ll'}(1), Q_{ll'}(2), Q_{ll'}(3), Q_{ll'}(4);\quad D_2', D_1', D_0';$$
$$D_2, D_1, E_{20}, E_{21}, \Delta;$$
$$C_L, H_c(3^-), H_c(5^-). \quad (5.1)$$

With the assumptions discussed in the Introduction we have

$$Q_{22}(2)=-9.80,\quad Q_{22}(4)=-15.80,\quad Q_{20}(2)=7.70,$$
$$D_2=-3.00,\quad E_{20}=1.00, \quad (5.2)$$
$$C_2=0.30,\quad C_4=0.20,\quad H_c(3^-)=6.41,\quad H_c(5^-)=9.27.$$

The spin-orbit coupling parameter in the $1p$ shell D_1 can be taken from Kurath's work[39] as

$$D_1\approx-4.00, \quad (5.3a)$$

for a hole at the end of the shell. E_{21} can be roughly estimated as

$$3\le E_{21}\le6, \quad (5.3b)$$

from the values of the first negative-parity states in O^{16} and O^{17}.

Using harmonic-oscillator wave functions, we obtain an estimate for the 2^2-2^2 pole-interaction parameter in the $1p$ shell of

$$Q_{11}(2)\approx0.70Q_{22}(2), \quad (5.4a)$$

assuming a long-range interaction potential of the form

$$\alpha_\lambda\propto r^\lambda. \quad (5.4b)$$

[34a] The experimental value of 0.25×10^{-49} cm^4 does not yield a set of parameters consistent with Eq. (4.16) and $Q_{L_1L_2}\approx Q_{L_3L_4}$. The employed value should, however, be well within the experimental errors.

[35] B. J. Raz, Phys. Rev. **120**, 169 (1960).

[36] B. C. Carlson and I. Talmi, Phys. Rev. **96**, 436 (1954).

[37] G. Barton, D. N. Brink, and L. M. Delves, Nucl. Phys. **14**, 256 (1959).

[38] O. Koefod-Hansen and A. Winther, Kgl. Danske Videnskab. Selskab, Mat. Fys. Medd. **30**, 20 (1956).

[39] D. Kurath, Phys. Rev. **101**, 216 (1956).

Among the remaining parameters

$$Q_{12}(3),\ Q_{12}(1),\ Q_{10}(1),\ \Delta,\ E_{21},\ D_2',\ D_0',\ D_1', \quad (5.5)$$

both $Q_{10}(1)$ and D_0' influence only the $\frac{5}{2}^-$ states and affect the position of the lowest states only slightly. So we can take

$$D_0' \approx D_2' \quad (5.6)$$

and

$$Q_{10}(1) \approx -0.63 Q_{12}(1),$$

obtained from a long-range potential of the form (5.4b) and harmonic-oscillator wave functions.

It was found that for the parameters (5.5) with Eqs. (5.2), (5.3), (5.4a), and (5.6), the position of the three lowest levels and the measured $E1$, $E3$ transitions can be fitted for any value of $Q_{12}(1)$ between 0 and 20 MeV under reasonable adjustment of the remaining parameters. A value of $b^2 = 0.6\times 10^{-25}$ cm^2 required a value of

$$Q^{(1)} \approx 1.35\times 10^{-13} \approx 0.39\langle \mathrm{intr}|R_c|\mathrm{intr}\rangle, \quad (5.7)$$

for the dipole-core contribution to the transition rates in the cases investigated and a value of

$$Q^{(3)} \approx (12\pm 1.5)\times 10^{-38}\ \mathrm{cm}^3, \quad (5.8)$$

for the octupole part of the core. As the maximal $T=1$ contribution to the Ne^{20} ground state is 3.9% we obtain the estimate

$$Q^{(3)} \gtrsim Q^{(1)} Q^{(2)}/0.39 \approx 10.5\times 10^{-38},$$

in agreement with the required value of $Q^{(3)}$. The only further experimental information available is the $\frac{3}{2}^- \rightarrow \frac{1}{2}^-$ transition, which gives for the $M1$ contribution a value of

$$T(M1;\ \tfrac{3}{2}^- \rightarrow \tfrac{1}{2}^-) = (9.5\pm 0.4)\times 10^{13}\ \mathrm{sec}^{-1},$$

for the range of the $Q_{12}(1)$ values indicated above. This is by a factor 2.5 larger than the experimental value and the variation is too slow to use it to discriminate between the various sets of parameters.

VI. DISCUSSION

The results of this model for the positive-parity states are quite satisfactory (including the results for Ne^{21}) for the energies, quadrupole features, and magnetic moments in case (b). It seems, however, that the $M1$ transitions do not give such good agreement. In the case of Ne^{21}, e.g., the $\frac{5}{2}^+ \rightarrow \frac{3}{2}^+$ 0.35-MeV transition gives a lifetime which is smaller than the measured value by a factor of 2, although it is still within the limits of the experimental errors. Since the $\log ft$ values of the respective β-decays, which also depend on the matrix elements of **s**, though not as sensitively, show an agreement of better than 4% with the experimental results, it seems that the inclusion of exchange moments (Sachs[40]) is necessary to improve the situation. This contribution should be of the order of 30% of the nonexchange parts and have the right sign.

With the over-all fit of the available experimental data in case (b), we can assume that the possible 2-hole–1-particle contributions are small for the low-lying states and that the final wave functions are sufficiently correct.

No definite results can be established in the case of the negative-parity states even with the additional assumptions, though the preliminary results presented here show that the suggested structure for these states is able to reproduce the $E1$ and $E3$ transition data in a reasonable manner.

ACKNOWLEDGMENT

The author is indebted to Professor D. C. Peaslee for suggesting the problem, and his critical reading of the manuscript.

[40] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Cambridge, Massachusetts, 1953), p. 245.